GSE Honors Pre-Calculus EXAM Study Guide

NAME: Key

UNIT 1 Unit Circle and Graphs

1.) Find a (+) and (-) co-terminal angle for $\frac{-17\pi}{4}$: $\frac{-\pi}{4}$, $\frac{7\pi}{4}$
(−) (+)

2.) Evaluate $\sin(-\frac{7\pi}{6})$ $\frac{1}{2}$

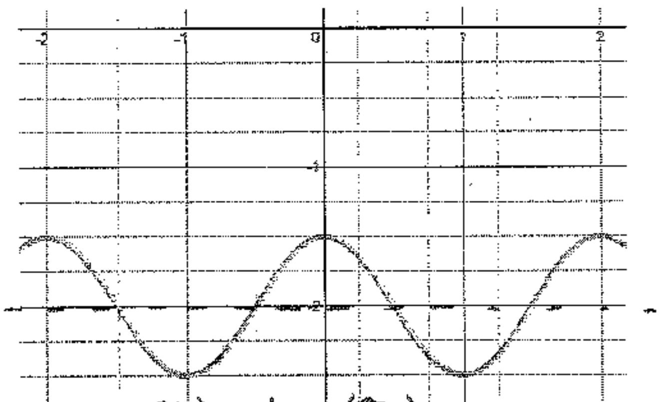
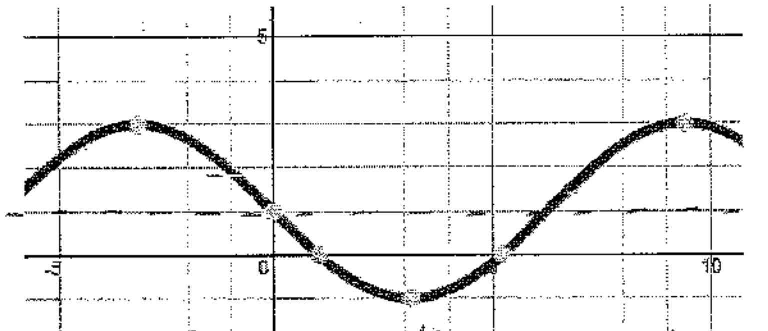
3.) Evaluate $\tan 300°$ $-\sqrt{3}$

4.) Identify the quadrant where $\sin\theta < 0$ and $\cos\theta < 0$. III
(−) (−)

5.) For the function $f(x) = -2sin\left(\frac{\pi}{2}x\right) - 5$, identify the following: amplitude, vertical shift, and period.
2 $y = -5$ 4

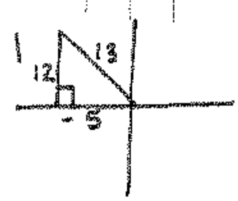
$\frac{2\pi}{\frac{\pi}{2}}$ = period

6.) Identify the equations of the following graphs:

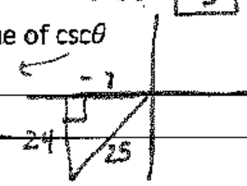



$f(x) = \frac{1}{2}\cos(\pi x) - 2$ $f(x) = -2\sin(\frac{\pi}{6}x) + 1$

7.) Let θ be an acute angle such that $sin\theta = \frac{12}{13}$ and cot < 0. Find the value of secθ
$\frac{1}{\cos\theta} = \boxed{-\frac{13}{5}}$

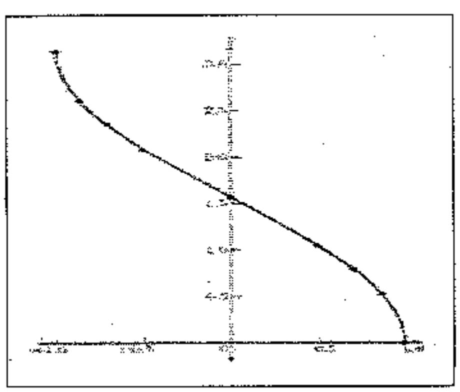
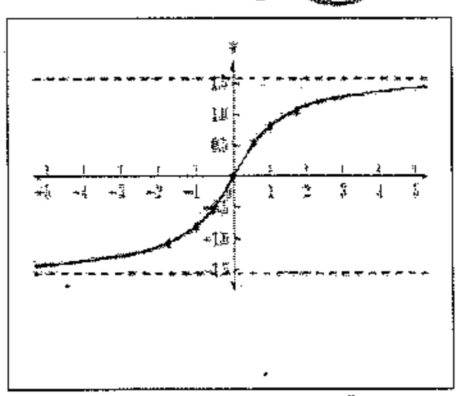
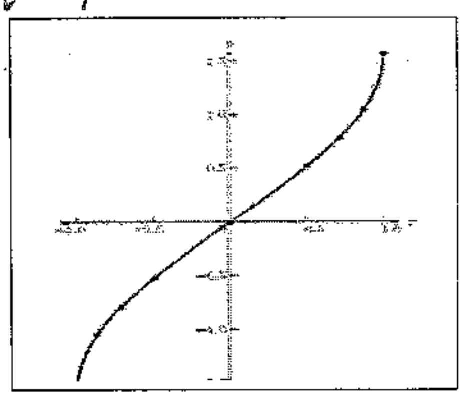


8.) Let θ be an acute angle such that $cos\theta = -\frac{7}{25}$ and tan > 0. Find the value of cscθ
III $\frac{1}{\sin\theta}$
$-\frac{25}{24}$



UNIT 2 Inverses

1.) Name the inverse functions and state their domain and range:

$f(x) = \cos^{-1}(x)$ D: $[-1, 1]$ R: $[0, \pi]$

$f(x) = \tan^{-1}(x)$ D: $\mathbb{R}$ R: $(-\frac{\pi}{2}, \frac{\pi}{2})$

$f(x) = \sin^{-1}(x)$ D: $[-1, 1]$ R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

2.) Evaluate: $Tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$
$\frac{\pi}{6}$

3.) Evaluate: $Cos^{-1}\left(\sin\left(\frac{7\pi}{6}\right)\right)$
$\frac{2\pi}{3}$

4.) Evaluate: $cos\left(tan^{-1}\frac{\sqrt{3}}{3}\right)$
$\frac{\sqrt{3}}{2}$

5.) Solve: $cos^{2}x - 3cosx + 2 = 0$
$(\cos x - 2)(\cos x - 1) = 0$
$\cos x = 2$ $\cos x = 1$
NP 0

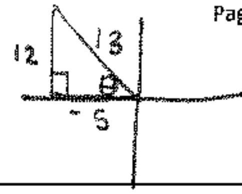
6.) Solve: $3\,tan^2 x - 1 = 0$
$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$
$\tan x = \pm\frac{\sqrt{3}}{3}$ $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

7.) Evaluate: $sec\left[arctan\left(\frac{8}{6}\right)\right]$ $1/\cos$

$\frac{10}{6}$ or $\frac{5}{3}$

8.) Evaluate: $csc\left[arccos\left(-\frac{5}{13}\right)\right]$ $1/\sin$

$13/12$

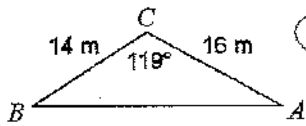


UNIT 3 Triangles

1.) Find both the area of the triangle and the length of AB.

$A = \frac{1}{2}(14)(16)(\sin 119°)$

$A = 97.9\ m^2$



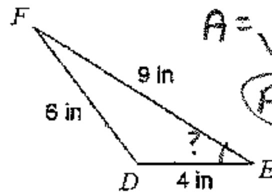
$\overline{AB}^2 = 14^2 + 16^2 - 2(14)(16)\cos 119°$

$\overline{AB} = 25.9m$

2.) Find both the area of the triangle and the $m\angle E$.

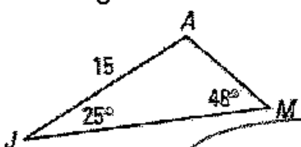
$A = \sqrt{9.5(.5)(3.5)(5.5)}$

$A = 9.56\ in^2$



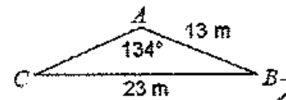
$6^2 = 4^2 + 9^2 - 2(4)(9)\cos\angle E$ $\angle E = 32.1°$

3.) Find the length of JM:



$\frac{15}{\sin 48°} \times \frac{\overline{JM}}{\sin 107°}$ $JM = 19.3$

4.) Solve triangle ABC below:

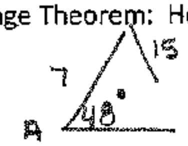


$\angle C = 24°$
$\angle B = 22°$

$\frac{23m}{\sin 134°} \times \frac{13m}{\sin\angle C}$

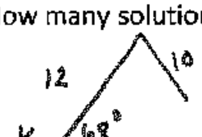
$\overline{CA}^2 = 23^2 + 13^2 - 2(23)(13)\cos 22°$ $\overline{CA} = 11.98$

5.) a) Hinge Theorem: How many solutions exist for ΔABC, given that $\angle A = 48°$, $a = 15$, and $b = 7$.



$(\sin 48)(7) = 5.2$ = height   hinge is bigger than height & fixed → 1 solutio

b) Hinge Theorem: How many solutions exist for ΔKLM, given that $\angle K = 68°$, $k = 10$, and $m = 12$

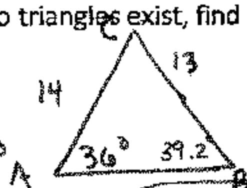


$(\sin 68)(12) = 11.1$ = height   hinge is less than height & fixed → 0 solutions

c) Hinge Theorem: How many solutions exist for ΔABC, given that $\angle A = 36°$, $a = 13$, and $b = 14$.
(If two triangles exist, find all missing sides and angles of both triangles)

$(\sin 36°)(14) = 8.2$ = height   hinge is more than height but less than fixed (2 solutions)

$\frac{13}{\sin 36°} \times \frac{c}{\sin 104.8}$



$\frac{13}{\sin 36°} = \frac{14}{\sin\angle B}$

$c = 21.4$ $\angle B = 39.2°$ $\angle C = 104.8$

$\frac{13}{\sin 36°} \times \frac{c}{\sin 3.2°}$

$\angle C = 3.2°$ $c = 1.23$

Triangle #2

UNIT 4 Use of Identities

1.) Simplify the following to *one* term: $\sec x - \sin x \tan x$

$\frac{1}{\cos x} - \frac{\sin x}{1} \cdot \frac{\sin x}{\cos x} \rightarrow \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} \rightarrow \frac{1 - \sin^2 x}{\cos x} \rightarrow \frac{\cos^{\cancel{2}} x}{\cancel{\cos x}} = \cos x$

2.) Use a sum or difference identity to evaluate the following: $\cos(-15°)$ → $\cos(30° - 45°)$ $\cos 30° \cos 45° + \sin 30° \sin 45°$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6} + \sqrt{2}}{4}$

3.) Solve: $\sin^2\theta - \frac{1}{4} = 0$

$\sqrt{\sin^2\theta} = \sqrt{\frac{1}{4}}$ $\sin\theta = \pm\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\sin\theta = \pm\frac{\sqrt{2}}{2}$ $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

4.) Solve: $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 2)(2\sin x + 1) = 0$

$2(\sin x + 1)(2\sin x + 1) = 0$

$\sin x = -1$ $\sin x = -1/2$

$\frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

5.) Verify: $\frac{\cancel{\sec^2\theta - 1}}{\tan^2\theta} = 1$

$\frac{\cancel{\tan^2\theta}}{\cancel{\tan^2\theta}} = 1$ $1 = 1$ ✓

6.) Verify: $\frac{\cot\theta}{\cancel{1 + \cot^2\theta}} = \sin\theta\cos\theta$ $\csc^2\theta$

$\frac{\cos\theta}{\cancel{\sin\theta}} \cdot \frac{\sin^{\cancel{2}}\theta}{1} = \cos\theta\sin\theta = \sin\theta\cos\theta$ ✓

$\cos(u-v) = \cos u\cos v + \sin u \sin v$

7.) Find the exact value of the trigonometric function $sec(u-v)$ given that $\sin u = -\frac{3}{5}$ and $\cos v = -\frac{7}{25}$ (Both u and v are in quadrant III)

$\left(-\frac{4}{5}\right)\left(-\frac{7}{25}\right) + \left(-\frac{3}{5}\right)\left(-\frac{24}{25}\right) = \frac{28}{125} + \frac{72}{125} = \frac{100}{125}$ $\frac{125}{100}$ or $\frac{5}{4}$

8.) Find the exact value of the trigonometric function $cos(u-v)$ given that $\cos u = \frac{3}{5}$ and lies in quadrant IV and $\tan v = -\sqrt{3}$ and in quadrant II.

sin / cos

$\left(\frac{3}{5}\right)\left(\frac{-1}{2}\right) + \left(\frac{-4}{5}\right)\left(\frac{\sqrt{3}}{2}\right) = -\frac{3}{10} - \frac{4\sqrt{3}}{10}$

$(-1)^2 + (\sqrt{3})^2 = c^2 \rightarrow c = \sqrt{2}$

$\frac{-3-4\sqrt{3}}{10}$

UNIT 5 Matrices

$A = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 7 \\ -3 & 2 \\ 4 & -1 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 1 & 2 \\ -3 & 0 & 4 \\ 5 & -7 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 0 & -3 & 3 \\ 2 & -1 & 5 \end{bmatrix}$ $E = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

1.) Using the given matrices above, find the following:

a.) $D \times B$

$\begin{bmatrix} 21 & -9 \\ 25 & 7 \end{bmatrix}$

b.) E^{-1}

$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

c.) $|A|$

NP

d.) $A - B$

NP

e.) $|C|$

65

2.) Find the area of ΔOLA given $O(2,-8)$, $L(-1,-4)$, $A(3,5)$.

$A = 21.5$

3.) Solve the following using a matrix equation.

In one week, a trophy store sold 32 trophies for a total of $580. Large trophies cost $25 each and small trophies cost $15 each. How many of each type of trophy were sold?

$25x + 15y = 580$
$x + y = 32$

10 = large trophies
22 = small trophies

4.) Solve the following using a matrix equation.

Kali and Eduardo each improved their yards by planting grass sod and ornamental grass. They bought their supplies from the same store. Kali spent $204 on 14 ft² of grass sod and 9 bunches of ornamental grass. Eduardo spent $140 on 7 ft² of grass sod and 14 bunches of ornamental grass. Find the cost of one ft² of grass sod and the cost of one bunch of ornamental grass.

$14x + 9y = \$204$
$7x + 14y = \$140$

$\rightarrow \begin{bmatrix} 14 & 9 \\ 7 & 14 \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 204 \\ 140 \end{bmatrix}$

$x = \$12$
$y = \$4$

5.) A parabola passes through points (-5, 81), (-1, 17), and (2, 11). Create a system of equations that represent each set of points. Then solve the system using matrices. Find the equation of the parabola.

$a(-5)^2 + b(-5) + c = 81$ $a(-1)^2 + b(-1) + c = 17$ $a(2)^2 + b(2) + c = 11$

$[2, -4, 11]$

$y = 2x^2 - 4x + 11$

6.) A parabola passes through points (1, -2), (-3, 10), and (4, 31). Create a system of equations that represent each set of points. Then solve the system using matrices. Find the equation of the parabola.

$a(1)^2 + b(1) + c = -2$ $a(-3)^2 + b(-3) + c = 10$ $a(4)^2 + b(4) + c = 31$

$[2, 1, -5]$

$y = 2x^2 + 1x - 5$

UNIT 6 Conics

1.) Describe how to tell the four different conic sections apart when given the equations both in the general form and in standard form. Circle: x^2 & y^2 have the same coefficients

ellipse/hyperbola: x^2 & y^2 have different coefficients

parabola: either x^2 or y^2 but not both.

ellipse (equation has a +)

hyperbola (equation has a −)

2.) What is the distance from the center to the foci of the following: $\frac{(x-2)^2}{49}+\frac{(y-4)^2}{16}=1$

$49-16=c^2$

$\sqrt{33}=c\approx\pm5.74$

3.) Convert the following from general to standard form: $x^2-4y^2+8x-8y+8=0$ $x^2+8x-4y^2-8y=-8$

Then, state the coordinates of the vertices and foci. $\frac{(x+4)^2}{4}-\frac{(y+1)^2}{1}=1$ $(x^2+8x+16)-4(y^2+2y+1)=-8+16-4$

center $(-4,-1)$ $4+1=c^2$ $\sqrt{5}=\sqrt{c^2}$

coordinates $(-6,-1),(-2,-1)$ foci $(-1.76,-1),(-6.23,-1)$ $\frac{(x+4)^2}{4}-\frac{4(y+1)^2}{4}=\frac{4}{4}$

4.) Write the equation of a parabola with a focus at (-3, 5) and a vertex of (-3, 2).

$p=3$ $(x+3)^2=12(y-2)$

5.) Find the equation of a hyperbola that has foci @ $(\pm5,0)$ and vertices @ $(\pm3,0)$ $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

$9+b^2=25$ $b=4$ $\frac{x^2}{9}-\frac{y^2}{4}=1$

$b^2=25-9$ $b^2=16$

6.) Identify the conic using the following equation: $25x^2+12y^2-150x-144y+357=0$

$25x^2-150x+12y^2-144y=-357$

UNIT 7 Vectors $25(x^2-6x+9)+12(y^2-12y+36)=-357+225+300$ $\frac{25(x-3)^2}{300}+\frac{12(y-6)^2}{300}=\frac{300}{300}$

$\frac{(x-3)^2}{12}+\frac{(y-6)^2}{25}=1$ ellipse

1.) Find the magnitude and direction of the following vectors:

a.) $\boldsymbol{a}=\langle-2,4\rangle$ b.) $\boldsymbol{b}=\langle2,-4\rangle$ c.) $\boldsymbol{c}=\langle-5,-3\rangle$ d.) $\boldsymbol{d}=\langle5,3\rangle$

$(-2)^2+(4)^2=c^2$ $(2)^2+(-4)^2=c^2$ $(-5)^2+(-3)^2=c^2$ $(5)^2+(3)^2=c^2$

$4+16=c^2$ $\tan^{-1}\left(\frac{4}{-2}\right)$ $4+16=c^2$ $\sqrt{20}=\sqrt{c^2}$ $\tan^{-1}\left(\frac{-4}{2}\right)$ $25+9=c^2$ $\sqrt{34}=\sqrt{c^2}$ $\sqrt{34}=\sqrt{c^2}$

$\sqrt{20}=\sqrt{c^2}$ $c=2\sqrt{5}$ @ $116.57°$ $c=2\sqrt{5}$ @ $296.67°$ $c=\sqrt{34}$ @ $210.96°$ $c=\sqrt{34}$ @ $30.96°$

$\tan^{-1}\left(\frac{-3}{-5}\right)=$ $\tan^{-1}\left(\frac{3}{5}\right)=$

2.) Find the component form of the following vectors:

a.) magnitude of 5 @ 225° b.) magnitude of 6 @ 127° c.) magnitude of 3 @ 290°

$\cos45°=\frac{x}{5}$ $\cos53°=\frac{x}{6}$ ~~cos 29~~ $\cos70°=\frac{x}{3}$

$x=3.53$ $\langle-3.53,-3.53\rangle$ $x=3.61$ $x=1.02$

$\sin45°=\frac{y}{5}$ $\sin53°=\frac{y}{6}$ $\sin70°=\frac{y}{3}$

$y=3.53$ $\langle-3.61,4.79\rangle$ $y=4.79$ $\langle1.02,-2.82\rangle$ $y=2.82$

3.) If a= $\langle5,3\rangle$ and c= $\langle-4,2\rangle$, then find $2c-3a$

$2\langle-4,2\rangle-3\langle5,3\rangle$

$\langle-8,4\rangle+\langle-15,-9\rangle$

$\langle-23,-5\rangle$

4.) If m= $\langle-2,4\rangle$ and n= $\langle1,-5\rangle$, then find $\frac{1}{2}m+2n$

$\frac{1}{2}\langle-2,4\rangle+2\langle1,-5\rangle$

$\langle-1,2\rangle+\langle2,-10\rangle$

$\langle1,-8\rangle$

5.) A plane takes off from Palm Springs Airport at an angle of 120° and traveling 200mph. A 30 mph south wind is blowing (toward North), and is pushing the plane off-course. Find the groundspeed and true course of the plane.

x	y
-100	173.2
0	30

$\cos60°=\frac{x}{200}$ $x=100$ $\langle-100,203.2\rangle$ $\tan^{-1}\left(\frac{203.2}{-100}\right)$

$\sin60°=\frac{y}{200}$ $y=173.2$ $(-100)^2+(203.2)^2=c^2$ @ $116.2°$ $c=226.5$ mph

6.) The captain of a ship discovers that he must divert his course in order to avoid several icebergs. He is heading due east when he abruptly turns at an angle of 40°. After traveling 2 miles on this bearing, he turns at an angle of 100° and continues at this angle for 8.8 miles until he reaches his destination. How far out of his way does the captain have to travel to get to his destination?

x	y
1.53	1.28
-1.52	8.7

$\langle.01,9.98\rangle$ $\tan^{-1}\left(\frac{9.98}{.01}\right)$

$(.01)^2+(9.98)^2=c^2$

$.0001+99.6004=c^2$ $c=9.98$ @ 89.9