

Solving quadratic equations by graphing and factoring

Warm Up

Find the x -intercept of each function.

1. $f(x) = -3x + 9$ 3 2. $f(x) = 6x + 4 - \frac{2}{3}$

Factor each expression.

3. $3x^2 - 12x$ $3x(x - 4)$ 4. $x^2 - 9x + 18$ $(x - 6)(x - 3)$

5. $x^2 - 49$ $(x - 7)(x + 7)$

Objectives

Solve quadratic equations by graphing or factoring.

Determine a quadratic function from its roots.

Vocabulary

zero of a function

root of an equation

binomial

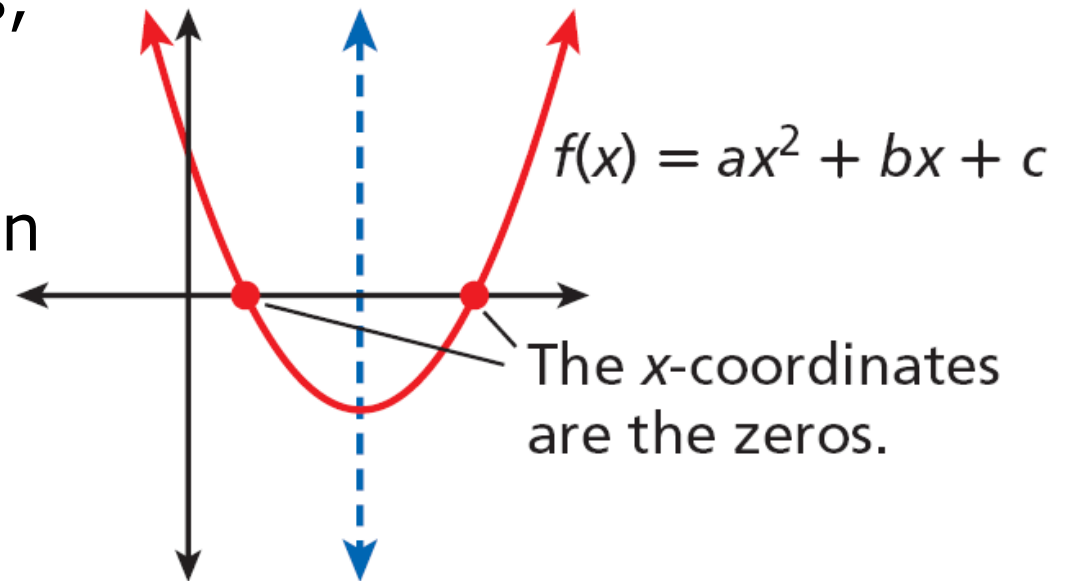
trinomial

When a soccer ball is kicked into the air, how long will the ball take to hit the ground? The height h in feet of the ball after t seconds can be modeled by the quadratic function $h(t) = -16t^2 + 32t$. In this situation, the value of the function represents the height of the soccer ball. When the ball hits the ground, the value of the function is zero.



A **zero of a function** is a value of the input x that makes the output $f(x)$ equal zero. The zeros of a function are the x -intercepts.

Unlike linear functions, which have no more than one zero, quadratic functions can have two zeros, as shown at right. These zeros are always symmetric about the axis of symmetry.



Helpful Hint

Recall that for the graph of a quadratic function, any pair of points with the same y -value are symmetric about the axis of symmetry.

Example 1: Finding Zeros by Using a Graph or Table

Find the zeros of $f(x) = x^2 - 6x + 8$ by using a graph and table.

Method 1 Graph the function $f(x) = x^2 - 6x + 8$. The graph opens upward because $a > 0$. The y-intercept is 8 because $c = 8$.

Find the vertex: $x = -\frac{b}{2a} = -\frac{(-6)}{2(1)} = 3$

The x-coordinate of the vertex is $-\frac{b}{2a}$.

Example 1 Continued

Find the zeros of $f(x) = x^2 - 6x + 8$ by using a graph and table.

Find $f(3)$: $f(x) = x^2 - 6x + 8$

$$f(3) = (3)^2 - 6(3) + 8 \quad \textit{Substitute 3 for x.}$$

$$f(3) = 9 - 18 + 8$$

$$f(3) = -1$$

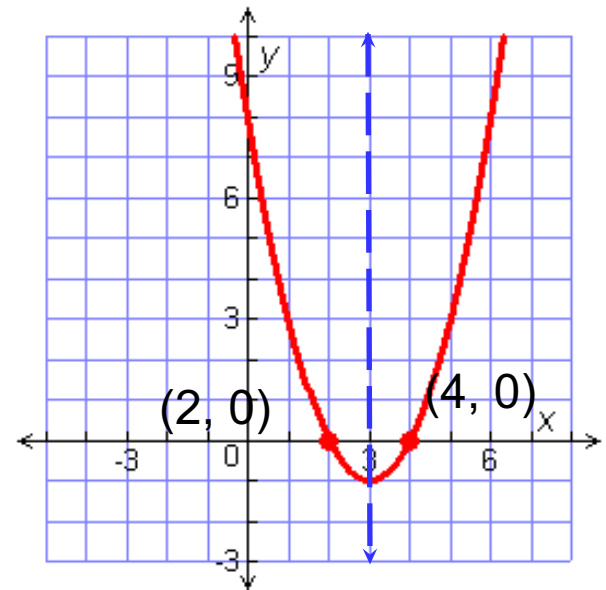
The vertex is $(3, -1)$

Example 1 Continued

Plot the vertex and the y -intercept.
Use symmetry and a table of values
to find additional points.

x	1	2	3	4	5
$f(x)$	3	0	-1	0	3

The table and the graph
indicate that the zeros are
2 and 4.



Example 1 Continued

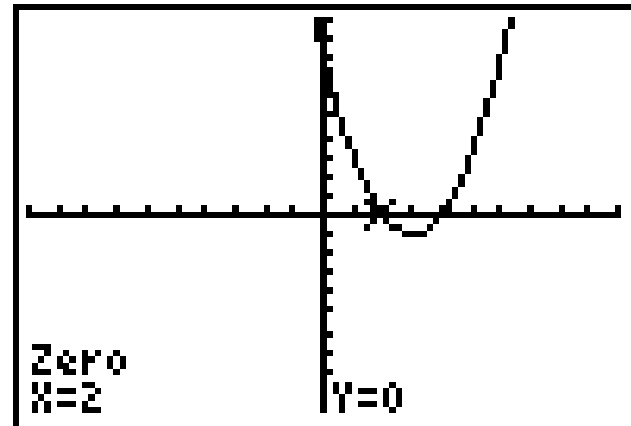
Find the zeros of $f(x) = x^2 - 6x + 8$ by using a graph and table.

Method 2 Use a calculator.

Enter $y = x^2 - 6x + 8$ into a graphing calculator.

X	Y1	
1	3	
2	0	
3	-1	
4	0	
5	3	
6	8	
7	15	

X=3



Both the table and the graph show that $y = 0$ at $x = 2$ and $x = 4$. These are the zeros of the function.

Check It Out! Example 1

Find the zeros of $g(x) = -x^2 - 2x + 3$ by using a graph and a table.

Method 1 Graph the function $g(x) = -x^2 - 2x + 3$. The graph opens downward because $a < 0$. The y -intercept is 3 because $c = 3$.

Find the vertex: $x = -\frac{b}{2a} = -\frac{(-2)}{2(-1)} = -1$ *The x -coordinate of the vertex is $-\frac{b}{2a}$.*

Check It Out! Example 1 Continued

Find the zeros of $g(x) = -x^2 - 2x + 3$ by using a graph and table.

$$\text{Find } g(1): g(x) = -x^2 - 2x + 3$$

$$g(-1) = -(-1)^2 - 2(-1) + 3 \quad \textit{Substitute } -1 \textit{ for } x.$$

$$g(-1) = -1 + 2 + 3$$

$$g(-1) = 4$$

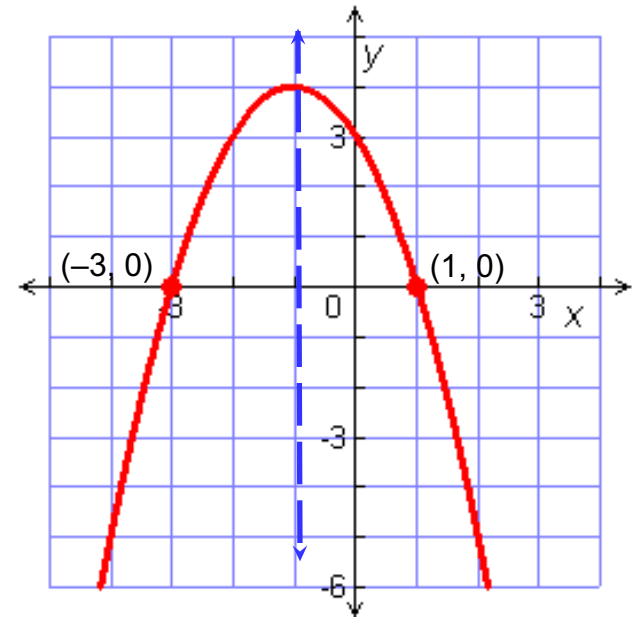
The vertex is $(-1, 4)$

Check It Out! Example 1 Continued

Plot the vertex and the y-intercept.
Use symmetry and a table of values
to find additional points.

x	-3	-2	-1	0	1
$f(x)$	0	3	4	3	0

The table and the graph
indicate that the zeros are
-3 and 1.



Check It Out! Example 1 Continued

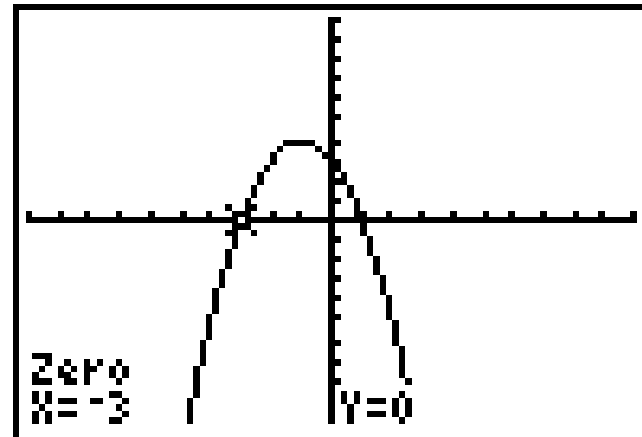
Find the zeros of $f(x) = -x^2 - 2x + 3$ by using a graph and table.

Method 2 Use a calculator.

Enter $y = -x^2 - 2x + 3$ into a graphing calculator.

X	Y ₁	
-4	-5	
-3	0	
-2	3	
-1	4	
0	3	
1	0	
2	-5	

X = -1



Both the table and the graph show that $y = 0$ at $x = -3$ and $x = 1$. These are the zeros of the function.

You can also find zeros by using algebra. For example, to find the zeros of $f(x) = x^2 + 2x - 3$, you can set the function equal to zero. The solutions to the related equation $x^2 + 2x - 3 = 0$ represent the zeros of the function.

The solution to a quadratic equation of the form $ax^2 + bx + c = 0$ are *roots*. The **roots of an equation** are the values of the variable that make the equation true.

You can find the roots of some quadratic equations by factoring and applying the Zero Product Property.

Reading Math

- Functions have *zeros* or *x-intercepts*.
- Equations have *solutions* or *roots*.

Zero Product Property

For all real numbers a and b ,

WORDS	NUMBERS	ALGEBRA
If the product of two quantities equals zero, at least one of the quantities equals zero.	$3(0) = 0$ $0(4) = 0$	If $ab = 0$, then $a = 0$ or $b = 0$.

Example 2A: Finding Zeros by Factoring

Find the zeros of the function by factoring.

$$f(x) = x^2 - 4x - 12$$

$$x^2 - 4x - 12 = 0$$

Set the function equal to 0.

$$(x + 2)(x - 6) = 0$$

Factor: Find factors of -12 that add to -4 .

$$x + 2 = 0 \text{ or } x - 6 = 0$$

Apply the Zero Product Property.

$$x = -2 \text{ or } x = 6$$

Solve each equation.

Example 2A Continued

Find the zeros of the function by factoring.

Check

Substitute each value into original equation.

$$x^2 - 4x - 12 = 0$$

$(-2)^2 - 4(-2) - 12$	0
$4 + 8 - 12$	0
0	0 ✓

$$x^2 - 4x - 12 = 0$$

$(6)^2 - 4(6) - 12$	0
$36 - 24 - 12$	0
0	0 ✓

Example 2B: Finding Zeros by Factoring

Find the zeros of the function by factoring.

$$g(x) = 3x^2 + 18x$$

$$3x^2 + 18x = 0$$

Set the function to equal to 0.

$$3x(x+6) = 0$$

Factor: The GCF is 3x.

$$3x = 0 \text{ or } x + 6 = 0$$

Apply the Zero Product Property.

$$x = 0 \text{ or } x = -6$$

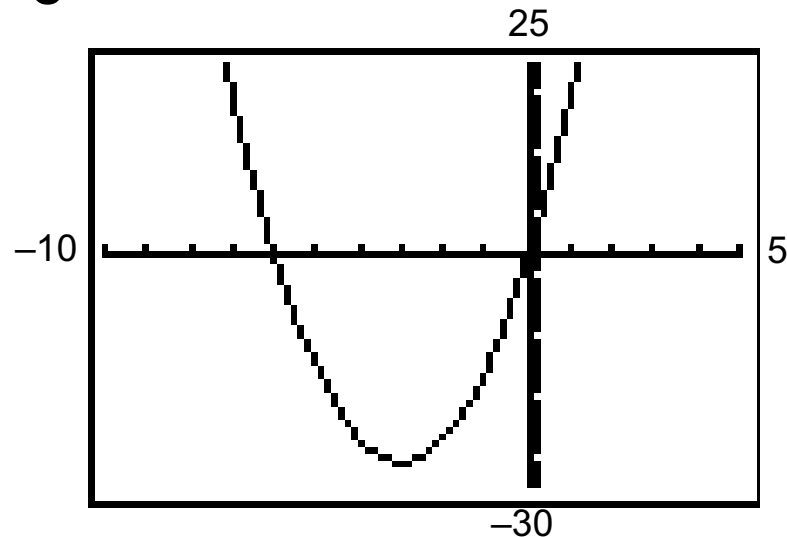
Solve each equation.

Example 2B Continued

Check Check algebraically and by graphing.

$$\begin{array}{r|l} 3x^2 + 18x = 0 & \\ \hline 3(0)^2 + 18(0) & 0 \\ 0 + 0 & 0 \\ 0 & 0 \quad \checkmark \end{array}$$

$$\begin{array}{r|l} 3x^2 + 18x = 0 & \\ \hline 3(-6)^2 + 18(-6) & 0 \\ 108 - 108 & 0 \\ 0 & 0 \quad \checkmark \end{array}$$



Check It Out! Example 2a

Find the zeros of the function by factoring.

$$f(x) = x^2 - 5x - 6$$

$$x^2 - 5x - 6 = 0$$

Set the function equal to 0.

$$(x + 1)(x - 6) = 0$$

Factor: Find factors of -6 that add to -5 .

$$x + 1 = 0 \text{ or } x - 6 = 0$$

Apply the Zero Product Property.

$$x = -1 \text{ or } x = 6$$

Solve each equation.

Check It Out! Example 2a Continued

Find the zeros of the function by factoring.

Check

Substitute each value into original equation.

$$x^2 - 5x - 6 = 0$$

$(-1)^2 - 5(-1) - 6$	0
$1 + 5 - 6$	0
0	0 ✓

$$x^2 - 5x - 6 = 0$$

$(6)^2 - 5(6) - 6$	0
$36 - 30 - 6$	0
0	0 ✓

Check It Out! Example 2b

Find the zeros of the function by factoring.

$$g(x) = x^2 - 8x$$

$$x^2 - 8x = 0$$

Set the function to equal to 0.

$$x(x - 8) = 0$$

Factor: The GCF is x .

$$x = 0 \text{ or } x - 8 = 0$$

Apply the Zero Product Property.

$$x = 0 \text{ or } x = 8$$

Solve each equation.

Check It Out! Example 2b Continued

Find the zeros of the function by factoring.

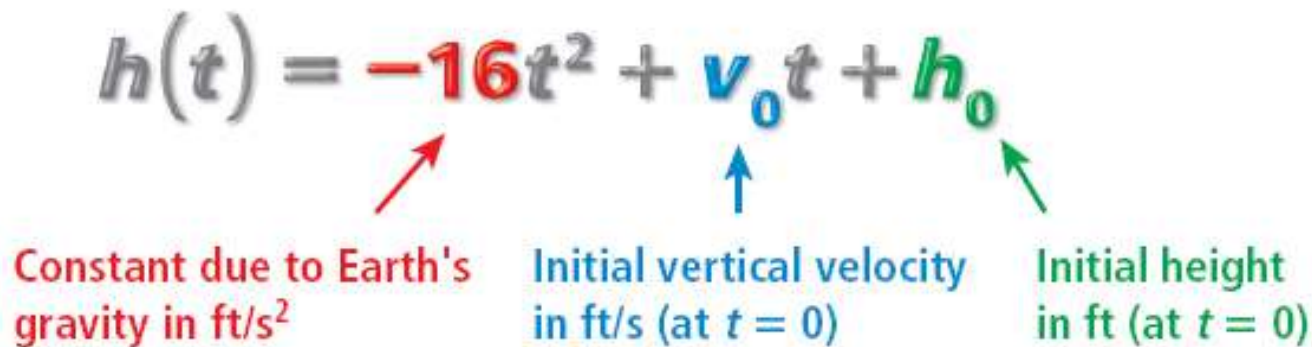
Check

Substitute each value into original equation.

$$\begin{array}{r|l} x^2 - 8x = 0 & \\ \hline (0)^2 - 8(0) & 0 \\ 0 - 0 & 0 \\ 0 & 0 \quad \checkmark \end{array}$$

$$\begin{array}{r|l} x^2 - 8x = 0 & \\ \hline (8)^2 - 8(8) & 0 \\ 64 - 64 & 0 \\ 0 & 0 \quad \checkmark \end{array}$$

Any object that is thrown or launched into the air, such as a baseball, basketball, or soccer ball, is a *projectile*. The general function that approximates the height h in feet of a projectile on Earth after t seconds is given.

$$h(t) = -16t^2 + v_0t + h_0$$


The diagram shows the equation $h(t) = -16t^2 + v_0t + h_0$ with three arrows pointing from descriptive text below to terms in the equation. A red arrow points from the text 'Constant due to Earth's gravity in ft/s²' to the term $-16t^2$. A blue arrow points from the text 'Initial vertical velocity in ft/s (at $t = 0$)' to the term v_0t . A green arrow points from the text 'Initial height in ft (at $t = 0$)' to the term h_0 .

Constant due to Earth's gravity in ft/s²

Initial vertical velocity in ft/s (at $t = 0$)

Initial height in ft (at $t = 0$)

Note that this model has limitations because it does not account for air resistance, wind, and other real-world factors.

Example 3: Sports Application

A golf ball is hit from ground level with an initial vertical velocity of 80 ft/s. After how many seconds will the ball hit the ground?

$$h(t) = -16t^2 + v_0t + h_0 \quad \textit{Write the general projectile function.}$$

$$h(t) = -16t^2 + 80t + 0 \quad \textit{Substitute 80 for } v_0 \textit{ and 0 for } h_0.$$

Example 3 Continued

The ball will hit the ground when its height is zero.

$$-16t^2 + 80t = 0$$

Set $h(t)$ equal to 0.

$$-16t(t - 5) = 0$$

Factor: The GCF is $-16t$.

$$-16t = 0 \text{ or } (t - 5) = 0$$

Apply the Zero Product Property.

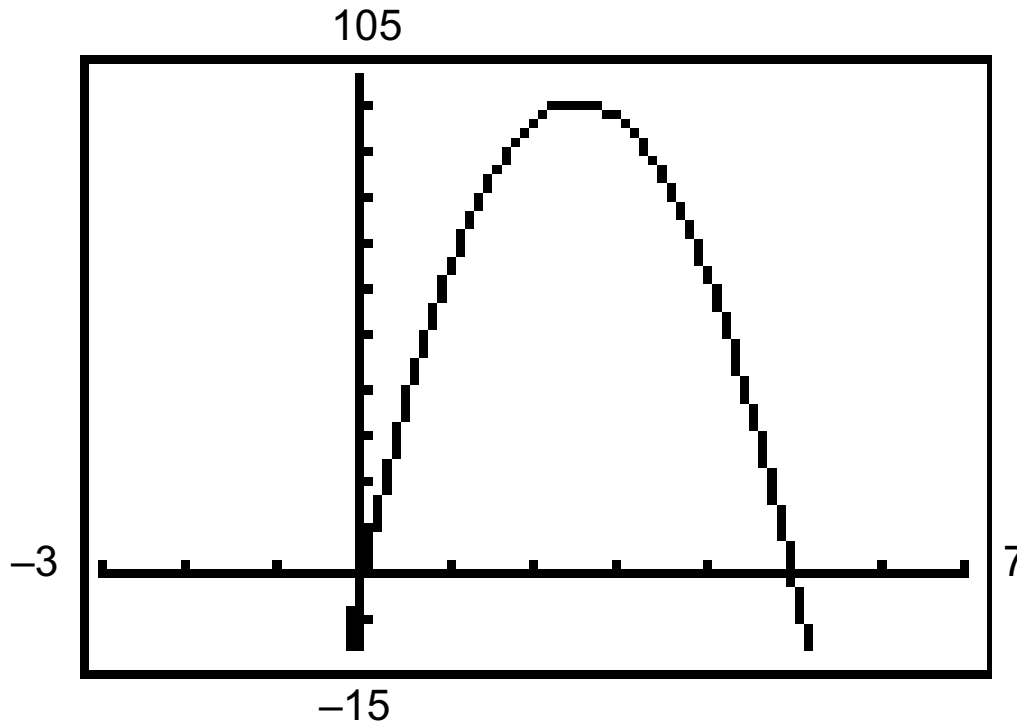
$$t = 0 \text{ or } t = 5$$

Solve each equation.

The golf ball will hit the ground after 5 seconds. Notice that the height is also zero when $t = 0$, the instant that the golf ball is hit.

Example 3 Continued

Check The graph of the function $h(t) = -16t^2 + 80t$ shows its zeros at 0 and 5.



Check It Out! Example 3

A football is kicked from ground level with an initial vertical velocity of 48 ft/s. How long is the ball in the air?

$$h(t) = -16t^2 + v_0t + h_0$$

Write the general projectile function.

$$h(t) = -16t^2 + 48t + 0$$

Substitute 48 for v_0 and 0 for h_0 .

Check It Out! Example 3 Continued

The ball will hit the ground when its height is zero.

$$-16t^2 + 48t = 0$$

Set $h(t)$ equal to 0.

$$-16t(t - 3) = 0$$

Factor: The GCF is $-16t$.

$$-16t = 0 \text{ or } (t - 3) = 0$$

Apply the Zero Product Property.

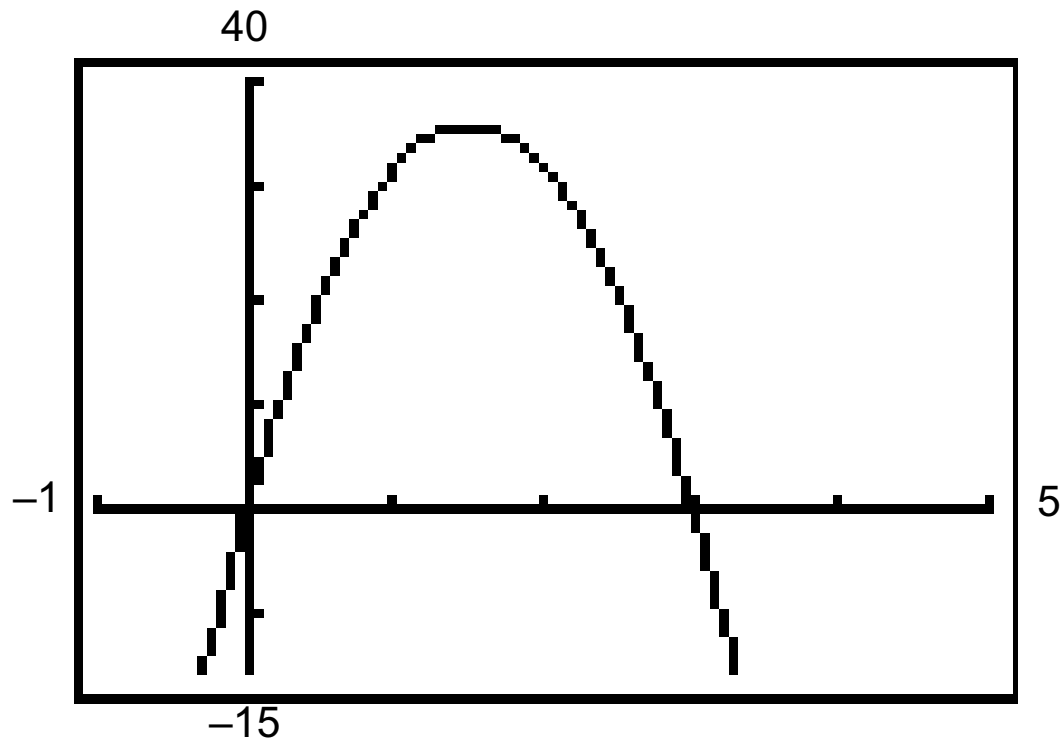
$$t = 0 \text{ or } t = 3$$

Solve each equation.

The football will hit the ground after 3 seconds. Notice that the height is also zero when $t = 0$, the instant that the football is hit.

Check It Out! Example 3 Continued

Check The graph of the function $h(t) = -16t^2 + 48t$ shows its zeros at 0 and 3.



Quadratic expressions can have one, two or three terms, such as $-16t^2$, $-16t^2 + 25t$, or $-16t^2 + 25t + 2$. Quadratic expressions with two terms are **binomials**. Quadratic expressions with three terms are **trinomials**. Some quadratic expressions with perfect squares have special factoring rules.

Special Products and Factors

Difference of Two Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Perfect-Square Trinomial

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

Example 4A: Find Roots by Using Special Factors

Find the roots of the equation by factoring.

$$4x^2 = 25$$

$$4x^2 - 25 = 0$$

Rewrite in standard form.

$$(2x)^2 - (5)^2 = 0$$

Write the left side as $a^2 - b^2$.

$$(2x + 5)(2x - 5) = 0$$

Factor the difference of squares.

$$2x + 5 = 0 \text{ or } 2x - 5 = 0$$

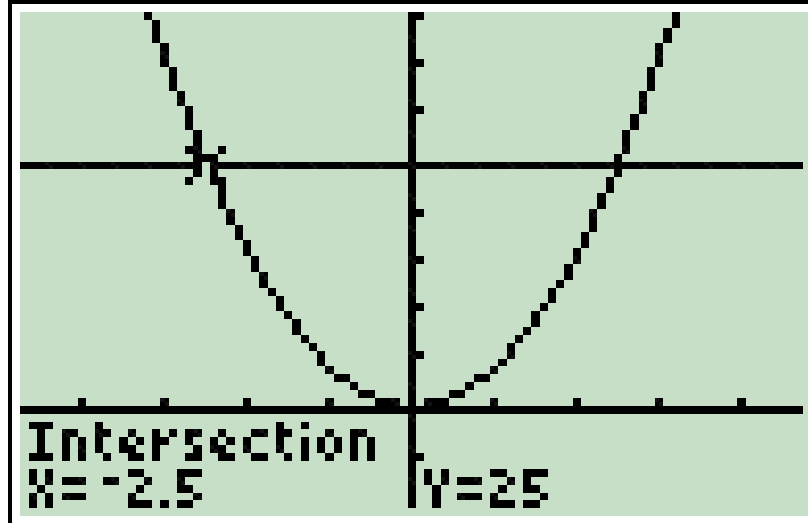
Apply the Zero Product Property.

$$x = -\frac{5}{2} \text{ or } x = \frac{5}{2}$$

Solve each equation.

Example 4 Continued

Check Graph each side of the equation on a graphing calculator. Let Y1 equal $4x^2$, and let Y2 equal 25. The graphs appear to intersect at $-\frac{5}{2}$ and $\frac{5}{2}$.



Example 4B: Find Roots by Using Special Factors

Find the roots of the equation by factoring.

$$18x^2 = 48x - 32$$

$$18x^2 - 48x + 32 = 0 \quad \text{Rewrite in standard form.}$$

$$2(9x^2 - 24x + 16) = 0 \quad \text{Factor. The GCF is 2.}$$

$$9x^2 - 24x + 16 = 0 \quad \text{Divide both sides by 2.}$$

$$(3x)^2 - 2(3x)(4) + (4)^2 = 0 \quad \text{Write the left side as } a^2 - 2ab + b^2.$$

$$(3x - 4)^2 = 0 \quad \text{Factor the perfect-square trinomial.}$$

$$3x - 4 = 0 \text{ or } 3x - 4 = 0 \quad \text{Apply the Zero Product Property.}$$

$$x = \frac{4}{3} \text{ or } x = \frac{4}{3} \quad \text{Solve each equation.}$$

Example 4B Continued

Check Substitute the root $\frac{4}{3}$ into the original equation.

$$\begin{array}{r|l} 18x^2 = 48x - 32 & \\ \hline 18\left(\frac{4}{3}\right)^2 & 48\left(\frac{4}{3}\right) - 32 \\ 18\left(\frac{16}{9}\right) & 64 - 32 \\ 32 & 32 \quad \checkmark \end{array}$$

Check It Out! Example 4a

Find the roots of the equation by factoring.

$$x^2 - 4x = -4$$

$$x^2 - 4x + 4 = 0$$

Rewrite in standard form.

$$(x - 2)(x - 2) = 0$$

Factor the perfect-square trinomial.

$$x - 2 = 0 \text{ or } x - 2 = 0$$

Apply the Zero Product Property.

$$x = 2 \text{ or } x = 2$$

Solve each equation.

Check It Out! Example 4a Continued

Check Substitute the root 2 into the original equation.

$$\begin{array}{r|l} x^2 - 4x = -4 & \\ \hline (2)^2 - 4(2) & -4 \\ 4 - 8 & -4 \\ -4 & -4 \checkmark \end{array}$$

Check It Out! Example 4b

Find the roots of the equation by factoring.

$$25x^2 = 9$$

$$25x^2 - 9 = 0$$

Rewrite in standard form.

$$(5x)^2 - (3)^2 = 0$$

Write the left side as $a^2 - b^2$.

$$(5x + 3)(5x - 3) = 0$$

Factor the difference of squares.

$$5x + 3 = 0 \text{ or } 5x - 3 = 0$$

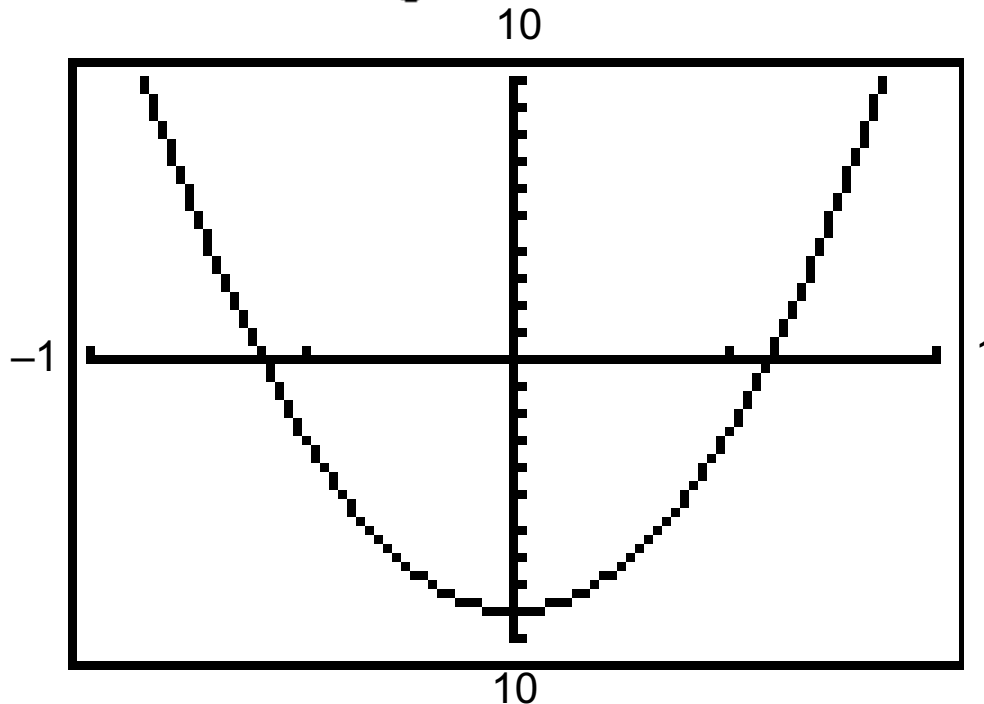
Apply the Zero Product Property.

$$x = -\frac{3}{5} \text{ or } x = \frac{3}{5}$$

Solve each equation.

Check It Out! Example 4b Continued

Check Graph the related function $f(x) = 25x^2 - 9$ on a graphing calculator. The function appears to have zeros at $-\frac{3}{5}$ and $\frac{3}{5}$.



If you know the zeros of a function, you can work backward to write a rule for the function

Example 5: Using Zeros to Write Function Rules

Write a quadratic function in standard form with zeros 4 and -7.

$$x = 4 \text{ or } x = -7$$

Write the zeros as solutions for two equations.

$$x - 4 = 0 \text{ or } x + 7 = 0$$

Rewrite each equation so that it equals 0.

$$(x - 4)(x + 7) = 0$$

Apply the converse of the Zero Product Property to write a product that equals 0.

$$x^2 + 3x - 28 = 0$$

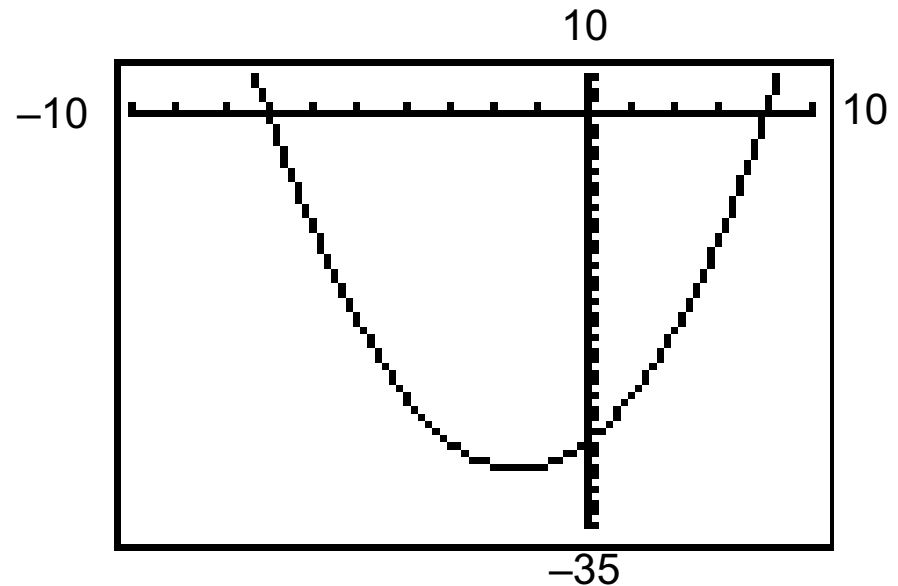
Multiply the binomials.

$$f(x) = x^2 + 3x - 28$$

Replace 0 with $f(x)$.

Example 5 Continued

Check Graph the function $f(x) = x^2 + 3x - 28$ on a calculator. The graph shows the original zeros of 4 and -7 .



Check It Out! Example 5

Write a quadratic function in standard form with zeros 5 and -5.

$$x = 5 \text{ or } x = -5$$

$$x + 5 = 0 \text{ or } x - 5 = 0$$

$$(x + 5)(x - 5) = 0$$

$$x^2 - 25 = 0$$

$$f(x) = x^2 - 25$$

Write the zeros as solutions for two equations.

Rewrite each equation so that it equals 0.

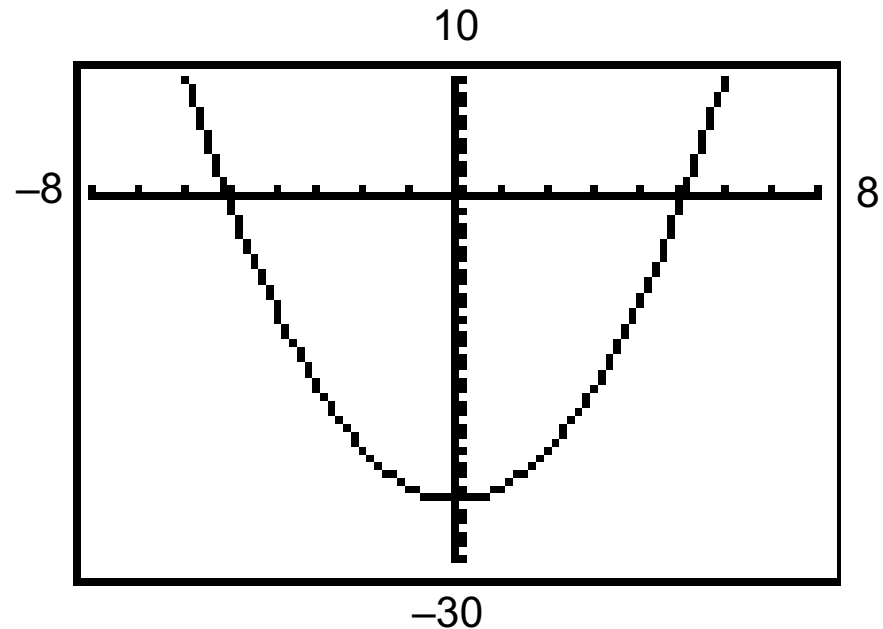
Apply the converse of the Zero Product Property to write a product that equals 0.

Multiply the binomials.

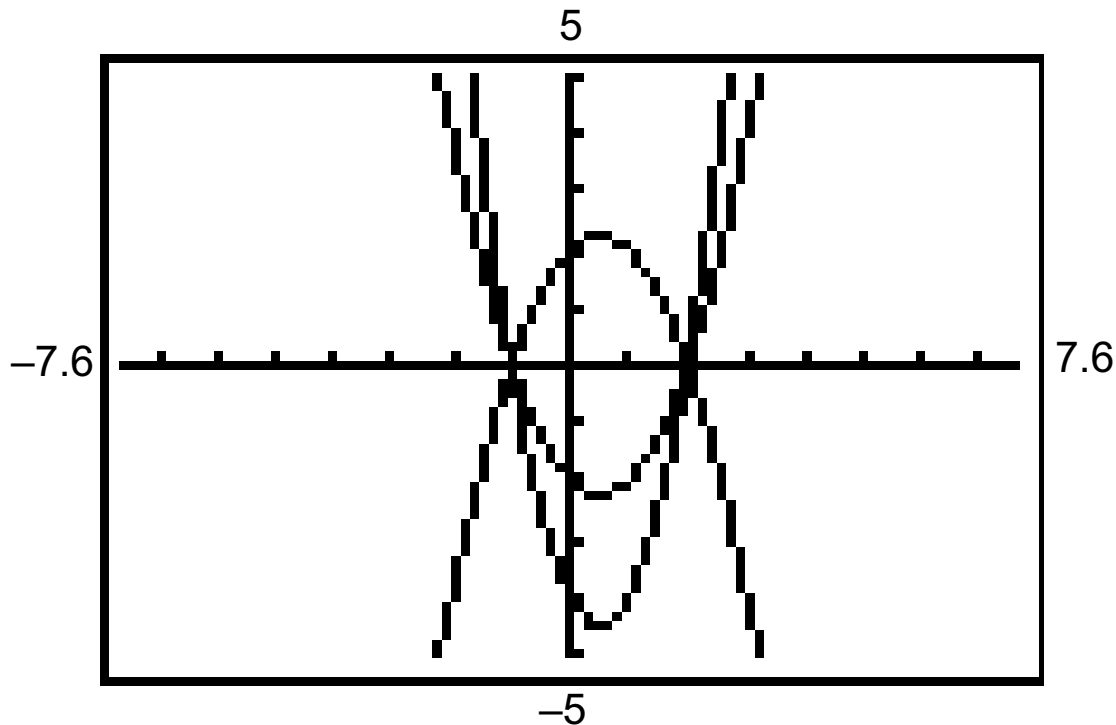
Replace 0 with $f(x)$.

Check It Out! Example 5 Continued

Check Graph the function $f(x) = x^2 - 25$ on a calculator. The graph shows the original zeros of 5 and -5 .



Note that there are many quadratic functions with the same zeros. For example, the functions $f(x) = x^2 - x - 2$, $g(x) = -x^2 + x + 2$, and $h(x) = 2x^2 - 2x - 4$ all have zeros at 2 and -1.



Lesson Quiz: Part I

Find the zeros of each function.

1. $f(x) = x^2 - 7x$ 0, 7

2. $f(x) = x^2 - 9x + 20$ 4, 5

Find the roots of each equation using factoring.

3. $x^2 - 10x + 25 = 0$ 5

4. $7x = 15 - 2x^2$ -5, $\frac{3}{2}$

Lesson Quiz: Part II

5. Write a quadratic function in standard form with zeros 6 and -1 .

Possible answer: $f(x) = x^2 - 5x - 6$

6. A rocket is launched from ground level with an initial vertical velocity of 176 ft/s. After how many seconds with the rocket hit the ground?

after 11 s