

Summer Assignment Notes

Topic #1: Piecewise Functions

Piecewise Functions: A piecewise function is a function which is defined symbolically using two or more formulas for different values in the domain.

Example:

$$f(x) = \begin{cases} x^2 + 4, & x < 5 \\ 0, & x = 5 \\ 2x, & x > 5 \end{cases}$$

Example of the domain values (set of x-values to use for a particular formula

Example of the "formula" used for a domain

How to read the function above:

"If x is a number smaller than 5, then use the formula, $f(x) = x^2 + 4$ "

"If x is 5, then use the formula $f(x) = 0$ "

"If x is a number larger than 5, then use the formula, $f(x) = 2x$ "

Note: The domain of $f(x)$ is all real numbers. Although each value of x has a specified formula, all values of x are accounted for amongst the formulas.

Example 1: Given the following piecewise function:

$$f(x) = \begin{cases} x^2 + 4, & x < 5 \\ 0, & x = 5 \\ 2x, & x > 5 \end{cases}$$

Find :

a) $f(3)$

b) $f(-2)$

c) $f(5)$

d) $f(8)$

Solution:

For $f(3)$, decide where 3 fits in the domain of $f(x)$. 3 is in the domain of the first "formula" ($x < 5$) so:

$$f(3) = 3^2 + 4 = 9 + 4 = 13$$

For $f(-2)$, decide where -2 fits in the domain of $f(x)$. -2 is in the domain $x < 5$.
Use, $f(x) = x^2 + 4$

$$f(-2) = (-2)^2 + 4 = 8$$

For $f(5)$, 5 is in the domain $x = 5$, therefore we should use $f(x) = 0$.

$$f(5) = 0.$$

For $f(8)$, 8 fits in the domain $x > 5$, therefore we should use the formula $f(x) = 2x$.

$$f(8) = 2(8) = 16.$$

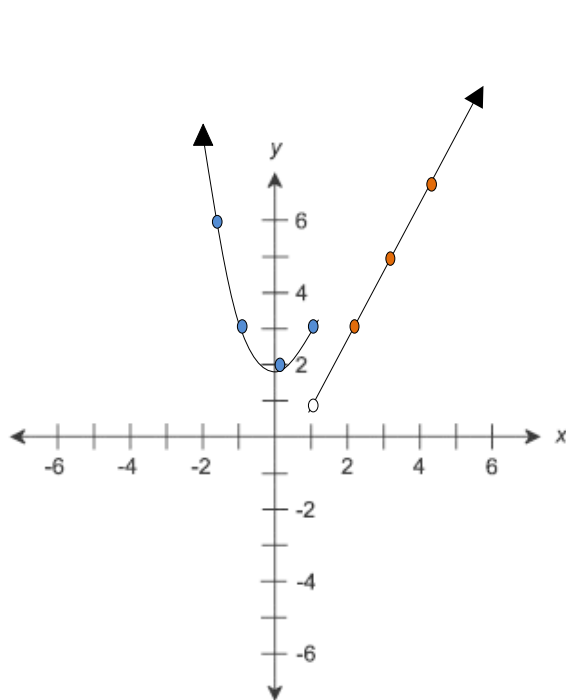
Example 2:

Graph the following piecewise function:

$$f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x - 1, & x > 1 \end{cases}$$

$$f(x) = x^2 + 2$$

$$f(x) = 4x - 1$$



x	y
-2	6
-1	3
0	2
1	3

Included
(closed
point)

x	y
1	1
2	3
3	5
4	7

Not
included
(open
point)

Note: $x = 1$ is in both tables, even though it only belongs in the domain of one of the formulas. You must plot the point for both, but one will be plotted as an open circle (meaning it is not included in

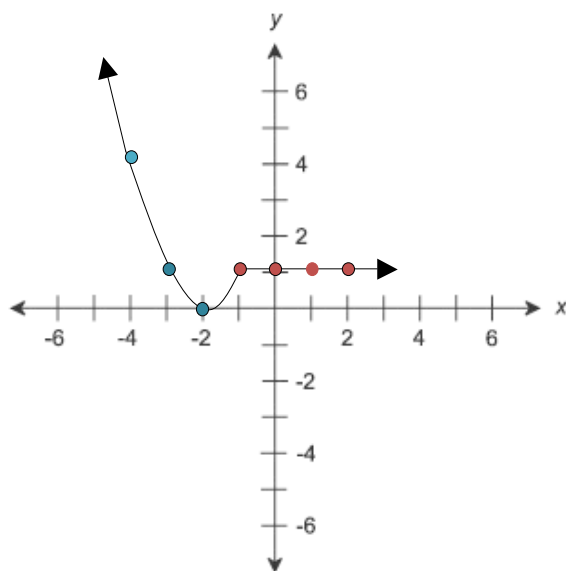
The example above is “not continuous.” There is a “break” in the graph when $x = 1$.

It is possible for a piecewise function to be continuous. In order for a piecewise function to be continuous, the point at which the graph switches from one formula to another must go to the same y-coordinate for this to happen.

For example:

Graph the following piecewise function:

$$f(x) = \begin{cases} (x+2)^2, & x < -1 \\ 1, & x \geq -1 \end{cases}$$



$$f(x) = (x+2)^2$$

x	y
-4	4
-3	1
-2	0
-1	1

Open circle
(not included
in domain)

$$f(x) = 1$$

x	y
-1	1
0	1
1	1
2	1

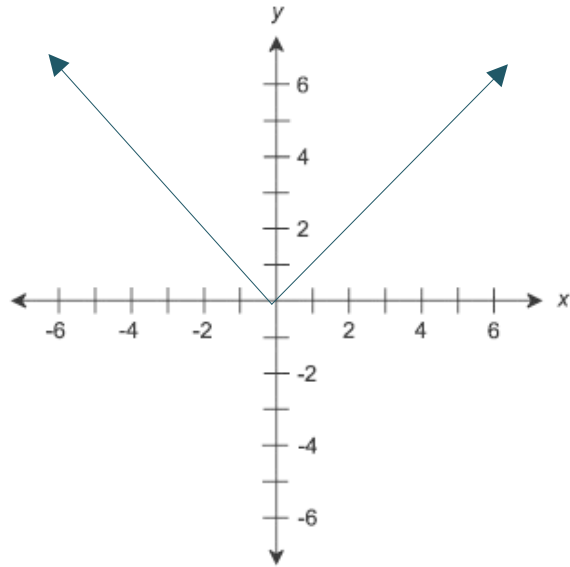
Closed Circle
(included in domain)

Note: Since the input $x = -1$ contains the same output for both formulas, the graph is "continuous"

Topic #2: Absolute Value Graphs as Piecewise Functions

Most basic absolute value function:

$$f(x) = |x|$$



Note: Each output (y-value) can be defined by two different inputs (except for 0). To be able to write the piecewise function, this is VERY IMPORTANT!

NOTE: In order to identify where your various formulas should start and end (the domain), you need to identify where the expression in the absolute value equals 0.

For the example above:

$x = 0$, so the piecewise function is:

$$f(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

These domain values are determined by the fact that when you set the expression in the absolute value = 0, that is the value of x .

Let's try a more difficult example:

Rewrite the following absolute value equation as a piecewise function.

$$f(x) = |x - 3|$$

$$x - 3 = 0$$

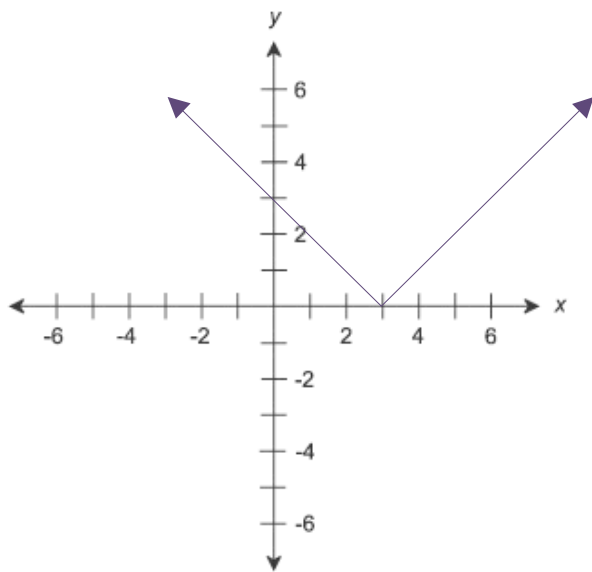
The graph changes when $x = 3$.

So when $x \geq 3$, the equation is the line $y = x - 3$ and when $x < 3$, the line is the "opposite of the equation above"... $y = -(x-3)$

$$f(x) = \begin{cases} x - 3, & x \geq 3 \\ -(x - 3), & x < 3 \end{cases}$$

OR

$$f(x) = \begin{cases} x - 3, & x \geq 3 \\ -x + 3, & x < 3 \end{cases}$$



Example 3: Write the equation $y = -|x - 3|$ as a piecewise function.

Identify where the graph changes directions by setting the expression in the absolute value = 0

$$x - 3 = 0$$

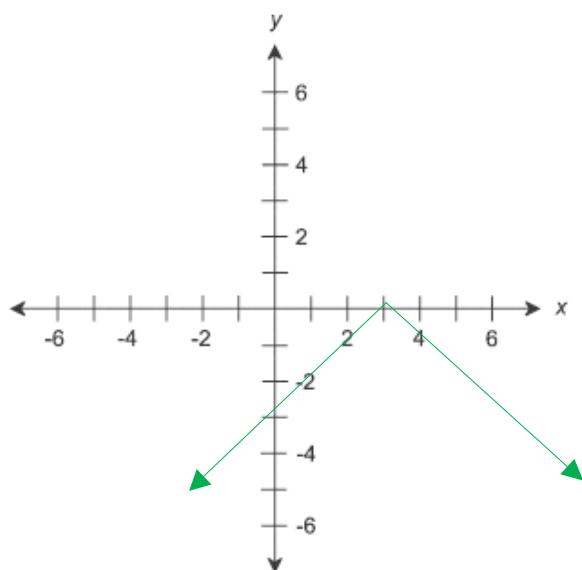
$$x = 3$$

For value $x \geq 3$, use the equation $y = -(x - 3)$. For the values $x < 3$, use the "opposite"... $y = -(-(x-3))$ which is simplified to $y = x - 3$.

$$y = \begin{cases} -(x-3), & x \geq 3 \\ x-3, & x < 3 \end{cases}$$

OR

$$y = \begin{cases} -x+3, & x \geq 3 \\ x-3, & x < 3 \end{cases}$$



Example 4: Write the following absolute value function as a piecewise function.

$$f(x) = |x - 3| - 2$$

Identify where the graph switches directions.

$$x - 3 = 0$$

$$x = 3$$

For the domain values, $x \geq 3$, the equation is

$$f(x) = x - 3 - 2$$

$$= x - 5.$$

For the values $x < 3$, the equations is:

$$f(x) = -(x - 3) - 2$$

$$= -x + 3 - 2$$

$$= -x + 1$$

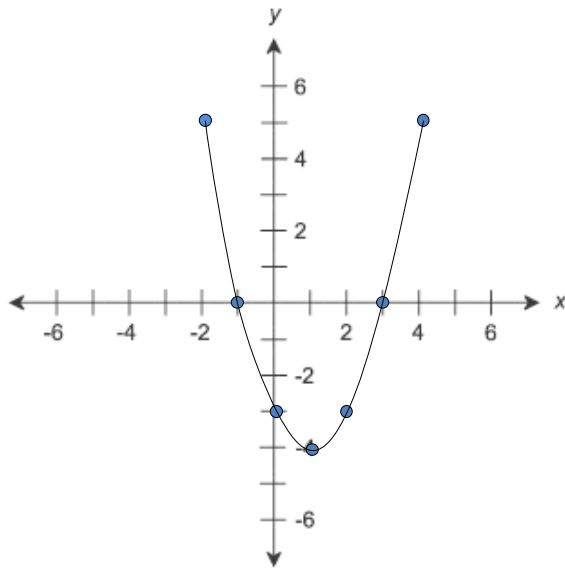
The piecewise function is:

$$f(x) = \begin{cases} x - 5, & x \geq 3 \\ -x + 1, & x < 3 \end{cases}$$

Example 5: Identify the piecewise function for the following absolute value equation.

$$f(x) = |x^2 - 2x - 3|$$

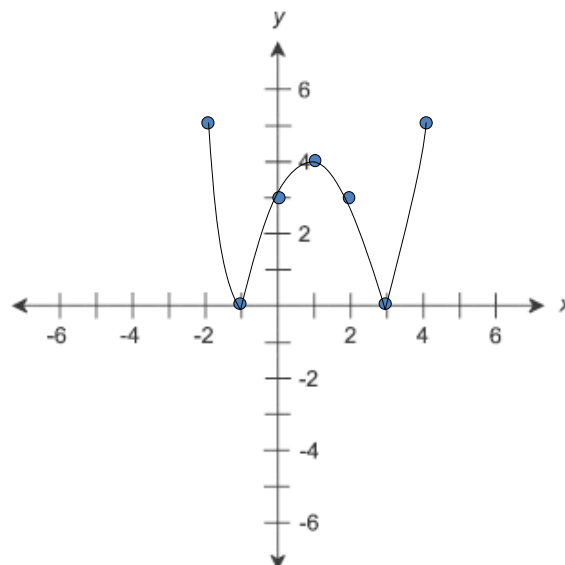
Remember: All absolute value equations will make negative values positive.
Let's look at the graph of $y = x^2 - 2x - 3$



x	y
-2	5
-1	0
0	-3
1	-4
2	-3
3	0
4	5

Now: Let's think about what the graph of $f(x)$ might look like.

$$f(x) = |x^2 - 2x - 3|$$



x	y
-2	5
-1	0
0	3
1	4
2	3
3	0
4	5

What would the piecewise function look like then?

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3, x = -1$$

$$f(x) = \begin{cases} x^2 - 2x - 3, & x < -1 \\ -(x^2 - 2x - 3), & -1 \leq x \leq 3 \\ x^2 - 2x - 3, & x > 3 \end{cases}$$

Topic #3: Factoring Polynomials

Review of Basic Factoring:

Step 1: Remember to look for a Greatest Common Factor first (in calculus, these may be more obscure than you think!)

Example:

$$2a^2 - 3a \rightarrow a(2a - 3)$$

$$(x + 3)^3(x + 2) - (x + 3)^2(3x + 1) \rightarrow (x + 3)^2 [(x + 3)(x + 2) - (3x + 1)]$$

Step 2: Look for a difference of two squares.

Example:

$$x^2 - 16 \rightarrow (x + 4)(x - 4)$$

$$25 - 9x^2 \rightarrow (5 - 3x)(5 + 3x)$$

Note: When the number is the first term of your binomial, the number is the first term in the factored form of a difference of two squares.

Step 3: Look for factoring a trinomial.

Example:

$$x^2 - 3x - 4 \rightarrow (x - 4)(x + 1)$$

$$x^2 - 7x + 12 \rightarrow (x - 3)(x - 4)$$

Note: You are looking for two numbers that multiply to the third term and add to the coefficient of the 2nd term.

Type 1: Factoring By Grouping

Example 1:

$$x^3 + 2x^2 \mid -6x - 12$$

$$x^2(x + 2) - 6(x + 2)$$

$$(x^2 - 6)(x + 2)$$

Note: Cut the polynomial in "half" and then factor out a GCF for the first half and a GCF for the 2nd half. Use "undistributive property" and write as two binomials.

Example 2:

$$3x^2 - 11x - 4$$

$$a \cdot c = -12$$

$$b = -11$$

Numbers that multiply to -12 and add to -11 are **-12 and 1**

$$3x^2 - 12x + 1x - 4$$

$$\begin{array}{l} 3x^2 - 12x \mid + 1x - 4 \\ 3x(x - 4) + 1(x - 4) \\ (3x + 1)(x - 4) \end{array}$$

Note: If there is no GCF, pull out a 1 as a "placeholder"

Putting it All Together:

Factor out a GCF, then look for another type of factoring. Continue factoring until you have broken the problem into its simplest pieces.

Example:

Factor:

$$x^3 + 4x^2 - 12x$$

$$x(x^2 + 4x - 12) \rightarrow \text{Factor out a GCF}$$

$$x(x + 6)(x - 2) \rightarrow \text{Factor the trinomial}$$

Example 2:

$$x^2(x - 2) + x(x - 2) - 12(x - 2)$$

$$(x - 2)(x^2 + x - 12)$$

$$(x - 2)(x + 4)(x - 3)$$

Topic #4: Parent Functions and Transformations

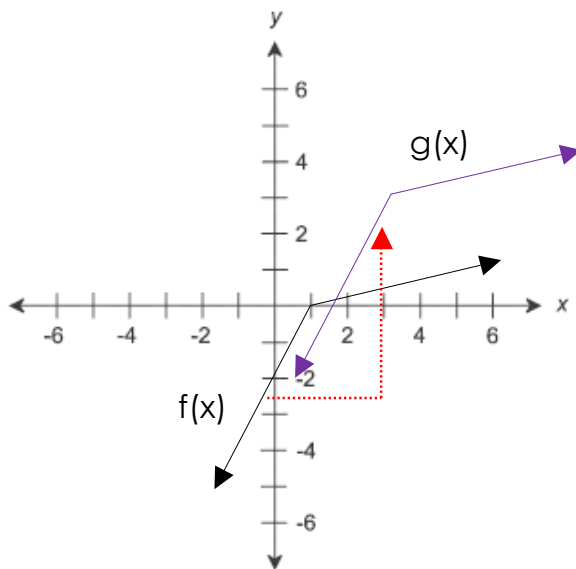
Remember from Algebra 2/Trigonometry:

If the original function is $f(x)$.

$f(x) + c$	→	shifts the graph up “c” units
$f(x) - c$	→	shifts the graph down “c” units
$f(x + c)$	→	shifts the graph left “c” units
$f(x - c)$	→	shifts the graph right “c” units
$af(x)$	→	stretches the graph by a factor of “a”
$-f(x)$	→	reflects the graph in the x-axis
$f(-x)$	→	reflects the graph in the y-axis

Example 1:

Given $f(x)$ graphed below, graph: $g(x) = f(x - 2) + 3$



Shift the graph **2 units right**
and **3 units up**

Example 2:

Describe the transformation of the graph of $f(x)$, if $g(x) = 3f(x) - 2$

Solution:

$g(x)$ takes the graph of $f(x)$ and stretches it by 3 and shifts the graph down 2 units.

Topic #5: Radicals and Exponents

Rules of Exponents:

$$x^m \cdot x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{mn}$$

$$x^{-m} = \frac{1}{x^m}$$

$$x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

Note: In calculus it is important that you are fluent going back and forth between positive and negative exponents as well as between radicals and fractional exponents!

Examples:

$$5x^{-2} = \frac{5}{x^2}$$

$$4\sqrt[5]{3x} = 4(3x)^{\frac{1}{5}}$$

Example 1: Simplify Using Only Positive Exponents

$$(16x^2y)^{\frac{3}{4}}$$

$$16^{\frac{3}{4}} x^{\frac{6}{4}} y^{\frac{3}{4}}$$

$$8x^{\frac{3}{2}}y^{\frac{3}{4}}$$

For now, leave fractional exponents unless it simplifies fully. These are FINE for calculus.

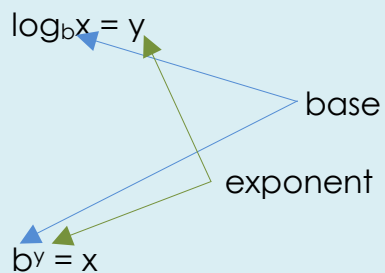
Example 2:

$$\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}} = \frac{\sqrt{4(x-4)}}{\sqrt[4]{(x-4)^3}} = \frac{2\sqrt{(x-4)}}{\sqrt[4]{(x-4)^3}} = \frac{2(x-4)^{\frac{1}{2}}}{(x-4)^{\frac{3}{4}}} = 2(x-4)^{\frac{1}{2}-\frac{3}{4}}$$

$$= 2(x-4)^{-\frac{1}{4}} = \frac{2}{(x-4)^{1/4}} = \frac{2}{\sqrt[4]{x-4}}$$

Topic #6: Logarithmic and Exponential Functions

Logarithm to Exponential Functions



$$\log_b(mn) = \log_b(m) + \log_b(n)$$

$$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$$

$$\log_b(m^n) = n\log_b m$$

Examples:

Rewrite the following in exponential form.

$$\log_3 x = 2$$

base exponent

$$3^2 = x$$

$$9 = x$$

Rewrite the following in logarithmic form.

$$5^z = y$$

$$\log_5 y = z$$

Note: If the log has “no base written”, the log is base 10.

Example: $\log(100)$

Example: Evaluate the following logarithms without a calculator. Use the properties of logarithms when necessary.

$$\log_2 8 = \log_2 2^3 = 3$$

$$\log_3 \left(\frac{1}{27} \right) = \log_3 3^{-3} = -3$$

$$\log_{12} 1 = \log_{12} 12^0 = 0$$

$$\log_{36} \sqrt{6} = \log_{36} (6)^{\frac{1}{2}} = \log_{36} (36^{1/2})^{1/2} = 1/4$$

$$\log 25 + \log 4 = \log (25 \cdot 4) = \log (100) = \log (10^2) = 2$$

$$\log_2 (\sqrt{2})^5 = 5 \log_2 (2)^{1/2} = 5(1/2) = 5/2$$

$y = e^x$ and $y = \ln(x)$ are inverses of one another as well. e is Euler's number. $e = 2.718\dots$ e is a transcendental number like $\sqrt{2}$ and π . It continues on forever
 $\ln$ is the natural log. It could be rewritten as $\log_e x$.

Examples:

$$\ln(e^4) = 4$$

$$\ln\left(\frac{1}{e}\right) = \ln(e^{-1}) = -1$$

$$\frac{1}{3} \ln \left(\frac{1}{e^3} \right) = \frac{1}{3} \ln (e^{-3}) = \left(\frac{1}{3} \right) (-3) = -1$$

Note: The domain and range of an exponential function, such as ($y = e^x$ and $y = 3^x$ (graphed below) are:

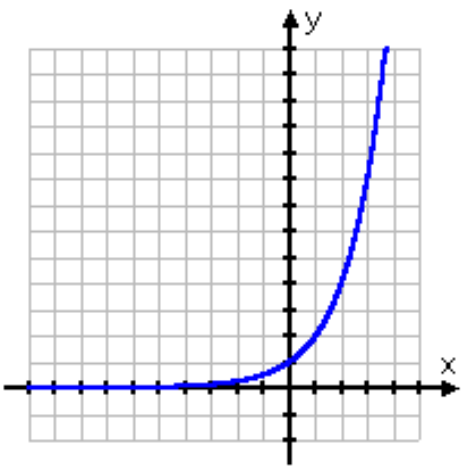
D: {x: all real numbers}

R: {y: $(0, \infty)$ }

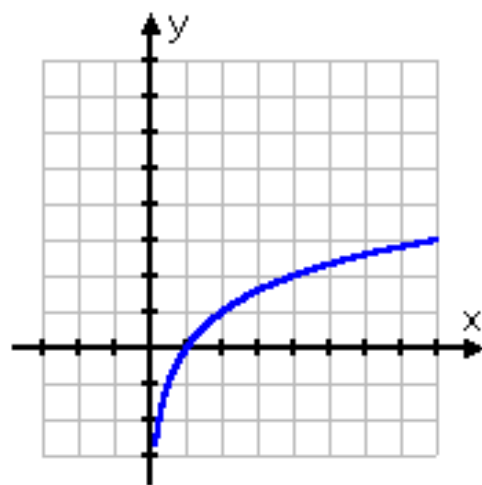
Since logarithms and exponential functions are inverse functions, they also switch their domains and ranges. Therefore, the domain and range of a logarithmic function (graphed below) are:

D: {x: $(0, \infty)$ }

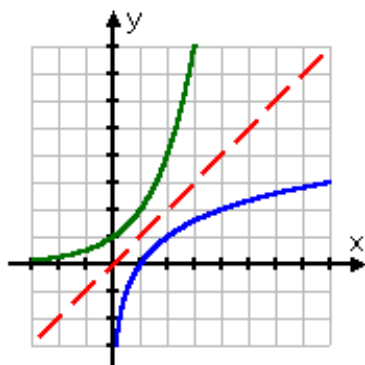
R: {y: all real numbers}



Exponential Graph



Logarithmic Graph



When solving exponential and logarithmic equations, use the inverse function to help you to solve. Using the inverses and properties of logarithms, you can solve exponential functions efficiently.

Example:

Solve for x:

$$\log_5(2x+5) = 2$$

$$(2x + 5) = 5^2$$

(Rewriting as an exponential function)

$$2x + 5 = 25$$

$$2x = 20$$

$$x = 10$$

$$3^x = 12$$

$$\ln(3^x) = \ln(12)$$

(Take the natural log of both sides)

$$x \ln(3) = \ln(12)$$

(Use the properties of exponents)

$$x = \frac{\ln 12}{\ln 3}$$

$$2.262$$

Note: In calculus, we will always round final answers to three decimal places.

$$2e^{2x-5} = 24$$

$$e^{2x-5} = 12$$

$$\ln(e^{2x-5}) = \ln(12)$$

$$2x - 5 = \ln(12)$$

$$2x = \ln(12) + 5$$

$$x = 3.742$$

Topic #7: Evaluating Functions

To evaluate a function, replace the variable with what it is equal to according to the function notation.

Example:

Given a function $f(x)$, evaluate $f(3)$ means to replace x with 3 and simplify.

Example:

$$\text{Given } f(x) = x^2 + 2$$

$$f(3) = (3)^2 + 2 = 11$$

$$f(z) = (z)^2 + 2 = z^2 + 2$$

$$f(a + 2) = (a + 2)^2 + 2 = a^2 + 4a + 4 + 2 = a^2 + 4a + 6$$

$$f(x+h) = (x + h)^2 + 2 = x^2 + 2xh + h^2 + 2$$

Topic #8: Composing and Decomposing Functions

When working with composing functions, things to remember:

Work right to left or inside out...

eg. $(f \circ g)(x) \rightarrow$ work right to left

$f(g(x)) \rightarrow$ work inside out

The "interior" function, you are going to use in the outer function and evaluate like we did for topic #7: evaluating functions.

Example:

Given $f(x) = \sin(x)$, $g(x) = x^2$

, $h(x) = 2x$, $k(x) = x + 1$

Evaluate $f(g(x))$

Evaluate $g(k(3))$

Evaluate $g(k(h(x)))$

Evaluate $f(g(k(x)))$

Solutions:

$$f(g(x)) = f(x^2) = \sin(x^2)$$

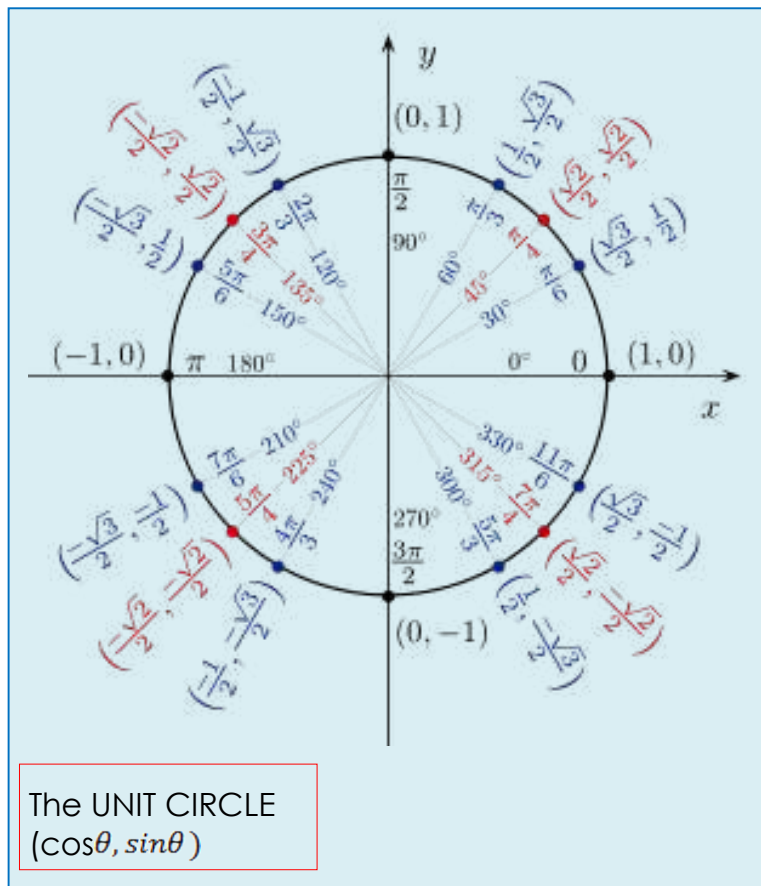
$$g(k(3)) = g(3 + 1) = g(4) = 4^2 = 16$$


$$g(k(h(x))) = g(k(2x)) = g(2x + 1) = (2x + 1)^2 = 4x^2 + 4x + 1$$

$$f(g(k(x))) = f(g(x+1)) = f((x+1)^2) = f(x^2 + 2x + 1) = \sin(x^2 + 2x + 1)$$

Topic #9: Trigonometry Review

The following should be memorized!



Identities to Know:

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

HINT: In calculus, we use radians the majority of the time so PLEASE be familiar with radians before school starts next year.

Solving Trig Equations:

Get the trig function alone. If you need to use substitution to solve, substitute the trig function for a variable.

Use inverse trig functions ($\arcsin x$, $\sin^{-1}x$, etc) to get the values. Determine all the values on the given interval of the domain.

Some problems are going to be standard values of $\sin\theta$, $\cos\theta$, $\tan\theta$. Do not use a calculator for these.

Example: Solve the following trigonometric equation on the interval $0 \leq \theta < 2\pi$

$$2\sin\theta - \sqrt{2} = 0$$

$$\sin\theta = \frac{\sqrt{2}}{2}$$

Sine Positive	All Positive
Tangent Positive	Cosine Positive

Quadrant Reference

On the unit circle, sine is positive in quadrants I and II. So we are looking for values in those two quadrants.

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

Example 2: Solve the following trigonometric equation for all values $0 \leq \theta < 2\pi$

$$2\cos^2\theta - 3\cos\theta + 1 = 0$$

Step 1: Use substitution $x = \cos\theta$

$$2x^2 - 3x + 1 = 0$$

$$(2x - 1)(x - 1) = 0$$

$$2x - 1 = 0$$

$$x = 1/2$$

$$\cos\theta = 1/2$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$x - 1 = 0$$

$$x = 1$$

$$\cos\theta = 1$$

$$\theta = 0$$

Topic #10: Writing Equations of Lines

$$(y - y_1) = m(x - x_1)$$

m = slope

(x_1, y_1) = a point on the line (this can be any point!)

So to write the equation you need to know the slope:
(WHICH IN CALCULUS WE WILL REFER TO AS RATE OF CHANGE!)

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

(In calculus this will be referred to as “average rate of change”)

Example:

Calculate the slope between $(0, 2)$ and $(-1, 3)$

$$m = \frac{3-2}{-1-0} = \frac{1}{-1} = -1$$

Example:

Write the equation of a line with a slope of $-\frac{1}{3}$ and goes through the point $(-1, 5)$

$$(y - y_1) = m(x - x_1)$$

$$y - 5 = -\frac{1}{3}(x + 1)$$

Topic #11: Domain and Range

Domain: values of x (the input) that will work in the function. Some functions have restrictions in their domain. Not all values of x can be used for all functions. Some restrictions result in **asymptotes and holes**. Other functions just start at certain x values and do not include values above or possibly below that starting x -value.

Range: Values of y (the output) that can be evaluated from the function. Functions that have restrictions on the range are exponential functions, quadratic functions and possibly other even powered polynomials.

Some functions to check for domain restrictions are:

Rational Functions

Radical Functions (specifically even roots)

Logarithmic Functions

Some functions to check for range restrictions are:

Quadratic Functions

Radical Functions

Rational Functions

Exponential Functions

Example:

What is the domain of the following function?

$$f(x) = \sqrt{2x + 4}$$

$$2x + 4 \geq 0$$

$$2x \geq -4$$

$$x \geq -2$$

Example:

What is the range of the following function?

$$f(x) = x^2 + 2x - 4$$

The vertex occurs at $x = -b/2a$
 $-2/2(1) = -1$

The y value at that point is $f(-1) = (-1)^2 + 2(-1) - 4 = 1 - 2 - 4 = -5$

Since this parabola opens upward, its range is $[-5, \infty)$.

Topic #12: Rational Equations/Expressions and Functions

Simplifying Rational Expressions:

Things to Remember:

Factor before starting any problem.

For addition and subtraction problems, make sure that you either get the same denominator or "get rid of the denominator."

Once you are left with either one fraction or just multiplication, you can begin to cancel factors that are in both the numerator.

For complex fractions, determine the LCD of both the numerator and the denominator. Multiply both fractions by that LCD.

Example 1:

Simplify:

$$\frac{x^2-3x}{2x^2+x-6} \div \frac{x^2-5x+6}{x^2-4} \rightarrow \frac{x^2-3x}{2x^2+x-6} * \frac{x^2-4}{x^2-5x+6} \rightarrow \frac{x(x-3)}{(2x-3)(x+2)} * \frac{(x+2)(x-2)}{(x-2)(x-3)}$$
$$\frac{\cancel{x}(\cancel{x-3})}{(2x-3)(\cancel{x+2})} * \frac{(\cancel{x+2})(\cancel{x-2})}{(\cancel{x-2})(\cancel{x-3})} = \boxed{\frac{x}{2x-3}}$$

Example 2:

Simplify:

$$\frac{3a+1}{a^2-1} - \frac{1}{a+1}$$
$$\frac{3a+1}{(a+1)(a-1)} - \frac{1}{a+1}$$
$$\frac{3a+1}{(a+1)(a-1)} - \frac{1}{a+1} \left(\frac{a-1}{a-1} \right)$$
$$\frac{3a+1-a+1}{(a+1)(a-1)} = \frac{2a+2}{(a+1)(a-1)} = \frac{\cancel{2(a+1)}}{(\cancel{a+1})(a-1)} = \boxed{\frac{2}{a-1}}$$

Example 3:
Simplify:

$$\frac{\frac{3x}{x+3}}{x^2-9} \cdot \frac{\frac{(x+3)(x-3)}{1}}{\frac{(x+3)(x-3)}{1}} = \frac{\frac{3x(x+3)(x-3)}{(x+3)(x-3)}}{\frac{(x+3)(x-3)}{(x+3)(x-3)}} = \frac{3x(x-3)}{x} = 3(x-3) \text{ or } 3x - 9$$

Solving Rational Equations:

Things to Remember:

Make sure you either get all fractions to have the same denominator or get rid of the denominator by multiplying every term by the Least Common Denominator.

Create an equation from the numerators.

Solve the equation created in step 2.

Check to make sure the value is in the domain of the rational equation. (*Check the denominators of the original equation and make sure they are not 0).

Example:
Solve for all values of x.

$$\frac{9}{x} + \frac{9}{x+2} = 12$$

$$\left(\frac{9}{x} + \frac{9}{x+2} = 12\right) \left(\frac{x(x+2)}{1}\right)$$

$$\frac{9x(x+2)}{x} + \frac{9x(x+2)}{x+2} = 12x(x+2)$$

$$9x+18+9x = 12x^2 + 24x$$

$$0 = 12x^2 + 6x - 18$$

$$0 = 6(2x^2 + x - 3)$$

$$0 = 6(2x + 3)(x - 1)$$

$$x = -3/2, x = 1$$

Verify that these are in the domain of the equation.

Check denominators: $x \neq 0, -2$

Graphing Rational Functions:

Zeros: To find the zeros, set the numerator equal to zero, factor and solve.

Y-intercept: To find the value of the y-intercept, find $f(0)$.

Vertical Asymptotes: To get the vertical asymptotes, first factor the numerator and denominators of the rational function. Cancel factors they share in common and then set the "new" denominator equal to zero.

Holes: To get the holes, factor the numerator and denominator. Cancel the factors that are in both numerator and denominator. (Set the factors that cancel equal to zero to find the holes of the graph). This will give you the x-values of the holes. To get the y-values, evaluate the "new, canceled" function with the x-value of the hole.

End Behavior: For the end behavior, there are three things that could happen.
The graph can have a horizontal asymptote at $y = 0$.
The graph can have a horizontal asymptote at another y-value.
The graph can have no asymptote.

****We will be working with end behavior a lot at the beginning of calculus!**

With the above information, you can get a relatively good idea of what the graph of the rational function will look like. We are missing a few things like maximum and minimum values. We will learn how to calculate these values in calculus next year.

Let's look at a few problems.

Finding Zeros (x-intercepts) of the Rational Functions

An x-intercept occurs when $y = 0$. In the case of a rational function, this is when the numerator of the fraction equals 0.

Example 1: Find the x-intercepts of the following function.

$$y = \frac{x^4(2x - 3)^7(x + 4)^8}{(x + 2)^3(x - 1)^{20}}$$

Just the numerator = 0 is important in this case.

$$0 = x^4(2x - 3)^7(x + 4)^8$$

$$0 = x^4$$

$$+ 4$$

$$x = 0,$$

$$0 = 2x - 3$$

$$0 = x$$

$$x = 3/2, x = -4$$

The x - intercepts are (0,0), (1.5,0) and (-4,0)

Example 2: Find the x-intercepts of the following function.

$$y = \frac{x^2 - 2x - 3}{x}$$

Set the numerator = 0

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3, x = -1$$

The x-intercepts are (3,0) and (-1,0)

Finding the y-intercept of the function:

The y-intercept occurs when $x = 0$.

Example 1:

$$y = \frac{x^4(2x - 3)^7(x + 4)^8}{(x + 2)^3(x - 1)^{20}}$$

Substitute $x = 0$

$$y = \frac{0^4(2(0) - 3)^7(0 + 4)^8}{(0 + 2)^3(0 - 1)^{20}}$$

$$y = 0$$

(0,0) is the y-intercept

Example 2:

$$y = \frac{x^2 - 2x - 3}{x}$$

Substitute $x = 0$

$$y = \frac{0^2 - 2(0) - 3}{0} = -3/0 = \text{undefined.}$$

There are no y-intercepts.

Finding the Vertical Asymptotes

A rational function has a vertical asymptote when the bottom of the fraction (the denominator) is equal to 0.

Example 1: Find the vertical asymptotes of the following function.

$$y = \frac{x^4(2x - 3)^7(x + 4)^8}{(x + 2)^3(x - 1)^{20}}$$

Set the denominator equal to 0.

$$(x + 2)^3 = 0$$

$$x = -2$$

$$(x - 1)^{20} = 0$$

$$x = 1$$

There are vertical asymptotes with equations $x = -2$ and $x = 1$.

Example 2: Find the vertical asymptotes of the following function.

$$y = \frac{x^2 - 2x - 3}{x}$$

Set the denominator = 0

$$x = 0$$

There is a vertical asymptote at $x = 0$.

Finding horizontal asymptotes

Before we begin finding vertical asymptotes, we need a couple definitions.

Definition 1:

Degree – If a polynomial has one variable, it is the highest exponent

Definition 2:

Leading Coefficient – The coefficient of the variable with the highest exponent

Think of a rational function as two polynomial functions being divided.

$$y = \frac{P(x)}{Q(x)}, \text{ where } P(x) \text{ and } Q(x) \text{ are both polynomial}$$

There are 3 cases:

Case #1: The degree of $P(x)$ is larger than the degree of $Q(x)$ (The numerator has a higher degree than the denominator) – In this case, there is no horizontal asymptote

Case #2: The degree of $Q(x)$ is larger than the degree of $P(x)$ (The degree of the denominator is larger than the degree of the numerator) – in this case the horizontal asymptote is $y = 0$

Case #3: The degree of $P(x)$ and $Q(x)$ are the same – in this case compare the leading coefficients of each of these polynomials

We are really going to look at examples of the last 2 cases. The first one is easy to determine.

Example 1: Find the horizontal asymptote of the following rational function.

$$y = \frac{3x^2 - 7x}{8 + 5x^4}$$

The largest degree in the numerator is 3 and the largest degree in the denominator is 4. This is case 2. The denominator will grow much faster as you get larger and larger values of x . So the horizontal asymptote is $y = 0$. (We will go into more detail with this in the school year!)

Example 2: Find the horizontal asymptote of the following rational function.

$$y = \frac{6x^3 + 2}{5x - 7x^2}$$

In this case, the largest degree is 3 in both the numerator and denominator.

The horizontal asymptote can be determined just by looking at the term with the largest degree in the numerator and denominator!

$$y = \frac{6x^3}{-7x^3} = \frac{6}{-7} = -\frac{6}{7}.$$

The horizontal asymptote is located at $y = -6/7$

Finding Holes:

Holes in rational functions occur when the function has factors that cancel from the numerator and denominator.

Example: Find the holes of the following graph.

$$y = \frac{(x+2)(x-3)(x+4)}{(x-2)(x-3)}$$

Since $x-3$ cancels, the hole is at $x-3=0$. So the hole is at $x = 3$.

To figure out the y -value of the hole, substitute 3 into the simplified equation.

$$y = \frac{(x+2)(x+4)}{(x-2)} = \frac{(3+2)(3+4)}{(3-2)} = (5)(7) = 35$$

The hole is at (3,35)

For the summer assignment, I am going to have you work on identifying features of the rational function. We will work on graphing these functions and others with and without a calculator the first 2 weeks of school.