TASK 8: Complex Operations Name: ________________________________

With vectors, each form (component form and magnitude-direction form) had advantages and disadvantages for operations such as addition, subtraction, and scalar multiplication. The same is true for complex numbers. Let's see what the advantages and disadvantages of each form are.

1. First, find the modulus and argument (magnitude and direction) of each of the following complex numbers, then write each in polar form. We will be using these complex numbers throughout the task, so be careful! ***Round angles to the nearest whole number***.

	$3 + 4i$	$2i$	$-5 - 12i$	$4 - 4i$
modulus				
argument				
polar form				

2. Use what you know about complex numbers to simplify each of these **addition** and **subtraction** expressions in standard form. Complete the table. The first one has been done for you.

expression in standard form	$(3 + 4i) + (2i)$	$(3 + 4i) + (-5 - 12i)$	$(-5 - 12i) - (4 - 4i)$
answer in standard form $a + bi$	$3 + 6i$		
modulus of answer	$3\sqrt{5}$		
argument of answer	$63°$		
answer in polar form $r \text{ cis } \theta$	$3\sqrt{5} \text{ cis } 63°$		
original in polar form	$5 \text{ cis } 53° + 2 \text{ cis } 90°$		

3. Use what you know about complex numbers to simplify each of these **multiplication** expressions in standard form. Complete the table. The first one has been done for you.

expression in standard form	$(3 + 4i)(2i)$	$(3 + 4i)(-5 - 12i)$	$(-5 - 12i)(4 - 4i)$
answer in standard form $a + bi$	$-8 + 6i$		
modulus of answer	10		
argument of answer	143°		
answer in polar form r cis θ	10 cis 143°		
original in polar form	(5 cis 53°)(2 cis 90°)		

4. Use what you know about complex numbers to simplify each of these **division** expressions in standard form. Complete the table. The first one has been done for you.

expression in standard form	$\frac{2i}{3 + 4i}$	$\frac{-5 - 12i}{3 + 4i}$
answer in standard form $a + bi$	$\frac{8}{25} + \frac{6}{25}i$	
modulus of answer	$\frac{2}{5}$	
argument of answer	37°	
answer in polar form r cis θ	$\frac{2}{5}$ cis 37°	
original in polar form	$\frac{2 \text{ cis } 90°}{5 \text{ cis } 53°}$	

As we saw with vectors, there are advantages and disadvantages to each form—component and magnitude-direction form. The same is true with complex numbers: some operations are easy to perform in standard (rectangular) form, whereas others are easy in polar form.

5. Investigate your tables from #2, #3, and #4, and see if you can find quick methods in either (or both) forms to perform each operation. Write instructions in each box explaining how to perform each operation. The first column has been completed for you.

	addition (#2)	subtraction (#2)
standard form $a + bi$	Easy! :) Add real parts. Add imaginary parts.	
polar form r cis θ	Convert to standard form first, then follow standard form instructions.	

	multiplication (#3)	division (#4)
standard form $a + bi$		
polar form r cis θ		

	exponentiation (#6-10)
standard form $a + bi$	
polar form r cis θ	

← We will complete this part of the table after the next part of our task.

Now let's take a look at how we can find powers of complex numbers.

6. Use your summary table from #5 to help you multiply (5 cis 20°)(5 cis 20°)(5 cis 20°).

7. Just as we can write (5)(5)(5) as 5^3, we can write (5 cis 20°)(5 cis 20°)(5 cis 20°) as $(5 \text{ cis } 20°)^3$. Using patterns you notice, explain how to find $(2 \text{ cis } 10°)^{17}$ without actually multiplying everything out. ____

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8. Explain why this process for finding powers of complex numbers in polar form makes sense based on the processes you wrote in #5 for adding and multiplying complex numbers. ____

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9. Go back to your table in #5 and complete the last column: exponentiation.

We know how to write radicals using exponent notation. For example, $\sqrt[4]{5} = 5^{1/4}$. *We can therefore use what we summarized in #8 to find* n^{th} *roots of complex numbers:*

10. Write $\sqrt[3]{3+4i}$ using exponent notation. Use the processes you summarized in #9 to find $\sqrt[3]{3+4i}$.

11. Use your summary table from #5 to help you perform the following operations:

a. $(3 + 6i) + (4 - 5i)$ ____

b. (6 cis 30°) – (4 cis 120°) ______________________________________

c. $(8 \text{ cis } 135°)(10 \text{ cis } 210°)$ ______________________________

d. $(4 - 3i)(-7 - 24i)$ ______________________________

e. $\dfrac{10 \text{ cis } 330°}{2 \text{ cis } 135°}$ ______________________________

f. $\dfrac{6 + 8i}{3 - 4i}$ ______________________________

Multiplication, division, and exponentiation can require an extra step to make the argument between 0° and 360°.

12. Use your summary table from #5 to help you multiply $(6 \text{ cis } 150°)(3 \text{ cis } 240°)$ quickly. Why is your answer not in typical polar form? ______________________________

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13. Adjust your answer to make the argument an angle between 0° and 360°. Explain your process. ______________________________

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14. Use your summary table from #5 to divide $\dfrac{6 \text{ cis } 150°}{3 \text{ cis } 240°}$.

Why is your answer not in typical polar form? ______________________________

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15. Adjust your answer to make the argument an angle between 0° and 360°.

Explain your process. ____

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16. Use your summary table from #5 to find $(2 \text{ cis } 120°)^{14}$.

Why is your answer not in typical polar form? _________________________________

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17. Adjust your answer to make the argument an angle between 0° and 360°.

Explain your process. ____

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18. In your table from #5, write the additional step you had to perform and when it would be necessary. ____

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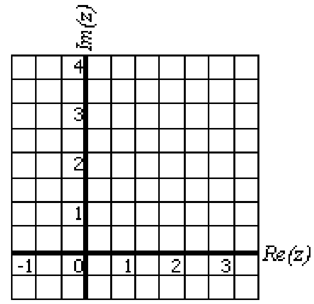
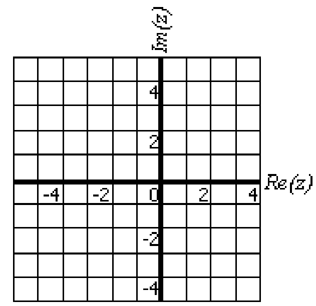
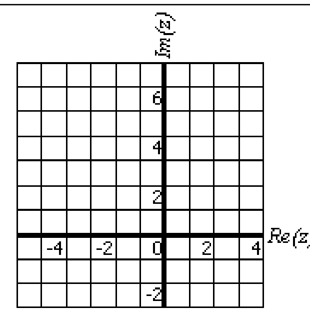
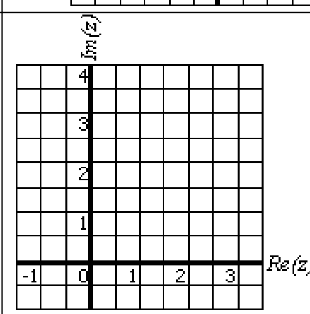
- This will make your summary tables complete.

19. Use your summary tables to represent each of the following problems on the next page and their answers on the complex plane:

question	show your work here	answer	on the complex plane
parts in standard form: $(-1 + 2i) + (3 + i)$ parts in polar form:		in standard form: in polar form:	
parts in standard form: $(-1 + 2i) - (3 + i)$ parts in polar form:		in standard form: in polar form:	
parts in standard form: $(-1 + 2i)(3 + i)$ parts in polar form:		in standard form: in polar form:	
parts in standard form: $(-1 + 2i) \div (3 + i)$ parts in polar form:		in standard form: in polar form:	
parts in standard form: $\sqrt{-1 + 2i}$ parts in polar form:		in standard form: in polar form: