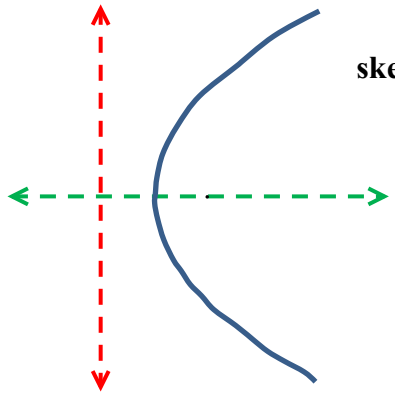
Our Only Focus: Parabolas Review

(Re)Presenting the…Parabola



Label the following features on the sketch to the left.

Vertex
Focus
Axis of Symmetry
Directrix
The directed distance p (label 2 different places)

Now answer the questions below.

(a) What relationship does the locus of points forming a parabola have with the focus and directrix?

(b) What relationship does the vertex have with the focus and the directrix?

(c) What relationship does the directed distance p have with the focus and the directrix?

Just as with circles, the most useable form for parabolas is standard form. Therefore, we need to know the following:

Standard Form of a Parabola and Related Information

With vertex (h, k) and directed distance from the vertex to the focus p:

$$\textit{Vertical Axis of Symmetry: } (x-h)^2 = 4p(y-k)$$

If p is positive, the parabola opens up; if p is negative, the parabola opens down.

$$\textit{Horizontal Axis of Symmetry: } (y-k)^2 = 4p(x-h)$$

If p is positive, the parabola opens to the right; if p is negative, the parabola opens to the left.

Let's try writing a few equations in standard form.

1. Write the equation of the parabola with a vertex at the origin and a focus at $(5, 0)$.

2. Write the equation of the parabola with focus at $(-3, 3)$ and directrix at $y = 9$.

3. Write the equation of the parabola that opens to the left, contains a distance of 5 between the focus and the directrix, and contains a vertex at $(9, 6)$.

Often you will be given an equation for a parabola that is not in standard form and you'll need to convert the equation to standard form. Consider the following equation of a parabola:

$$5y^2 - 6x + 10y - 7 = 0$$

This parabola has been written in general form. Using what we know about the coefficients from general form, we have $C = 5$, $D = -6$, $E = 10$, and $F = -7$. It's easy to see that the y term is squared, so either the parabola will open left or right, but beyond this, it's difficult to tell anything else about the relation. Therefore, we will have to convert to standard form by manipulating terms and completing the square:

$$5y^2 - 6x + 10y - 7 = 0$$

$$5y^2 + 10y = 6x + 7$$

$$5\left(y^2 + 2y + \left(\frac{2}{2}\right)^2\right) = 6x + 7 + 5\left(\frac{2}{2}\right)^2$$

$$5\left(y^2 + 2y + 1\right) = 6x + 12$$

$$5\left(y + 1\right)^2 = 6\left(x + 2\right)$$

$$\left(y + 1\right)^2 = \frac{6}{5}(x + 2)$$

So what do we now know? Well, we know the vertex of the parabola is at. $(-2, -1)$. We know the parabola opens to the right because p is positive. How do we know it's positive? Let's see…

Standard Form: $(y-k)^2 = 4p(x-h)$, so

$$4p = \frac{6}{5} \; so \; p = \frac{6}{20} = \frac{3}{10}$$

Therefore the focus is at $\left(-2+\frac{3}{10}, -1\right) = \left(\frac{-17}{10}, -1\right)$ and the directrix would be at $x = -2-\frac{3}{10}$ which simplifies to. $x = -\frac{23}{10}$

Convert the following equations of parabolas into standard form.

1. $x^2 + x - y = 5$

2. $2y^2 + 16y = -x - 27$

3. $x = -y^2 + 6y - 5$

Two more things …

Parabolas are from the past – they're not our focus now. But the next two conic sections are built upon your knowledge of these simplest of conics. Therefore, think about (and answer!) these two questions.

1. We know that the general form of a quadratic relation is

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

What relationship do the coefficients A and C have for a parabola?

2. Why was this activity named "our only focus"?