Honours GSE Algebra II/Advanced Algebra Name: ____________________________________
Unit 2: Operations with Polynomials
Finding Inverses Learning Task (Task 10) Date: ___________________Period: ___________

Build new functions from existing functions

MGSE9-12.F.BF.4 Find inverse functions.

MGSE9-12.F.BF.4a Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2(x^3)$ or f(x) = (x+1)/(x-1) for x ≠ 1.

Math Goals

In order to understand that a logarithm is an inverse of an exponential, a student should be able to calculate the inverses of simple functions. This task leads a student through the procedure of finding inverses of one-to-one functions. Logarithms will be discussed in unit 5.

Suppose we have a function *f* that takes x to y, so that

f(x) = y.

An inverse function, which we call f^{-1}, is another function that takes y back to x. So

$f^{-1}(y) = x$.

For f^{-1} to be an inverse of *f*, this needs to work for every x that *f* acts upon.

Key Point: The inverse of the function *f* is the function that sends each *f*(x) back to x. We denote the inverse of *f* by f^{-1}.

Working out f^{-1} by reversing the operations of *f*

One way to work out an inverse function is to reverse the operations that *f* carries out on aNumber.

Example: We shall set *f*(x) = 4x, so that *f* takes a number x and multiplies it by 4: *f*(x) = 4x (multiply by 4).

We want to define a function that will take 4 times x, and send it back to x. This is the same as saying that $f^{-1}(x)$ divides x by 4. So $f^{-1}(x) = \frac{x}{4}$ (divide by 4).

There is an important point about notation here. You should notice that $f^{-1}(x)$ does not mean $\frac{1}{f(x)}$

For this example, $\frac{1}{f(x)}$ would be $\frac{1}{4x}$ with the x in the denominator, and that is not the same.

Here is a slightly more complicated **example**. Suppose we have *f*(x) = 3x + 2 .

We can break up this function into a series of operations. First the function multiplies by 3, and then it adds on 2.

- x
- Times 3
- Then, add 2

To get back to x from *f*(x), we would need to reverse these operations. So we would need to take away 2, and then divide by 3. When we undo the operations, we have to reverse the order as well.

- x – 2
- Then, divide by 3

$f^{-1}(x) = \frac{x-2}{3}$

Here is one more **example** of how we can reverse the operations of a function to find its inverse.
Suppose we have $f(x) = 7 - x^3$.
It is easier to see the sequence of operations to be carried out on x if we rewrite the function as $f(x) = -x^3 + 7$.

So the first operation performed by f takes x and cubes it; then the result is multiplied by −1; and finally 7 is added on.

- x^3
- Times -1
- Plus, 7

So to get from $f(x)$ to x, we need to

- Subtract 7
- Then, divide by -1
- And, take the cube root

So, $f^{-1}(x) = \sqrt[3]{-x+7}$

Key Point
We can work out f^{-1} by reversing the operations of f. If there is more than one operation, then we must reverse the order as well as reversing the individual operations.

Exercises
Work out the inverses of the following functions:
(a) $f(x) = 6x$

(b) $f(x) = 3 + 4x^3$

(c) $f(x) = 1 - 3x$

<u>Using algebraic manipulation to work out inverse functions</u>
Another way to work out inverse functions is by using algebraic manipulation. We can demonstrate this using our second example, $f(x) = 3x + 2$.

Now the inverse function takes us from $f(x)$ back to x. If we set $y = f(x) = 3x + 2$, then f^{-1} is the function that takes y to x. So to work out f^{-1} we need to know how to get to x from y. If we rearrange the expression to solve for x
$y = 3x + 2$,
$y - 2 = 3x$
so that $x = \frac{y-2}{3}$ So, reversing x and y yields $f^{-1}(x) = \frac{x-2}{3}$

We can use the method of algebraic manipulation to work out inverses when we have slightly trickier functions than the ones we have seen so far. Let us take

$f(x) = \frac{3x}{2x-1}$, $x \neq ½$.

We have made the restriction $x \neq ½$ because at $x = ½$ the function does not have a value. This is because the denominator is zero when $x = ½$.

Now we set $y = \frac{3x}{2x-1}$. Multiplying both sides by $2x - 1$ we get $y(2x - 1) = 3x$, and then multiplying out the bracket gives $2yx - y = 3x$.

We want to rearrange this equation so that we can express x as a function of y, and to do this we take the terms involving x to the left-hand side, giving $2yx - 3x = y$.

Now we can then take out x as a factor on the left-hand side to get $x(2y - 3) = y$, and dividing throughout by $2y - 3$ we finally obtain $x = \frac{y}{2y-3}$

So the by reversing the x and y the inverse function is $f^{-1}(x) = \frac{x}{2x-3}$

In the last example, it would not have been possible to work out the inverse function by trying to reverse the operations of *f*. This example shows how useful it is to have algebraic manipulation to work out inverses.

Key Point

Algebraic manipulation is another method that can be used to work out inverse functions. The key points are to solve the function for x and then reverse the x and the y.

This last form is the way to find the inverse of an exponential function.

Exercises

Find the inverse of the following using this algebraic manipulation method.

(a) $f(x) = -5x - 1$

(b) $f(x) = \frac{3x+7}{2}$

(c) $f(x) = \frac{2x}{5x-1}$

(d) $f(x) = \frac{x-1}{2x+3}$

More in depth practice with inverses can be found at
http://www.khanacademy.org/math/trigonometry/functions_and_graphs/fucntion_inverses/v/introduction-to-function-inverses