AP Calculus AB

Name ___________________________________

Dates: Start ___________ Finish ___________

AP Calculus AB Summer Assignment 2017

Welcome to AP Calculus AB! This packet is a compilation of Advanced Algebra & Pre-Calculus topics that you will use continuously in AP Calculus AB. Most topics will have a brief description and a couple of examples to help you recall what you studied previously. These topics should not be new to you, with the possible exception of piecewise functions.

This review packet is due the ***second*** day of school, and it is ***for a grade***! You will be tested on this material on the Friday of the first week of school. Please try to work on this packet weekly throughout the summer to keep your math skills sharp. There are ten sections, so please do not save this for the last minute! Study groups are suggested, not only for this packet, but also throughout the year.

If you need a graphing calculator and do not have one, I recommend using www.desmos.com over the summer. We will have a class set of the TI-nSpire CX handhelds for use during the school year, so you do not need to purchase one. However, if you do wish to purchase your own graphing calculator, I recommend that you wait for the back-to-school sales. There are many excellent graphing calculators out there, so please consult with me if you have questions about which brand or model may work best for you.

Helpful links: (you may visit my teacher webpage and click through the links)

Piecewise Functions - https://www.mathsisfun.com/sets/functions-piecewise.html

Unit Circle - http://www.embeddedmath.com/downloads/index.php?item=unitcircle.php

Khan Academy videos - https://www.khanacademy.org/

BrightStorm videos - https://www.brightstorm.com/sample-video-lessons

I hope you have a wonderful summer vacation! If you have any questions about this packet, please e-mail me at laura.coons@henry.k12.ga.us. There are a couple of weeks where I will not be checking my school e-mail, but I will respond as soon as possible!

Take care, and I look forward to seeing you next school year!

Ms. Coons ☺

I. Functions & Composition of Functions

A function is a set of points (x, y) such that for every x, there is one and only one y. In short, in a function, the x-values cannot repeat while the y-values can. The notation for functions is either "$y\ =$" or "$f\ (x) =$. To evaluate a function for a given value, substitute the value in the function for x.

With composition of functions, you first substitute a value into one function, determine the answer, and then substitute that answer into a second function.

Examples:

In the function $f(x) = x^2 + 4x - 3$, find $f(3)$.

$$f(3) = (3)^2 + 4(3) - 3 = 9 + 12 - 3 = 18$$

For the functions $f(x) = x - 4$ and $g(x) = x^2 - 2x + 1$, find $g\big(f(a+1)\big)$.

$$f(a+1) = a + 1 - 4 = a - 3$$
$$g(a-3) = (a-3)^2 - 2(a-3) + 1$$
$$= a^2 - 6a + 9 - 2a + 6 + 1$$
$$g\big(f(a+1)\big) = a^2 - 8a + 16$$

Evaluate.

Let $f(t) = v_0 t + \frac{1}{2}at^2$, where $v_0\ and\ a$ are constants. Find:

1) $f\left(\frac{1}{2}\right) =$

2) $f(x-1) =$

Let $f(x) = x^2 - 3$ and $g(x) = \frac{1}{2}x + 4$. Find:

3) $f\big(g(4)\big) =$

4) $g\big(f(x-2)\big) =$

II. Domain & Range

Find the domain and range of each function. Use interval notation (parenthesis or square brackets).

1) $f(x) = x^2 + 2x - 4$

2) $f(x) = 2\sqrt{x} + 4$

3) $f(x) = \frac{x-4}{(x+1)(x-2)}$

III. Factoring

Special cases of factoring that may help:

Common factor: $x^3 + x^2 + x = x\left(x^2 + x + 1\right)$
Difference of squares: $x^2 - y^2 = (x+y)(x-y)$ or $x^{2n} - y^{2n} = \left(x^n + y^n\right)\left(x^n - y^n\right)$
Perfect squares: $x^2 + 2xy + y^2 = (x+y)^2$
Perfect squares: $x^2 - 2xy + y^2 = (x-y)^2$
Sum of cubes: $x^3 + y^3 = (x+y)\left(x^2 - xy + y^2\right)$ - Trinomial unfactorable
Difference of cubes: $x^3 - y^3 = (x-y)\left(x^2 + xy + y^2\right)$ - Trinomial unfactorable
Grouping: $xy + xb + ay + ab = x(y+b) + a(y+b) = (x+a)(y+b)$

Completely factor the following:

1) $3x^3 - 5x^2 + 2x$

2) $16x^4 - 24x^2 + 9$

3) $3x^8 - 3$

4) $x^3 - 9x^2 - 81x + 729$

5) $4x^4 + 7x^2 - 36$

IV. Linear Functions

You have learned the equation of a line with slope m and y-intercept b is given by $y = mx + b$. The slope of a line passing through two points (x_1, y_1) and (x_2, y_2) can be found by $m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

One other form that will be helpful in Calculus is point-slope form. The equation of a line passing through the points (x_1, y_1) and having a slope m is written as $y - y_1 = m(x - x_1)$.

Example. Find the equation of a line in slope-intercept form, with the given slope, passing through the given point.

$m = 3, (2, -5)$ Writing in point-slope form: $y + 5 = 3(x - 2)$

Solve for y: $y + 5 = 3x - 6$

$y = 3x - 11$

Find the equation of a line in slope-intercept form, with the given slope, passing through the given point.

1) $m = 6, (1, -7)$

2) $m = -\frac{1}{2}, (-8, 2)$

3) $m = \frac{2}{5}, (4, \frac{1}{5})$

Find the equation of a line in slope-intercept form, passing through the following points.

4) $(4, -6)$ and $(-2, 5)$

5) $(-7, 3)$ and $(3, -5)$

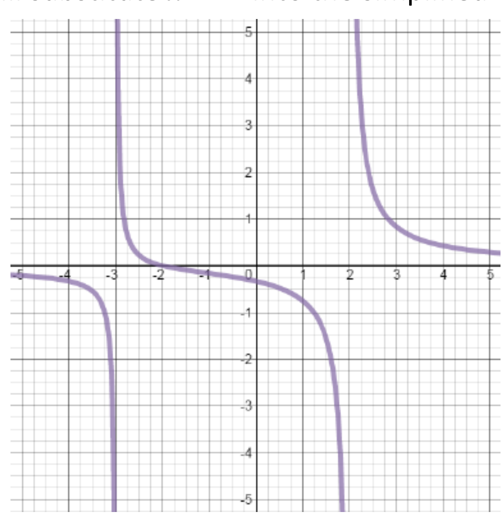
6) $\left(-2, \frac{2}{3}\right)$ and $\left(\frac{1}{2}, 1\right)$

V. Rational Functions – Asymptotes & Holes

You studied rational functions previously in Advanced Algebra. Rational functions have a variable in the denominator. Thus, rational functions may have one or more of the following: **vertical asymptotes, horizontal asymptotes, and holes**. To find **holes**, factor out any common factors of the numerator and denominator. Then, set these factors that cancelled to zero to find the x coordinate of the hole (see example below). Once you have simplified your function, set the factors in the denominator equal to zero to find the **vertical asymptotes**. Horizontal asymptotes are found by looking at the degrees of the numerator and denominator. If the function has a higher degree in the denominator (i.e. "bottom-heavy"), then the horizontal asymptote is $y = 0$. If the function has the same highest degree in both the numerator and denominator, then divide your leading coefficients to find the horizontal asymptote.

Example. Find the hole(s) & asymptote(s) of the function $f(x) = \frac{(x+2)(x-1)}{(x-1)(x+3)(x-2)}$. Then, graph the function.

Hole: The factor $(x - 1)$ is common to both the numerator and denominator. Thus, I know the hole will have an x-coordinate of $x = 1$. To find the y-coordinate, I will substitute $x = 1$ into the simplified function $f(x) = \frac{(x+2)}{(x+3)(x-2)}$. $f(1) = \frac{(1+2)}{(1+3)(1-2)} = -\frac{3}{4}$.



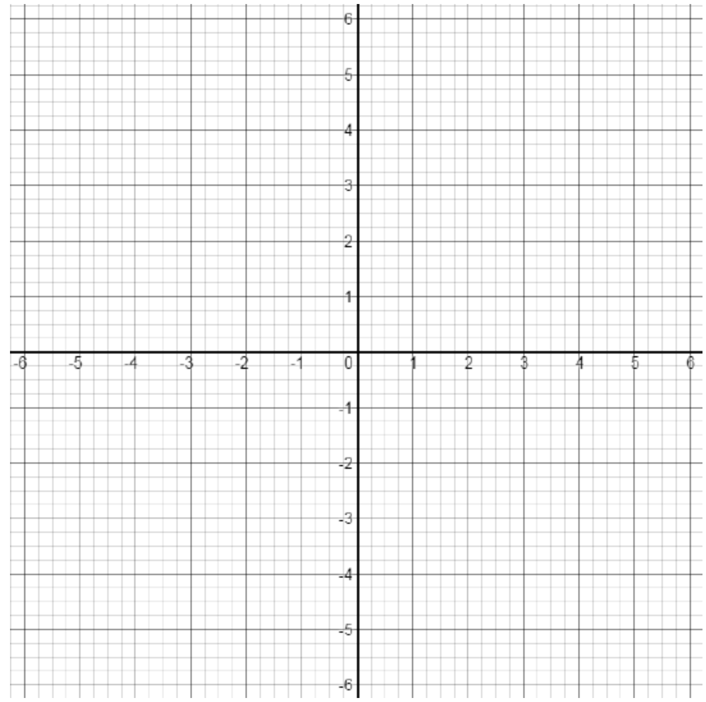
Thus, my hole is located at $(1, -\frac{3}{4})$.

Vertical Asymptotes: The vertical asymptotes will be $x = -3$ and $x = 2$, which are found by setting the factors in the denominator *of the simplified function* equal to zero and solving for x.

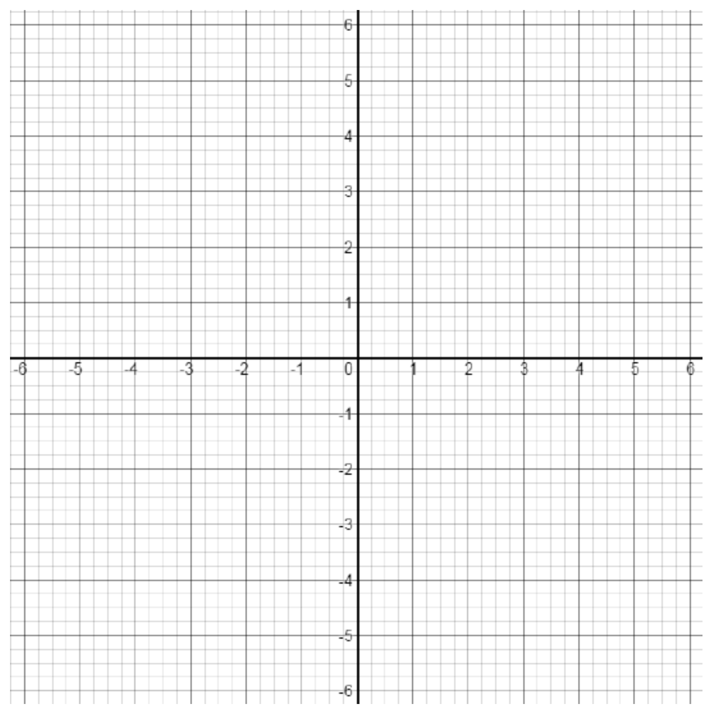
Horizontal Asymptote: The horizontal asymptote will be $y = 0$, because the denominator has a large degree than the numerator.

Find the hole(s) & asymptote(s) of the following functions. Then, **graph** the functions.

1) $f(x) = \frac{(x+3)(x+1)}{(x-2)(x+3)(x+4)}$



2) $f(x) = \frac{(x-2)(x-1)(x-2)}{(x-2)(x+3)(x-4)}$



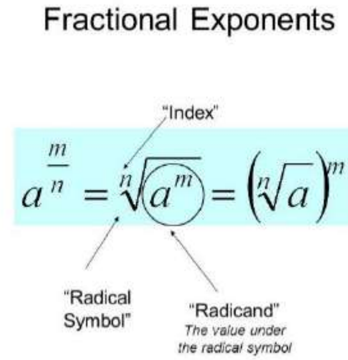
VI. Rational Exponents

Let's remember some rules:

Negative Exponents:

For any real base

$b \neq 0,\ b^{-n} = \dfrac{1}{b^n}$



Rules for Rational Exponents

The following rules hold for any nonzero real numbers a and b and rational numbers r and s for which the expressions represent real numbers.

1. $a^r a^s = a^{r+s}$	Product rule
2. $\dfrac{a^r}{a^s} = a^{r-s}$	Quotient rule
3. $(a^r)^s = a^{rs}$	Power of a power rule
4. $(ab)^r = a^r b^r$	Power of a product rule
5. $\left(\dfrac{a}{b}\right)^r = \dfrac{a^r}{b^r}$	Power of a quotient rule

Examples: Simplify and write with positive exponents.

$$(-5x^3)^{-2} = \frac{1}{(-5x^3)^2} = \frac{1}{(-5)^2x^6} = \frac{1}{25x^6}$$

Simplify and write with positive exponents.

1) $-12^2\mathrm{x}^{-5}$

2) $(-12\mathrm{x}^5)^{-2}$

3) $(-32\mathrm{x}^{-5})^{-3/5}$

4) $\frac{1}{4}(16\mathrm{x}^2)^{-3/4}(32\mathrm{x})$

5) $\dfrac{(\mathrm{x}^2-1)^{-1/2}}{(\mathrm{x}^2-1)^{1/2}}$

VII. Piecewise Functions

https://www.mathsisfun.com/sets/functions-piecewise.html

A piecewise function is essentially a function broken up into different pieces depending on the input value of x.

For problems 1-3, evaluate each piecewise function at the given values of the independent variable.

1. $f(x)=\begin{cases} 6x-1 & \text{if } x<0 \\ 7x+3 & \text{if } x\geq 0 \end{cases}$ a. $f(-3)$ b. $f(0)$ c. $f(4)$

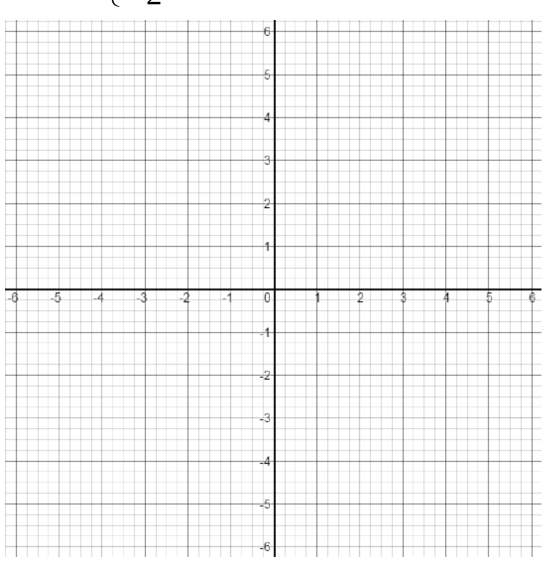
2. $f(x)=\begin{cases} \dfrac{x^2-9}{x+2} & \text{if } x\leq -1 \\ 6 & \text{if } x>-1 \end{cases}$ a. $f(-3)$ b. $f(1)$

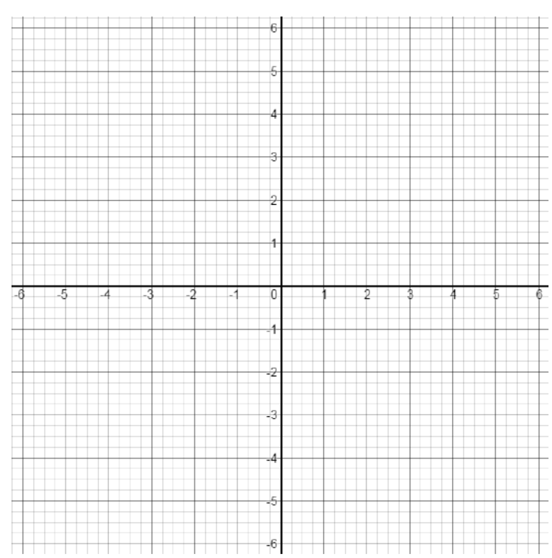
3. $f(x)=\begin{cases} 2+x & \text{if } x<-4 \\ -x & \text{if } -4\leq x\leq 2 \\ \dfrac{1}{3}x & \text{if } x>2 \end{cases}$ a. $f(2)$ b. $f(3)$

Graph the following piecewise functions:

9. $f(x)=\begin{cases} -2x & \text{if } x<-1 \\ 3x-1 & \text{if } -1\leq x\leq 2 \\ -\dfrac{1}{2}x & \text{if } x>2 \end{cases}$

10. $f(x)=\begin{cases} 2+x & \text{if } x<-2 \\ -x & \text{if } -2\leq x\leq 1 \\ 0 & \text{if } x>1 \end{cases}$

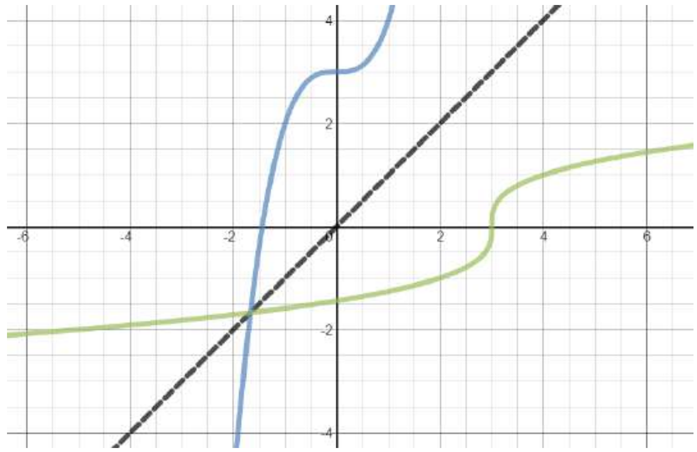




VIII. Inverse Functions

To find the inverse of a function, replace $f(x) =$ with $y =$, swap the x and y, solve for y, and replace the new $y =$ with $f^{-1}(x) =$.

Example:



$f(x) = \sqrt[3]{x-3}$

$y = \sqrt[3]{x-3}$

$x = \sqrt[3]{y-3}$

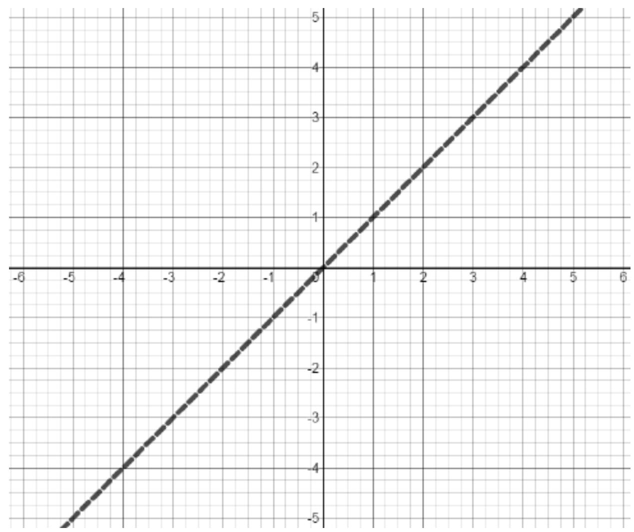
$x^3 = y - 3$

$y = x^3 + 3$

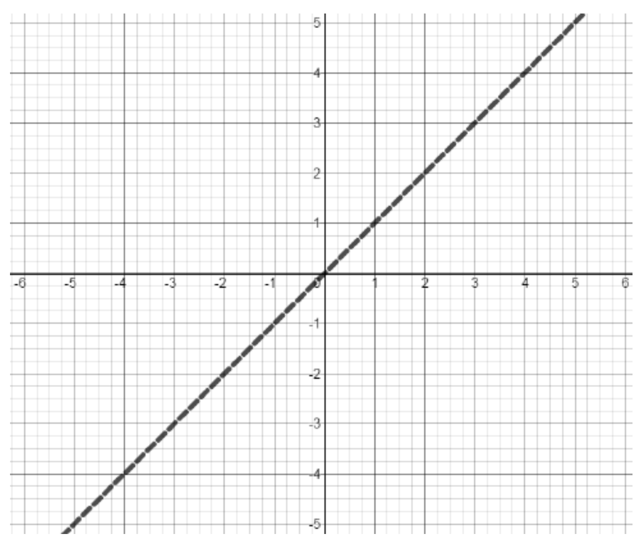
$f^{-1}(x) = x^3 + 3$

Find the inverse of each function below. Then, graph the function and its inverse. Remember, a function and its inverse should be symmetric about the Identity Function, $f(x) = x$.

1) $f(x) = \sqrt[3]{\frac{-x+2}{2}}$



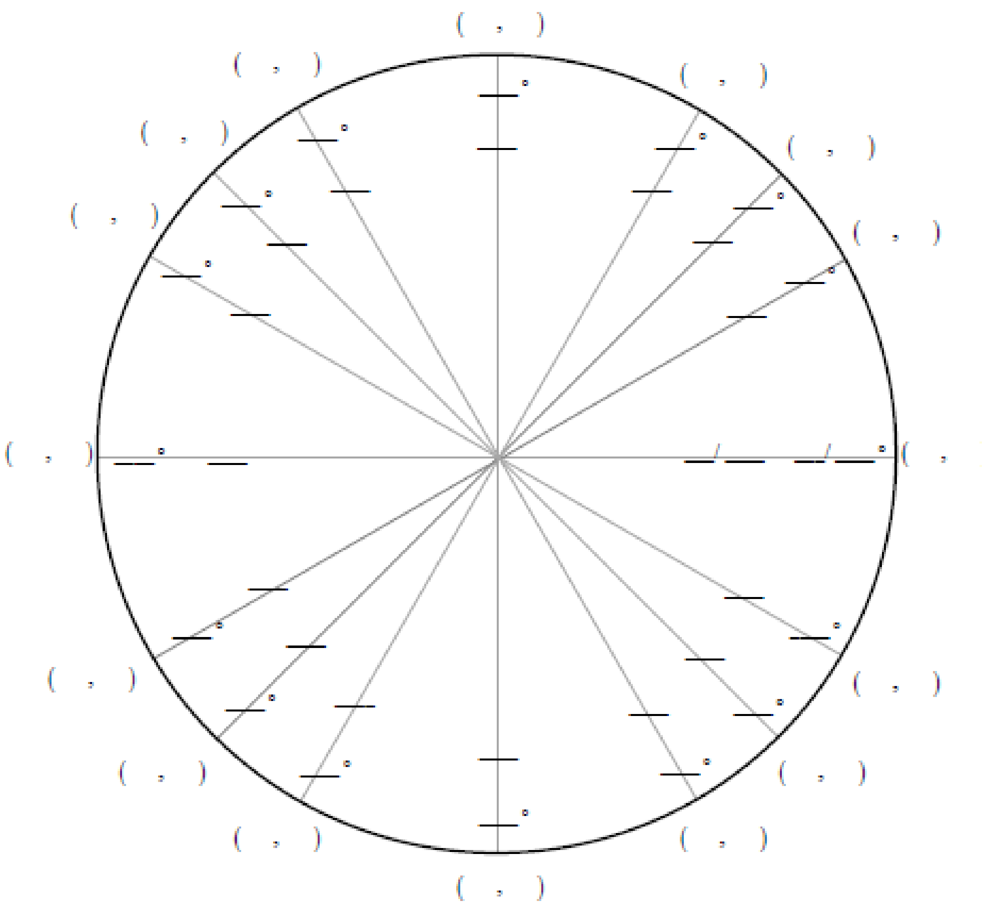
2) $f(x) = \frac{4}{x+1} + 2$



IX. Unit Circle Trig

For Calculus, you should have the unit circle memorized and be able to reference it quickly.

Fill in the unit circle below:



State the values of the following:

1) $\sin\frac{\pi}{3}$

2) $\cos\pi$

3) $\tan\frac{\pi}{4}$

4) $\csc\frac{3\pi}{2}$

5) $\sec\frac{\pi}{2}$

6) $\cot\frac{\pi}{6}$

X. Inverse Trig

Inverse trig functions are written either as $\mathrm{Sin}^{-1}(x)$ or $\mathrm{Arcsin}(x)$. The capital letters indicate principle values, which are defined between the values indicated below.

$\mathrm{Sin}^{-1}(x)$: between $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (QI & QIV on the unit circle)

$\mathrm{Cos}^{-1}(x)$: between $[0, \pi]$ (QI & QII on the unit circle)

$\mathrm{Tan}^{-1}(x)$: between $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (QI & QIV on the unit circle)

Example:

Find the value of each expression in radian angle measure.

$\mathrm{Sin}^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$Arccos\left(Tan\left(-\frac{\pi}{4}\right)\right) = Arccos(-1) = \pi$

Solve:

Find the value of each expression in radian angle measure.

1) $Arcsin(1)$

2) $Sec^{-1}(-2\ )$

3) $\cot\left(\mathrm{Cos}^{-1}\left(\frac{12}{13}\right)\right)$

4) $tan\left(Cos^{-1}\left(\frac{1}{2}\right) - \ Sin^{-1}\left(\frac{1}{2}\right)\right)$

5) $tan\left(Csc^{-1}(-2) + Cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$