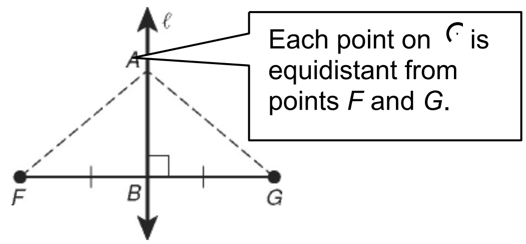
Study Guide –Quiz

Perpendicular and Angle Bisectors

Theorem	Example
Perpendicular Bisector Theorem If a point is on the perpendicular bisector of a segment, then it is **equidistant,** or the same distance, from the endpoints of the segment.	 **Given:** ℓ is the perpendicular bisector of $\overline{FG}$. **Conclusion:** $AF = AG$

The **Converse of the Perpendicular Bisector Theorem** is also true. If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

You can write an equation for the perpendicular bisector of a segment. Consider the segment with endpoints $Q(-5, 6)$ and $R(1, 2)$.

Step 1 Find the midpoint of $\overline{QR}$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-5+1}{2}, \frac{6+2}{2}\right)$$

$$= (-2, 4)$$

Step 2 Find the slope of the $\perp$ bisector of $\overline{QR}$.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-6}{1-(-5)} \quad \text{Slope of } \overline{QR}$$

$$= -\frac{2}{3}$$

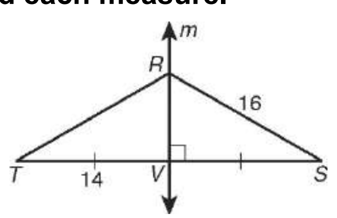
So the slope of the $\perp$ bisector of $\overline{QR}$ is $\frac{3}{2}$.

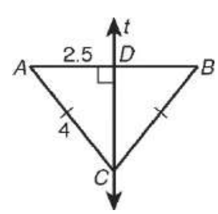
Step 3 Use the point-slope form to write an equation.

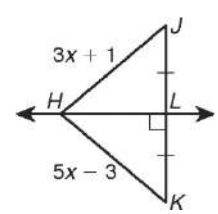
$y - y_1 = m(x - x_1)$ Point-slope form

$y - 4 = \frac{3}{2}(x+2)$ Slope $= \frac{3}{2}$; line passes through $(-2, 4)$, the midpoint of $\overline{QR}$.

Find each measure.







1. $RT =$ ________________ 2. $AB =$ ________________ 3. $HJ =$ ________________

Write an equation in point-slope form for the perpendicular bisector of the segment with the given endpoints.

4. $A(6, -3)$, $B(0, 5)$

5. $W(2, 7)$, $X(-4, 3)$

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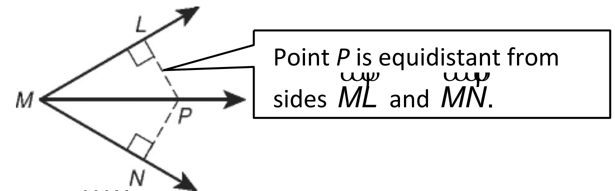
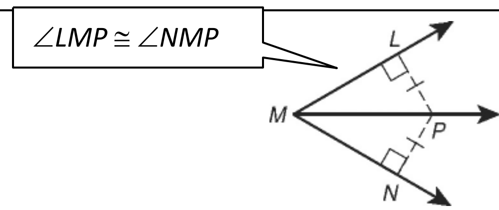
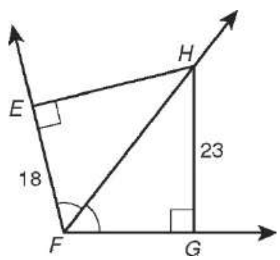
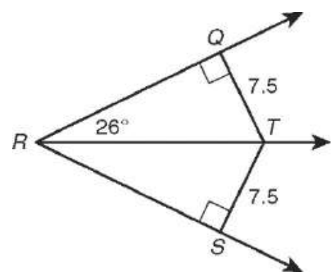
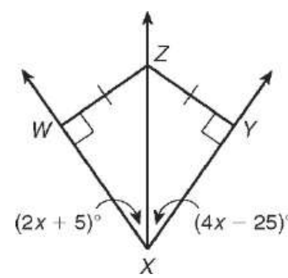
Perpendicular and Angle Bisectors continued

Theorem	Example
Angle Bisector Theorem If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.	 **Given:** $\overrightarrow{MP}$ is the angle bisector of $\angle LMN$. **Conclusion:** $LP = NP$
Converse of the Angle Bisector Theorem If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.	 **Given:** $LP = NP$ **Conclusion:** $\overrightarrow{MP}$ is the angle bisector of $\angle LMN$.

Find each measure.





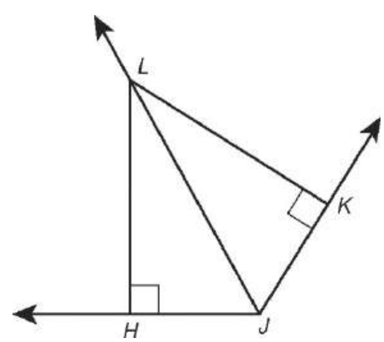


6. *EH* ______________________

7. m$\angle QRS$ ______________________

8. m$\angle WXZ$ ______________________

Use the figure for Exercises 9–11.



9. Given that $\overrightarrow{JL}$ bisects $\angle HJK$ and $LK = 11.4$, find LH.

____

10. Given that $LH = 26$, $LK = 26$, and m$\angle HJK = 1228$, find m$\angle LJK$.

____

11. Given that $LH = LK$, m$\angle HJL = (3y + 19)8$, and m$\angle LJK = (4y + 5)8$, find the value of y.

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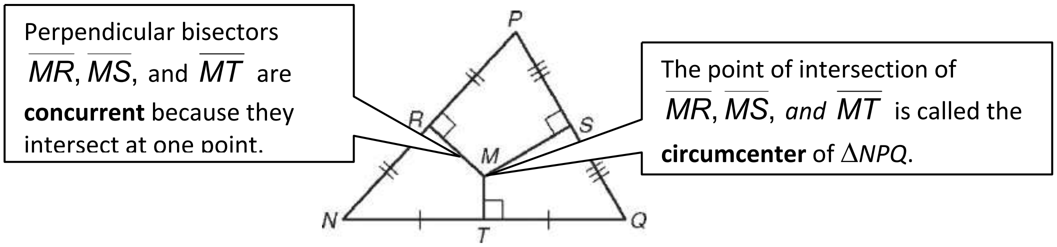
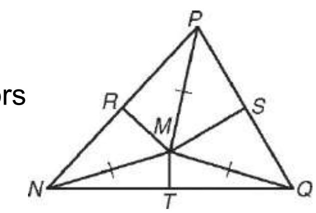
Bisectors of Triangles

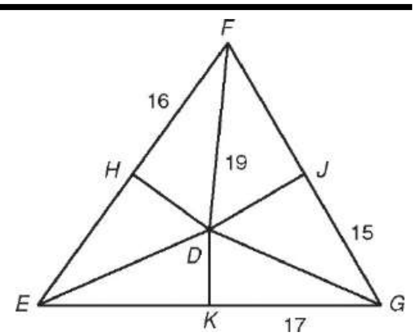


Theorem	Example
Circumcenter Theorem The circumcenter of a triangle is equidistant from the vertices of the triangle.	**Given:** $\overline{MR}, \overline{MS}$, and $\overline{MT}$ are the perpendicular bisectors of $\triangle NPQ$. **Conclusion:** $MN = MP = MQ$ 

If a triangle on a coordinate plane has two sides that lie along the axes, you can easily find the circumcenter. Find the equations for the perpendicular bisectors of those two sides. The intersection of their graphs is the circumcenter.

$\overline{HD}, \overline{JD}$, and $\overline{KD}$ are the perpendicular bisectors of $\triangle EFG$.

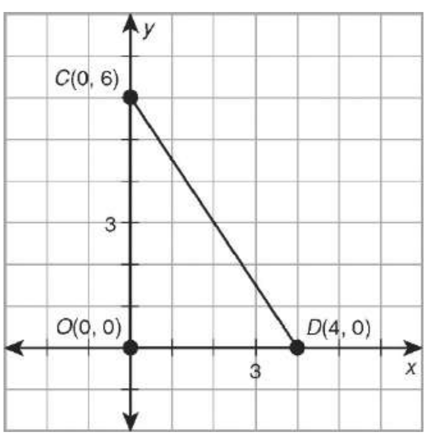
Find each length.



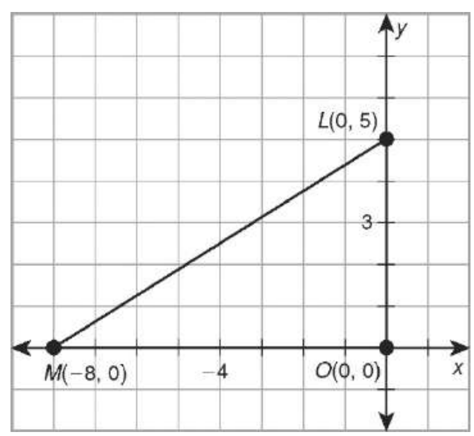
1. *DG* ______________________
2. *EK* ______________________
3. *FJ* ______________________
4. *DE* ______________________

Find the circumcenter of each triangle.

5.



6.



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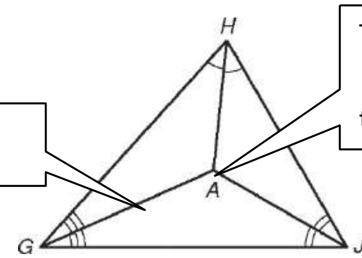
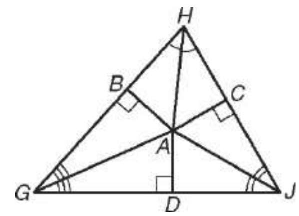
Bisectors of Triangles continued

Angle bisectors of ΔGHJ intersect at one point.

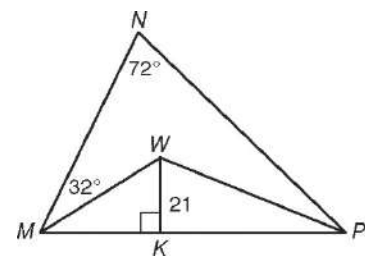


The point of intersection of $\overline{AG}$, $\overline{AH}$, *and* $\overline{AJ}$ is called the **incenter** of ΔGHJ.

Theorem	Example
Incenter Theorem The incenter of a triangle is equidistant from the sides of the triangle.	**Given:** $\overline{AG}$, $\overline{AH}$, and $\overline{AJ}$ are the angle bisectors of ΔGHJ. **Conclusion:** $AB = AC = AD$ 

$\overline{WM}$ and $\overline{WP}$ are angle bisectors of ΔMNP, and $WK = 21$.

Find $m\angle WPN$ and the distance from W to $\overline{MN}$ and $\overline{NP}$.



$m\angle NMP = 2m\angle NMW$	Def. of $\angle$ bisector
$m\angle NMP = 2(328) = 648$	Substitute.
$m\angle NMP + m\angle N + m\angle NPM = 1808$	n Sum Thm.
$648 + 728 + m\angle NPM = 1808$	Substitute.
$m\angle NPM = 448$	Subtract 1368 from each side.
$m\angle WPN = \frac{1}{2} m\angle NPM$	Def. of $\angle$ bisector
$m\angle WPN = \frac{1}{2}(448) = 228$	Substitute.

The distance from W to $\overline{MN}$ and $\overline{NP}$ is 21 by the Incenter Theorem.

***$\overline{PC}$* and *$\overline{PD}$* are angle bisectors of ΔCDE. Find each measure.**

7. the distance from P to $\overline{CE}$ 8. $m\angle PDE$

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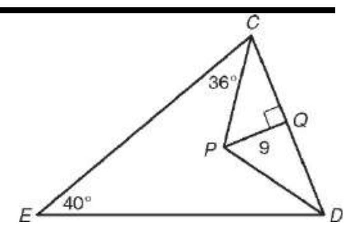

***$\overline{KX}$* and *$\overline{KZ}$* are angle bisectors of ΔXYZ. Find each measure.**

9. the distance from K to $\overline{YZ}$ 10. $m\angle KZY$

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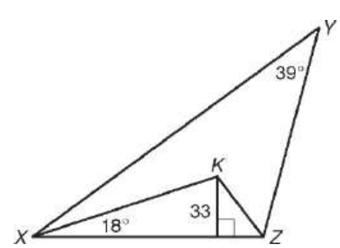

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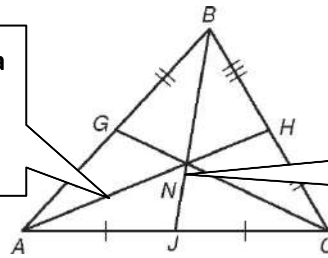
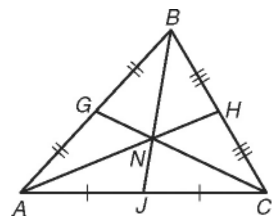
Medians and Altitudes of Triangles

$\overline{AH}, \overline{BJ}$, and $\overline{CG}$ are **medians of a triangle.** They each join a vertex and the midpoint of the opposite side.



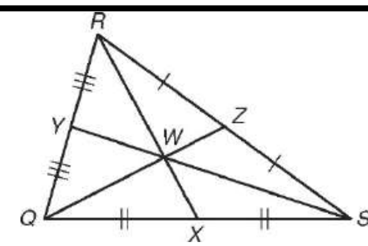
The point of intersection of the medians is called the **centroid** of n*ABC*.

Theorem	Example
Centroid Theorem The centroid of a triangle is located $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.	 **Given:** $\overline{AH}, \overline{CG}$, and $\overline{BJ}$ are medians of n*ABC*. **Conclusion:** $AN = \frac{2}{3}AH$, $CN = \frac{2}{3}CG$, $BN = \frac{2}{3}BJ$

In △*ABC* above, suppose $AH = 18$ and $BN = 10$. You can use the Centroid Theorem to find *AN* and *BJ*.

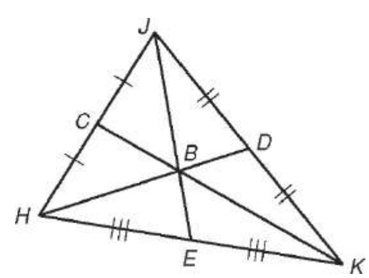
$AN = \frac{2}{3}AH$	Centroid Thm.	$BN = \frac{2}{3}BJ$	Centroid Thm.
$AN = \frac{2}{3}(18)$	Substitute 18 for *AH*.	$10 = \frac{2}{3}BJ$	Substitute 10 for *BN*.
$AN = 12$	Simplify.	$15 = BJ$	Simplify.

In △*QRS*, $RX = 48$ and $QW = 30$. Find each length.



1. *RW* ______________________
2. *WX* ______________________
3. *QZ* ______________________
4. *WZ* ______________________

In △*HJK*, $HD = 21$ and $BK = 18$. Find each length.



5. *HB* ______________________
6. *BD* ______________________
7. *CK* ______________________
8. *CB* ______________________

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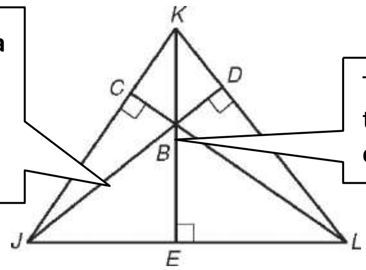
Reteach

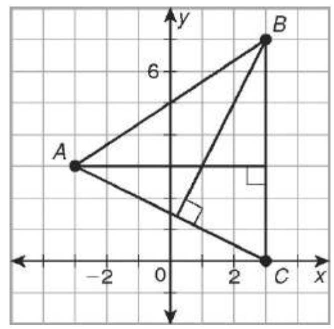
Medians and Altitudes of Triangles ***continued***

$\overline{JD}$, $\overline{KE}$, and $\overline{LC}$ are **altitudes of a triangle.** They are perpendicular segments that join a vertex and the line containing the side opposite the vertex.



The point of intersection of the altitudes is called the **orthocenter** of $\triangle JKL$.

Find the orthocenter of $\triangle ABC$ with vertices $A(-3, 3)$, $B(3, 7)$, and $C(3, 0)$.



Step 1 Graph the triangle.

Step 2 Find equations of the lines containing two altitudes.

The altitude from A to $\overline{BC}$ is the horizontal line $y = 3$.

The slope of $\overleftrightarrow{AC} = \dfrac{0-3}{3-(-3)} = -\dfrac{1}{2}$, so the slope of the altitude from B to $\overline{AC}$ is 2. The altitude must pass through $B(3, 7)$.

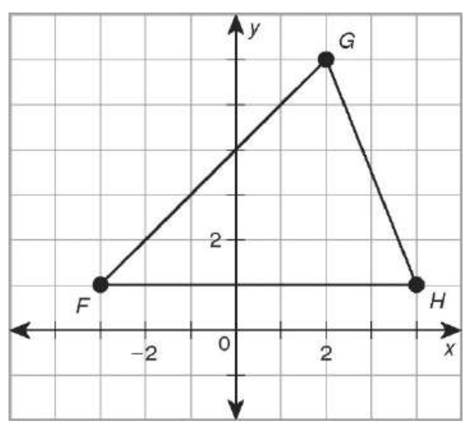
$y - y_1 = m(x - x_1)$ Point-slope form

$y - 7 = 2(x - 3)$ Substitute 2 for m and the coordinates of $B(3, 7)$ for (x_1, y_1).

$y = 2x + 1$ Simplify.

Step 3 Solving the system of equations $y = 3$ and $y = 2x + 1$, you find that the coordinates of the orthocenter are $(1, 3)$.

Triangle *FGH* has coordinates $F(-3, 1)$, $G(2, 6)$, and $H(4, 1)$.



9. Find an equation of the line containing the altitude from G to $\overline{FH}$.

____

10. Find an equation of the line containing the altitude from H to $\overline{FG}$.

____

11. Solve the system of equations from Exercises 9 and 10 to find the coordinates of the orthocenter.

____

Find the orthocenter of the triangle with the given vertices.

12. $N(-1, 0)$, $P(1, 8)$, $Q(5, 0)$

____

13. $R(-1, 4)$, $S(5, -2)$, $T(-1, -6)$

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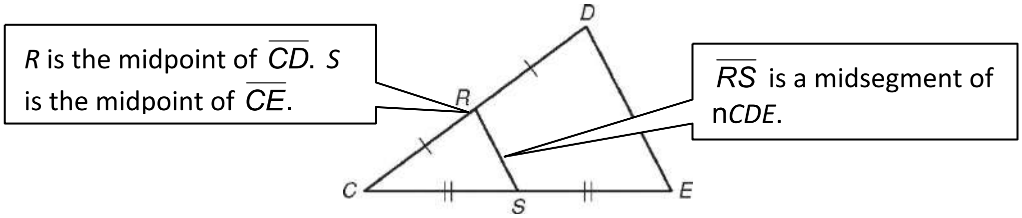
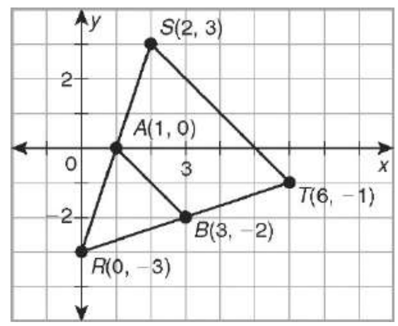
The Triangle Midsegment Theorem

A **midsegment** of a triangle joins the midpoints of two sides of the triangle. Every triangle has three midsegments.



Use the figure for Exercises 1–4. $\overline{AB}$ is a midsegment of △*RST*.



1. What is the slope of midsegment $\overline{AB}$ and the slope of side $\overline{ST}$?

   ____

2. What can you conclude about $\overline{AB}$ and $\overline{ST}$?

   ____

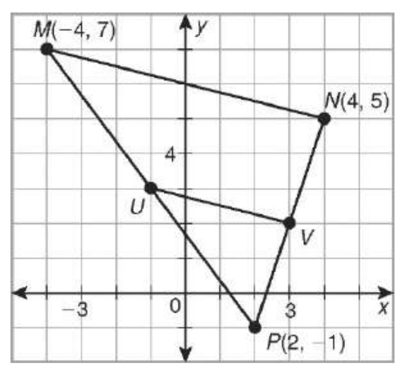
3. Find *AB* and *ST*.

   ____

4. Compare the lengths of $\overline{AB}$ and $\overline{ST}$.

   ____

Use △*MNP* for Exercises 5–7.



5. $\overline{UV}$ is a midsegment of △*MNP*. Find the coordinates of *U* and *V*.

   ____

6. Show that $\overline{UV}$ i $\overline{MN}$.

   ____

   ____

   ____

7. Show that $UV = \frac{1}{2}MN$.

   ____

   ____

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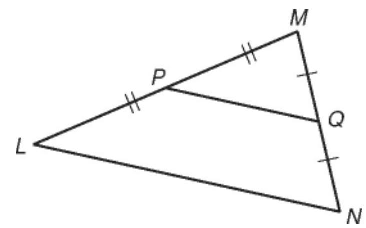
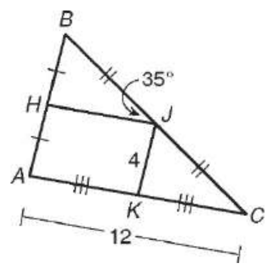
The Triangle Midsegment Theorem continued

Theorem	Example
Triangle Midsegment Theorem A midsegment of a triangle is parallel to a side of the triangle, and its length is half the length of that side.	 **Given:** $\overline{PQ}$ is a midsegment of n*LMN.* **Conclusion:** $\overline{PQ}$ i $\overline{LN}$, $PQ = \frac{1}{2}LN$

You can use the Triangle Midsegment Theorem to find various measures in $\triangle ABC$.



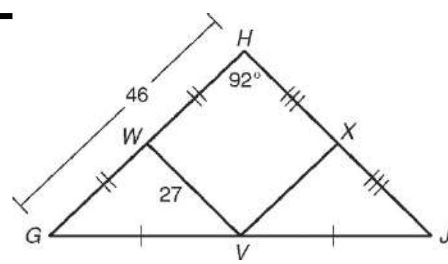
$HJ = \frac{1}{2}AC$ n Midsegment Thm.

$HJ = \frac{1}{2}(12)$ Substitute 12 for *AC*.

$HJ = 6$ Simplify.

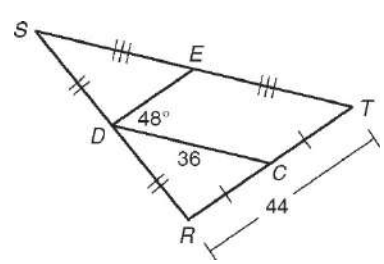
$JK = \frac{1}{2}AB$	n Midsegment Thm.	$\overline{HJ}$ i $\overline{AC}$	Midsegment Thm.
$4 = \frac{1}{2}AB$	Substitute 4 for *JK.*	$m\angle BCA = m\angle BJH$	Corr. ? Thm.
$8 = AB$	Simplify.	$m\angle BCA = 358$	Substitute 358 for $m\angle BJH$.

Find each measure.



8. $VX =$ ______________________
9. $HJ =$ ______________________
10. $m\angle VXJ =$ ______________________
11. $XJ =$ ______________________

Find each measure.



12. $ST =$ ______________________
13. $DE =$ ______________________
14. $m\angle DES =$ ______________________
15. $m\angle RCD =$ ______________________