Pythagorean Theorem Review

5 QUESTIONS

If a triangle is formed with the given side lengths, which would be a **right triangle**?

A. 5 in, 6 in, and 7 in

B. 9 in, 12 in, and 15 in

C. 25 in, 30 in, and 25 in

D. 12 in, 18 in, and 30 in

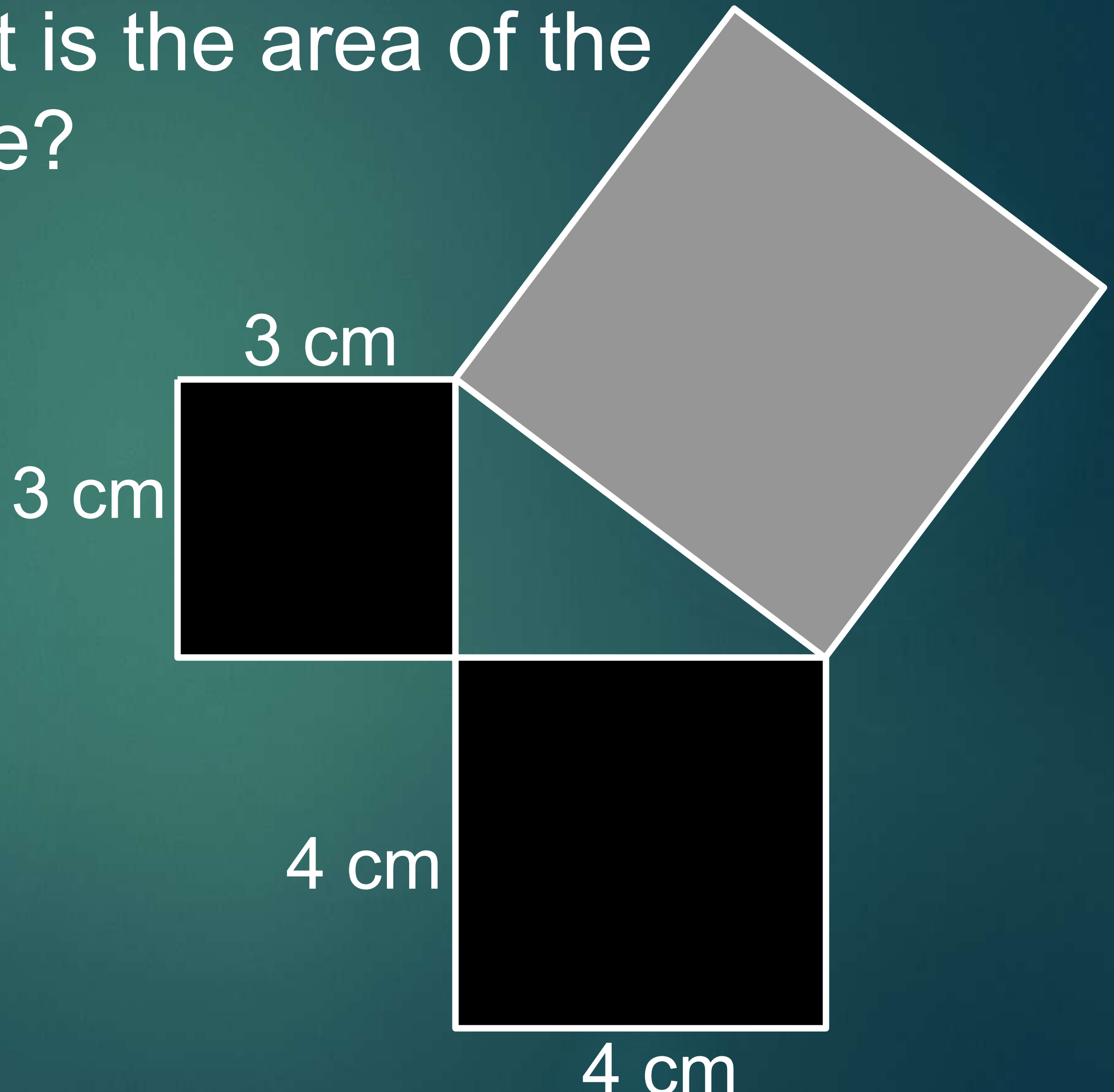
The three squares below form a right triangle. What is the area of the shaded square?

A. 5 cm^2

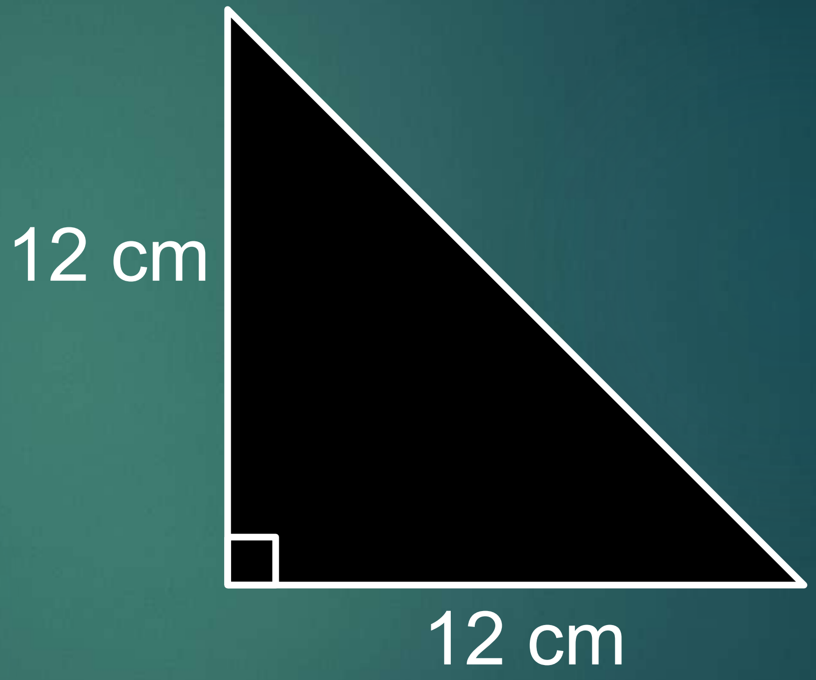
B. 9 cm^2

C. 25 cm^2

D. 16 cm^2



Find the length of the missing side. Round your answer to the nearest whole number.

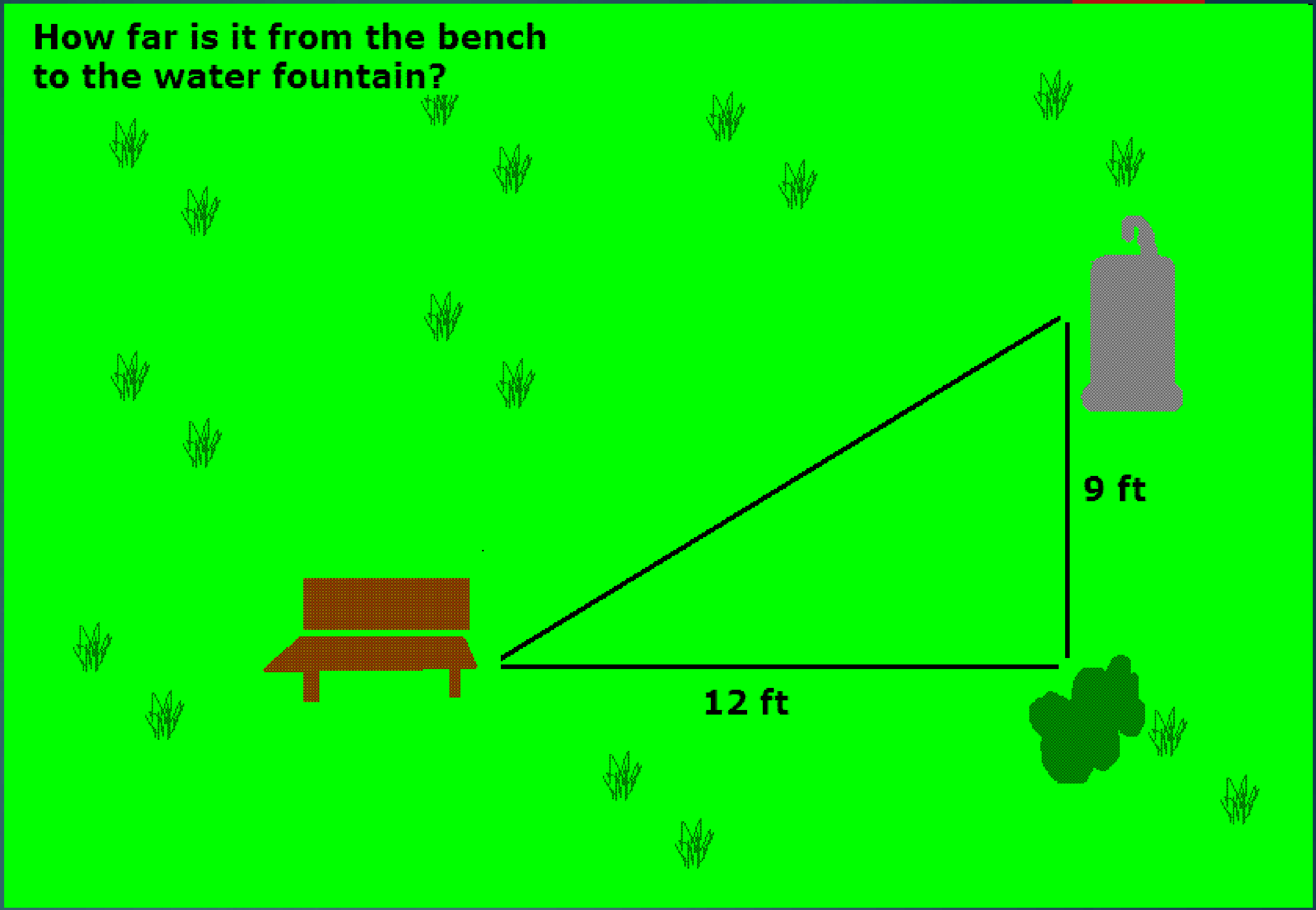


A. 17 cm

B. 24 cm

C. 144 cm

D. 288 cm



A. 21 cm

B. 15 cm

C. 25 cm

D. 225 cm

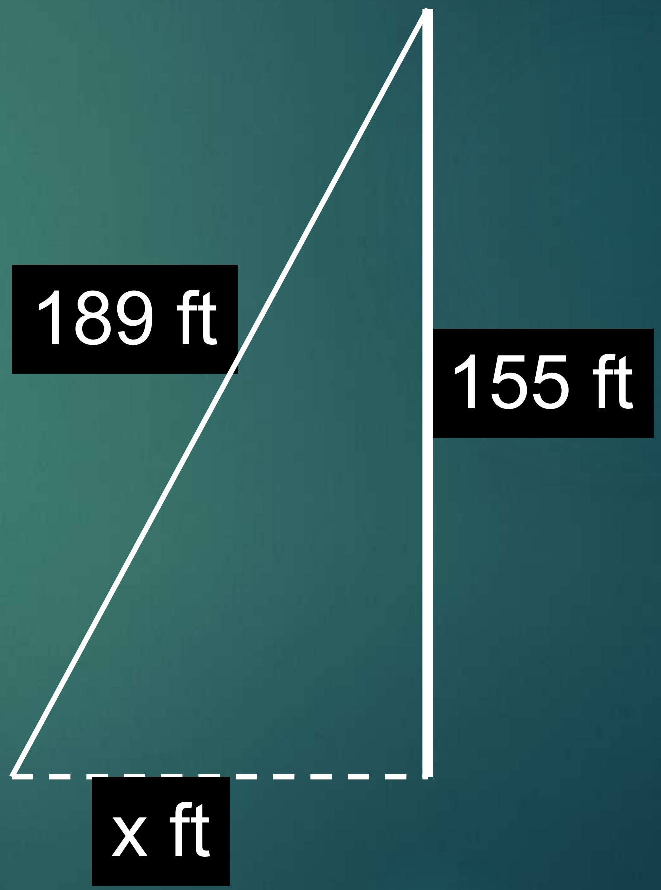
The Zilker Park Christmas Tree is 155 ft. tall and is made of 39 strands of lights that are 189 ft long. How far from the base of the tree is each strand attached?

A. 17 ft

B. 88 ft

C. 108 ft

D. 244 ft

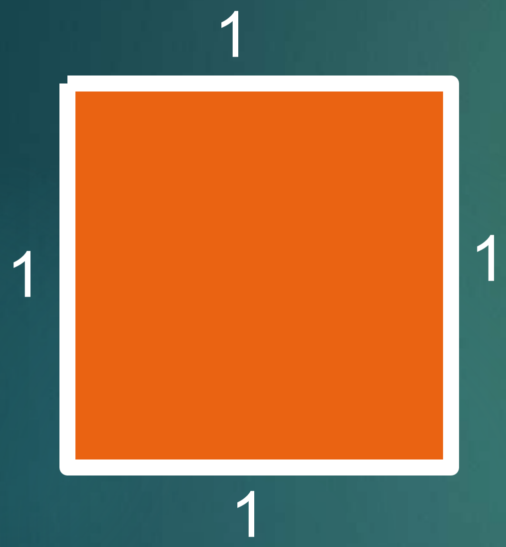


Two Special Right Triangles

45°- 45°- 90°

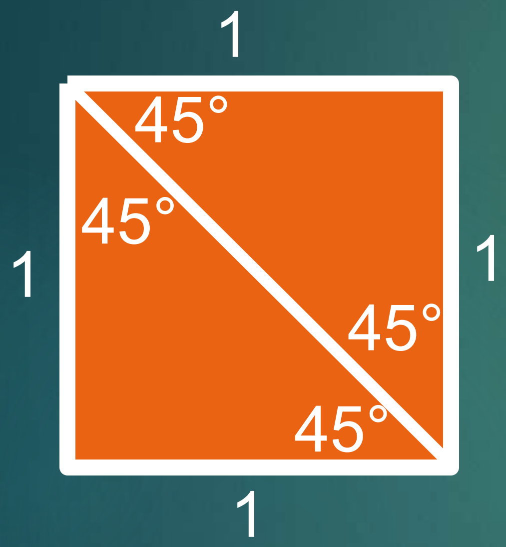
30°- 60°- 90°

45°- 45°- 90°



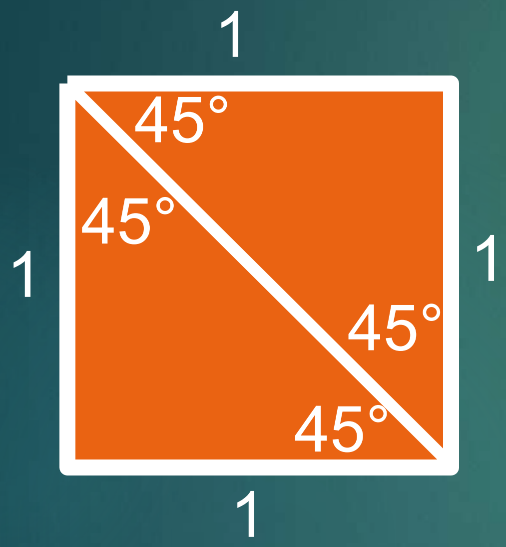
The 45-45-90 triangle is based on the square with sides of 1 unit.

45°- 45°- 90°



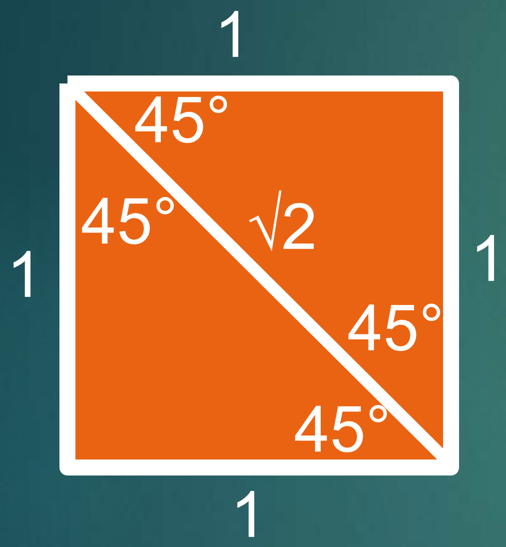
If we draw the diagonals we form two 45-45-90 triangles.

45°- 45°- 90°



Using the Pythagorean Theorem we can find the length of the diagonal.

45°- 45°- 90°



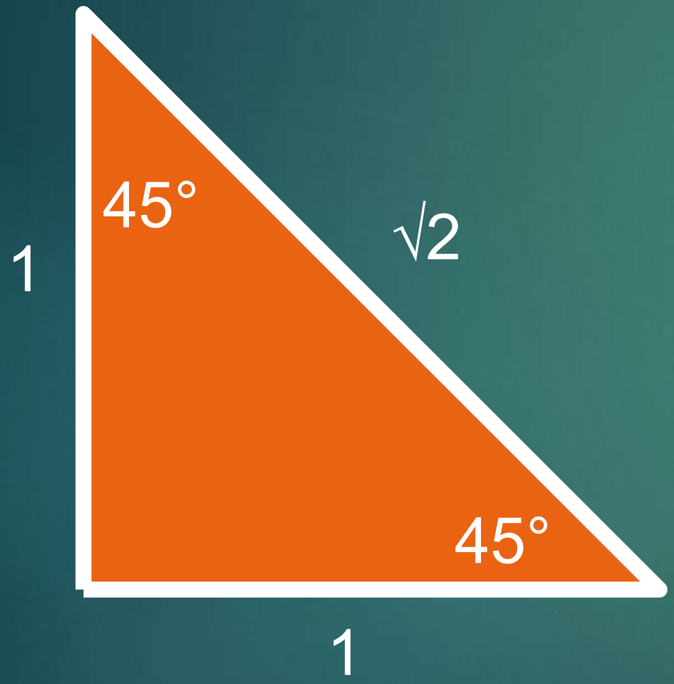
$$1^2 + 1^2 = c^2$$

$$1 + 1 = c^2$$

$$2 = c^2$$

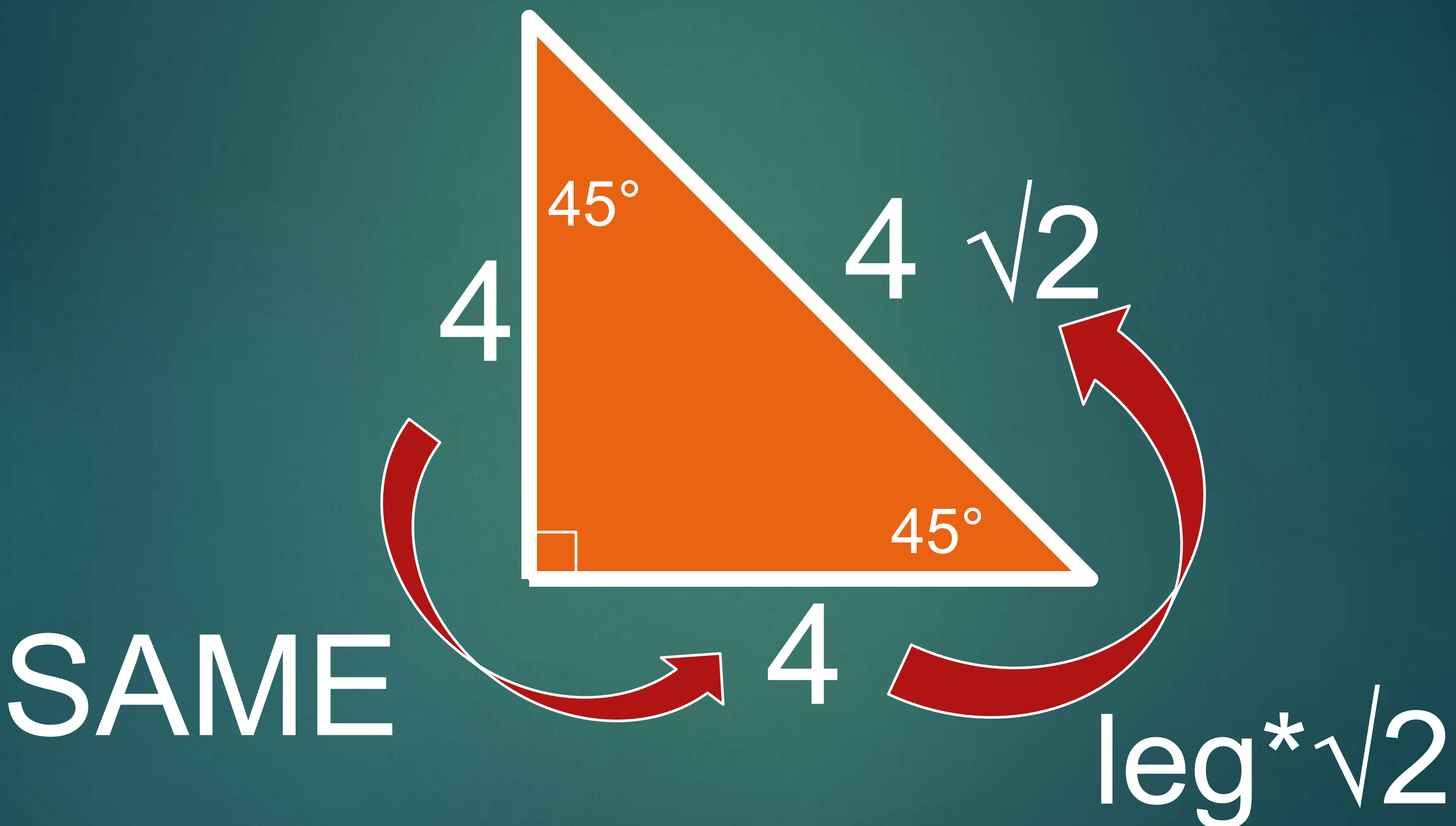
$$\sqrt{2} = c$$

45°- 45°- 90°

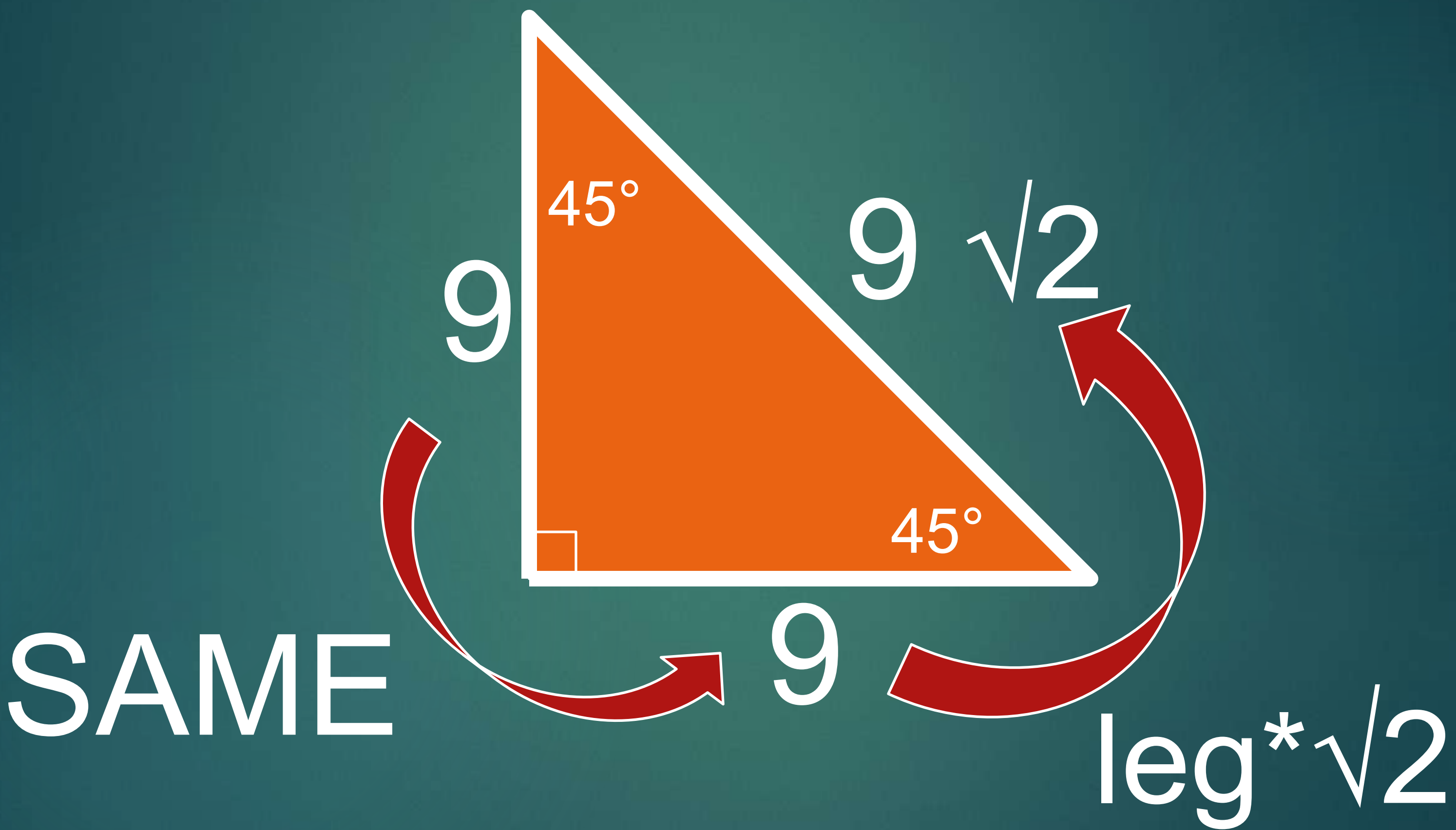


Conclusion: the ratio of the sides in a 45-45-90 triangle is 1-1-$\sqrt{2}$

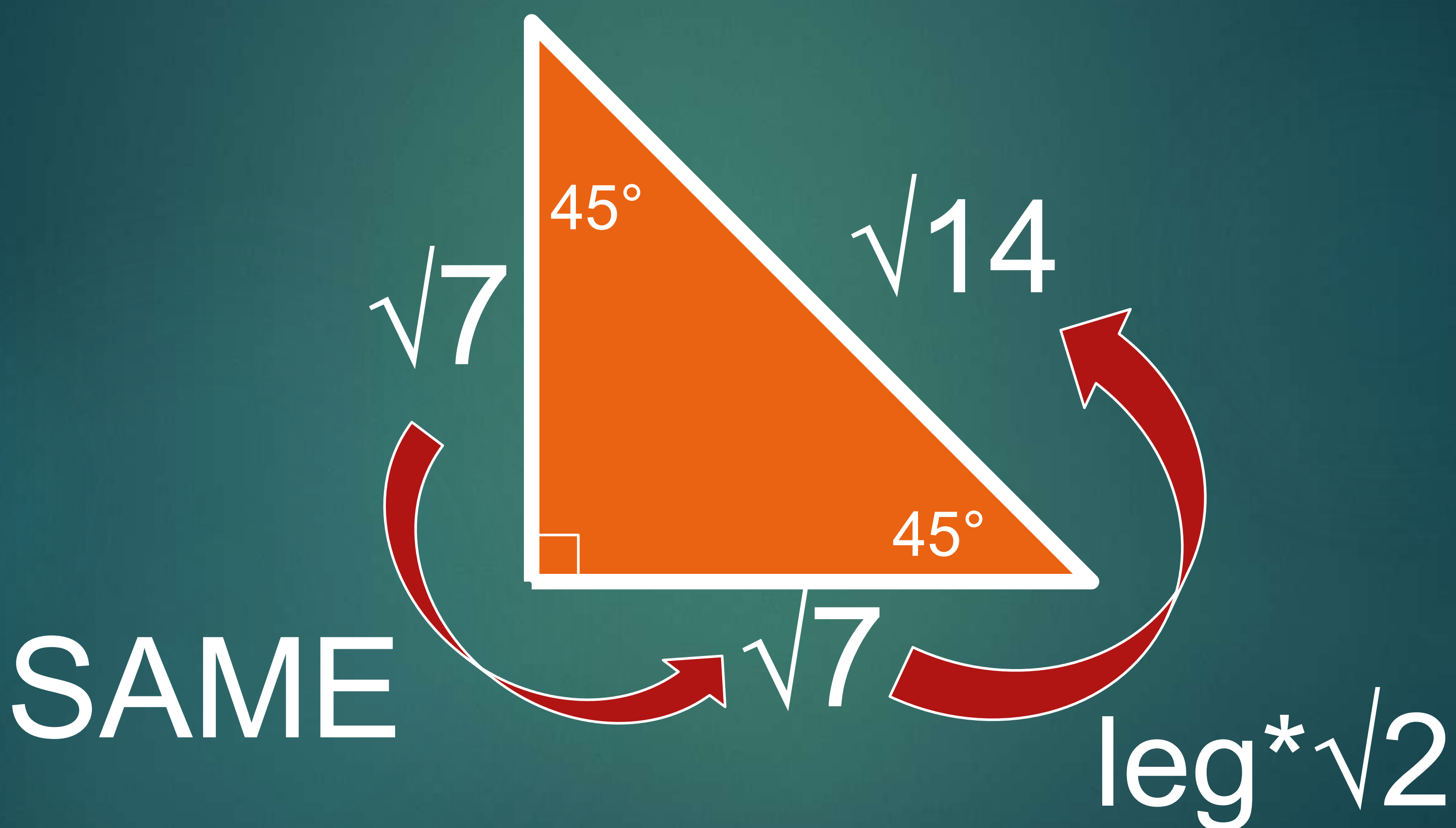
45°- 45°- 90° Practice



45°- 45°- 90° Practice



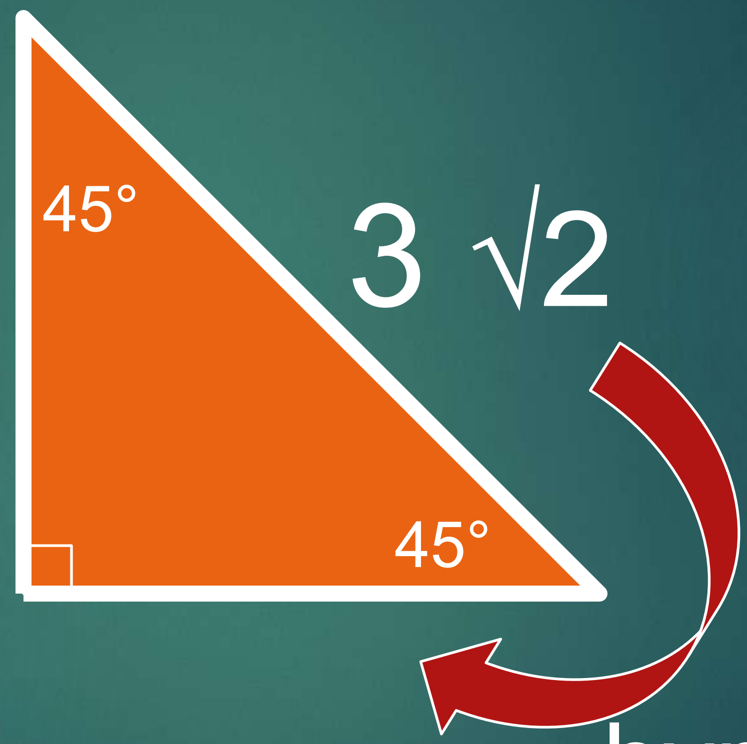
45°- 45°- 90° Practice



45°- 45°- 90° Practice

Now Let's Go Backward

45°- 45°- 90° Practice



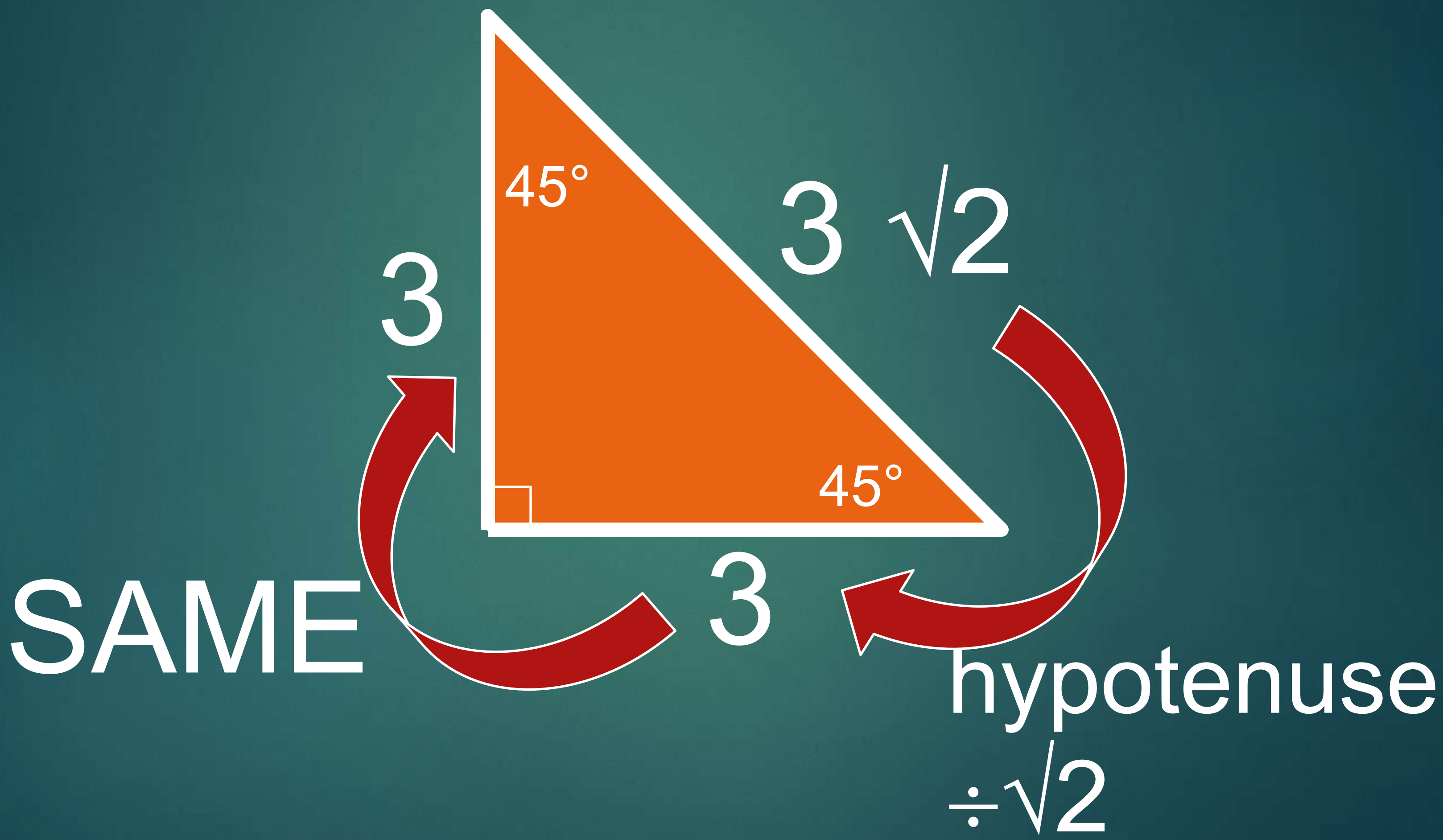
hypotenuse

÷√2

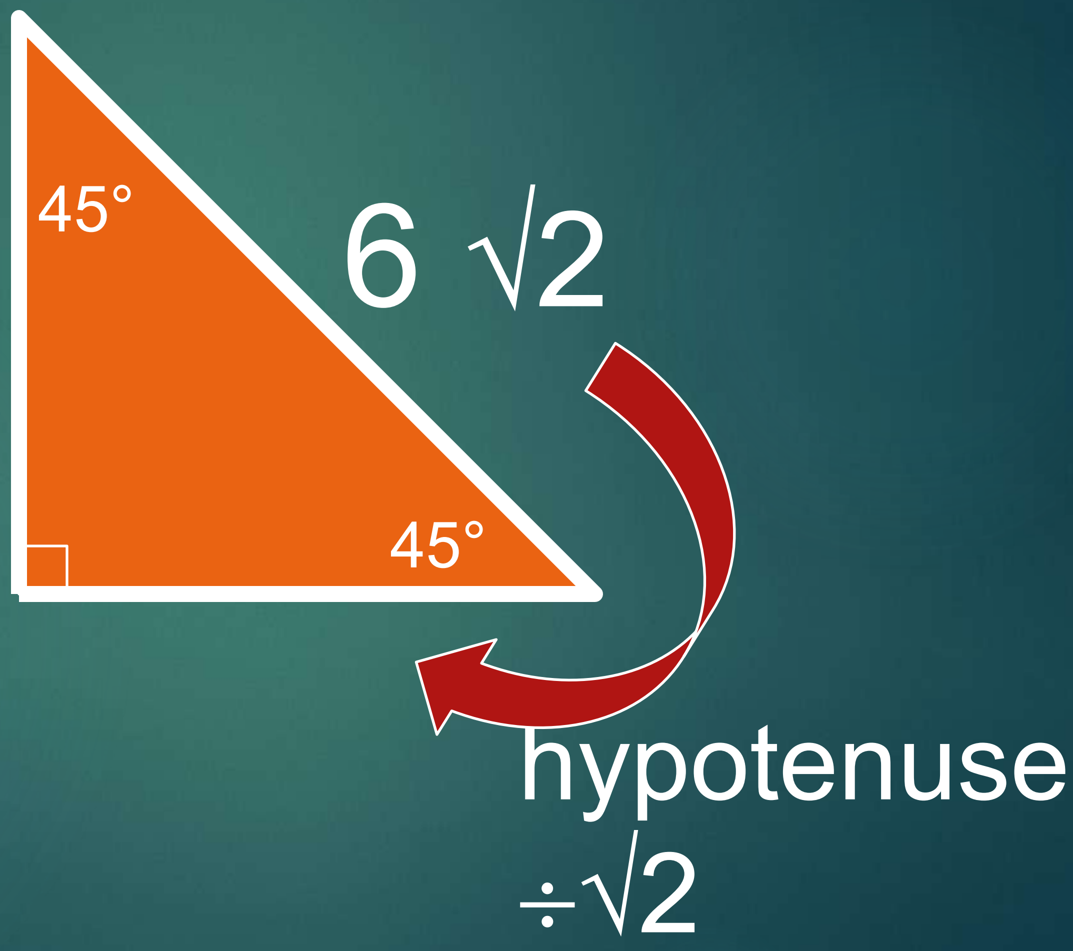
45°- 45°- 90° Practice

$$\frac{3\sqrt{2}}{\sqrt{2}} = 3$$

45°- 45°- 90° Practice



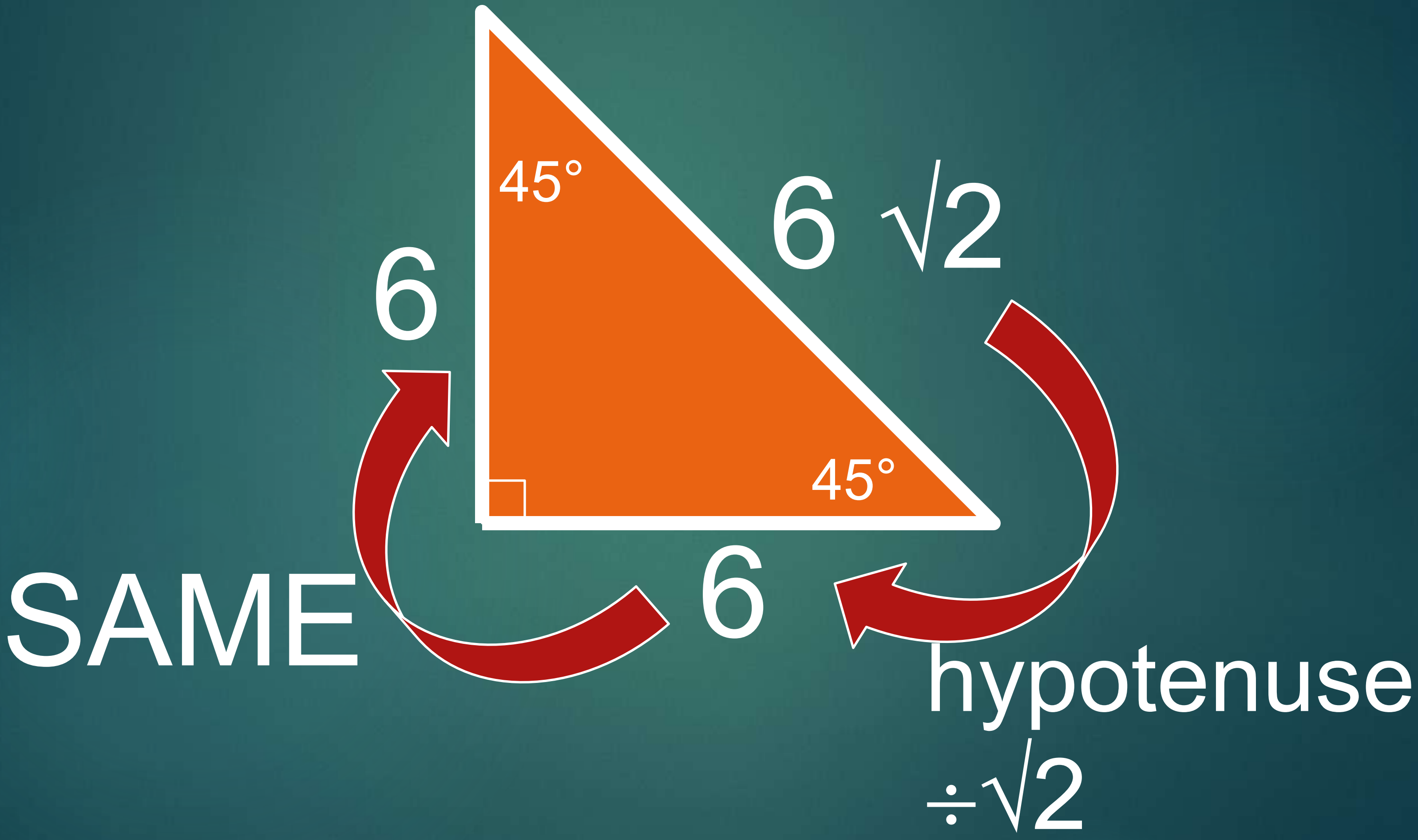
45°- 45°- 90° Practice



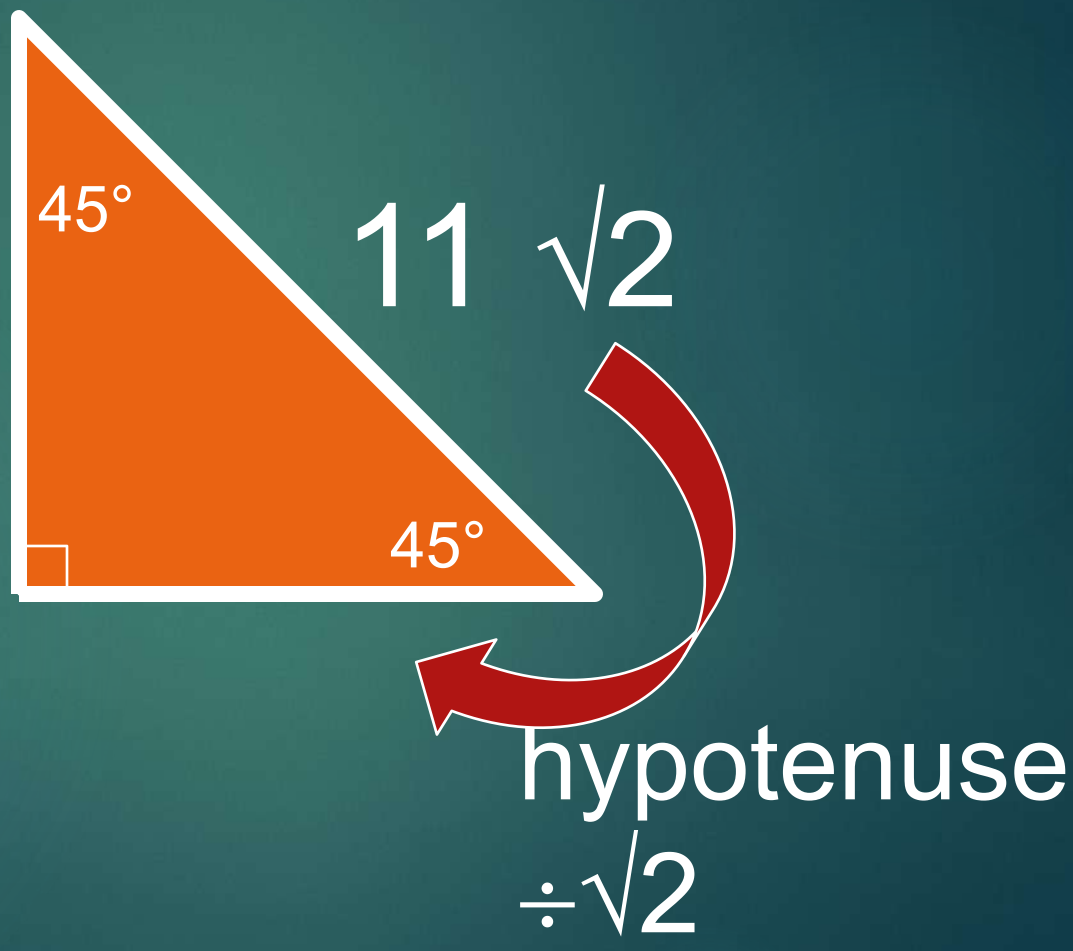
45°- 45°- 90° Practice

$$\frac{6\sqrt{2}}{\sqrt{2}} = 6$$

45°- 45°- 90° Practice



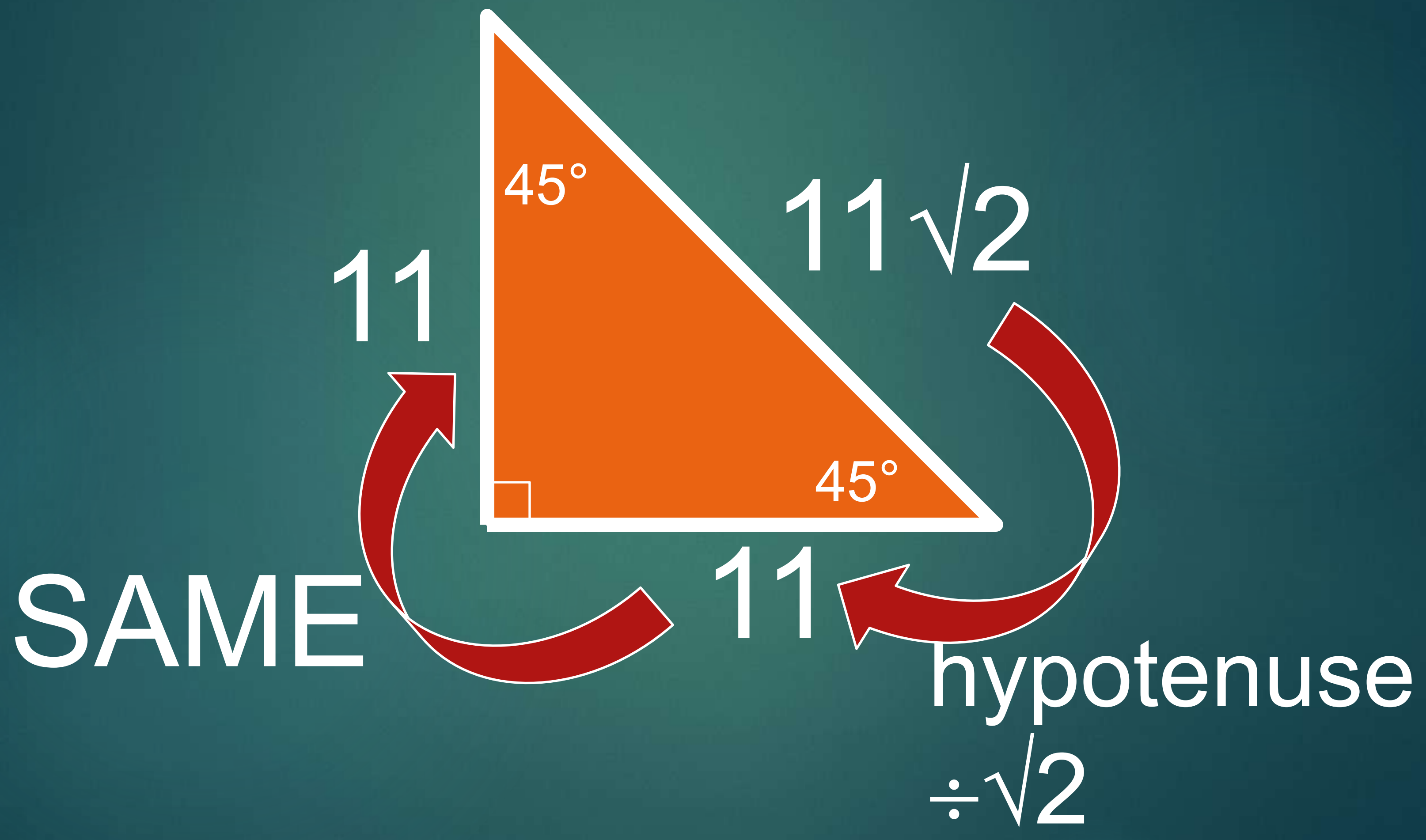
45°- 45°- 90° Practice



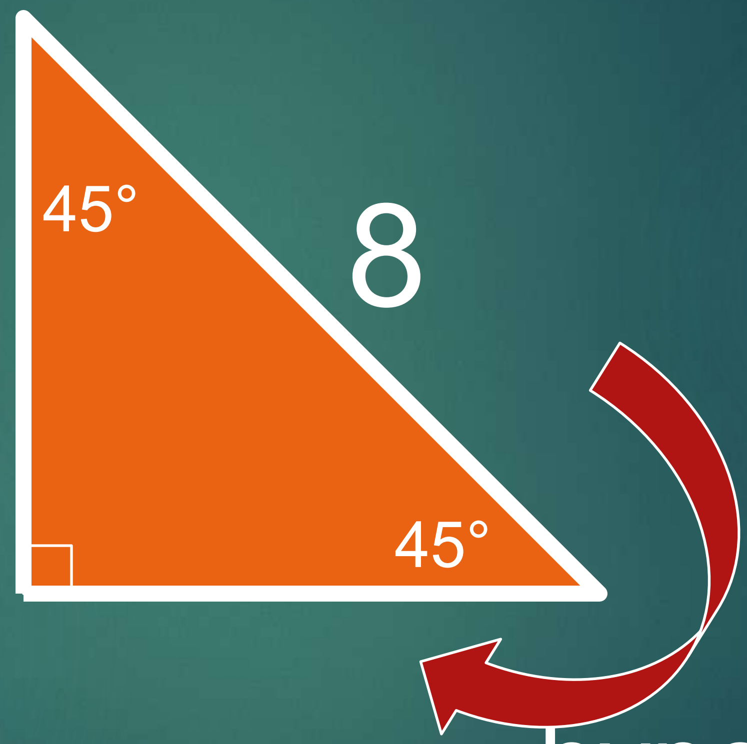
45°- 45°- 90° Practice

$$\frac{11\sqrt{2}}{\sqrt{2}} = 11$$

45°- 45°- 90° Practice



45°- 45°- 90° Practice



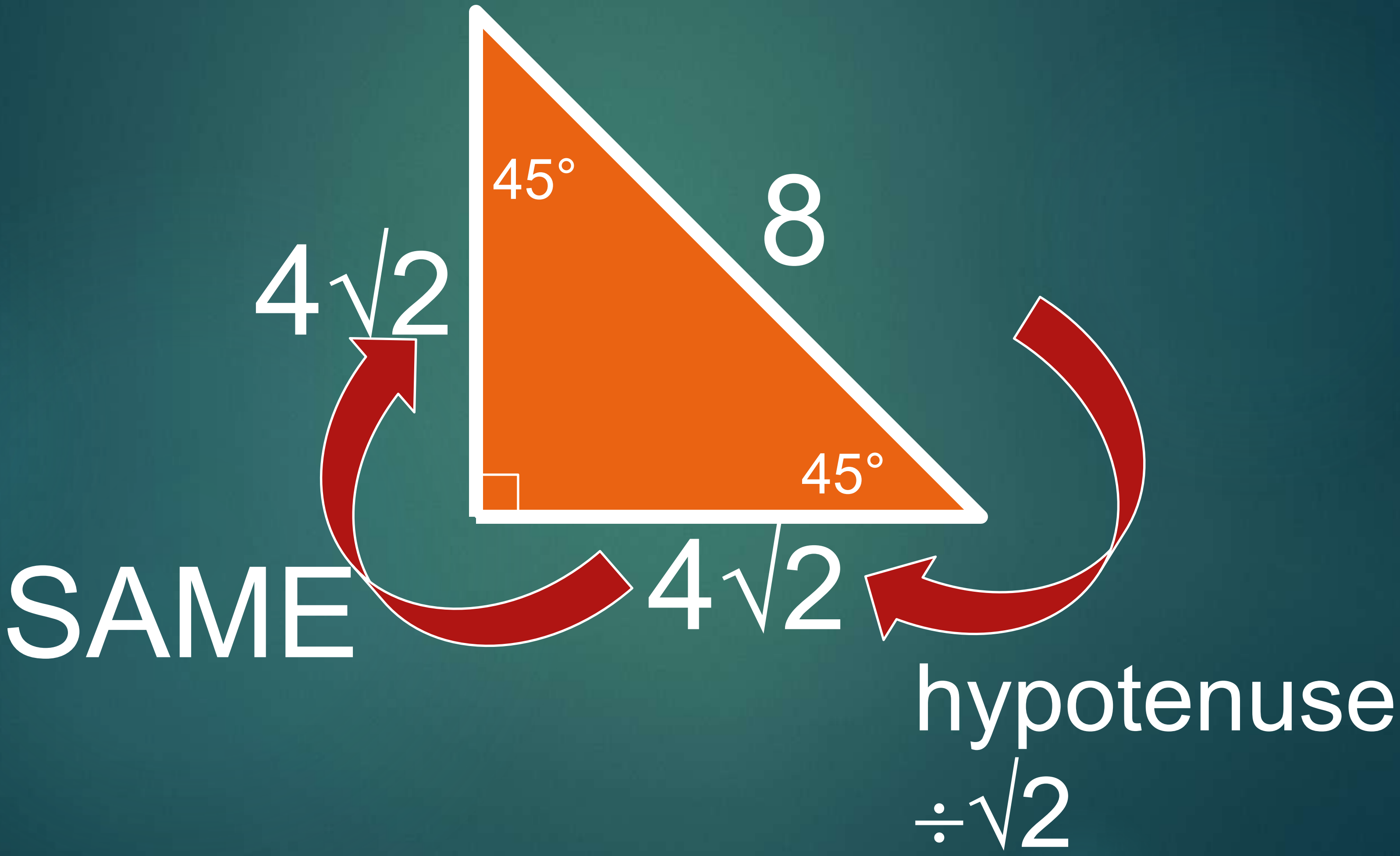
hypotenuse

$\div\sqrt{2}$

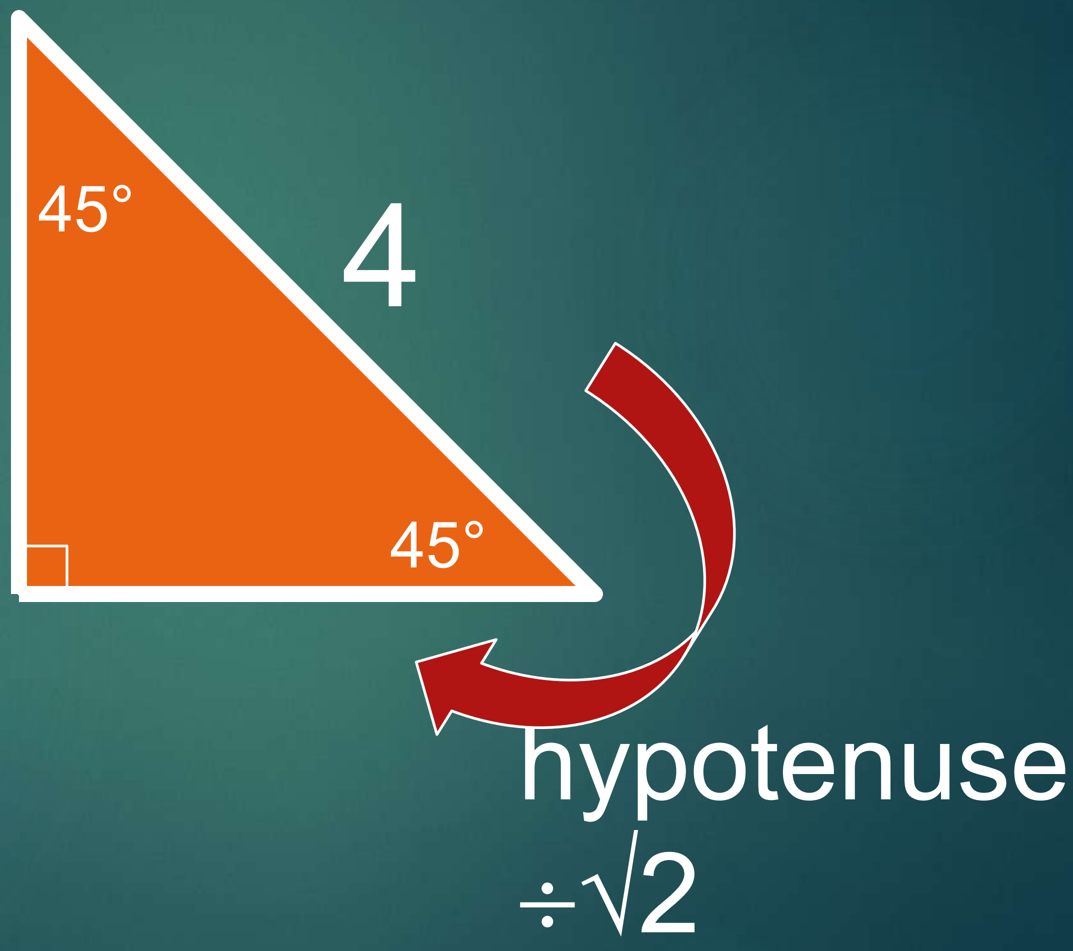
45°- 45°- 90° Practice

$$\frac{8}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

45°- 45°- 90° Practice



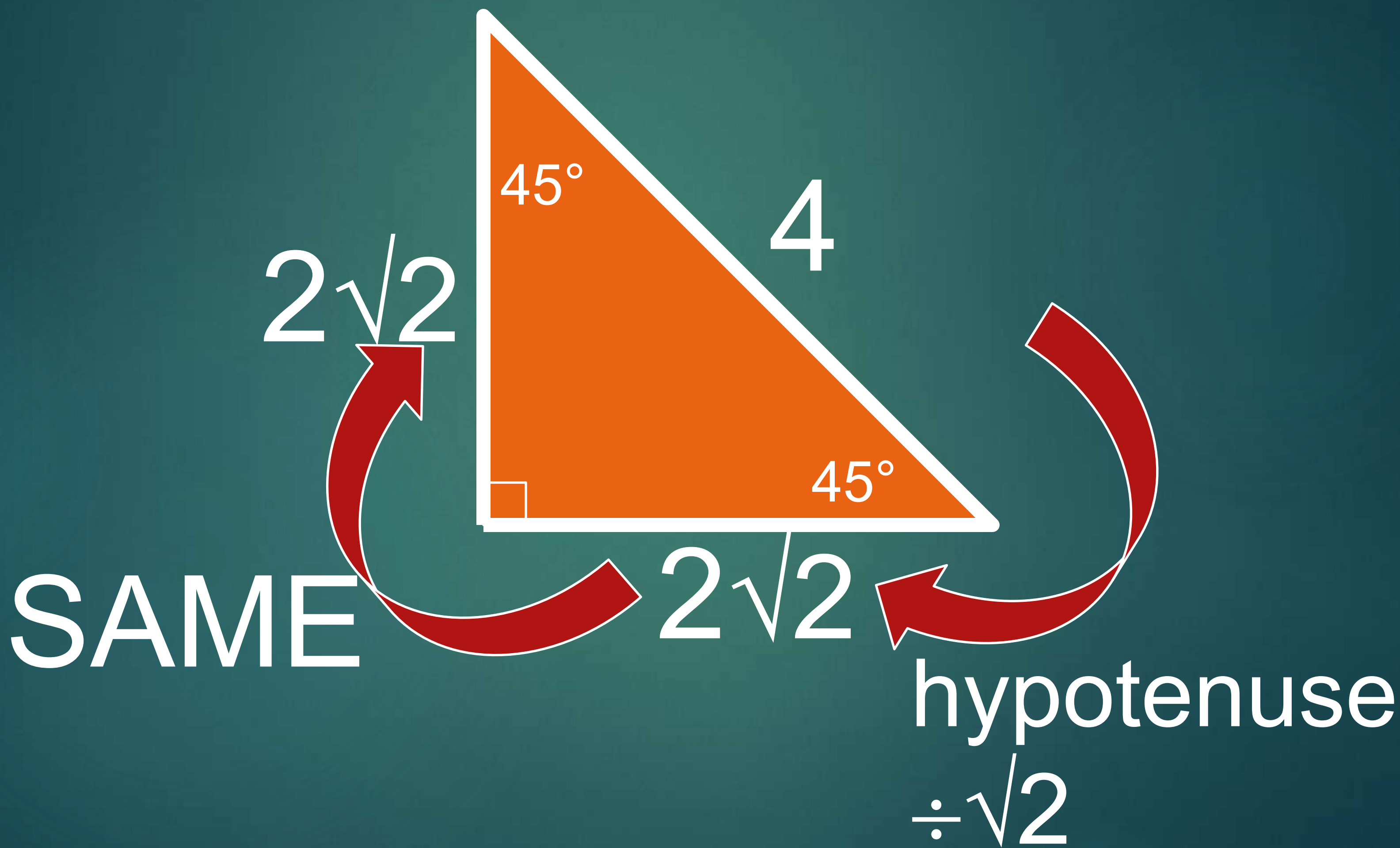
45°- 45°- 90° Practice



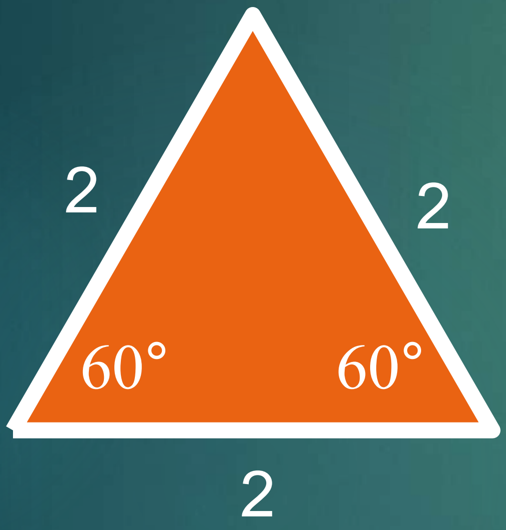
45°- 45°- 90° Practice

$$\frac{4}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

45°- 45°- 90° Practice

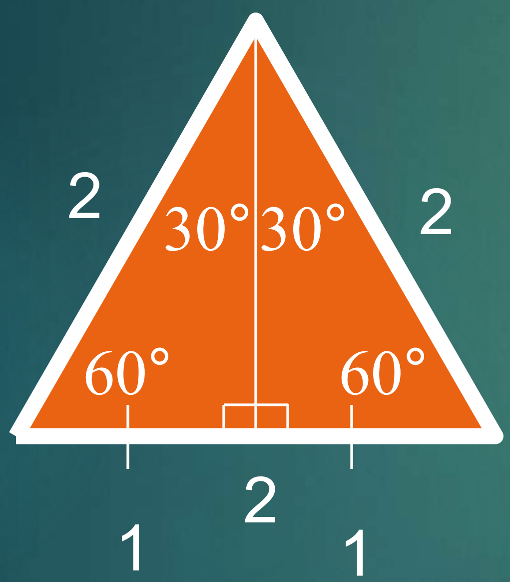


30°- 60°- 90°



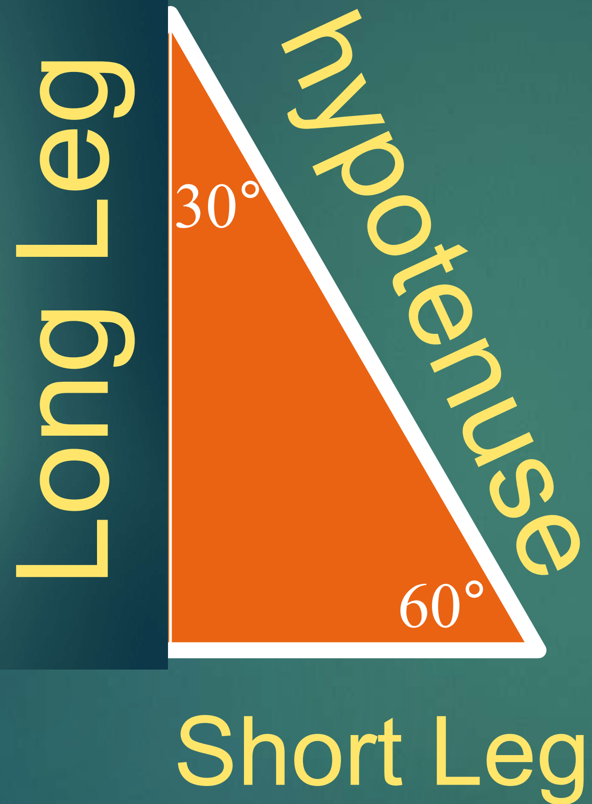
The 30-60-90 triangle is based on an equilateral triangle with sides of 2 units.

30°- 60°- 90°



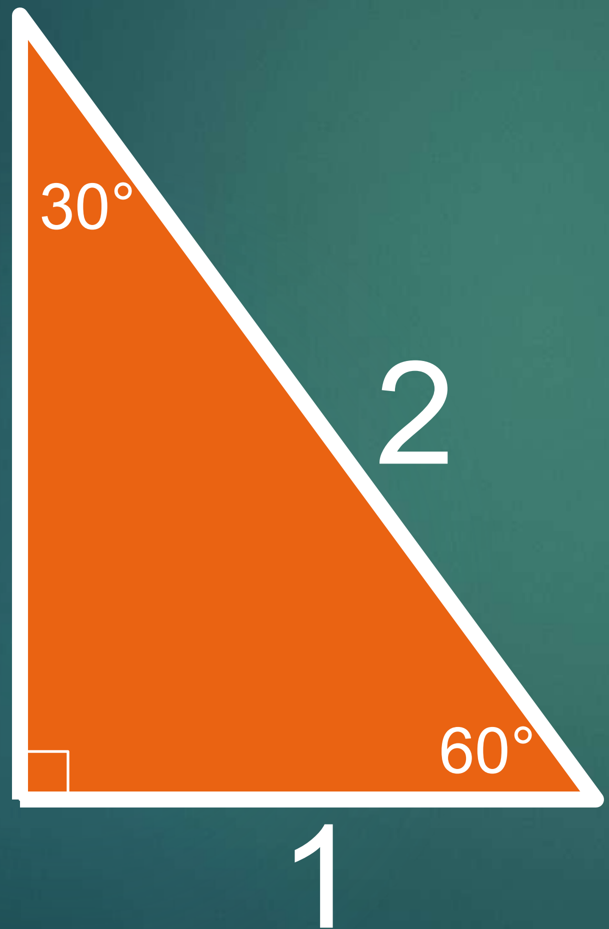
The altitude (also the angle bisector and median) cuts the triangle into two congruent triangles.

30°- 60°- 90°



This creates the 30-60-90 triangle with a hypotenuse a short leg and a long leg.

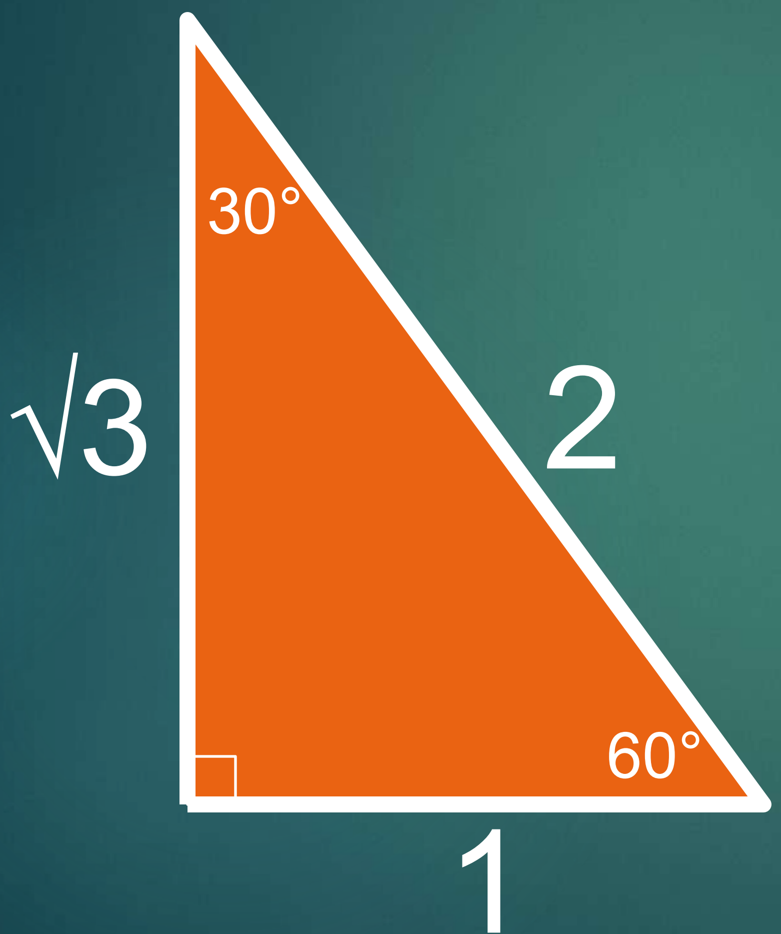
30°- 60°- 90° Practice



We saw that the hypotenuse is twice the short leg.

We can use the Pythagorean Theorem to find the long leg.

30°- 60°- 90° Practice



$A^2 + B^2 = C^2$

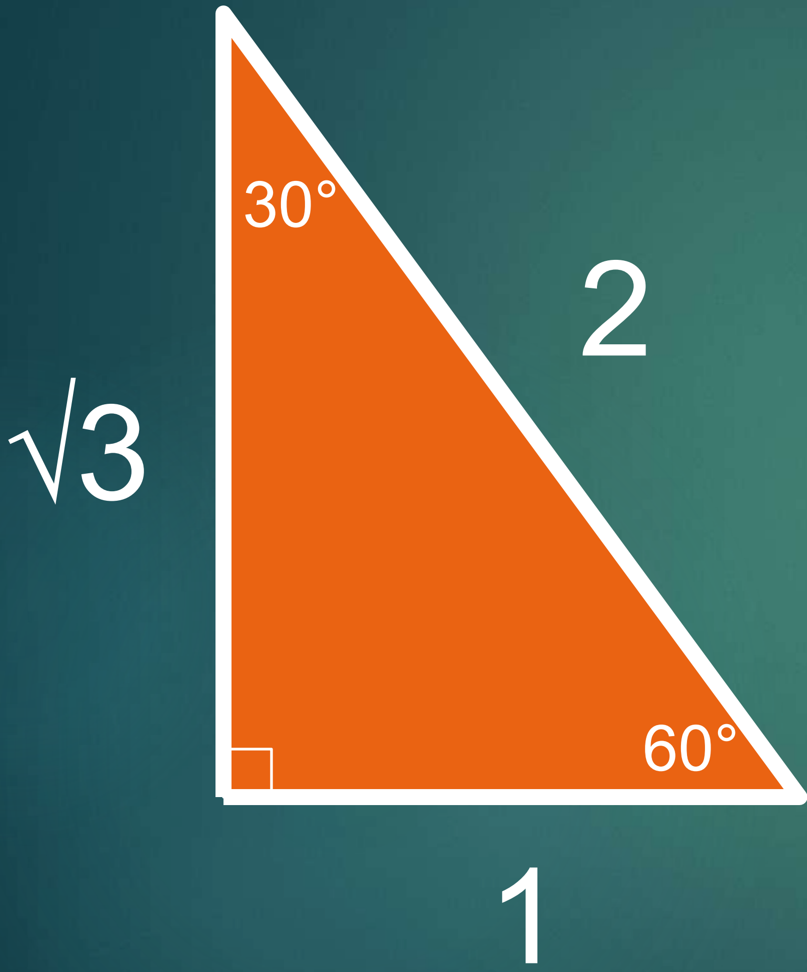
$A^2 + 1^2 = 2^2$

$A^2 + 1 = 4$

$A^2 = 3$

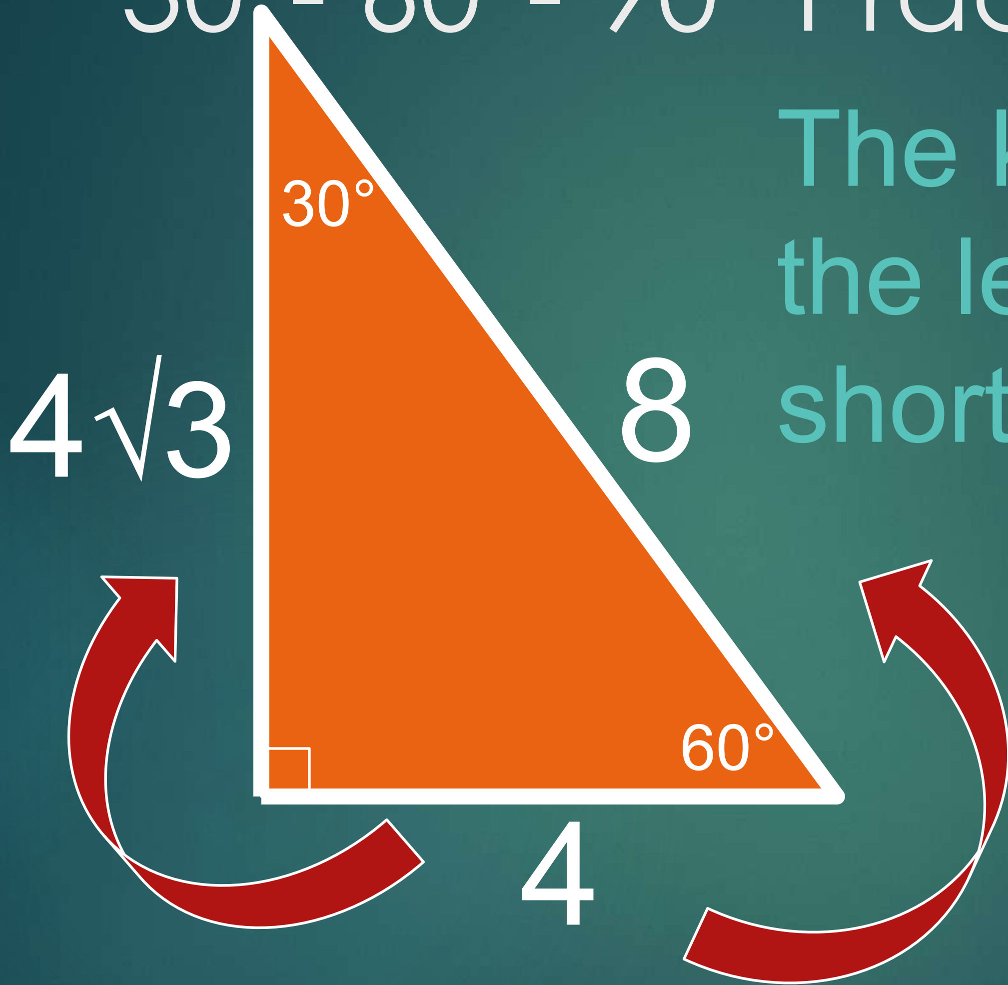
$A = \sqrt{3}$

30°- 60°- 90°



Conclusion: the ratio of the sides in a 30-60-90 triangle is 1- 2 - √3

30°- 60°- 90° Practice

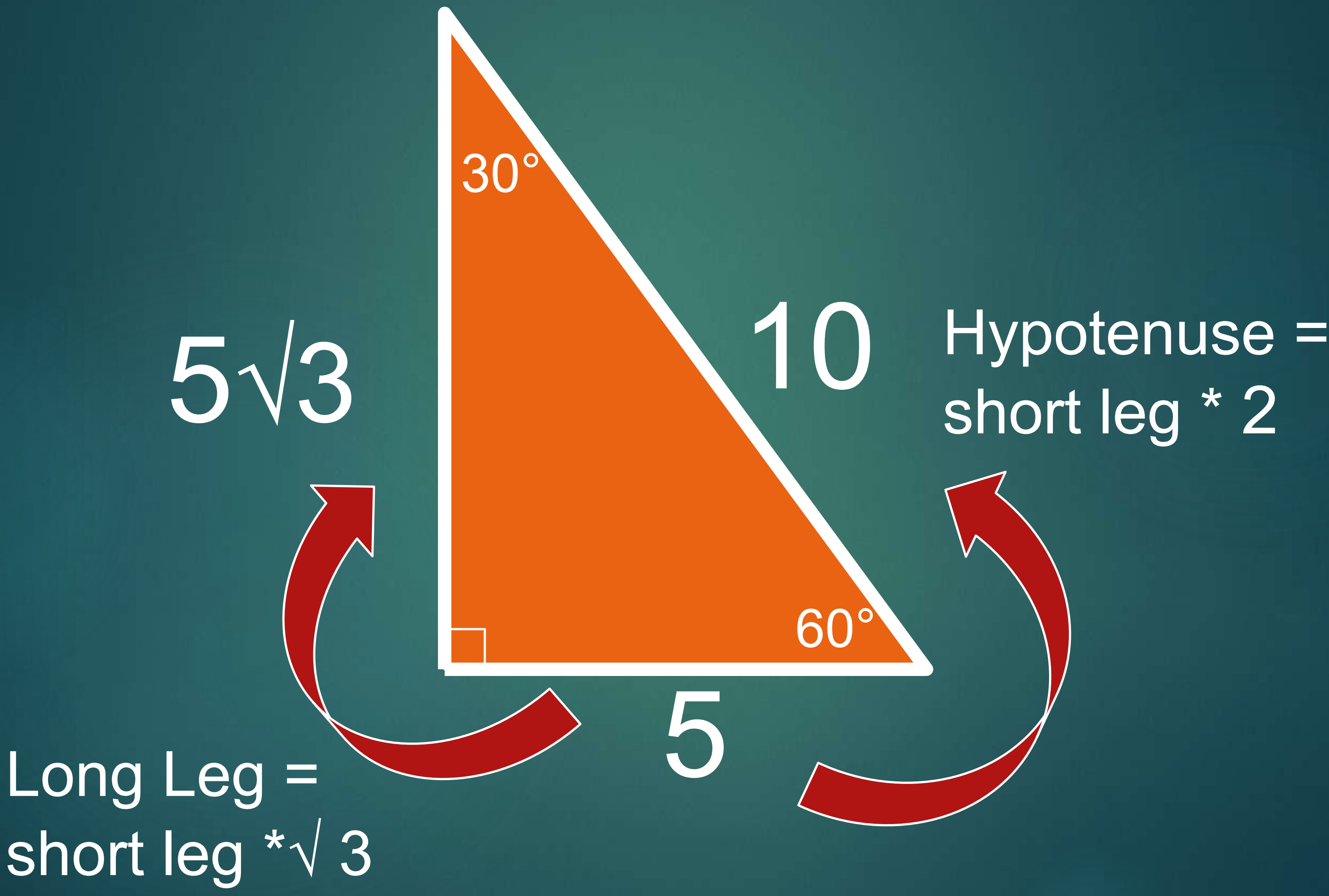


The key is to find the length of the short side.

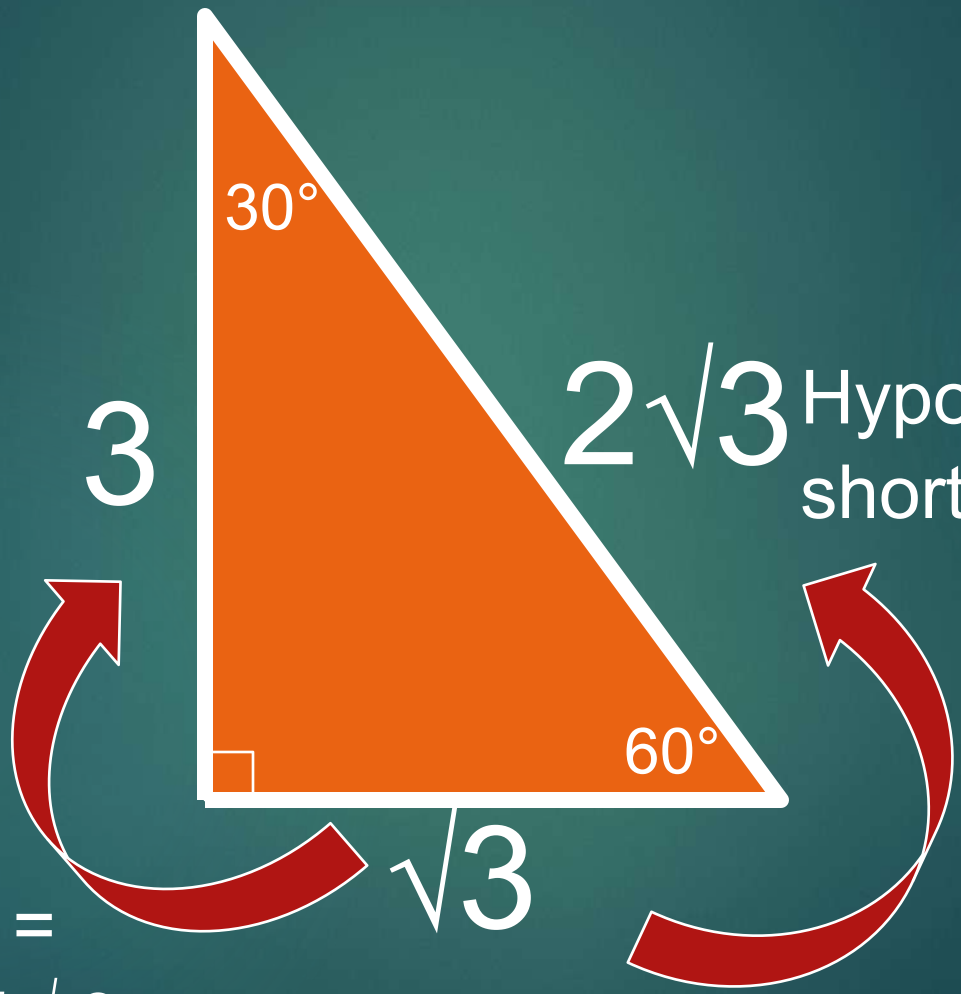
Hypotenuse = short leg * 2

Long Leg = short leg *√ 3

30°- 60°- 90° Practice



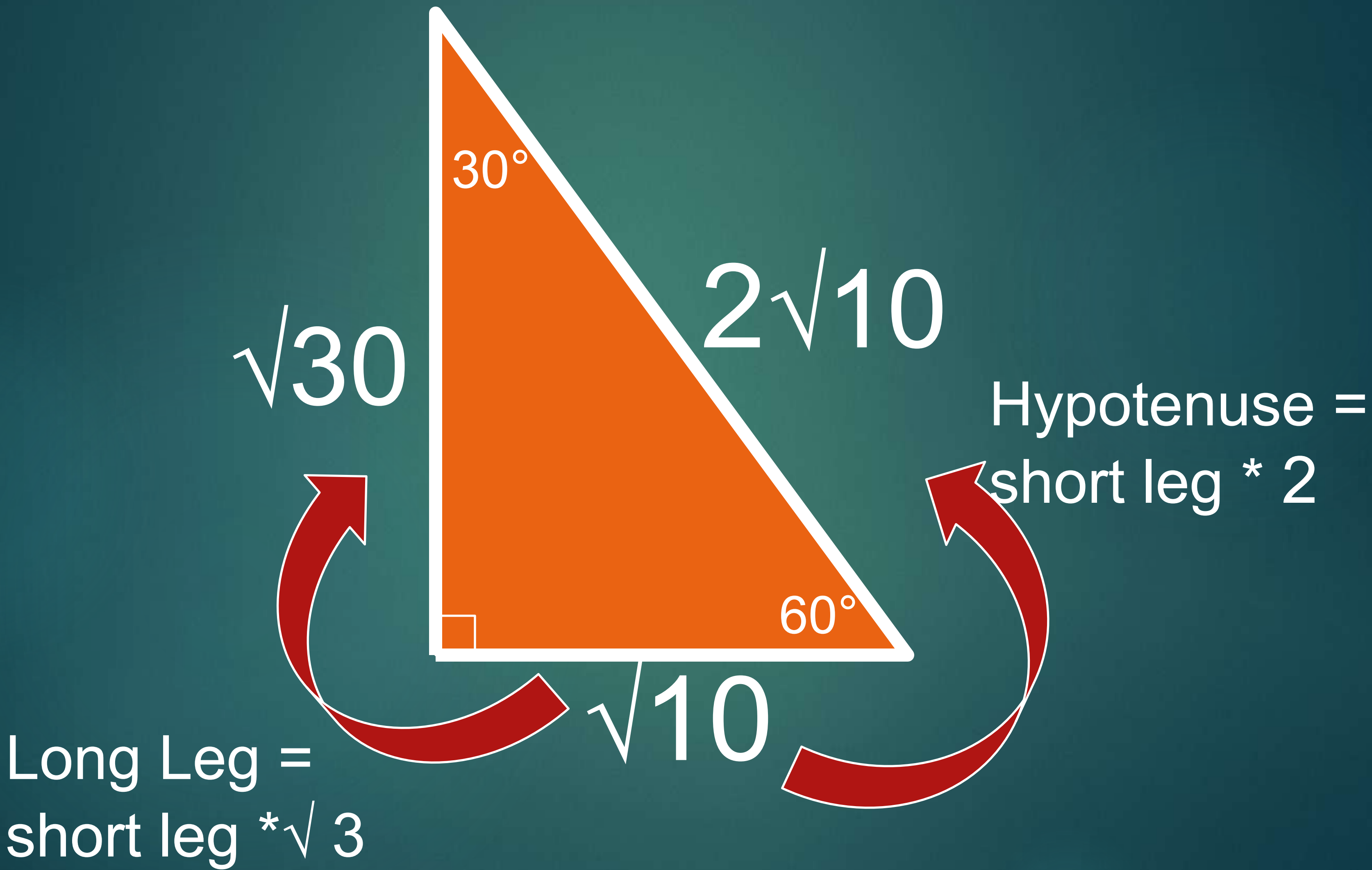
30°- 60°- 90° Practice



Hypotenuse =
short leg * 2

Long Leg =
short leg *√ 3

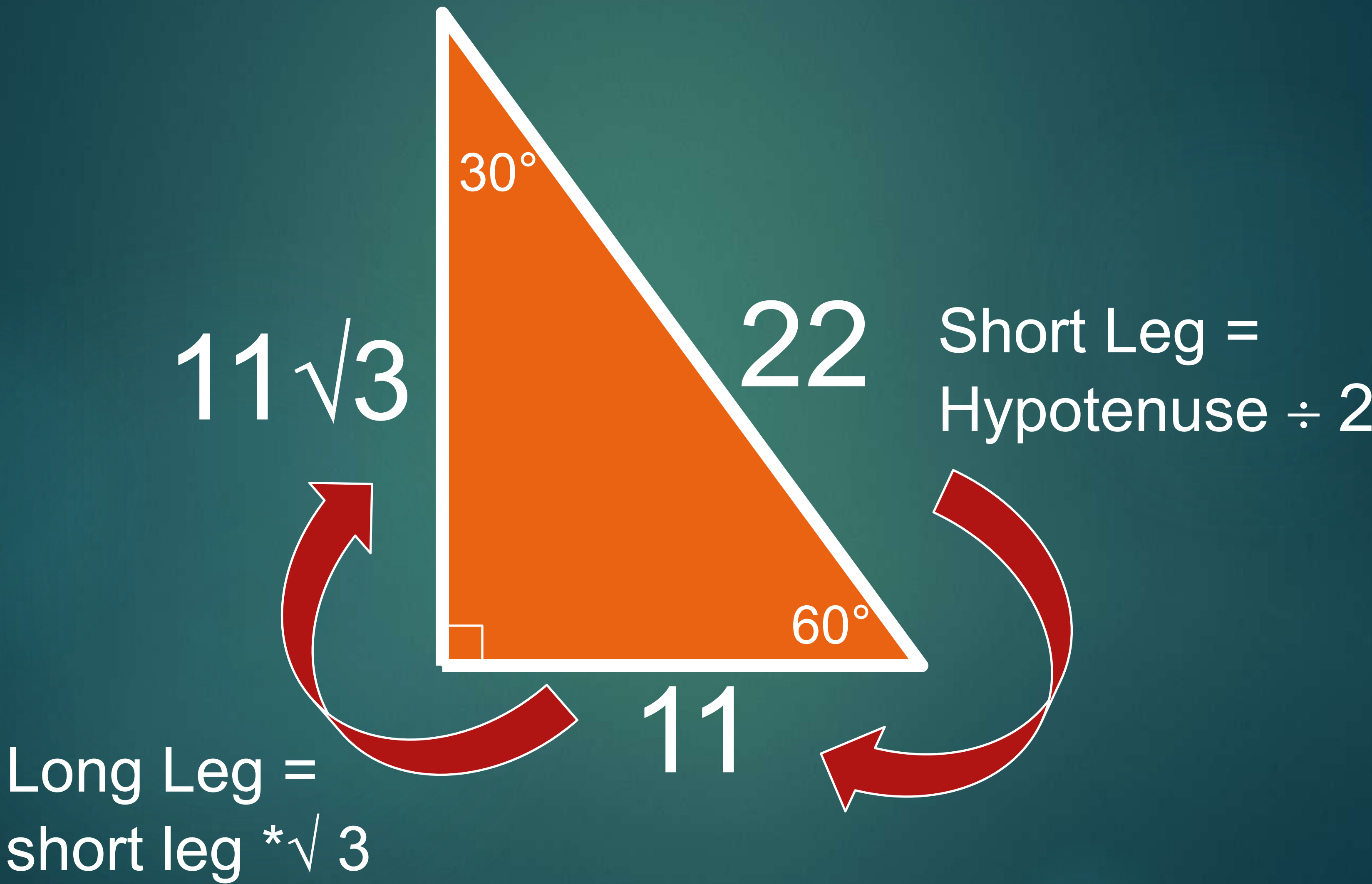
30°- 60°- 90° Practice



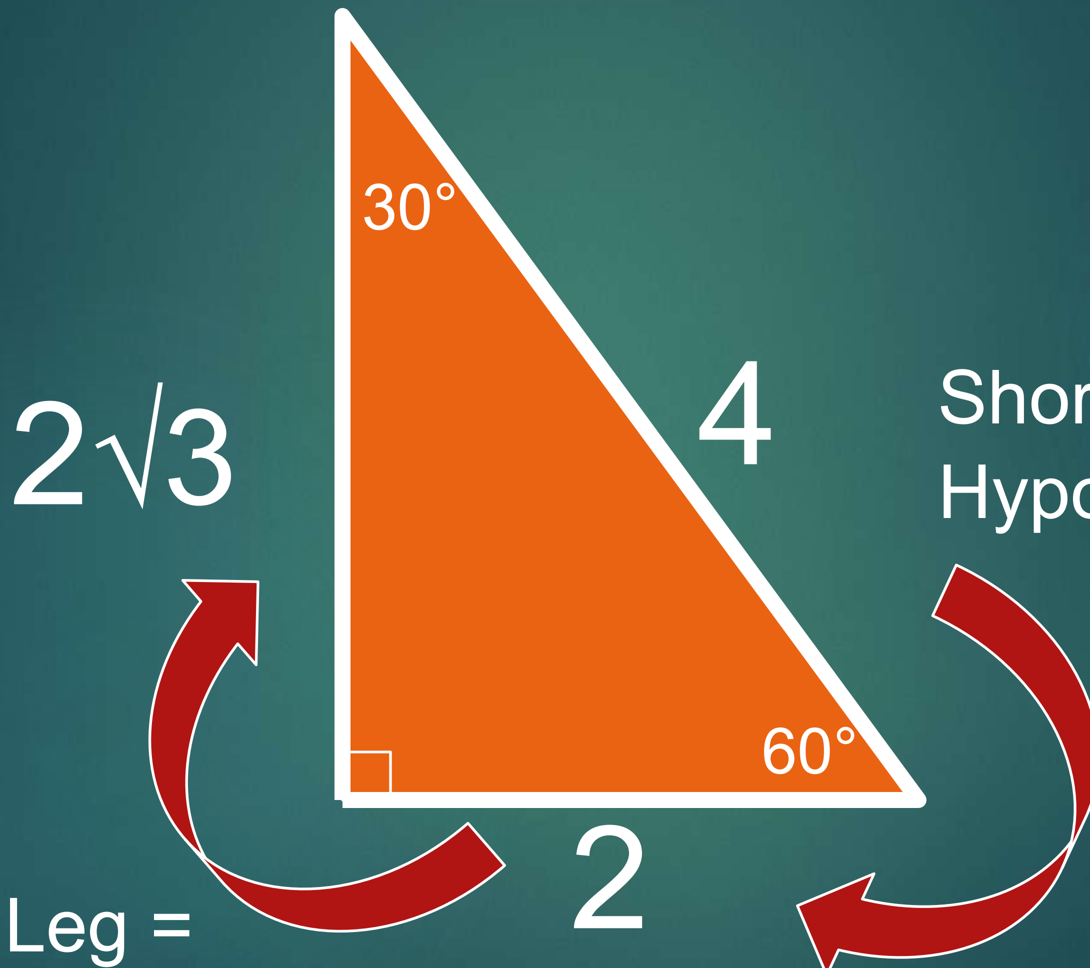
30°- 60°- 90° Practice

Now Let's Go Backward

30°- 60°- 90° Practice



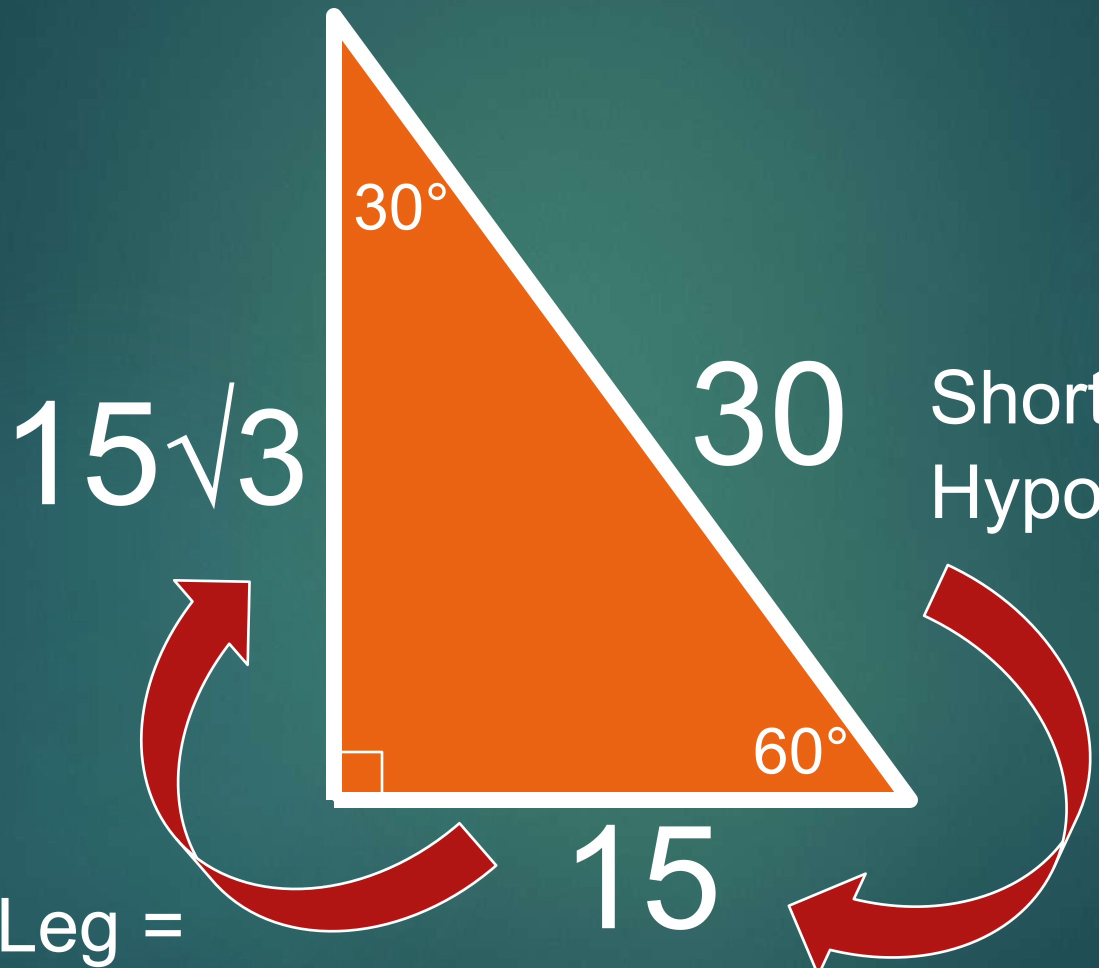
30°- 60°- 90° Practice



Short Leg =
Hypotenuse ÷ 2

Long Leg =
short leg *√ 3

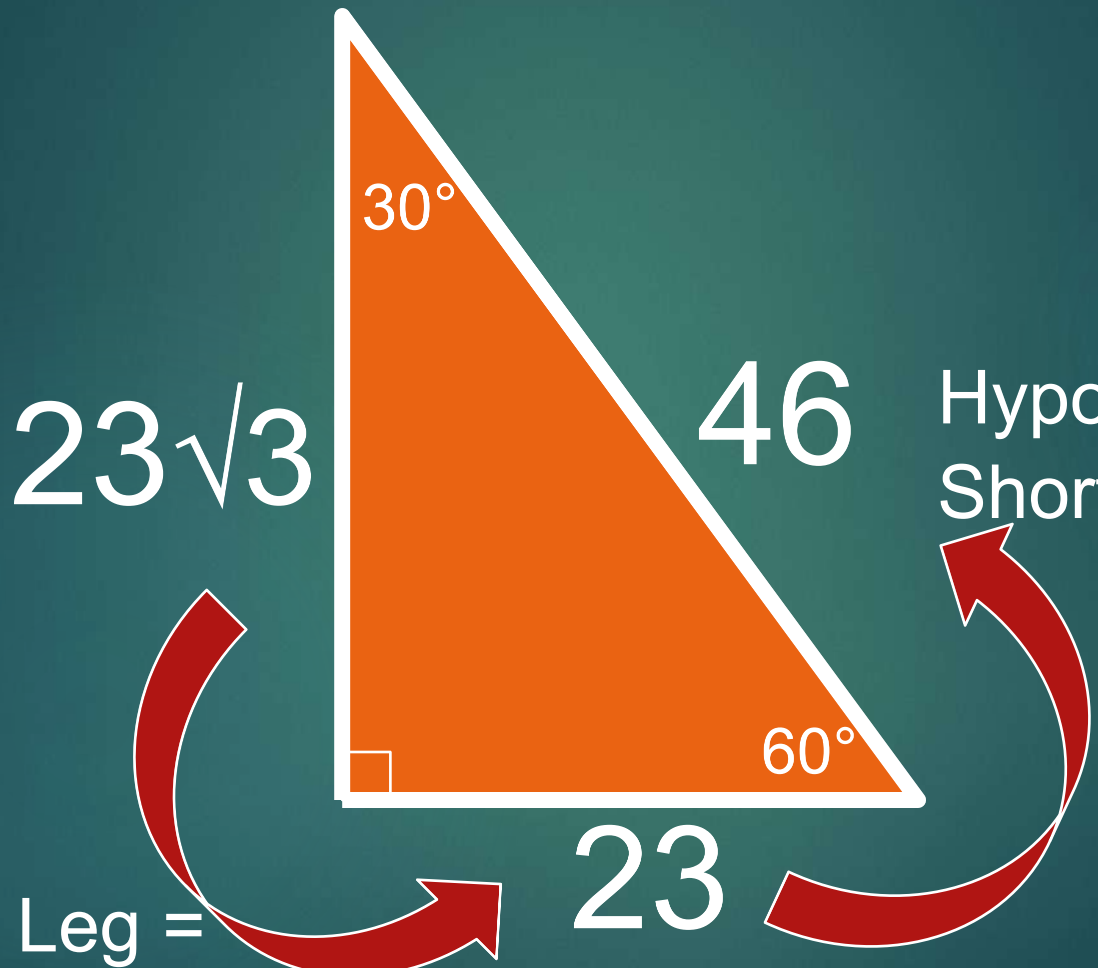
30°- 60°- 90° Practice



Short Leg = Hypotenuse ÷ 2

Long Leg = short leg *√ 3

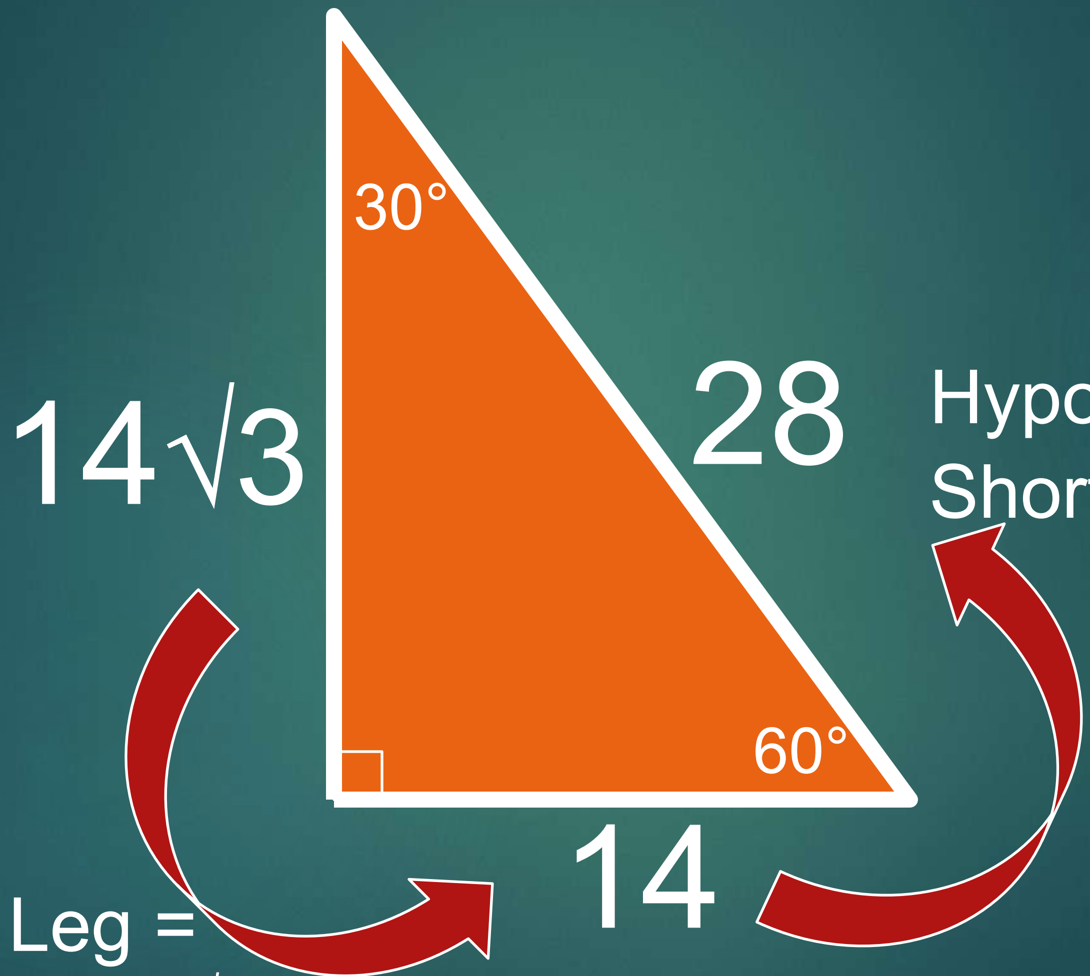
30°- 60°- 90° Practice



Hypotenuse =
Short Leg * 2

Short Leg =
Long leg ÷ √ 3

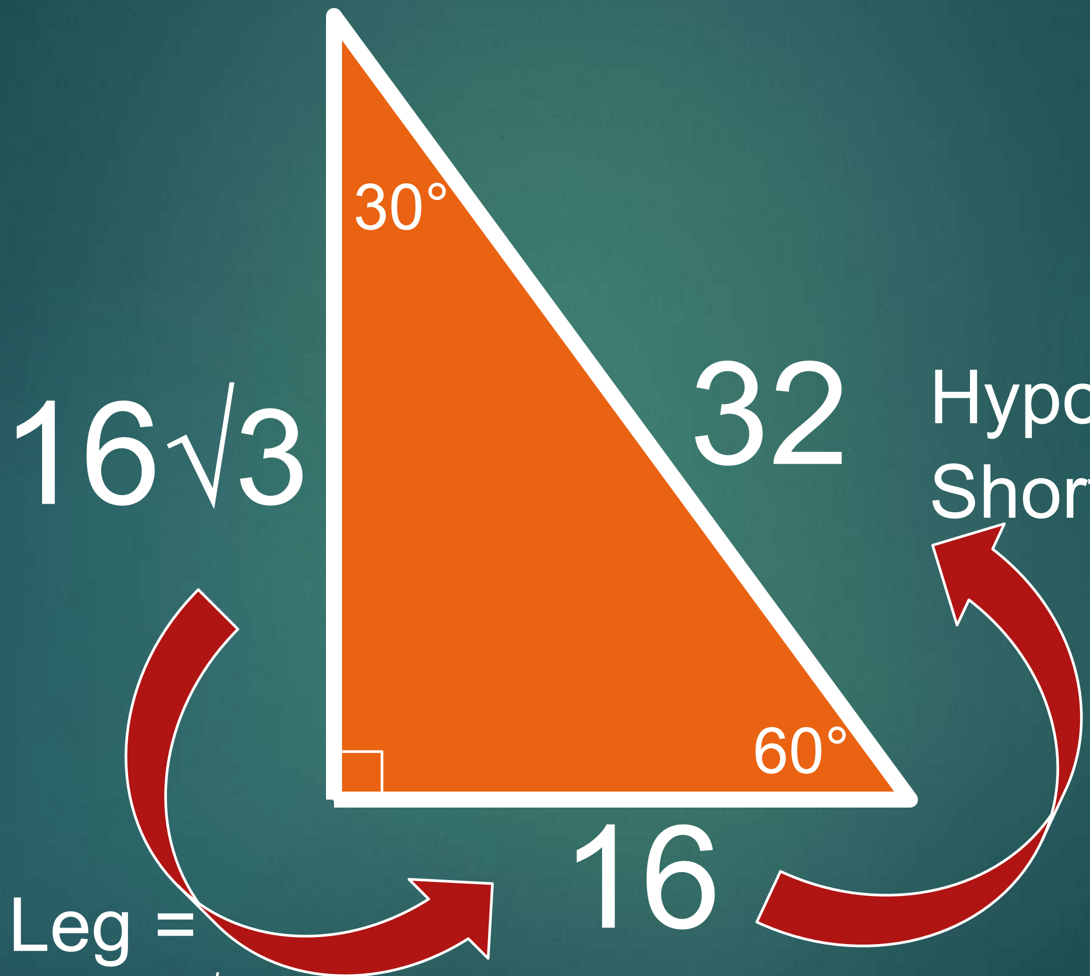
30°- 60°- 90° Practice



Hypotenuse =
Short Leg * 2

Short Leg =
Long leg ÷ √ 3

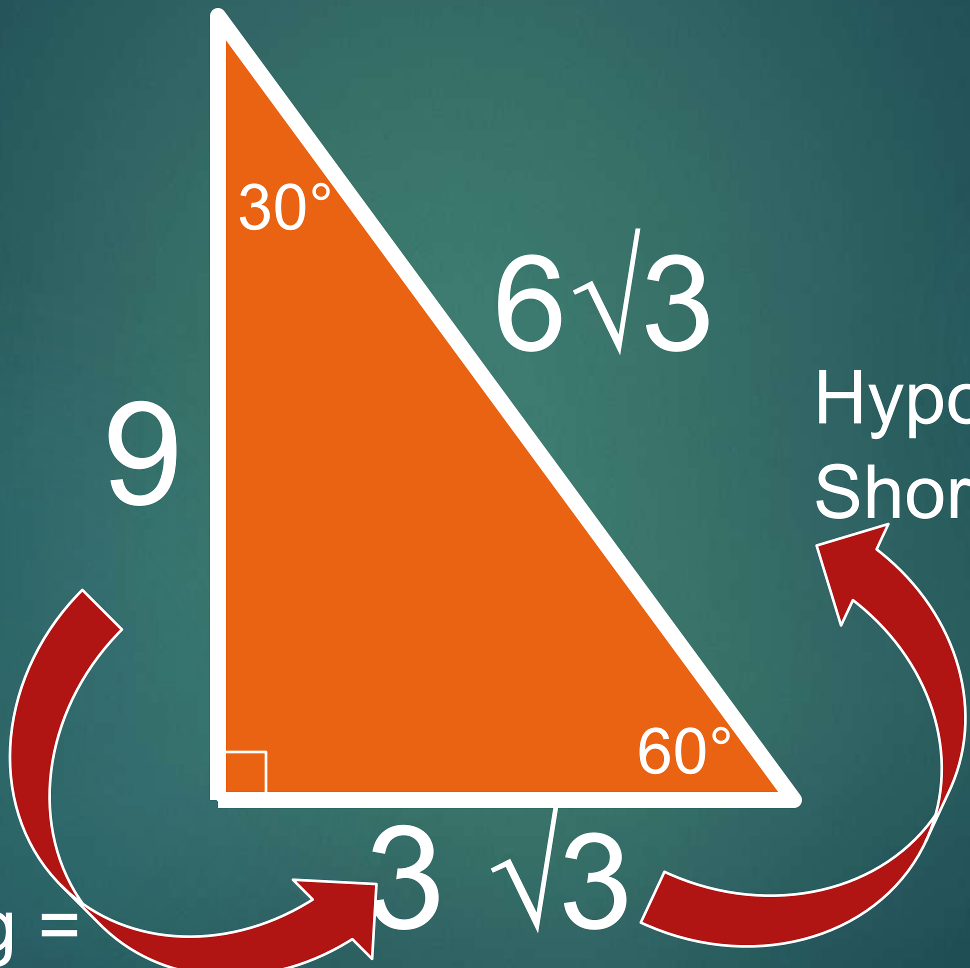
30°- 60°- 90° Practice



Hypotenuse =
Short Leg * 2

Short Leg =
Long leg ÷ √ 3

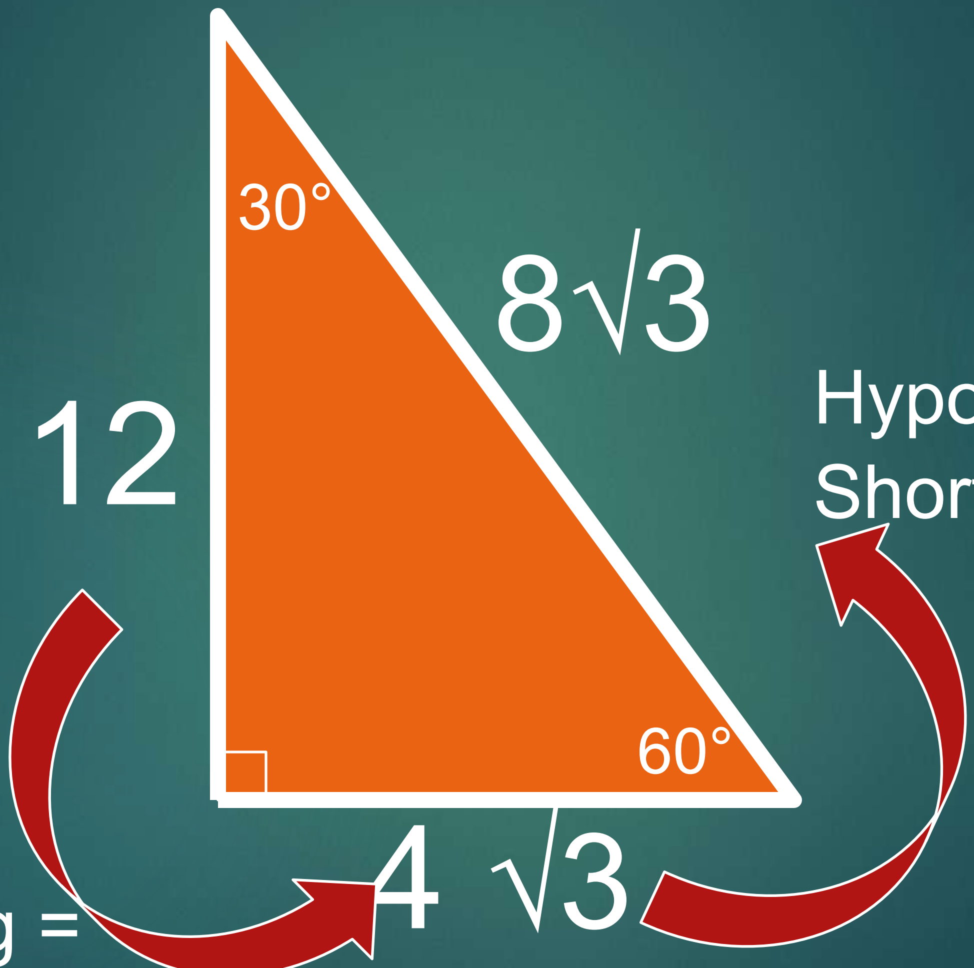
30°- 60°- 90° Practice



Hypotenuse =
Short Leg * 2

Short Leg =
Long leg ÷ √ 3

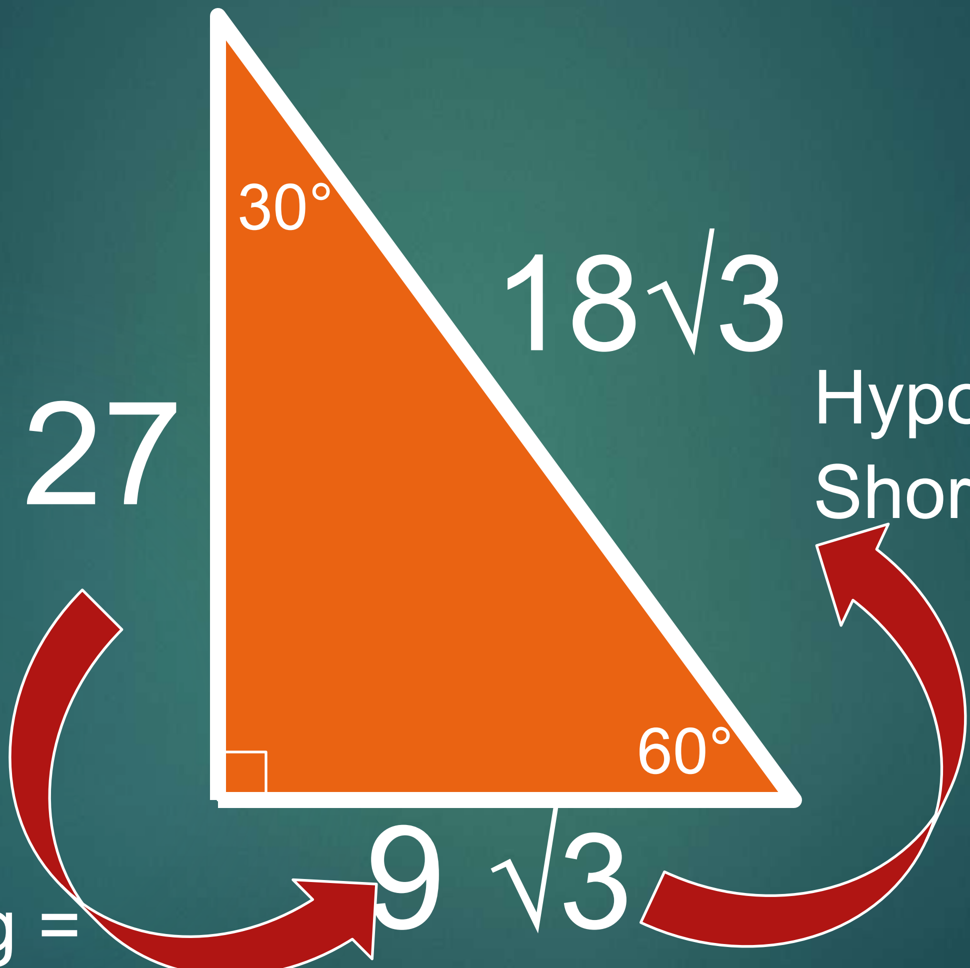
30°- 60°- 90° Practice



Hypotenuse =
Short Leg * 2

Short Leg =
Long leg ÷ √ 3

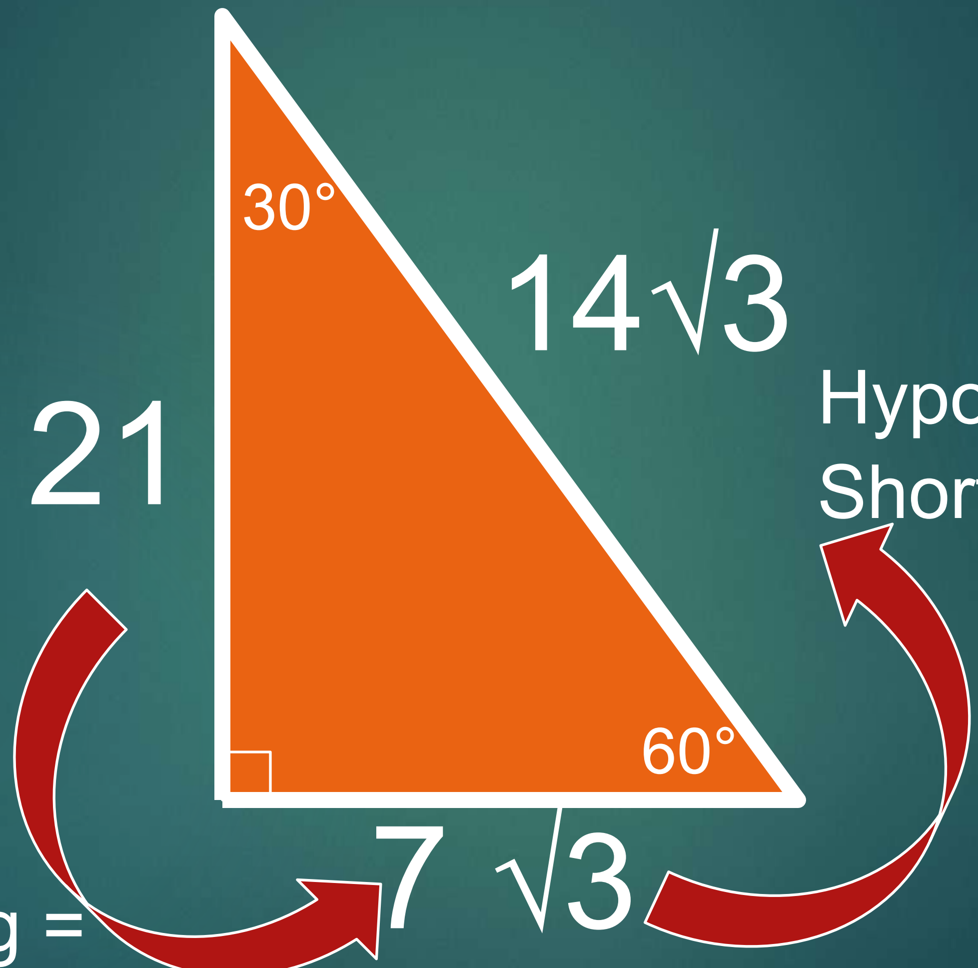
30°- 60°- 90° Practice



Hypotenuse =
Short Leg * 2

Short Leg =
Long leg ÷ √ 3

30°- 60°- 90° Practice



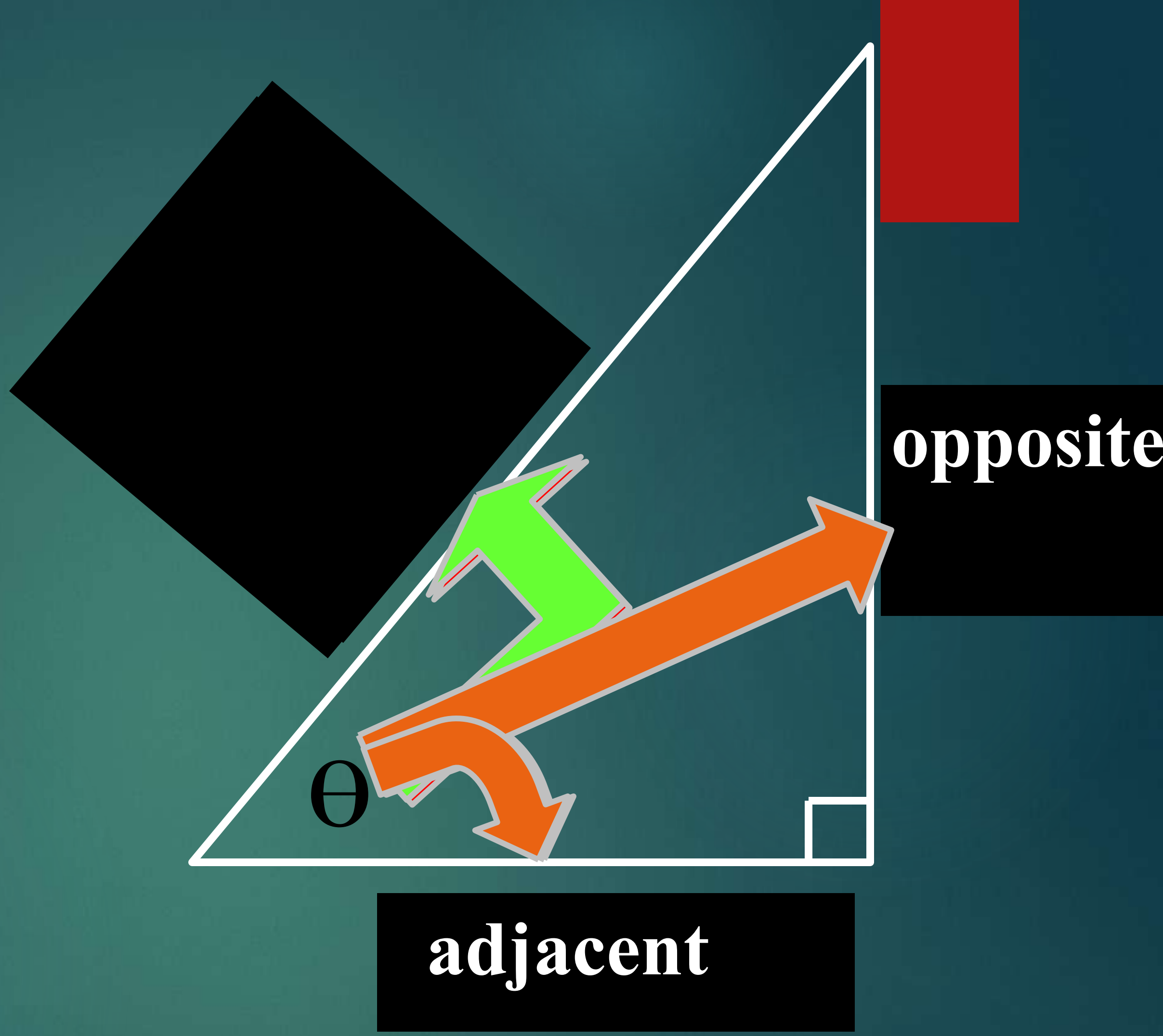
Hypotenuse =
Short Leg * 2

Short Leg =
Long leg ÷ √ 3

$$Sin = \frac{Opp}{Hyp}$$

$$Cos = \frac{Adj}{Hyp}$$

$$Tan = \frac{Opp}{Adj}$$

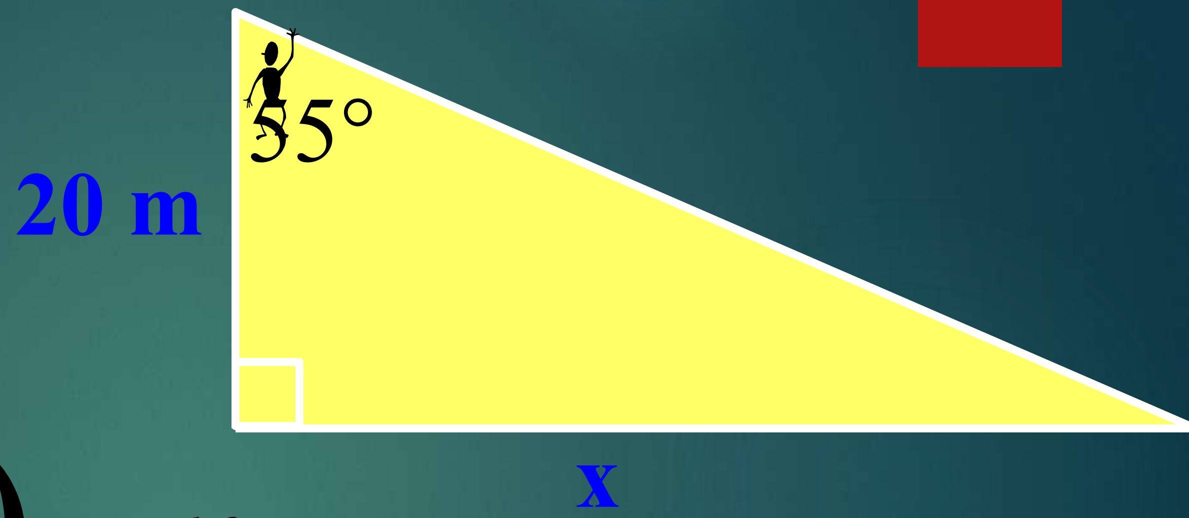


Finding a side.

(Figuring out which ratio to use and getting to use a trig button.)

Ex: 1 Figure out which ratio to use. Find x. Round to the nearest tenth.

$$\tan(55) = \frac{x}{20}$$



$$20\tan(55) = x$$

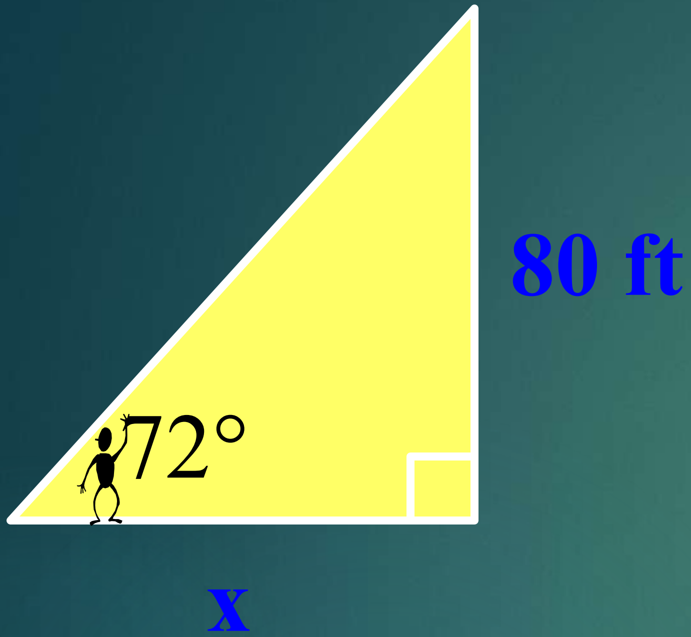
20 | tan | 55 |)

$$x \approx 28.6\text{ m}$$

Shrink yourself down and stand where the angle is.

Now, figure out which trig ratio you have and set up the problem.

Ex: 2 Find the missing side. Round to the nearest tenth.



$$\tan(72) = \frac{80}{x}$$

$$x\tan(72) = 80$$

$$x = \frac{80}{(\tan(72))}$$

80	÷	(	tan	72	)	)	=

Shrink yourself down and stand where the angle is.

$$x \approx 26 \text{ ft}$$

Now, figure out which trig ratio you have and set up the problem.

Ex: 3 Find the missing side. Round to the nearest tenth.

$$\sin(24) = \frac{x}{283}$$

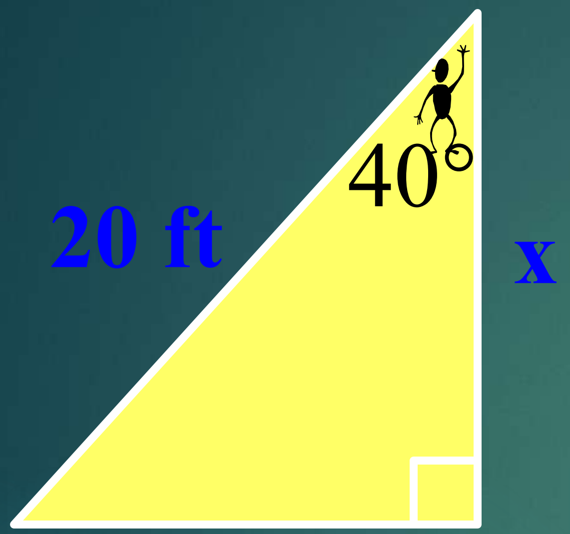
Shrink yourself down and stand where the angle is.

$$283\sin(24) = x$$

Now, figure out which trig ratio you have and set up the problem.

$$x \approx 115.1\,\text{m}$$

Ex: 4 Find the missing side. Round to the nearest tenth.



$$\cos(40) = \frac{x}{20}$$

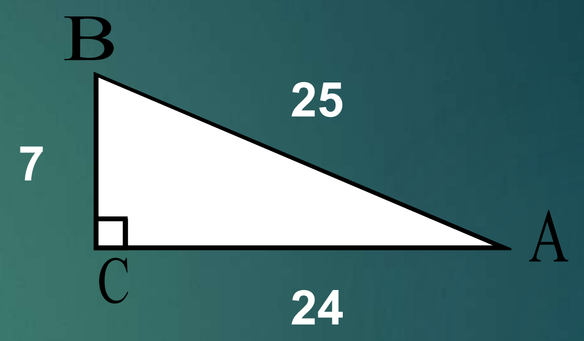
$$20\cos(40) = x$$

$$x \approx 15.3 \text{ ft}$$

Problem-Solving Strategies

You are given all 3 sides of the triangle.

Find the two non-right angles.



1. Use 2 different trig ratios to get each of the angles.

$$CosA = \frac{24}{25} \qquad TanB = \frac{24}{7}$$

$$A = Cos^{-1}\left(\frac{24}{25}\right) \qquad B = Tan^{-1}\left(\frac{24}{7}\right)$$

$$A = 16.3° \qquad B = 73.7°$$

Finding the Sides of a Triangle

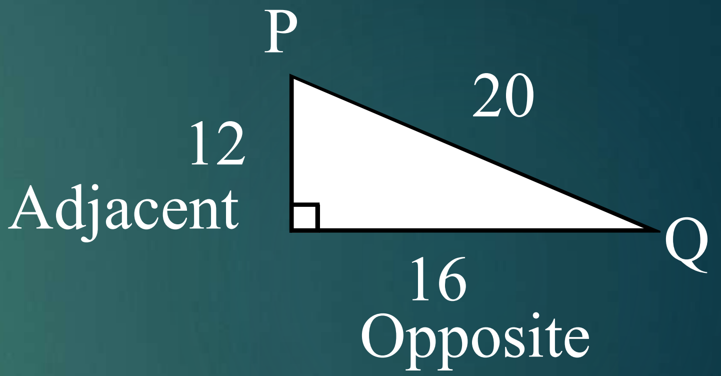
Remember:
SOHCAHTOA

$$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$$

Review: Trig Ratios

First we will find the Sine, Cosine and Tangent ratios for Angle P.

Next we will find the Sine, Cosine, and Tangent ratios for Angle Q



$\text{Sin P} = \frac{16}{20}$

$\text{Cos P} = \frac{12}{20}$

$\text{Tan P} = \frac{16}{12}$

$\text{Sin Q} = \frac{12}{20}$

$\text{Cos Q} = \frac{16}{20}$

$\text{Tan Q} = \frac{12}{16}$

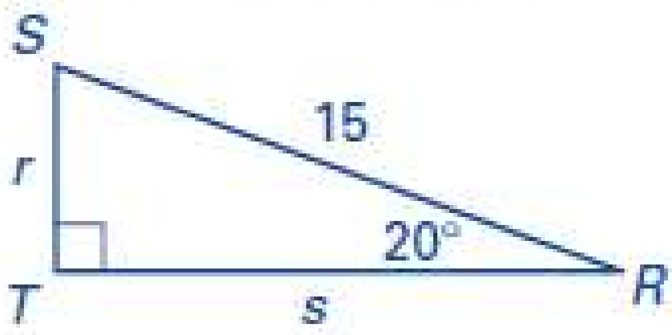
Remember SohCahToa

Solving Right Triangles

Every right triangle has one right angle, two acute angles, one hypotenuse, and two legs.

To ***Solve a Right Triangle*** **means to determine the measures of <u>all six</u> parts.**

Solve the right triangle. Round decimals to the nearest tenth.



$$\angle S = 90° - 20°$$

$$\angle S = 70°$$

$$\sin 20° = \frac{r}{15}$$

$$15(\sin 20°) = r$$

$$r \approx 5.1303$$

$$r \approx 5.1$$

$$\cos 20° = \frac{s}{15}$$

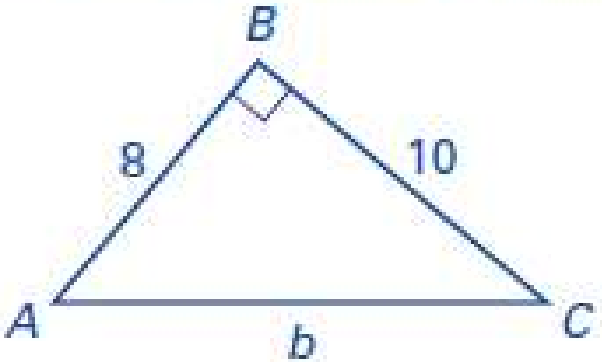
$$15(\cos 20°) = s$$

$$s \approx 14.0954$$

$$s \approx 14.1$$

But what if you don't know either of the acute angles? To solve those triangle we must use *Inverse Trig Functions*

Solve the right triangle. Round decimals to the nearest tenth.



$$\tan < A = \frac{10}{8} = 1.25$$

$$< A = \tan^{-1}(1.25)$$

$$< A = 51.3°$$

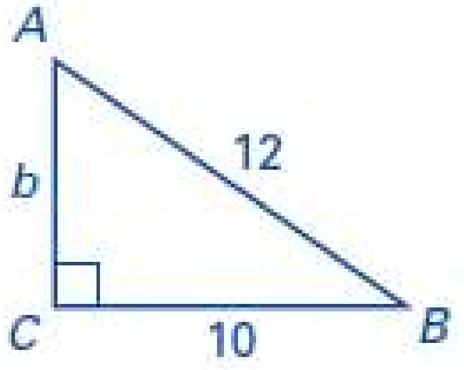
$$< B = 90° - 51.3° = 38.7°$$

$$b^2 = 8^2 + 10^2$$

$$b = \sqrt{64 + 100}$$

$$b = \sqrt{164} \approx 12.8$$

Solve the right triangle. Round decimals to the nearest tenth.



$$12^2 = 10^2 + b^2$$

$$b = \sqrt{144 - 100} = \sqrt{44}$$

$$b \approx 6.6332 \approx 6.6$$

$$\cos B = \frac{10}{12} = \frac{5}{6}$$

$$\cos B = 0.8$$

$$B = \cos^{-1}(0.8)$$

$$B \approx 36.869^\circ \approx 36.9^\circ$$

$$m < A = 90^\circ - 36.9^\circ$$

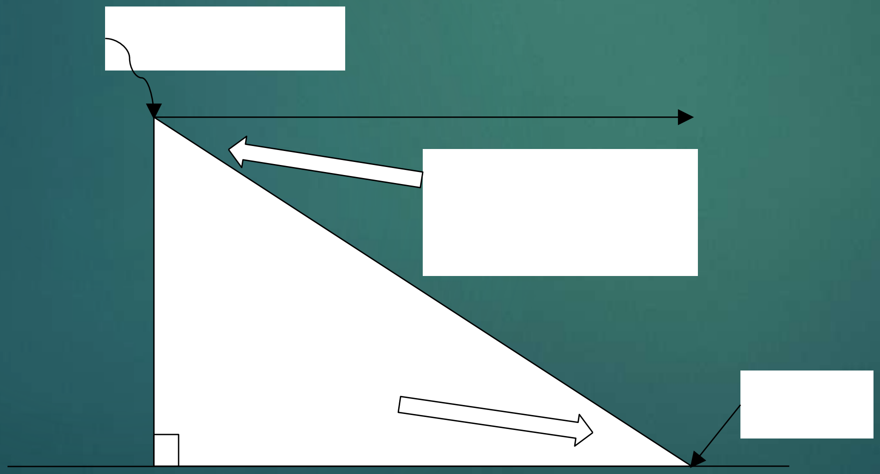
$$m < A \approx 53.1^\circ$$

Angle of Elevation/Depression

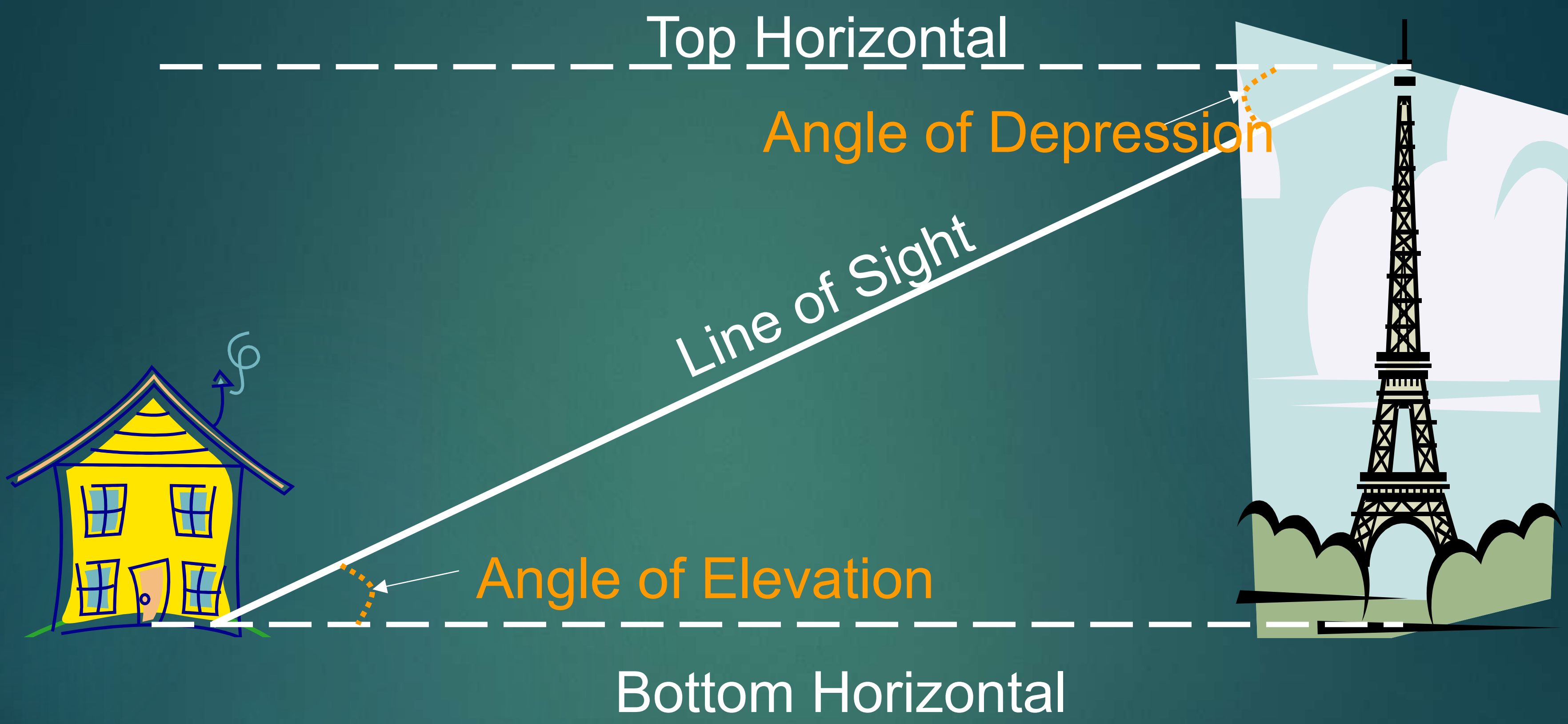
Sometimes when we use right triangles to model real-life situations, we use the terms angle of elevation and angle of depression.

If you are standing on the ground and looking up at a hot air balloon, the angle that you look up from ground level is called the **angle of elevation**.

If someone is in the hot air balloon and looks down to the ground to see you, the angle that they have to lower their eyes, from looking straight ahead, is called the **angle of depression**.



Angles of Elevation and Depression

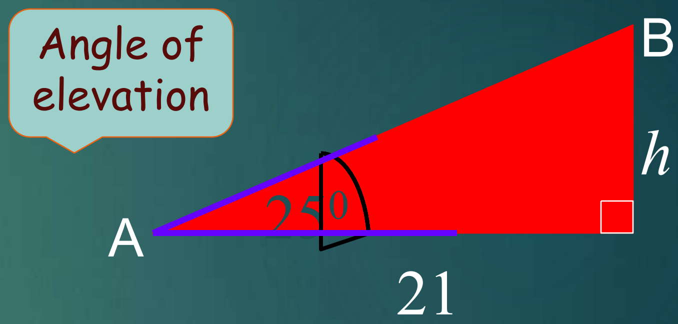


Since the two horizontal lines are parallel, by Alternate Interior Angles the angle of depression *must be equal* to the angle of elevation.

Example 1

The angle of elevation of building **A** to building **B** is **25^0**. The distance between the buildings is **21** meters. Calculate how much taller Building **B** is than building A

Step 1: Draw a right angled triangle with the given information.



Step 2: Take care with placement of the angle of elevation

$$\tan 25^\circ = \frac{h}{21}$$

Step 3: Set up the trig equation.

$$h = 21 \times \tan 25^\circ$$

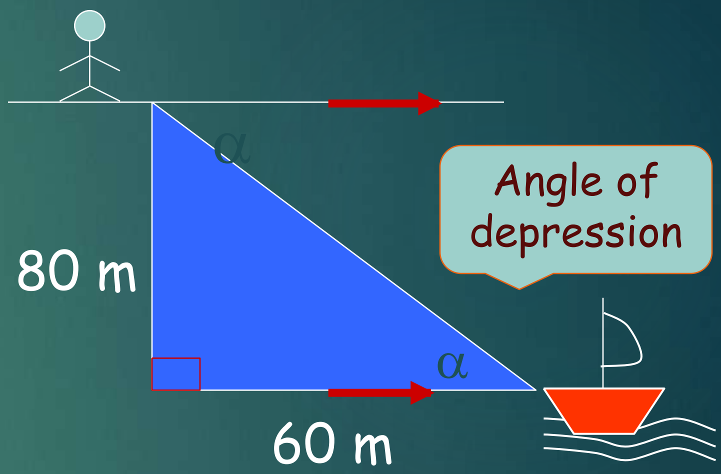
$$h = 9.8\,m$$

Step 4: Solve the trig equation.

Example 2

A boat is **60** meters out to sea. Madge is standing on a cliff **80** meters high. What is the angle of depression from the top of the cliff to the boat?

Step 1: Draw a right angled triangle with the given information.



Step 2: Alternate interior angles place α inside the triangle.

Step 3: Decide which trig ratio to use.

$$\tan\alpha = \frac{80}{60}$$

Step 4: Use calculator to find the value of the unknown.

$$\alpha = \tan^{-1}\left(\frac{80}{60}\right)$$

$$\alpha = 53.1^{\circ}$$

Example 3

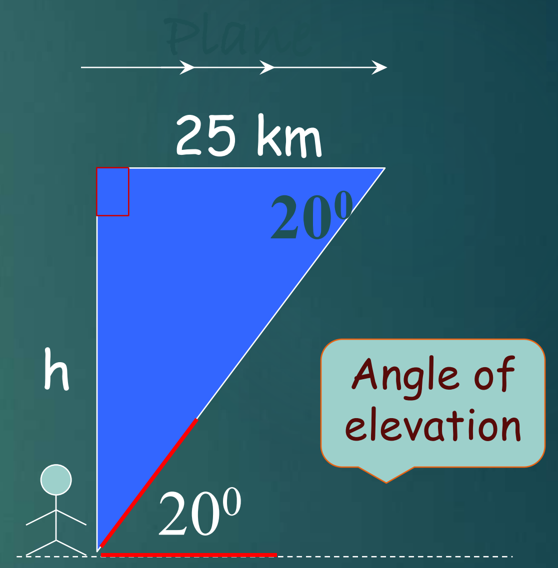
Marty is standing on level ground when he sees a plane directly overhead. The angle of elevation of the plane after it has travelled **25** km is **20^0**. Calculate the altitude of the plane at this time.

Step 1: Draw a right angled triangle with the given information.

Step 2: Alternate interior angles places 20^0 inside the triangle.

Step 3: Decide which trig ratio to use.

Step 4: Use calculator to find the value of the unknown.



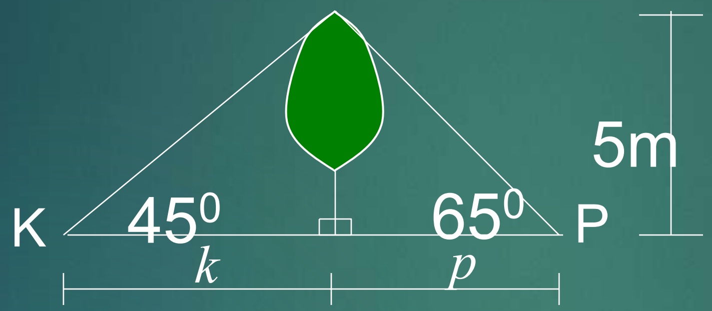
$$\tan 20^0 = \frac{h}{25}$$

$$h \approx 9.099$$

$$h \approx 9\ km \quad (nearest\ km)$$

Example 4

Kate and Petra are on opposite sides of a tree. The angle of elevation to the top of the tree from Kate is **45°** and from Petra is **65°**. If the tree is **5 m** tall, who is closer to the tree, Kate or Petra?



Kate

$$\tan 45° = \frac{5}{k}$$

$$k = \frac{5}{\tan 45°}$$

$$k = 5\,m$$

Petra

$$\tan 65° = \frac{5}{p}$$

$$p = \frac{5}{\tan 65°}$$

$$p = 2.3\,m$$

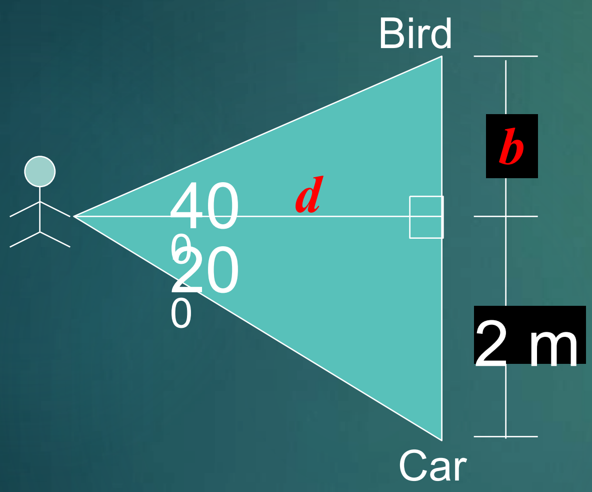
Answer

Therefore, Petra is closer to the tree, since the distance is shorter.

Example 5

Maryann is peering outside her window. From her window she sees her car and a bird hovering above her car. The angle of depression of Maryann's car is **20^0** whilst the angle of elevation to the bird is **40^0**. If Maryann's window is **2m** off the ground, what is the bird's altitude at that moment?

Step 1: Draw a diagram



Step 2: Set up the trig equations in two parts. Find d first, then b.

Step 3: Solve the equations and answer the question.

$$\tan 20^\circ = \frac{2}{d}$$

$$d = \frac{2}{\tan 20^\circ}$$

$$d \approx 5.5\ m$$

$$\tan 40^\circ \approx \frac{b}{5.5}$$

$$b \approx 5.5 \tan 40^\circ$$

$$b \approx 4.6\ m$$

Therefore,

The bird is 6.6 m (2 + 4.6) from the ground at that moment.

Your Turn 1

You sight a rock climber on a cliff at a **32°** angle of elevation. The horizontal ground distance to the cliff is **1000 ft**. Find the line of sight distance to the rock climber.

$$\text{Cos}\,\angle 32° = \frac{1000}{x}$$

$$x = \frac{1000}{\text{Cos}\,\angle 32°}$$

$$x \approx 1179 \text{ ft}$$

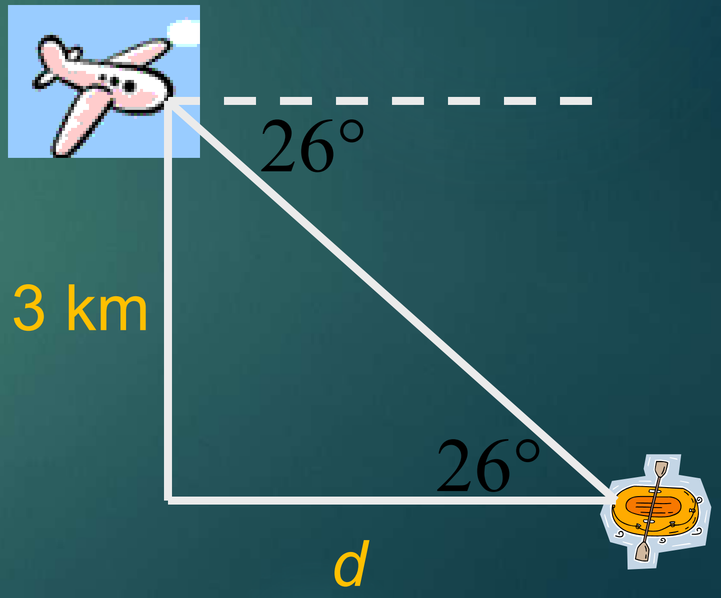
Your Turn 2:

An airplane pilots sights a life raft at a **26°** angle of depression. The airplane's altitude is **3 km**. What is the airplane's surface distance ***d*** from the raft?

$$\text{Tan } \angle 26° = \frac{3}{d}$$

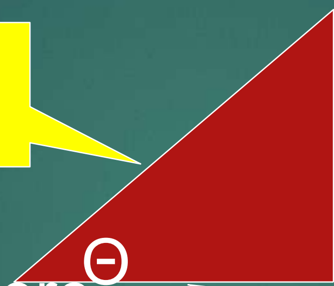
$$d = \frac{3}{\text{Tan } \angle 26°}$$

$$d \approx 6.2 \text{ km}$$



Angles of Depression or Elevation

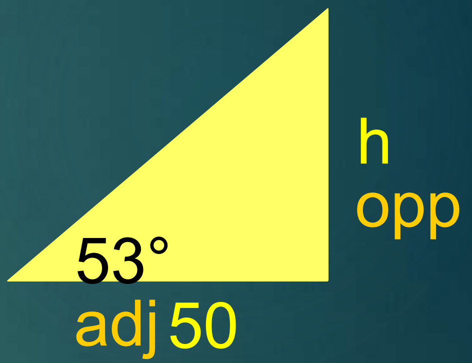
- Step 1: Draw ***this triangle*** to fit problem



- Step 2: Label sides from angle's view
- Step 3: Identify trig function to use
- Step 4: Set up equation
- Step 5: Solve for variable
  - Use inverse trig functions for an angle

Example 1

Before the Mast: **At a point on the ground 50 feet from the foot of the flagpole, the angle of elevation to the top of the pole is 53°. Find the height of the flagpole.**



Step 1: Draw a triangle to fit problem

Step 2: Label sides from angle's view

Step 3: Identify trig function to use

Step 4: Set up equation

Step 5: Solve for variable

$$\tan 53° = \frac{h}{50}$$

$$50 \tan 53° = h = 66.35 \text{ feet}$$

S → O / H
C → A / H
T → O / A

Example 2

Job Site **A 20-foot ladder leans against a wall so that the base of the ladder is 8 feet from the base of the building. What angle does the ladder make with the ground?**

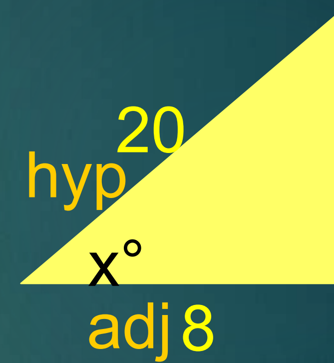
Step 1: Draw a triangle to fit problem

Step 2: Label sides from angle's view

Step 3: Identify trig function to use

Step 4: Set up equation

Step 5: Solve for variable



$$\cos x^\circ = \frac{8}{20}$$

$$\cos^{-1}(8/20) = x = 66.42^\circ$$

S → O / H
C → A / H
T → O / A

Example 3

CIRCUS ACTS At the circus, a person in the audience watches the high-wire routine. A 5-foot-6-inch tall acrobat is standing on a platform that is 25 feet off the ground. How far is the audience member from the base of the platform, if the angle of elevation from the audience member's line of sight to the top of the acrobat is 27°?

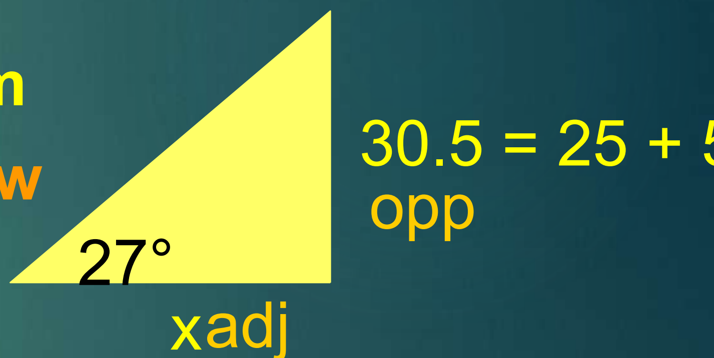
Step 1: Draw a triangle to fit problem

Step 2: Label sides from angle's view

Step 3: Identify trig function to use

Step 4: Set up equation

Step 5: Solve for variable



$$\tan 27° = \frac{30.5}{x}$$

$$x \tan 27° = 30.5$$

$$x = (30.5) / (\tan 27°)$$

$$x = 59.9$$

S → O / H

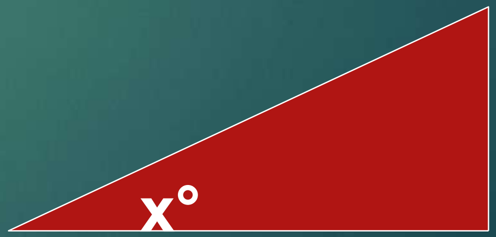
C → A / H

T → O / A

Example 4

DIVING **At a diving competition, a 6-foot-tall diver stands atop the 32-foot platform. The front edge of the platform projects 5 feet beyond the ends of the pool. The pool itself is 50 feet in length. A camera is set up at the opposite end of the pool even with the pool's edge. If the camera is angled so that its line of sight extends to the top of the diver's head, what is the camera's angle of elevation to the nearest degree?**

Answer: about 39.4°



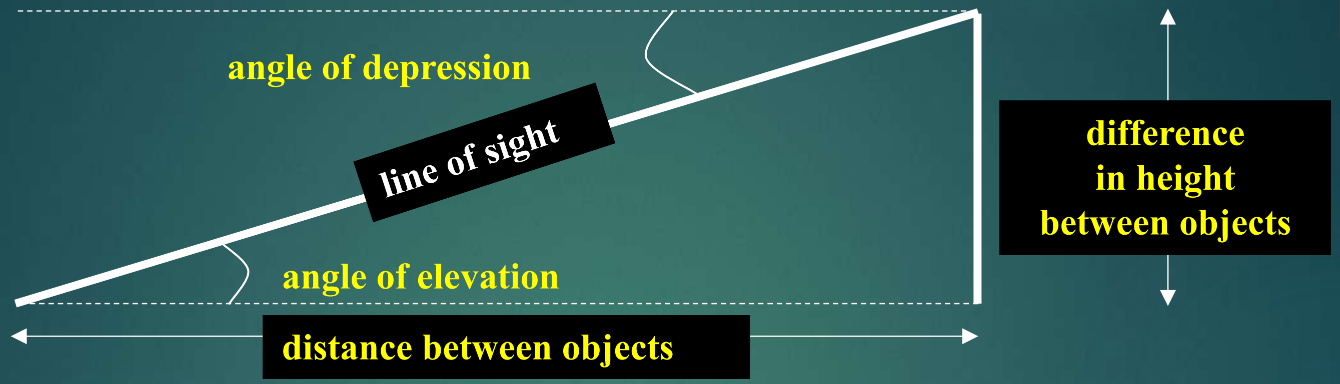
Example 5

SHORT-RESPONSE TEST ITEM

A roller coaster car is at one of its highest points. It drops at a 63° angle for 320 feet. How high was the roller coaster car to the nearest foot before it began its fall?

Answer: The roller coaster car was about 285 feet above the ground.

Angles of Elevation and Depression



1. The angle of elevation from point A to the top of the press box is 49°. If point A is 400 ft from the base of the press box, how high is the press box?

2. Find the angle of elevation of the sun when a 12.5 ft post casts a 18 ft shadow?

3. A ladder leaning up against a barn makes an angle of 78° with the ground when the ladder is 5 feet from the barn. How long is the ladder?

Angles of Elevation and Depression

1. The angle of elevation from point A to the top of the press box is 49°. If point A is 400 ft from the base of the press box, how high is the press box?

   Side; $\tan 49° = h/400$ $400 \tan 49° = h = 460.1$ ft

2. Find the angle of elevation of the sun when a 12.5 ft post casts a 18 ft shadow?

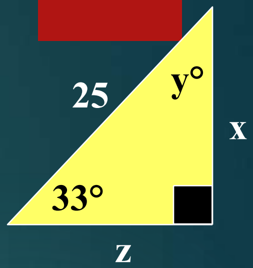
   Angle; $\tan x° = 12.5/18$ $x = \tan^{-1}(12.5/18) = 34.78°$

3. A ladder leaning up against a barn makes an angle of 78° with the ground when the ladder is 5 feet from the barn. How long is the ladder?

   Side; $\cos 78° = 5/L$ $L = 5/\cos 78° = 24.05$ ft

4. From the top of the 120 foot fire tower a ranger observes a fire

Quiz 2 Need-to-Know



- Sin (angle) = Opposite / Hypotenuse
- Cos (angle) = Adjacent / Hypotenuse
- Tan (angle) = Opposite / Adjacent

- To find an angle use inverse Trig Function
  - Trig Fnc^{-1} (some side / some other side) = angle

- To Solve Any Trig Word Problem
  - Step 1: Draw a triangle to fit problem
  - Step 2: Label sides from angle's view
  - Step 3: Identify trig function to use
  - Step 4: Set up equation
  - Step 5: Solve for variable

Angle of Elevat
or of Depressio

angle goes here

Summary & Homework

- **Summary:**
  - Trigonometry can be used to solve problems related to angles of elevation and depression
  - Angle always goes in lower left corner

- **Homework:** pg xxx, 5, 6, 8, 9, 17-19

THE END