A – Polynomial Addition, Subtraction & Multiplication

These first three operations of polynomials combine what you already know about combining like terms and distributing (FOIL).

To add polynomials, you combine like terms. Putting like terms together may help.

$$(5x^2 - 3xy - 12y^2) + (-9x^2 - 5xy + 3y^2) = \begin{array}{r} 5x^2 - 3xy - 12y^2 \\ \underline{-9x^2 - 5xy + 3y^2} \\ -4x^2 - 8xy - 9y^2 \end{array}$$

To subtract polynomials, distribute the minus sign and then treat the problem as an addition problem, combining like terms.

$$(5x^2 - 3xy - 12y^2) - (-9x^2 - 5xy + 3y^2) = \begin{array}{r} 5x^2 - 3xy - 12y^2 \\ -\underline{(-9x^2 - 5xy + 3y^2} \end{array} \begin{array}{c} \rightarrow \\ \rightarrow \\ \end{array} \begin{array}{r} 5x^2 - 3xy - 12y^2 \\ \underline{9x^2 + 5xy - 3y^2} \\ 14x^2 + 2xy - 15y^2 \end{array}$$

To multiply polynomials, use FOIL or the distributive property to distribute each term in one factor to each term in the other(s). Combine like terms.

$$(2x - 3)(x^2 - 5x + 4) = \begin{array}{r} 2x^3 - 10x^2 + 8x \\ \underline{- 3x^2 + 15x - 12} \\ 2x^3 - 13x^2 + 23x - 12 \end{array}$$

Note: $(7a + 5b)^2$ is NOT $49a^2 + 25b^2$. It is $(7a + 5b)(7a + 5b) = \begin{array}{r} 49a^2 + 35ab \\ \underline{35ab + 25b^2} \\ 49a^2 + 70ab + 25b^2 \end{array}$

Try these:

1. $(5x^3 - x^2 + 8x + 2) + (3x^3 - 9x^2 - 4x + 12)$
2. $(5x^3 - x^2 + 3x + 8x + 2) - (3x^3 - 9x^2 - 4x + 12)$
3. $(5x - 4)(2x^3 - 3x^2 + x + 2)$
4. $(4x - 3)^2$

Also work with your partner to correct mistakes on QUIZ 5.

B – Long Division of Polynomials

To divide polynomials, you do many of the same steps as in long division of numbers. You focus on just the leading terms of the divisor and dividend and let the other terms take care of themselves. As with number division, after you multiply, you subtract and bring down the next part. (Remember with subtracting polynomials, you must distribute the minus to each term.)

$$
\begin{array}{r|l}
 & 4x^3 + 5x^2 + 3x + 2 \\
\hline
x-2 & 4x^4 - 3x^3 - 7x^2 - 4x - 9 \\
 & \underline{4x^4 - 8x^3} \\
 & \qquad 5x^3 - 7x^2 \\
 & \qquad \underline{5x^3 - 10x^2} \\
 & \qquad\qquad 3x^2 - 4x \\
 & \qquad\qquad \underline{3x^2 - 6x} \\
 & \qquad\qquad\qquad 2x - 9 \\
 & \qquad\qquad\qquad \underline{2x - 4} \\
 & \qquad\qquad\qquad\quad -5
\end{array}
$$

The quotient is written $4x^3 + 5x^2 + 3x + 2 - \frac{5}{x-2}$.

Use long division to complete these:

1. $(x^4 - 60x^2 - 256) \div (x + 8)$ ….put in zeros for missing terms!
2. $(15x^4 - 8x^3 - 22x^2 + 11x - 4) \div (3x - 4)$
3. $(3x^4 + 2x^3 + x^2 - 2x - 4) \div (x^2 - 1)$

Also work with your partner to correct the long division problems on QUIZ 5 and QUIZ 6.

C – Synthetic Division of Polynomials

Synthetic division assumes the “place value” of each term of a polynomial and drops the variables so we just focus on the operations with the coefficients.

$$
\begin{array}{r|l}
 & 4x^3 + 5x^2 + 3x + 2 \\ \hline
x-2 & 4x^4 - 3x^3 - 7x^2 - 4x - 9 \\
 & \underline{4x^4 - 8x^3} \\
 & \quad 5x^3 - 7x^2 \\
 & \quad \underline{5x^3 - 10x^2} \\
 & \qquad 3x^2 - 4x \\
 & \qquad \underline{3x^2 - 6x} \\
 & \qquad\quad 2x - 9 \\
 & \qquad\quad \underline{2x - 4} \\
 & \qquad\qquad -5
\end{array}
$$

becomes

$$
\begin{array}{r|rrrrr}
 & 4 & 5 & 3 & 2 & \\ \hline
-2 & 4 & -3 & -7 & -4 & -9 \\
 & & \underline{-8} & & & \\
 & & 5 & & & \\
 & & & \underline{-10} & & \\
 & & & 3 & & \\
 & & & & \underline{-6} & \\
 & & & & 2 & \\
 & & & & & \underline{-4} \\
 & & & & & -5
\end{array}
$$

You can “collapse” the problem and write all the arithmetic in one row like this:

$$
\begin{array}{r|rrrrr}
-2 & 4 & -3 & -7 & -4 & -9 \\
 & & -8 & -10 & -6 & -4 \\ \hline
 & 4 & 5 & 3 & 2 & -5
\end{array}
$$

$(4x^3 - 38x + 6) \div (x - 3)$.

If we put 4 -28 6 as the dividend, it will look like the dividend is $4x^2 - 38x + 6$, so we must think of the problem as $(4x^3 + 0x^2 - 38x + 6) \div (x - 3)$. $(x - 3)$ is the divisor, but we write it as 3, because we want the "zero" to be on the outside, not the factor. When we solve $x - 3 = 0$ for x, $x = 3$.

$$
\begin{array}{l|rrrr}
\underline{3}\,] & 4 & 0 & -38 & 6 \\
 & \downarrow & 12 & 36 & -6 \\
\hline
 & 4 & 12 & -2 & 0
\end{array}
\quad \rightarrow \quad 4x^2 + 12x - 2
$$

Try these, using synthetic division:

1. $(3x^3 + 17x^2 + 6x - 20) \div (x + 5)$
2. $(x^4 - 2x^3 + 3x^2 - 14x + 20) \div (x - 2)$
3. $(x^4 - 60x^2 - 256) \div (x + 8)$

Work with your partner to correct problems on QUIZ 6.

D - POLYNOMIAL IDENTITIES

Perfect Square Trinomial

(A perfect square plus another perfect square PLUS a middle term that equals two times the product of the two square roots)

$a^2 + 2ab + b^2 = (a + b)(a + b)$ or $(a + b)^2$

OR

$a^2 - 2ab + b^2 = (a - b)(a - b)$ or $(a - b)^2$

Take the square root of first and third terms. These will be the terms of your answer; keep the same sign as the first sign. Check to make sure that the middle term is two times the product of the two terms in your answer. Notice if you FOIL the answer, the middle terms are the same, so there are two of them or two times the product of the two terms.

$49a^2 + 70ab + 25b^2 = (7a + 5b)(7a + 5b)$ or $(7a + 5b)^2$

The square root of $(49a^2)$ is $7a$ and the square root of $(25b^2)$ is $5b$ and the $70ab$ is two times the product of $7a$ and $5b$. $2(7a)(5b) = 70ab$.

Notice: $(7a + 5b)(7a + 5b) = 49a^2 + 35ab$

$$\begin{array}{r} 49a^2 + 35ab \\ \underline{ 35ab + 25b^2} \\ 49a^2 + 70ab + 25b^2 \end{array}$$

Difference of Two Squares

(A perfect square minus another perfect square)

$a^2 - b^2 = (a - b)(a + b)$

Take the square root of both terms. One factor is addition; the other is subtraction. Notice if you FOIL the answer, the middle terms cancel out.

$a^2 + ab - ab - b^2$ and you're left with the difference of the squares.

$4a^2 - 9b^2 = (2a - 3b)(2a + 3b)$

The square root of $(4a^2)$ is $2a$ and the square root of $(9b^2)$ is $3b$.

Notice $(2a - 3b)(2a + 3b) = 4a^2 + 6ab$

$$\underline{ - 6ab - 9b^2}$$

$4a^2 \qquad - 9b^2$

Sum of Two Squares

(A perfect square plus another perfect square)

$$a^2 + b^2 = (a - bi)(a + bi)$$

Take the square root of both terms. One factor is addition; other is subtraction. Notice if you FOIL the answer, the middle terms cancel out, but the back term is negative. To make it positive, we make the back terms imaginary.

$a^2 + ab - ab - b^2i^2$. Since $-b^2i^2 = +b^2$, you're left with the sum of squares.

$49a^2 + 25b^2 = (7a - 5bi)(7a + 5bi)$

The square root of $(49a^2)$ is 7a and the square root of $(25b^2)$ is 5b.

Notice: $(7a - 5ib)(7a + 5ib) = 49a^2 + 35iab$

$$\begin{array}{lr} & - 35iab - 25i^2b^2 \\ \hline 49a^2 & + 25b^2 \end{array}$$ but, $- 25i^2 = +25$

Sum/Difference of Two Cubes

(A perfect cube plus or minus another perfect cube

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2) \text{ OR } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Take the cube root of both terms. These will be the terms in the first set of parenthesis, with the same sign as the problem. In the second set of parenthesis, square the first term, change the sign, multiply the two terms together, +, square the second term.

$8a^3 + 27 = (2a + 3)(4a^2 - 6a + 9)$.

Because the cube root of $8a^3 = 2a$ and the cube root of 27 is 3.

Now square 2a for the first term, $4a^2$; change the sign (-); multiply 2a times 3 for the second term, 6a; put a plus sign; square 3 for the third term, 9.

Notice: $(2a + 3)(4a^2 - 6a + 9) = 8a^3 - 12a^2 + 18a$

$$\begin{array}{lr} & 12a^2 - 18a + 27 \\ \hline 8a^3 & + 27 \end{array}$$

Use identities to rewrite the following. Start with a GCF if applicable.

4. $27x^3 - 125$
5. $4x^2 + 100x + 625$
6. $16x^2 - 25$
7. $49x^2 + 81y^2$
8. $25x^2 + 49y^2$
9. $81x^2 - 144x + 64$
10. $8a^3 - 1000$
11. $25x^2 + 49y^2$
12. $b^2 - 121c^2$
13. $5x^2 + 320y^2$
14. $-9x^2 - 30x - 25$
15. $4a^3 - 4$

Work with your partner to make corrections to identities on QUIZ 7.

E – PASCAL'S TRIANGLE

By multiplying binomials by powers earlier, we discovered the following:

$(x + y)^2 = 1x^2 + 2xy + 1y^2$ 1 2 1

$(x + y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$ 1 3 3 1

$(x + y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$ 1 4 6 4 1

Notice that the powers of the first term of the binomial goes down by one as you move to the right in the answer, while the powers of the back term of the binomial go down as you move to the left in your answer. The coefficients of each term seem to follow Pascal's Triangle.

$(3x – 2y)^4$. The terms will have coefficients 1 4 6 4 1.
The (3x) term goes from $(3x)^4$ to $(3x)^0$ as the (-2y) term goes from $(-2y)^0$ to $(-2y)^{4.}$

$(3x + 2y)^4 = \underline{1}\ (3x)^4(-2y)^0 + \underline{4}\ (3x)^3(-2y)^1 + \underline{6}\ (3x)^2(-2y)^2 + \underline{4}\ (3x)^1(-2y)^3 + \underline{1}\ (3x)^0(-2y)^4$

$= 1\ (81x^4)(1) + 4(27x^3)(-2y) + 6(9x^2)(4y^2) + 4(3x)(-8y^3) + 1\ (1)(16y^{4)}$

$= 81x^4 - 216x^3y + 216x^2y^2 - 96xy^3 + 16y^4$

Expand each of the following using Pascal's Triangle.

1. $(x – 5)^5$

2. $(x – 3y)^5$

3. $(10x – y)^4$

4. $(3x – 5y)^4$

Work with your partner to make corrections to the last two problems on QUIZ 7.

F – ENRICHMENT

1. Divide using long division. (Recall that synthetic division only works with linear divisors.)

   $(2x^4 - 3x^3 - 11x^2 + 3x + 9) \div (x^2 - 4x + 3)$.

   Factor the divisor and your quotient to find all four factors of the dividend.

   All four factors are linear, so use synthetic division to show that each factor gives a zero remainder. (You can set the factors equal to 0 to find the number to put on the outside for synthetic division if you need to.)

2. Completely factor $16x^4 - 256y^4$. Keep factoring until you cannot factor the factors any more.

3. Completely factor $x^6 - y^6$, as indicated.

   Factor it as the difference of cubes: $(x^2)^3 - (y^2)^3$ and factor the factors again.

   Then, factor it as the difference of squares: $(x^3)^2 - (y^3)^2$ and factor the factors again.

   Verify that the answers are equivalent; they won't look the same!

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