Honours Algebra II/Advanced Algebra
Unit 7: Inferences and Conclusions from Data
Pennies Learning Task (Task 10)

STANDARDS ADDRESSED IN THIS TASK:
MGSE9-12.S.IC. 1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
MGSE9-12.S.IC. 2 Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation.
MGSE9-12.S.IC. 4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling

Part 1:
Sampling Distribution of a Sample Mean from a Normal Population

1) The scores of individual students on the ACT entrance exam have a normal distribution with mean 18.6 and standard deviation 5.9.
a) Use your calculator to simulate the scores of 25 randomly selected students who took the ACT. Record the mean and standard deviations of these 25 people in the table below. Repeat, simulating the scores of 100 people. (To do this, use the following command: *randNorm(μ,σ, n)*$\rightarrow$*L_1*.)

	Mean	**Standard Deviation**
Population	18.6	5.9
25 people		
100 people		

b) As a class, compile the means for the sample of 25 people.

Determine the mean and standard deviation of this set of means. That is, calculate $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$.
How does the mean of the sample means compare with the population mean?
How does the standard deviation of the sample means compare with the population standard deviation?

c) Describe the plot of this set of means. How does the plot compare with the normal distribution?

Name: ______________________________________Date: ____________ Period: __

d) As a class, compile the means for the sample of 100 people.

Determine the mean and standard deviation of this set of means. That is, calculate $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$.
How does the mean of the sample means compare with the population mean?
How does the standard deviation of the sample means compare with the population standard deviation?

e) Describe the plot of this set of means. How does the plot compare with the normal distribution?

f) Determine formulas for the mean of the sample means, $\mu_{\bar{x}}$, and the standard deviation of the sample means, $\sigma_{\bar{x}}$. Compare with a neighbor.

g) Just as we saw with proportions, the sample mean is an *unbiased estimator* of the population mean.

The Sampling Distribution of a Sample Mean: Choose a simple random sample of size *n* from a large population with mean *μ* and standard deviation *σ*. Then

- The mean of the sampling distribution of $\bar{x}$ is ____ and
- The standard deviation of the sampling distribution of $\bar{x}$ is _________.

h) Again, we must be cautious about when we use the formula for the standard deviation of $\bar{x}$. What was the rule when we looked at proportions? It is the same here.

i) Put these latter facts together with your response to part b to complete the following statement:

Choose a simple random sample of size n from a population that has a **normal distribution** with mean μ and standard deviation σ. Then the sample mean $\overline{x}$ has a ____________distribution with mean ________ and standard deviation ____________.

This problem centered on a population that was known to be normally distributed. What about populations that are not normally distributed? Can we still use the facts above? Let's investigate!

Part 2:
How Old are Your Pennies?[1]

a) Make a frequency table of the year and the age of the 25 pennies you brought to class like the one on the right. Do this on your own paper. Find the average age of the 25 pennies. Record the mean age as $\overline{x}(25)$.

Year	*Age*	*Frequency*
2012	0	3
2011	1	6
2010	2	3
...	...	...

$\overline{x}(25) =$ __________

b) Put your 25 pennies in a cup or bag and randomly select 5 pennies. Find the average age of the 5 pennies in your sample, and record the mean age as $\overline{x}(5)$. Replace the pennies in the cup, and repeat.

$\overline{x}_1(5)=$ _______________ $\overline{x}_2(5)=$ _______________

c) Repeat the process two more times, this time removing 10 pennies at a time. Calculate the average age of the sample of 10 pennies and record as $\overline{x}(10)$.

$\overline{x}_1(10)=$ _______________ $\overline{x}_2(10)=$ _______________

d) Make a penny dotplot of your 25 pennies.

Look at the shape of the final dotplot. Describe the distribution of the pennies' ages.

e) Make a second penny dotplot of the means for the sample size 5.

What is the shape of the dotplot for the distribution of $\overline{x}(5)$? How does it compare with the original distribution of pennies' ages?

f) Make a third penny dotplot of the means for the sample size 10.

What is the shape of the dotplot for the distribution of $\overline{x}(10)$? How does it compare with the previous plots?

g) Finally, make a fourth penny dotplot. Use this to record the means for the sample size of 25.

Describe the shape of the dotplot.

[1] The *Workshop Statistics* books provide an alternatives to actually working with pennies; however, it is not possible to recreate the methods here. It requires assigning three-digit numbers to penny ages and using a random number generator to "sample" pennies.

h) Now, find the mean and standard deviation of the sample means. Record your results in the chart below.

	Mean	Standard Deviation	Shape of the Distribution
"Population"			
Samples of 5			
Samples of 10			
Samples of 25			

i) Previously, we stated that if samples were taken from a normal distribution, that the mean and standard deviation of the sampling distribution of sample means was also normal with $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$. In this activity, we did not begin with a normal distribution. However, compare the means for the samples of 5, 10, and 25 with the overall mean of the pennies. Then compare the standard deviations with the standard deviation of all the pennies. Do these formulas appear to hold despite the population of penny ages being obviously non-normal? Explain.

j) Suppose that the U.S. Department of Treasury estimated that the average age of pennies presently in circulation is 12.25 years with a standard deviation of 9.5. Determine the theoretical means and standard deviations for the sampling distributions of sample size 5, 10, and 25.

	Mean	Standard Deviation
Population	12.25	9.5
Samples of 5		
Samples of 10		
Samples of 25		

k) This brings us to the **Central Limit Theorem (CLT) for Sample Means:**

Choose a simple random sample of size n from any population, regardless of the original shape of the distribution, with mean μ and finite standard deviation σ. When n is large, the sampling distribution of the sample mean $\bar{x}$ is approximately normal with mean ______ and standard deviation ________. Note: The statement "when n is large" seems a bit ambiguous. A good rule of thumb is that the sample size should be at least 30, as we can see in the dotplots above. When the sample sizes were sample, i.e. 5 and 10, the plots were still quite right skewed.