Honours GSE Advanced Algebra
Unit 5: Exponential and Logarithmic Functions
What is a Logarithm? (Spotlight Task) Task 5

Name: ____________________________________

Date:_________________ Period: ___________

As a society, we are accustomed to performing an action and then undoing or reversing that action. Identify the action that undoes each of those named.
Putting on a jacket

Opening a door

Walking forward

Depositing money in a bank

In mathematics we also find it useful to be able to undo certain actions.
What action undoes adding 5 to a number?

What action undoes multiplying a number by 4?

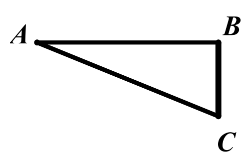
What action undoes squaring a number?

We say that addition and subtraction are inverse operations because one operation undoes the other. Multiplication and division are also inverse operations; squaring and taking the square root are inverse operations.

Inverse operations in mathematics help us solve equations. Consider the equation $2x + 3 = 35$. This equation implies some number (represented by x) has been multiplied by 2; then 3 has been added to the product for a result of 35. To determine the value of x, we subtract 3 from 35 to undo adding 3. This means that 2x must equal to 32. To undo multiplying the number by 2, we divide 32 by 2 and find the number represented by x is 16.

Explain how inverse operations are used in the solution of the following problems.

In right triangle ABC with right angle B, if BC is 8 cm and AC is 17 cm, determine the measure of angle A.



If $\sqrt{x+8} = 10$, determine the value of x.

Solve $x^3 = 27$ for x.

Solve $2x = 10$ for x.

In problem 8 of Task 3, "Bacteria in the Swimming Pool," we obtained the equation $1500(2)^t = 3000000$ to solve for t. This equation is equivalent to equivalent to $2^t = 2000$. Why? While in Task 3 we had no algebraic way to solve this equation because we lacked a strategy to isolate the exponent t. Our goal in this current task is to continue our idea of "undoing" to solve an equation; specifically, we need to find an action that will undo raising 2 to a power. This action needs to report the exponent to which 2 has been raised in order to obtain 2000. In order to rewrite $2^t = 2000$ so *t* is isolated, we need to define logarithms. Logarithms allow us to rewrite an exponential equation so that the exponent is isolated. Specifically, if $a = b^c$, then "c is the logarithm with base b of a" and is written as $\log_b a = c$. (We read "$\log_b a = c$" as "log base b of a is c.")

Using logarithms we can write $2^t = 2000$ as $\log_2 2000 = t$. These two expressions are equivalent, and in the expression $\log_2 2000 = t$ we have *t* isolated. Although this is a good thing, we still need a way to evaluate the expression $\log_2 2000$. We know it equals the exponent to which 2 must be raised in order to obtain a value of 2000, but we still don't know how to calculate this value. Hang on…we will get there in the next task! First some preliminary work must be done!

Let's look at a few examples:
***$10^2 = 100$* is equivalent to *$\log_{10}100 = 2$*.** Notice that 10 is the *base* in both the exponential form and the logarithmic form. Also notice that the logarithm is the exponent to which 10 is raised to obtain 100.

Evaluate *$\log_4 64$*. This question asks for the exponent to which 4 is raised to obtain 64. In other words, 4 to what power equals 64? ________
Consider the following problem: $\log_2 n = 4$. This equation is equivalent to $2^4 = n$; thus $n = 16$.

The relationship between exponents and logarithms must be understood clearly. The following practice problems will help you gain this understanding.

Rewrite each exponential equation as a logarithmic equation.
$6^2 = 36$
$10^3 = 1000$
$25^{\frac{1}{2}} = 5$

Rewrite each logarithmic equation as an exponential equation.
$\log_4 16 = 2$
$\log_6 1 = 0$
$\log_3 n = t$

Evaluate each of the following.
$\log_{10}(0.1)$ $\quad \log_3 81$ $\quad \log_2 \frac{1}{16}$ $\quad \log_5 5$

Between what two whole numbers is the value of $\log_3 18$?

Between what two whole numbers is the value of $\log_2 50$?

Solve each logarithmic equation for x.
$\log_9 81 = x$ $\quad \log_2 32 = x$ $\quad \log_7 1 = x$ $\quad \log_8 x = 3$ $\quad \log_5(3x + 1) = 2$

$\log_6(4x - 7) = 0$

Hopefully you now have an understanding of the relationship between exponents and logarithms. In logarithms, just as with exponential expressions, any positive number can be a base except 1 (we will explore this fact later). Logarithms which use 10 for the base are called common logarithms and are expressed simply as *log x*. It is not necessary to write the base. Calculators are programmed to evaluate common logarithms.

Use your calculator to evaluate log 78. First think about what this expression means.

Understanding logarithms can help solve more complex exponential equations. Consider solving the following equation for x: $10^x = 350$

We know that $10^2 = 100$ and $10^3 = 1000$ so x should be between 2 and 3. Rewriting $10^x = 350$ as the logarithmic equation $x = \log 350$, we can use the calculator to determine the value of x to the nearest hundredth.

Solve each of the following for x using logarithms. Determine the value of x to the nearest hundredth.
$10^x = 15$
$10^x = 0.3458$
$3(10^x) = 2345$
$-2(10^x) = -6538$
Logarithms that use the irrational number e as a base are of particular importance in many applications. Recall an irrational number is represented by a non-terminating, non-repeating decimal number. The value of e is 2.718281828…. The function $y = \log_e x$ is the natural logarithmic function and has a base of e. The shorthand for $y = \log_e x$ is $y = \ln x$. Calculators are also programmed to evaluate natural logarithms. Consider $\ln 34$ which means the exponent to which the base e must be raised to obtain 34. The calculator evaluates $\ln 34$ as approximately 3.526. This value makes sense because $e^{3.526}$ is approximately 33.9877, a value very close to 34!

Evaluate $\ln 126$. Use an exponential expression to confirm your solution makes sense.

Evaluate $\ln e$. Explain why your answer makes sense.

If $\ln x = 7$, determine the value of x to the nearest hundredth. HINT: Write the logarithmic equation in exponential form.

If $e^x = 85$, determine the value of x to the nearest hundredth. HINT: Write the exponential equation in logarithmic form.