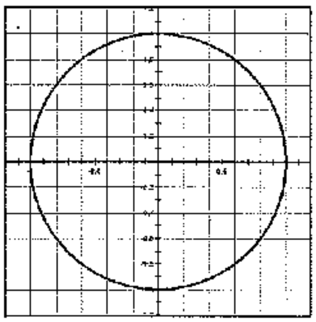
Right Triangles and the Unit Circle:

1. The circle below is referred to as a "unit circle." Why is this the circle's name?



Part I

2. Using a protractor, measure a 30° angle with vertex at the origin, initial side on the positive x-axis and terminal side above the x-axis. (Any angle with vertex at the origin, initial side on the positive x-axis and measured counterclockwise from the x-axis is called an angle in standard position.) Label the point where the terminal side intersects the circle as "A". Approximate the coordinates of point A using the grid.

3. Now, drop a perpendicular segment from the point you just put on the circle to the x-axis. You should notice that you have formed a right triangle. How long is the hypotenuse of your triangle? Using trigonometric ratios, specifically sine and cosine, determine the lengths of the two legs of the triangle. How do these lengths relate the coordinates of point A? How should these lengths relate to the coordinates of point A? Now use the properties of special right triangles to determine the lengths of the two legs. How do these lengths relate to the lengths found using trigonometric ratios? Which length is the exact solution and which is an estimate?

4. Using a Mira or paper folding, reflect this triangle across the y-axis. Label the resulting image point as point B. What are the coordinates of point B? How do these coordinates relate to the coordinates of point A? What obtuse angle (standard position angle) was formed with the positive x-axis (the initial side) as a result of this reflection? What is the reference angle (acute angle made with the terminal side and the x axis) for this angle?

5. Which of your two triangles can be reflected to create the angle in the third quadrant with a 30° reference angle? What is this angle measure? Complete this reflection. Mark the lengths of the three legs of the third quadrant triangle on your graph. What are the coordinates of the new point on circle? Label the point C.

6. Reflect the triangle in the first quadrant over the x-axis. What is this angle measure? Complete this reflection. Mark the lengths of the three legs of the triangle formed in quadrant four on your graph. What are the coordinates of the new point on circle? Label this point D.

7. Let's look at what you know so far about coordinates on the unit circle. Complete the table.

θ	x-coordinate (estimate)	y-coordinate (estimate)	x-coordinate (exact)	y-coordinate (exact)
30°				
150°				
210°				
330°				

Notice that all of your angles so far have a reference angle of 30°.

Use a calculator to verify your conclusions for cos 30° and sin 30°. Use your calculator to find trig values of other angles.

Based on these relationships, on the unit circle, x = ____________ and y = ____________.
This is a special case of the general trigonometric coefficients (rcosθ, rsinθ) where r = 1.

Part II

8. Now, let's look at the angles on the unit circle that have 45^0 reference angles. What are these angle measures?

9. Mark the first quadrant angle from #8 on the unit circle. Draw the corresponding right triangle as you did in Part I. What type of triangle is this? Use the Pythagorean Theorem or the properties of special right triangles to determine the lengths of the legs of the triangle. Confirm that these lengths match the coordinates of the point where the terminal side of the 45° angle intersects the unit circle using the grid on your graph of the unit circle.

10. Using the process from Part I, draw the right triangle for each of the angles you listed in #8. Determine the lengths of each leg and match each length to the corresponding x- or y-coordinate on the unit circle. List the coordinates on the circle for each of these angles in the table.

θ	x-coordinate (estimate)	y-coordinate (estimate)	x-coordinate (exact)	y-coordinate (exact)
45^0				
135^0				
225^0				
315^0				

Part III

11. At this point, you should notice a pattern between the length of the horizontal leg of each triangle and one of the coordinates on the unit circle. Which coordinate on the unit circle is given by the length of the horizontal leg of the right triangles?

12. Which coordinate on the unit circle is given by the length of the vertical leg of the right triangles?

13. Is it necessary to draw all four of the triangles with the same reference angle to determine the coordinates on the unit circle? What relationship(s) can you use to determine the coordinates instead?

14. Use special right triangles to determine the exact (x, y) coordinates where each angle with a 60° reference angle intersects the unit circle. Sketch each angle on the unit circle and clearly label the coordinates. Record your answers in the table.

θ	x-coordinate (estimate)	y-coordinate (estimate)	x-coordinate (exact)	y-coordinate (exact)
60°				
120°				
240°				
300°				

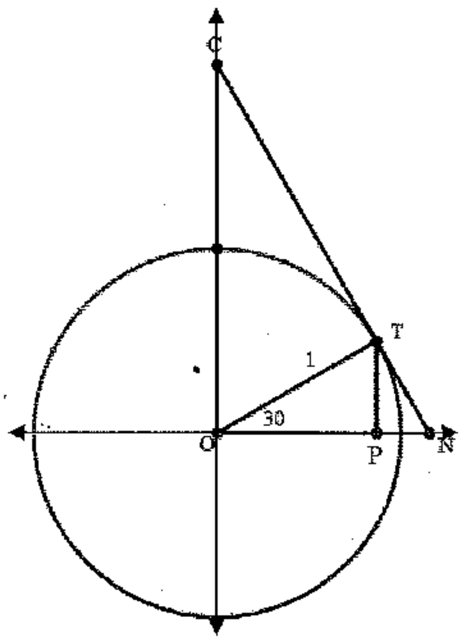
15. Think about what happens as you get to angles greater than 360°. How can you predict the value of the sine of 420°? What about the cosine of 600°?

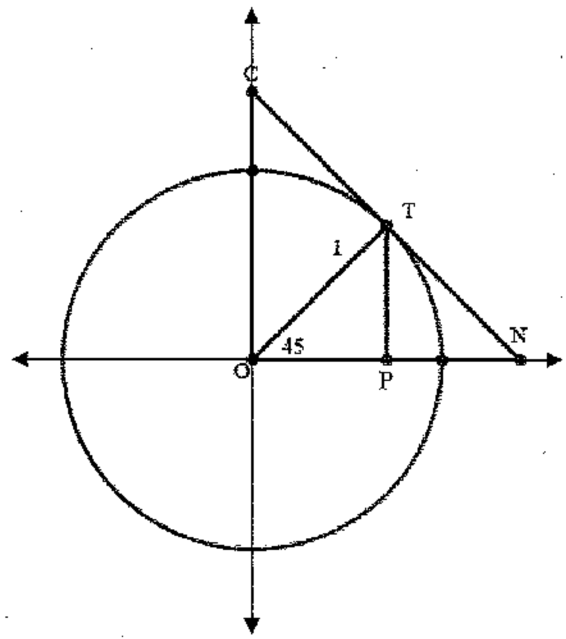
16. Does the same thing happen for negative angles? What is the largest negative angle that has the same sign and cosine as 120°?

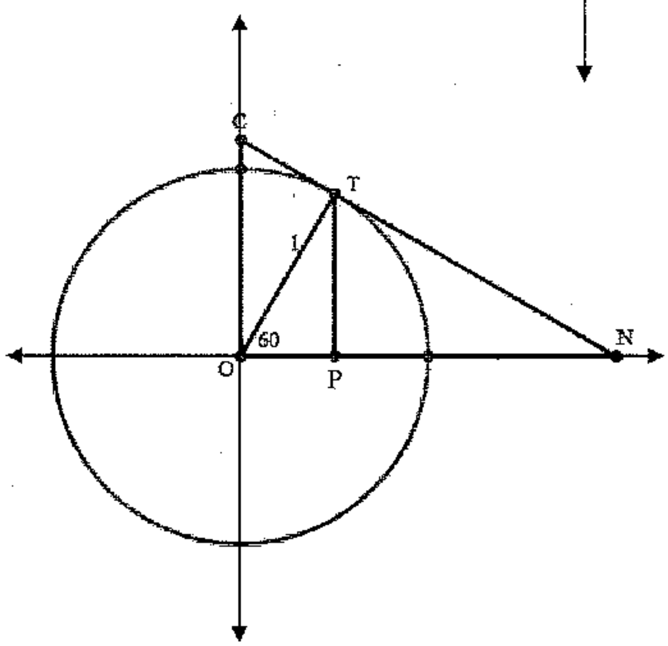
Part IV

17. In the figures below segment TP is perpendicular to segment ON. Line CN is tangent to circle O at T. N is the point where the line intersects the x-axis and C is the where the line intersects the y-axis. (Only segment CN is shown.)

Use your knowledge of sine and cosine to determine the length of segment TN. Use exact answers, no decimal approximations.







18. Using your understanding of the unit circle and $tangent\ \theta = \frac{sin\theta}{cos\theta}$, to complete the chart below for the indicated angles.

θ	Sin θ	Cos θ	Tan θ
30°			
45°			
60°			

19. How are these values of tangent related to the length you found in #4?

20. What would happen to the length of TN if the angle was changed to 0°?

21. What would happen to the length of TN if the angle was changed to 90°?

22. Based on the chart above fill in the sine, cosine, and tangent values for all the angles with the indicated reference angle.

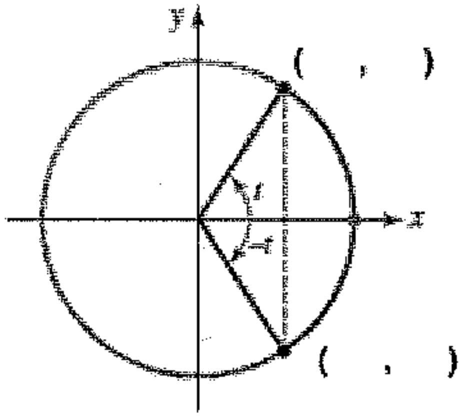
	θ	Sin θ	Cos θ	Tan θ
x	***30º***			
$\pi - x$				
$\pi + x$				
$2\pi - x$				

	θ	Sin θ	Cos θ	Tan θ
x	***45º***			
$\pi - x$				
$\pi + x$				
$2\pi - x$				

	θ	Sin θ	Cos θ	Tan θ
x	***60º***			
$\pi - x$				
$\pi + x$				
$2\pi - x$				

Part V

23. The symmetry of the unit circle is useful is generating the trigonometric values for an infinite number of angles. It is also useful in illustrating whether the sine and cosine functions are even or odd. Let's take 30° as an example. In the picture below $t = 30^\circ$. Fill in the missing information.



24. Based on the graphs of sine and cosine that we studied in Advanced Algebra, we know that sine is an odd function and cosine is an even function. What about the graphs help us to know this?

25. How can we use the information in #23 to further show that sine is an odd function and cosine is an even function?