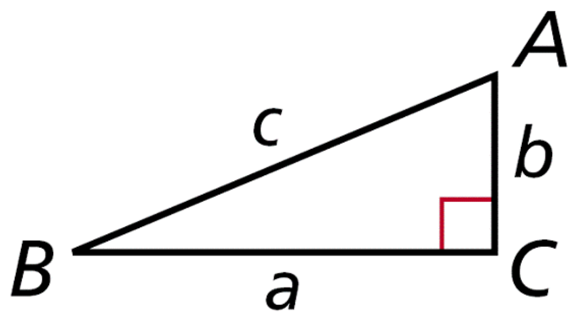
Solving right triangles

Warm Up

Use ΔABC for Exercises 1–3.



1. If $a = 8$ and $b = 5$, find c. $\sqrt{89}$

2. If $a = 60$ and $c = 61$, find b. 11

3. If $b = 6$ and $c = 10$, find sin B. 0.6

Find AB.

4. $A(8, 10)$, $B(3, 0)$ $5\sqrt{5}$

5. $A(1, -2)$, $B(2, 6)$ $\sqrt{65}$

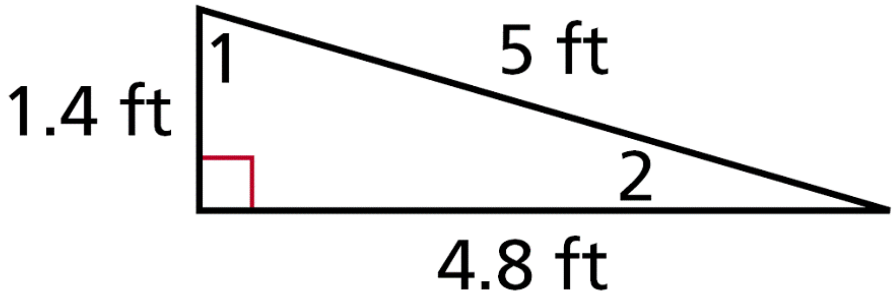
Objective

Use trigonometric ratios to find angle measures in right triangles and to solve real-world problems.

San Francisco, California, is famous for its steep streets. The steepness of a road is often expressed as a *percent grade*. Filbert Street, the steepest street in San Francisco, has a 31.5% grade. This means the road rises 31.5 ft over a horizontal distance of 100 ft, which is equivalent to a 17.5° angle. You can use trigonometric ratios to change a percent grade to an angle measure.

Example 1: Identifying Angles from Trigonometric Ratios

Use the trigonometric ratio $\cos A = \frac{24}{25}$ to determine which angle of the triangle is $\angle A$.



$\cos A = \frac{\text{adj. leg}}{\text{hyp.}}$ *Cosine is the ratio of the adjacent leg to the hypotenuse.*

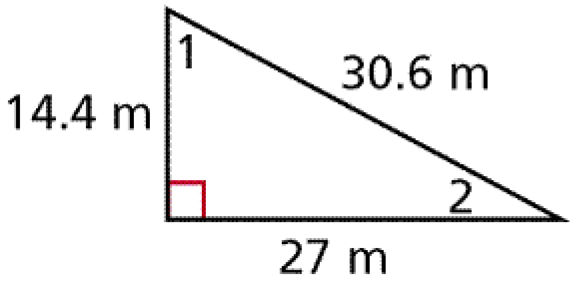
$\cos\angle 1 = \frac{1.4}{5} = \frac{7}{25}$ *The leg adjacent to ∠1 is 1.4. The hypotenuse is 5.*

$\cos\angle 2 = \frac{4.8}{5} = \frac{24}{25}$ *The leg adjacent to ∠2 is 4.8. The hypotenuse is 5.*

Since cos A = cos∠2, ∠2 is ∠A.

Check It Out! Example 1a

Use the given trigonometric ratio to determine which angle of the triangle is $\angle A$.



$$\sin A = \frac{8}{17}$$

$$\sin A = \frac{\text{opp. leg}}{\text{hyp.}}$$

Sine is the ratio of the opposite leg to the hypotenuse.

$$\sin\angle 1 = \frac{27}{30.6} = 0.88$$

The leg adjacent to ∠1 is 27. The hypotenuse is 30.6.

$$\sin\angle 2 = \frac{14.4}{30.6} = 0.47$$

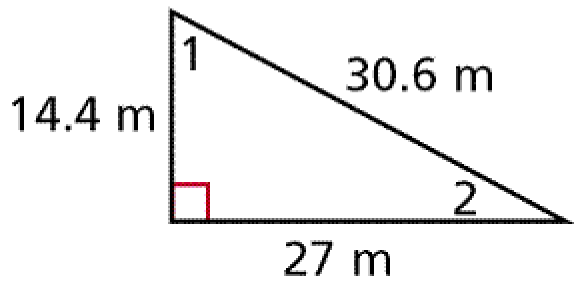
The leg adjacent to ∠2 is 14.4. The hypotenuse is 30.6.

Since sin∠A = sin∠2, ∠2 is ∠A.

Check It Out! Example 1b

Use the given trigonometric ratio to determine which angle of the triangle is $\angle A$.

tan A = 1.875



$$\tan A = \frac{\text{opp. leg}}{\text{adj. leg}}$$

Tangent is the ratio of the opposite leg to the adjacent leg.

$$\tan\angle 1 = \frac{27}{14.4} = 1.875$$

The leg opposite to $\angle 1$ is 27. The leg adjacent is 14.4.

$$\tan\angle 2 = \frac{14.4}{27} = 0.53$$

The leg opposite to $\angle 2$ is 14.4. The leg adjacent is 27.

Since $\tan\angle A = \tan\angle 1$, $\angle 1$ is $\angle A$.

In Lesson 8-2, you learned that $\sin 30° = 0.5$. Conversely, if you know that the sine of an acute angle is 0.5, you can conclude that the angle measures 30°. This is written as $\sin^{-1}(0.5) = 30°$.

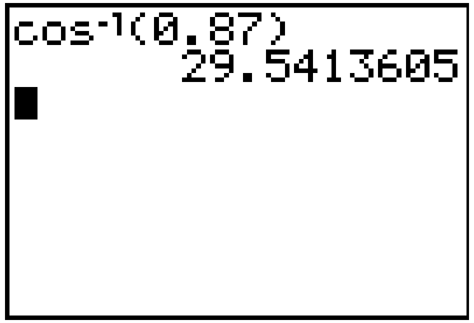
If you know the sine, cosine, or tangent of an acute angle measure, you can use the inverse trigonometric functions to find the measure of the angle.

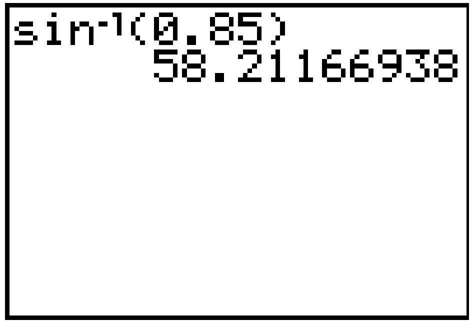
Inverse Trigonometric Functions
If $\sin A = x$, then $\sin^{-1} x = m\angle A$.
If $\cos A = x$, then $\cos^{-1} x = m\angle A$.
If $\tan A = x$, then $\tan^{-1} x = m\angle A$.

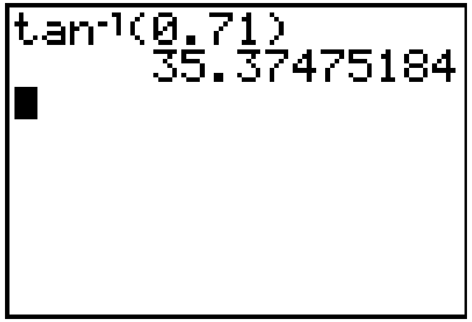
Example 2: Calculating Angle Measures from Trigonometric Ratios

Use your calculator to find each angle measure to the nearest degree.

A. $\cos^{-1}(0.87)$ **B.** $\sin^{-1}(0.85)$ **C.** $\tan^{-1}(0.71)$





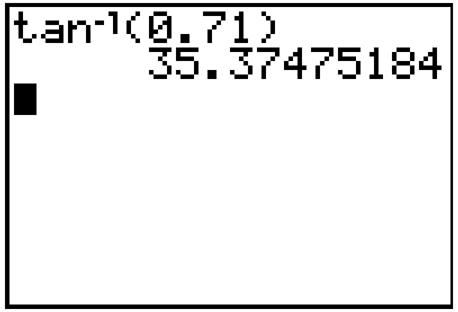


$\cos^{-1}(0.87) \approx 30°$ $\sin^{-1}(0.85) \approx 58°$ $\tan^{-1}(0.71) \approx 35°$

Check It Out! Example 2

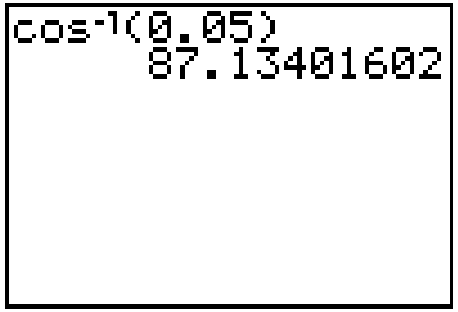
Use your calculator to find each angle measure to the nearest degree.

a. $\tan^{-1}(0.75)$



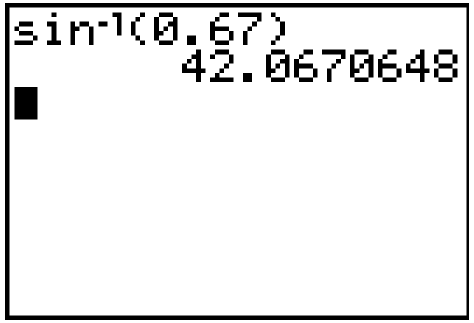
$\tan^{-1}(0.75) \approx 35°$

b. $\cos^{-1}(0.05)$



$\cos^{-1}(0.05) \approx 87°$

c. $\sin^{-1}(0.67)$

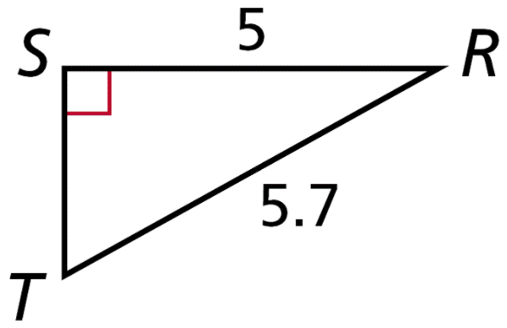


$\sin^{-1}(0.67) \approx 42°$

Using given measures to find the unknown angle measures or side lengths of a triangle is known as *solving a triangle*. To solve a right triangle, you need to know two side lengths or one side length and an acute angle measure.

Example 3: Solving Right Triangles

Find the unknown measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.



Method 1: By the Pythagorean Theorem,

$$RT^2 = RS^2 + ST^2$$

$$(5.7)^2 = 5^2 + ST^2$$

So $ST = \sqrt{7.49} \approx 2.74$.

$$m\angle R = \cos^{-1}\left(\frac{5}{5.7}\right) \approx 29°$$

Since the acute angles of a right triangle are complementary, $m\angle T \approx 90° - 29° \approx 61°$.

Example 3 Continued

Method 2:

$$m\angle R = \cos^{-1}\left(\frac{5}{5.7}\right) \approx 29°$$

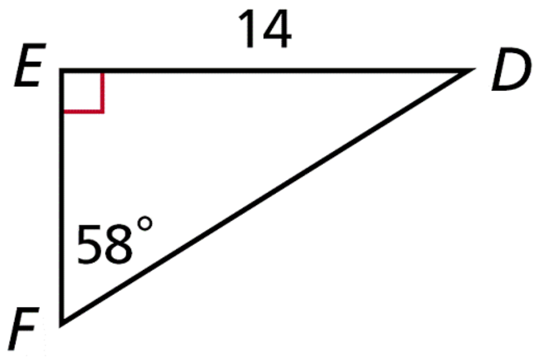
Since the acute angles of a right triangle are complementary, m∠*T* ≈ 90° – 29° ≈ 61°.

$\sin R = \frac{ST}{5.7}$, so *ST* = 5.7 sin*R.*

$$ST \approx 5.7 \sin\left[\cos^{-1}\left(\frac{5}{5.7}\right)\right] \approx 2.74$$

Check It Out! Example 3

Find the unknown measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.



Since the acute angles of a right triangle are complementary, m∠D = 90° – 58° = 32°.

$\tan 32° = \frac{EF}{14}$, so EF = 14 tan 32°. $EF \approx 8.75$

$DF^2 = ED^2 + EF^2$

$DF^2 = 14^2 + 8.75^2$

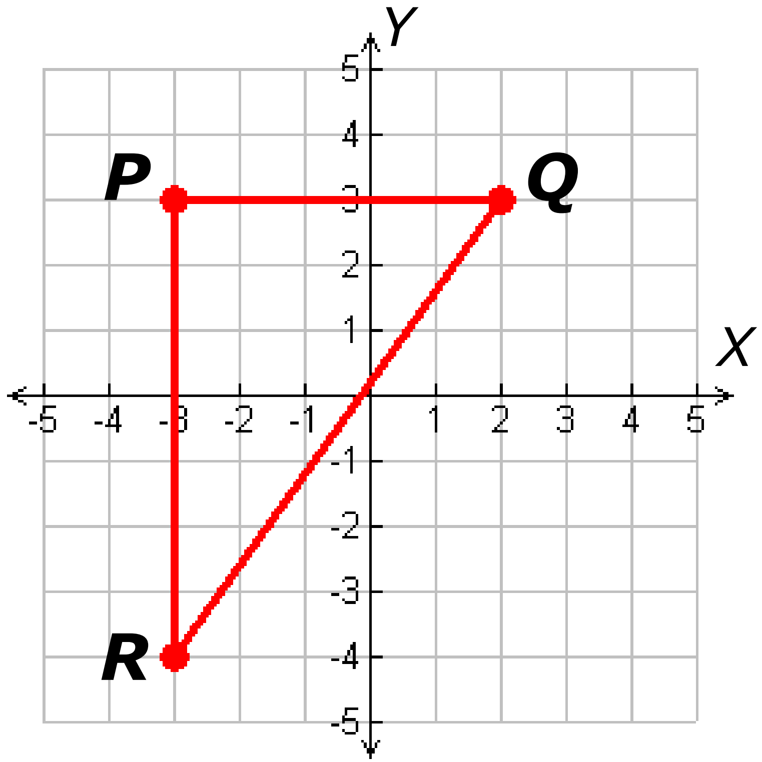
$DF \approx 16.51$

Example 4: Solving a Right Triangle in the Coordinate Plane

The coordinates of the vertices of ΔPQR are $P(-3, 3)$, $Q(2, 3)$, and $R(-3, -4)$. Find the side lengths to the nearest hundredth and the angle measures to the nearest degree.

Example 4 Continued

Step 1 Find the side lengths. Plot points *P*, *Q*, and *R*.



$PR = 7$ $PQ = 5$

By the Distance Formula,

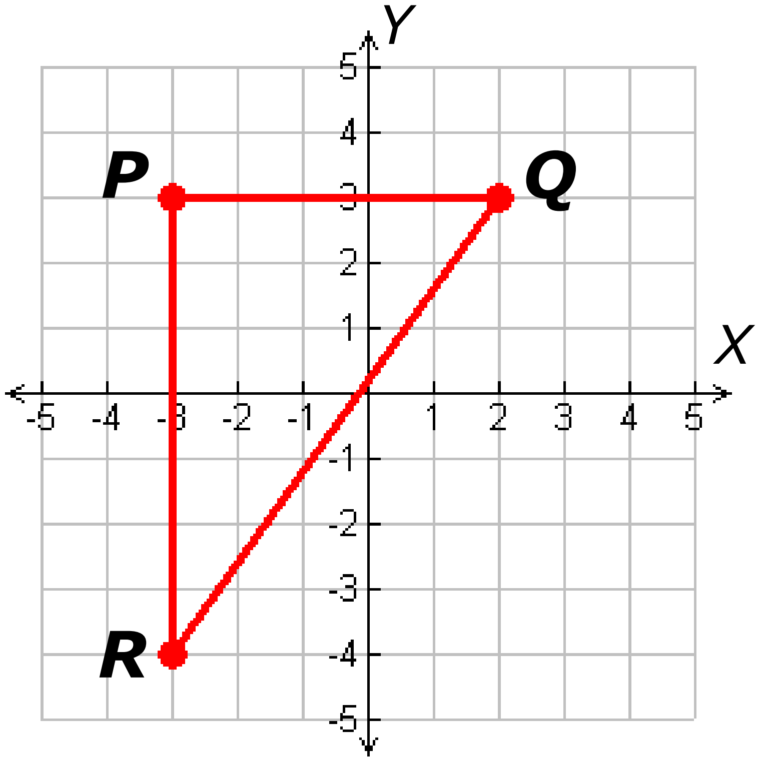
$$QR = \sqrt{(-3-2)^2 + (-4-3)^2}$$

$$= \sqrt{(-5)^2 + (-7)^2}$$

$$= \sqrt{25 + 49} = \sqrt{74} \approx 8.60$$

Example 4 Continued

Step 2 Find the angle measures.



$m\angle P = 90°$ *$\overline{PQ}$ and $\overline{PR}$ are $\perp$.*

$\overline{PR}$ is opp. $\angle Q$,

and $\overline{PQ}$ is adj. to $\angle Q$.

$$m\angle Q = \tan^{-1}\left(\frac{7}{5}\right) \approx 54°$$

The acute $\angle$s of a rt. Δ are comp.

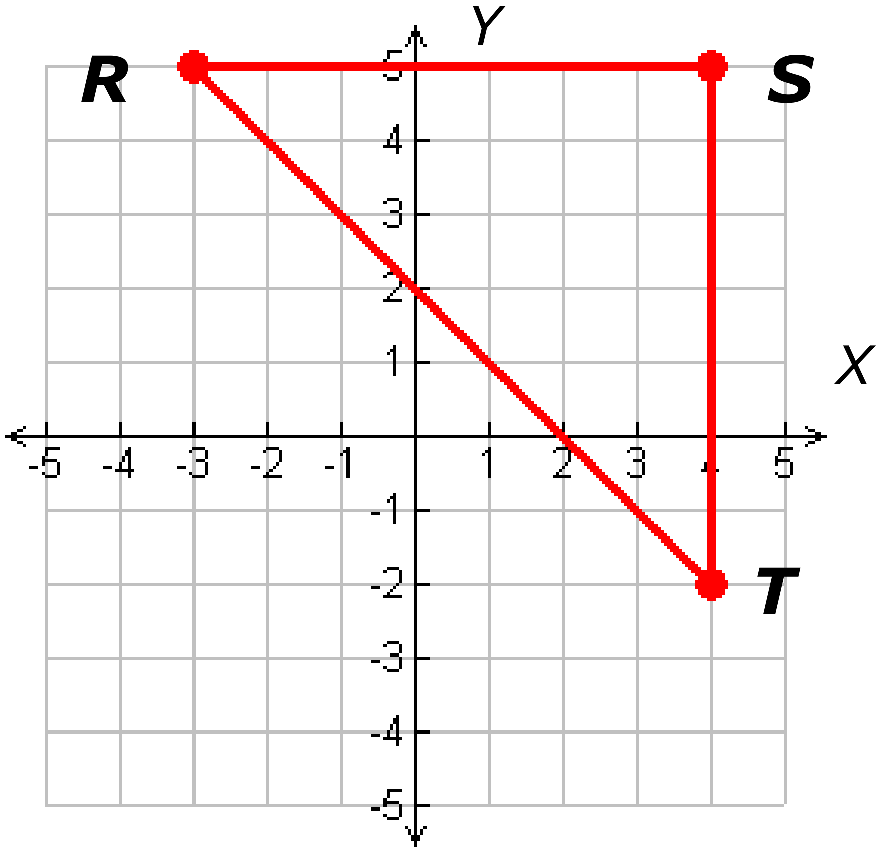
$m\angle R \approx 90° - 54° \approx 36°$

Check It Out! Example 4

The coordinates of the vertices of ΔRST are $R(-3, 5)$, $S(4, 5)$, and $T(4, -2)$. Find the side lengths to the nearest hundredth and the angle measures to the nearest degree.

Check It Out! Example 4 Continued

Step 1 Find the side lengths. Plot points *R*, *S*, and *T*.



$RS = ST = 7$

By the Distance Formula,

$$RT = \sqrt{(4-(-3))^2 + (-2-5)^2}$$
$$= \sqrt{(7)^2 + (-7)^2}$$
$$= \sqrt{49+49} = 7\sqrt{2} \approx 9.90$$

Check It Out! Example 4 Continued

Step 2 Find the angle measures.

$m\angle S = 90°$ — $\overline{RS}$ *and* $\overline{ST}$ *are* $\perp$.

$m\angle T = \tan^{-1}\left(\frac{7}{7}\right) = 45°$ — $\overline{RS}$ *is opp.* $\angle T$, *and* $\overline{ST}$ *is adj.* $\angle T$.

$m\angle R \approx 90° - 45° \approx 45°$ — *The acute* $\angle$*s of a rt.* Δ *are comp.*

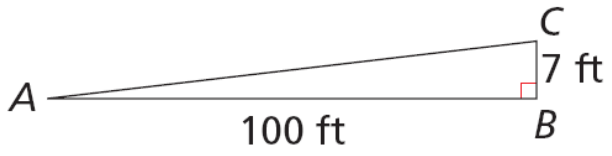
Example 5: Travel Application

A highway sign warns that a section of road ahead has a 7% grade. To the nearest degree, what angle does the road make with a horizontal line?

$$7\% = \frac{7}{100}$$

Change the percent grade to a fraction.

A 7% grade means the road rises (or falls) 7 ft for every 100 ft of horizontal distance.



Draw a right triangle to represent the road.

$$m\angle A = \tan^{-1}\left(\frac{7}{100}\right) \approx 4°$$

$\angle A$ is the angle the road makes with a horizontal line.

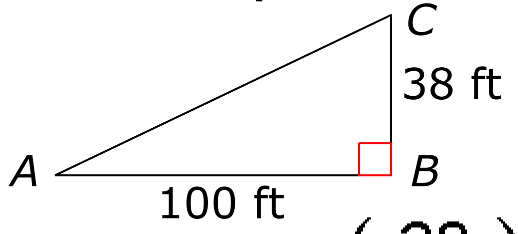
Check It Out! Example 5

Baldwin St. in Dunedin, New Zealand, is the steepest street in the world. It has a grade of 38%. To the nearest degree, what angle does Baldwin St. make with a horizontal line?

$$38\% = \frac{38}{100}$$

Change the percent grade to a fraction.

A 38% grade means the road rises (or falls) 38 ft for every 100 ft of horizontal distance.



Draw a right triangle to represent the road.

$$m\angle A = \tan^{-1}\left(\frac{38}{100}\right) \approx 21°$$

$\angle A$ is the angle the road makes with a horizontal line.

Lesson Quiz: Part I

Use your calculator to find each angle measure to the nearest degree.

1. $\cos^{-1}(0.97)$ 14°

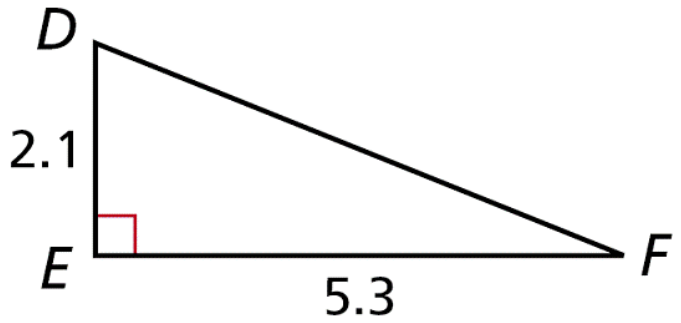
2. $\tan^{-1}(2)$ 63°

3. $\sin^{-1}(0.59)$ 36°

Lesson Quiz: Part II

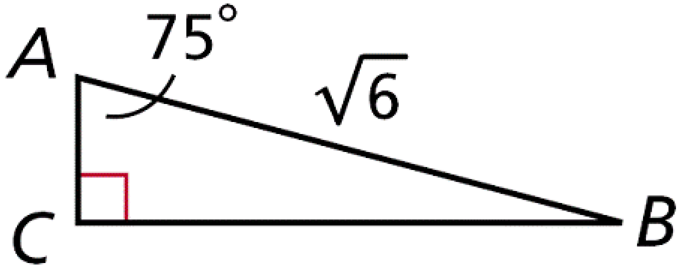
Find the unknown measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.

4.



$DF \approx 5.7$; $m\angle D \approx 68°$; $m\angle F \approx 22°$

5.



$AC \approx 0.63$; $BC \approx 2.37$; $m\angle B = 15°$

Lesson Quiz: Part III

6. The coordinates of the vertices of ΔMNP are $M(-3, -2)$, $N(-3, 5)$, and $P(6, 5)$. Find the side lengths to the nearest hundredth and the angle measures to the nearest degree.

$MN = 7$; $NP = 9$; $MP \approx 11.40$; $m\angle N = 90°$; $m\angle M \approx 52°$; $m\angle P \approx 38°$