

UNIT 1 Transformations, Congruence and Similarity

Transformations:

Translations - SLIDE - every point of the object must be moved in the same direction and for the same distance. The notation will look like this: $(x, y) \rightarrow (x + 3, y - 2)$. Three units right, two units down. The x coordinate is moved right or left, the y coordinate is moved up or down.

Rotation - TURN - rotating a figure about a point

$90^\circ (x, y) \rightarrow (-y, x)$

$180^\circ (x, y) \rightarrow (-x, -y)$

$270^\circ (x, y) \rightarrow (y, -x)$

$-90^\circ (x, y) \rightarrow (y, -x)$

Reflection - FLIP - flipping the object about a line of reflection. If the degrees are positive, the rotation is performed counterclockwise; if they are negative, the rotation is clockwise. The figure will not change size or shape, but, unlike a translation, will change direction.

Over x axis $(x, y) \rightarrow (x, -y)$

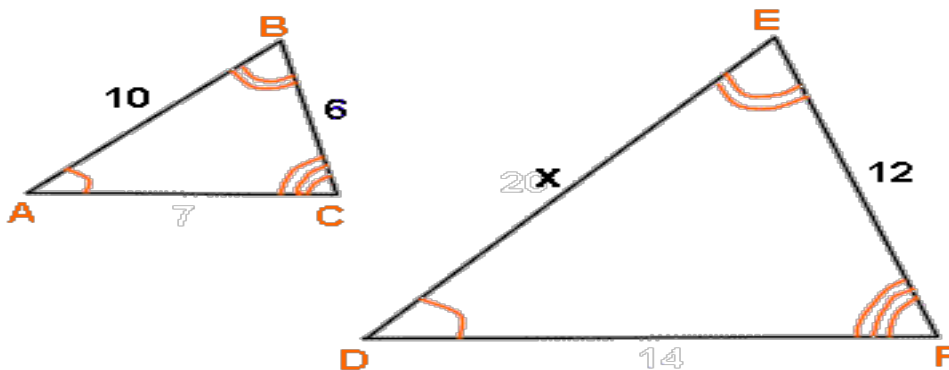
Over y axis $(x, y) \rightarrow (-x, y)$

$Y = x (x, y) \rightarrow (y, x)$

Dilation - ZOOM in or out- changing the size of a figure without changing its essential shape. *Multiply the pre-image by the absolute value of the scale factor.*

Congruence -same size and shape. This means that the corresponding sides are equal and the corresponding angles are equal.

Similar - Two triangles are **similar** if and only if the corresponding sides are in proportion and the corresponding angles are congruent.



Create a proportion matching the corresponding sides.

Two possible answers:

| Small triangle on top | Large triangle on top |
|-------------------------------|-------------------------------|
| $\frac{10}{x} = \frac{6}{12}$ | $\frac{x}{10} = \frac{12}{6}$ |
| $x = 20$ | $x = 20$ |

UNIT 2 Exponents

| Laws | Rule | Example |
|----------------------------|---|--|
| Product Rule | Keep base, + exponents $(x^a)(x^b) = x^{a+b}$ | $(x^2)(x^6)(b^2)$ $= x^8b^2$ |
| Quotient Rule | Keep base, - exponents $(x^a) \div (x^b) = x^{a-b}$ | $\frac{a^2b^{-1}}{a^{-2}b^3}$ $= a^4b^{-4}$ |
| Power of a Power | Keep base, x exponent $(x^a)^b = x^{ab}$ | $(y^3)^5$ $= y^{15}$ |
| Power of a Product | Distribute exponent $(xyz)^a = x^ay^az^a$ | $(-2x^3y^2)^3$ $= -2^3x^9y^6$ $= -8x^9y^6$ |
| Power of a Quotient | Distribute exponent $(\frac{x}{y})^a = \frac{x^a}{y^a}$ | $(\frac{3x^2}{4})^3$ $= \frac{27x^6}{12}$ |
| Zero Exponent | Any base with exponent 0 is equal to 1 $(x)^0 = 1$ | $(3xy)^0$ $= 1$ |
| Negative Exponents | Negative exponent can be rewritten with it's reciprocal $x^{-1} = \frac{1}{x}$ | $3^{-4}x^2y^3$ $= \frac{x^2y^3}{3^4}$ |

Scientific notation is a compact way of writing very large and very small numbers.

Standard form: 4508900000 Scientific Notation form: 4.5089×10^9

For **adding** and **subtracting** only when they have the same base and exponent. Since all numbers in scientific notation have the same base (10), we need only worry about the exponents. To be added or subtracted, two numbers in scientific notation must be manipulated so that their bases have the same exponent--this will ensure that corresponding digits in their coefficients have the same place value.

Example: $(4.215 \times 10^{-2}) + (3.2 \times 10^{-4}) = (4.215 \times 10^{-2}) + (0.032 \times 10^{-2}) = 4.247 \times 10^{-2}$

Example: $(8.97 \times 10^4) - (2.62 \times 10^3) = (8.97 \times 10^4) - (0.262 \times 10^4) = 8.71 \times 10^4$

Scientific Notation continued.....

For **multiplication**: multiply the principal parts, and simply **add** the exponents of 10.

$$\text{Example: } (3.4 \times 10^6)(4.2 \times 10^3) = (3.4)(4.2) \times 10^{(6+3)} = 14.28 \times 10^9 = 1.4 \times 10^{10}$$

$$\text{Example: } (6.73 \times 10^{-5})(2.91 \times 10^2) = (6.73)(2.91) \times 10^{(-5+2)} = 19.58 \times 10^{-3} = 1.96 \times 10^{-2}$$

For **division**: divide the principal parts, and simply **subtract** the exponents of 10.

$$\text{Example: } (6.4 \times 10^6)/(8.9 \times 10^2) = (6.4)/(8.9) \times 10^{(6-2)} = 0.719 \times 10^4 = 7.2 \times 10^3$$

$$\text{Example: } (3.2 \times 10^3)/(5.7 \times 10^{-2}) = (3.2)/(5.7) \times 10^{3-(-2)} = 0.561 \times 10^5 = 5.6 \times 10^4$$

RATIONAL & IRRATIONAL NUMBERS – be able to place rational and irrational numbers on a number line, and estimate rational and irrational numbers.

A **rational number** is a number that can be expressed as a fraction or ratio. ^[L]_[SEP]The numerator and the denominator of the fraction are both integers.

When the fraction is divided out, it becomes a terminating or repeating decimal.

Examples: whole numbers, integers, fractions, decimals that repeat or terminate

0, 4, -6, $\frac{2}{7}$, .875 (terminates) .3333333 (repeats)

An **irrational number** cannot be expressed as a fraction. ^[L]_[SEP]Irrational numbers cannot be represented as terminating or repeating decimals. ^[L]_[SEP]Irrational numbers are non-terminating, non-repeating decimals.

Examples: $\sqrt{2}$ (non perfect square roots) π (pi), .12122122212222....(keeps going on forever)

PERFECT SQUARE ROOTS & PERFECT CUBE ROOTS

Perfect squares

$$\begin{aligned}\sqrt{1} &= 1 \text{ since } 1^2 = 1 \\ \sqrt{4} &= 2 \text{ since } 2^2 = 4 \\ \sqrt{9} &= 3 \text{ since } 3^2 = 9 \\ \sqrt{16} &= 4 \text{ since } 4^2 = 16 \\ \sqrt{25} &= 5 \text{ since } 5^2 = 25 \\ \sqrt{36} &= 6 \text{ since } 6^2 = 36 \\ \sqrt{49} &= 7 \text{ since } 7^2 = 49 \\ \sqrt{64} &= 8 \text{ since } 8^2 = 64 \\ \sqrt{81} &= 9 \text{ since } 9^2 = 81 \\ \sqrt{100} &= 10 \text{ since } 10^2 = 100\end{aligned}$$

| CUBE | RESULT |
|--------|--------|
| 0^3 | 0 |
| 1^3 | 1 |
| 2^3 | 8 |
| 3^3 | 27 |
| 4^3 | 64 |
| 5^3 | 125 |
| 6^3 | 216 |
| 7^3 | 343 |
| 8^3 | 512 |
| 9^3 | 729 |
| 10^3 | 1000 |

$$\sqrt[3]{1} = 1$$

$$\sqrt[3]{8} = 2$$

$$\sqrt[3]{27} = 3$$

$$\sqrt[3]{64} = 4$$

$$\sqrt[3]{125} = 5$$

$$\sqrt[3]{216} = 6$$

$$\sqrt[3]{343} = 7$$

$$\sqrt[3]{512} = 8$$

$$\sqrt[3]{729} = 9$$

$$\sqrt[3]{1000} = 10$$

Linear equations with ONE, NO SOLUTION & INFINITELY MANY SOLUTIONS

It

is possible to have more than solution in other types of equations that are not linear, but it is also possible to have no solutions or infinite solutions. No solution would mean that there is no answer to the equation. It is impossible for the equation to be true no matter what value we assign to the variable. Infinite solutions would mean that any value for the variable would make the equation true.

One solution: $2x + 7 = 23$ (subtract 7 on both sides) $2x = 16$ (divide by 2) $x = 8$

No solution: $3x + 5 = 3x - 8$ (subtract 3x on both sides) $5 = -8$ false statement ---NO SOLUTION

$x + 2x + 1 = 3(x+2) + 3$ distribute by multiplying 3 to the parenthesis

$x + 2x + 1 = 3x + 6 + 3$ combine like terms

$3x + 1 = 3x + 9$ subtract 3x on both sides

$1 = 9$ false statement NO SOLUTION

Infinitely Many Solution: $4x + 9 = 4x + 9$ (subtract 4x on both sides) $9 = 9$ True statement INFINITE MANY

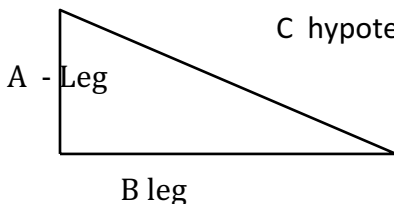
$x + 2x + 3 + 3 = 3(x + 2)$ distribute by multiplying 3 to the parenthesis

$x + 2x + 3 + 3 = 3x + 6$ combine like terms

$3x + 6 = 3x + 6$ both sides of the equation are the equal 😊. True statement = infinite many solutions!

UNIT 3 Geometric Applications of Exponents

Pythagorean theorem works only on right triangles: $a^2 + b^2 = c^2$

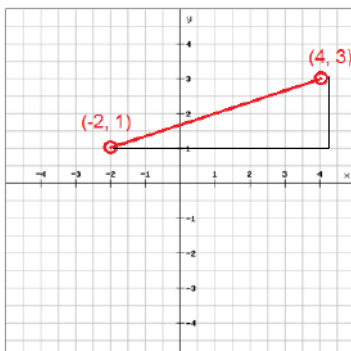


$$3^2 + 4^2 = ? \quad a = 3 \text{ (leg)} \quad b = 4 \text{ (leg)} \quad c = ? \text{ (hypotenuse)}$$

$$9 + 16 = ?$$

$$\sqrt{25} = 5$$

On the coordinate plane make the line segment into a right triangle, then use the Pythagorean theorem.



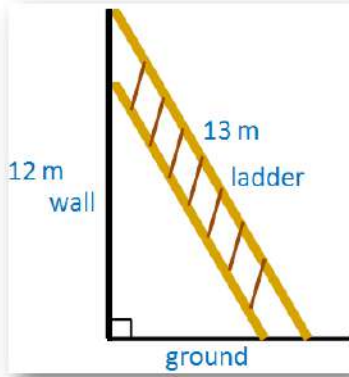
$12^2 + 4^2 = C^2$ there are 12 spaces on the bottom of triangle and 4 spaces on the side of the triangle.

$$144 + 16 = C^2$$

$$\sqrt{160} = C^2$$

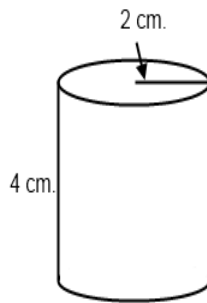
$$12.6 \approx C$$

REAL World Application of Pythagorean theorem:

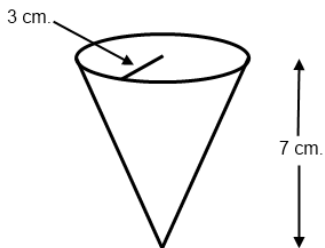


$$\begin{aligned}
 A^2 + B^2 &= C^2 \\
 12^2 + B^2 &= 13^2 \\
 144 + B^2 &= 169 \\
 -144 &\quad -144 \\
 B^2 &= \sqrt{25} \\
 B &= 5
 \end{aligned}$$

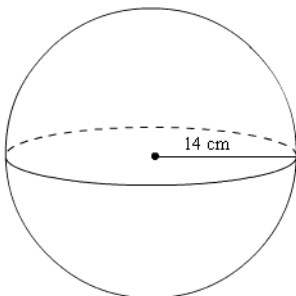
VOLUME of Cylinders, Cones & Spheres volume is always measured in cubic units.



Cylinder Volume
 Formula $V = \pi r^2 h$
 $V = 3.14 * 2^2 * 4$
 $V = 50.24 \text{ cm}^3$



Cone Volume Formula
 $V = \frac{1}{3} \pi r^2 h$
 $V = \frac{1}{3} * 3.14 * 3^2 * 7$
 $V = \frac{1}{3} * 197.82$ (divide by 3)
 $V = 65.94 \text{ cm}^3$



Sphere Volume Formula
 $V = \frac{4}{3} \pi r^3$
 $V = \frac{4}{3} * 3.14 * 14^3$
 $V = 4 * 3.14 * 2744$
 $V = 34464.64 \div 3 = 11488.2 \text{ cm}^3$

Unit 4 FUNCTIONS

Function - is a **relation** between a **set** of inputs and a set of outputs with the property that each input is related to exactly one output.

Relation is any set of ordered pairs.

All functions are relations, but not all relations are functions.

FUNCTION

NOT a FUNCTION

| $X + 6 = Y$ | |
|-------------|----|
| X | Y |
| 0 | 6 |
| 1 | 7 |
| 2 | 8 |
| 3 | 9 |
| 4 | 10 |

FUNCTION

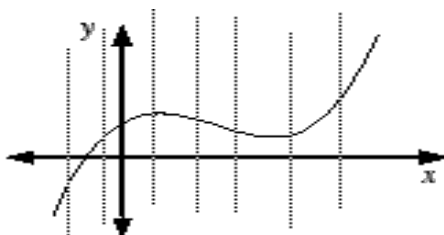
← This function table has the rule, "Add 6 to a number".

← Each row has an "X" number and a "Y" number that go together.

The "X" number is the input. Add 6 to this number. The "Y" number is the output. This is the answer to the X number plus 6.

Vertical Line Test

The graph is a function since there are no vertical lines that hit the graph more than once.



Which ones are functions?

Which ones are not functions?

The domain is the set of all first elements of ordered pairs (*x*-coordinates), also called input, independent.
 The range is the set of all second elements of ordered pairs (*y*-coordinates), also called output, dependent.

Function notation- $f(x)$ a fancy way of saying y .

$F(x) = 4x - 2$

$X = 2, 7, 12$

can you find $f(x)$ when $x = -13$ $f(x) = \underline{\hspace{2cm}}$

$F(x) = 6, 26, 46$

↙ ↘
 ↖ ↗
 Output value input value

When comparing functions look at the rate of change or slope and the *y*-intercept. $y = 2x + 4$ to $y = 8x - 5$ which has the greatest rate of change? $\underline{\hspace{2cm}}$ which has the greatest *y*-intercept? $\underline{\hspace{2cm}}$ where do the lines intersect? $\underline{\hspace{2cm}}$