

# Solving Equations

A **variable** is a letter or a symbol used to represent a value that can change.

A **constant** is a value that does not change.

A **numerical expression** contains only constants and operations.

An **algebraic expression** may contain variables, constants, and operations.

An **equation** is a mathematical statement that two expressions are equal.

A **solution of an equation** is a value of the variable that makes the equation true.

To find solutions, *isolate the variable*. A variable is isolated when it appears by itself on one side of an equation, and not at all on the other side.

# Solving Equations by Adding Or Subtracting

Isolate a variable by using inverse operations which "undo" operations on the variable.

An equation is like a balanced scale. To keep the balance, perform the same operation on both sides.

## **Inverse Operations**

<b>Operation</b>	<b>Inverse Operation</b>
Addition	Subtraction
Subtraction	Addition

## Example 1A: Solving Equations by Using Addition

**Solve the equation. Check your answer.**

$$y - 8 = 24$$

$$\begin{array}{r} + 8 \\ + 8 \\ \hline \end{array}$$

$$y = 32$$

*Since 8 is subtracted from  $y$ , add 8 to both sides to undo the subtraction.*

**Check**

$$\begin{array}{r|l} y - 8 = 24 & \\ \hline 32 - 8 & 24 \\ 24 & 24 \checkmark \end{array}$$

*To check your solution, substitute 32 for  $y$  in the original equation.*

## Check It Out! Example 1a

**Solve the equation. Check your answer.**

$$-6 = k - 6$$

$$\begin{array}{rcl} \underline{+ 6} & \underline{+ 6} & \text{Since 6 is subtracted from } k, \text{ add 6 to} \\ 0 = k & & \text{both sides to undo the subtraction.} \end{array}$$

<b>Check</b>	$-6 = k - 6$
	$-6 \quad   \quad 0 - 6$
	$-6 \quad   \quad -6 \quad \checkmark$

*To check your solution, substitute 0 for  $k$  in the original equation.*

## Check It Out! Example 1b

Solve the equation. Check your answer.

$$16 = m - 9$$

$$\begin{array}{r} + 9 \\ 25 = m \end{array}$$

*Since 9 is subtracted from  $m$ , add 9 to both sides to undo the subtraction.*

**Check**

16	=	$m$	-	9
16		25	-	9
16		16		✓

*To check your solution, substitute 25 for  $m$  in the original equation.*

## Example 2A: Solving Equations by Using Subtraction

**Solve the equation. Check your answer.**

$$m + 17 = 33$$

$$\begin{array}{r} -17 \\ m + 17 = 33 \\ \hline m = 16 \end{array}$$

*Since 17 is added to  $m$ , subtract 17 from both sides to undo the addition.*

**Check**

$m + 17 = 33$	
$16 + 17$	$33$
$33$	$33$ ✓

*To check your solution, substitute 16 for  $m$  in the original equation.*



## Check It Out! Example 2a

**Solve the equation. Check your answer.**

$$-5 = k + 5$$

$$\begin{array}{r} -5 \\ -10 = k \end{array} \quad \begin{array}{r} -5 \\ \end{array} \quad \text{Since 5 is added to } k, \text{ subtract 5 from both sides to undo the subtraction.}$$

**Check**

$-5 = k + 5$
$-5 \quad   \quad -10 + 5$
$-5 \quad   \quad -5 \checkmark$

*To check your solution, substitute  $-10$  for  $k$  in the original equation.*

## Check It Out! Example 2b

**Solve the equation. Check your answer.**

$$6 + t = 14$$

$$\underline{-6}$$

$$\underline{-6}$$

$$t = 8$$

*Since 6 is added to  $t$ , subtract 6 from both sides to undo the addition.*

**Check**

6 + $t$ = 14	
6 + 8	14
14	14 ✓

*To check your solution, substitute 8 for  $t$  in the original equation.*

Remember that subtracting is the same as adding the opposite. When solving equations, you will sometimes find it easier to add an opposite to both sides instead of subtracting.

## Example 3A: Solving Equations by Adding the Opposite

Solve  $-\frac{5}{11} + p = -\frac{2}{11}$ .

$$\begin{array}{r} + \frac{5}{11} \\ \hline \end{array}$$

$$\begin{array}{r} + \frac{5}{11} \\ \hline \end{array}$$

Since  $-\frac{5}{11}$  is added to  $p$ , add  $\frac{5}{11}$  to both sides.

$$p = \frac{3}{11}$$

**Check**  $-\frac{5}{11} + p = -\frac{2}{11}$

$$\begin{array}{r|l} -\frac{5}{11} + \frac{3}{11} & -\frac{2}{11} \\ \hline -\frac{2}{11} & -\frac{2}{11} \end{array} \checkmark$$

To check your solution, substitute  $\frac{3}{11}$  for  $p$  in the original equation.

# Solving Equations by Multiplying or Dividing

Solving an equation that contains multiplication or division is similar to solving an equation that contains addition or subtraction. Use inverse operations to undo the operations on the variable.

<b>Inverse Operations</b>	
<b>Operation</b>	<b>Inverse Operation</b>
Multiplication	Division
Division	Multiplication

## Example 4A: Solving Equations by Using Multiplication

**Solve the equation.**

$$-8 = \frac{j}{3}$$

$$(3)(-8) = (3)\left(\frac{j}{3}\right)$$

$$-24 = j$$

*Since  $j$  is divided by 3,  
multiply both sides by 3  
to undo the division.*

**Check**

$-8$	$=$	$\frac{j}{3}$
<hr/>		
$-8$	$ $	$\frac{-24}{3}$
$-8$	$ $	$-8$ ✓

*To check your solution,  
substitute  $-24$  for  $j$  in the  
original equation.*

**Check It Out! Example 4a**

**Solve the equation. Check your answer.**

$$\frac{p}{5} = 10$$

$$(5)\left(\frac{p}{5}\right) = (5)(10)$$

$$p = 50$$

*Since p is divided by 5,  
multiply both sides by 5  
to undo the division.*

**Check**

$\frac{p}{5}$	$=$	10
<hr/>		
50		10
<hr/>		
5		
10		10

*To check your solution,  
substitute 50 for p in the  
original equation.*



**Check It Out! Example 4b**

**Solve the equation. Check your answer.**

$$-13 = \frac{y}{3}$$

$(3)(-13) = (3)\left(\frac{y}{3}\right)$  *Since y is divided by 3, multiply both sides by 3 to undo the division.*

$-39 = y$

**Check**

$-13$	$=$	$\frac{y}{3}$
$-13$		$\frac{-39}{3}$
$-13$		$-13$

*To check your solution, substitute  $-39$  for  $y$  in the original equation.*



## Example 5A: Solving Equations by Using Division

**Solve the equation. Check your answer.**

$$9y = 108$$

$$\frac{9y}{9} = \frac{108}{9}$$

$$y = 12$$

*Since  $y$  is multiplied by 9,  
divide both sides by 9 to  
undo the multiplication.*

**Check**  $9y = 108$

$9(12)$	$108$
$108$	$108$

*To check your solution,  
substitute 12 for  $y$  in the  
original equation.*

## Check It Out! Example 5a

**Solve the equation. Check your answer.**

$$16 = 4c$$

$$\frac{16}{4} = \frac{4c}{4}$$

$$4 = c$$

*Since  $c$  is multiplied by 4,  
divide both sides by 4 to  
undo the multiplication.*

**Check**      $16 = 4c$

16		$4(4)$
16		$16$ ✓

*To check your solution,  
substitute 4 for  $c$  in the  
original equation.*

Remember that dividing is the same as multiplying by the reciprocal. When solving equations, you will sometimes find it easier to multiply by a reciprocal instead of dividing. This is often true when an equation contains fractions.

## Example 6A: Solving Equations That Contain Fractions

**Solve the equation.**

$$\frac{5}{6}w = -20$$

$$\left(\frac{6}{5}\right)\frac{5}{6}w = \left(\frac{6}{5}\right)(-20)$$

$$w = -24$$

*The reciprocal of  $\frac{5}{6}$  is  $\frac{6}{5}$ . Since  $w$  is multiplied by  $\frac{5}{6}$ , multiply both sides by  $\frac{6}{5}$ .*

**Check**  $\frac{5}{6}w = -20$

$\left(\frac{5}{6}\right)(-24)$	$-20$
$-20$	$-20$ ✓

*To check your solution, substitute  $-24$  for  $w$  in the original equation.*

# Example 6B: Solving Equations That Contain Fractions

Solve the equation.

$$\frac{3}{16} = \frac{1}{8} z$$

$$(8) \left( \frac{3}{16} \right) = (8) \left( \frac{1}{8} \right) z$$

$$\frac{3}{2} = z$$

The reciprocal of  $\frac{1}{8}$  is 8. Since  $z$  is multiplied by  $\frac{1}{8}$ , multiply both sides by 8.

Check

$$\frac{3}{16} = \frac{1}{8} z$$

$\left( \frac{3}{16} \right)$	$\left( \frac{1}{8} \right) \left( \frac{3}{2} \right)$
$\frac{3}{16}$	$\frac{3}{16}$

✓

To check your solution, substitute  $\frac{3}{2}$  for  $z$  in the original equation.

# Solving Equations with Variables on One Side

## Example 1A: Solving Two-Step Equations

**Solve  $18 = 4a + 10$ .**

$$\begin{array}{r} 18 = 4a + 10 \\ -10 \quad -10 \\ \hline 8 = 4a \end{array}$$

*First  $a$  is multiplied by 4. Then 10 is added. Work backward: Subtract 10 from both sides.*

$$\begin{array}{r} 8 = 4a \\ \hline 4 \quad 4 \end{array}$$

*Since  $a$  is multiplied by 4, divide both sides by 4 to undo the multiplication.*

$$2 = a$$

## Example 1B: Solving Two-Step Equations

**Solve  $5t - 2 = -32$ .**

$$\begin{array}{rcl} 5t - 2 & = & -32 \\ + 2 & & + 2 \\ \hline \end{array}$$

$$5t = -30$$

$$\begin{array}{rcl} 5t & = & -30 \\ \hline 5 & & 5 \end{array}$$

$$t = -6$$

*First  $t$  is multiplied by 5. Then 2 is subtracted. Work backward: Add 2 to both sides.*

*Since  $t$  is multiplied by 5, divide both sides by 5 to undo the multiplication.*

## Check it Out! Example 1a

**Solve  $-4 + 7x = 3$ .**

$$\begin{array}{r} -4 + 7x = 3 \\ + 4 \qquad + 4 \\ \hline 7x = 7 \end{array}$$

$$\begin{array}{r} 7x = 7 \\ \hline 7 \quad 7 \\ \hline x = 1 \end{array}$$

*First  $x$  is multiplied by 7. Then  $-4$  is added. Work backward: Add 4 to both sides.*

*Since  $x$  is multiplied by 7, divide both sides by 7 to undo the multiplication.*



## Check it Out! Example 1b

**Solve**  $\frac{n}{7} + 2 = 2$ .

$$\begin{array}{r} \frac{n}{7} + 2 = 2 \\ -2 \quad -2 \\ \hline \end{array}$$

$$\frac{n}{7} = 0$$

$$(7)\frac{n}{7} = (7)0$$

$$n = 0$$

*First  $n$  is divided by 7. Then 2 is added. Work backward: Subtract 2 from both sides.*

*Since  $n$  is divided by 7, multiply both sides by 7 to undo the division.*

## Example 2A: Solving Two-Step Equations That Contain Fractions

**Solve**  $\frac{y}{8} - \frac{3}{4} = \frac{7}{12}$ .

**Method 1** Use fraction operations.

$$\frac{y}{8} - \frac{3}{4} = \frac{7}{12}$$

$$+ \frac{3}{4} \quad + \frac{3}{4}$$

$$\frac{y}{8} = \frac{16}{12}$$

$$8 \left( \frac{y}{8} \right) = 8 \left( \frac{16}{12} \right)$$

*Since  $\frac{3}{4}$  is subtracted from  $\frac{y}{8}$ , add  $\frac{3}{4}$  to both sides to undo the subtraction.*

*Since  $y$  is divided by 8, multiply both sides by 8 to undo the division.*

## Example 2A Continued

**Solve**  $\frac{y}{8} - \frac{3}{4} = \frac{7}{12}$  .

**Method 1** Use fraction operations.

$$8\left(\frac{y}{8}\right) = 8\left(\frac{16}{12}\right)$$

$$y = \frac{8 \cdot 16}{12} \quad \textit{Simplify.}$$

$$y = \frac{32}{3}$$

## Example 2A Continued

**Solve**  $\frac{y}{8} - \frac{3}{4} = \frac{7}{12}$ .

**Method 2** Multiply by the LCD to clear the fractions.


$$24 \left( \frac{y}{8} - \frac{3}{4} \right) = 24 \left( \frac{7}{12} \right)$$

*Multiply both sides by 24, the LCD of the fractions.*

$$24 \left( \frac{y}{8} \right) - 24 \left( \frac{3}{4} \right) = 24 \left( \frac{7}{12} \right)$$

*Distribute 24 on the left side.*

$$3y - 18 = 14$$

$$\underline{\quad +18 \quad +18 \quad}$$

$$3y = 32$$

*Simplify.*

*Since 18 is subtracted from 3y, add 18 to both sides to undo the subtraction.*

## Example 2A Continued

**Solve**  $\frac{y}{8} - \frac{3}{4} = \frac{7}{12}$  .

**Method 2** Multiply by the LCD to clear the fractions.

$$\frac{3y}{3} = \frac{32}{3}$$

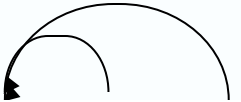
$$y = \frac{32}{3}$$

*Since  $y$  is multiplied by 3, divide both sides by 3 to undo the multiplication.*

## Check It Out! Example 2a

**Solve**  $\frac{2x}{5} - \frac{1}{2} = 5$ .

**Method 2** Multiply by the LCD to clear the fractions.


$$10 \left( \frac{2x}{5} - \frac{1}{2} \right) = 10(5)$$

*Multiply both sides by 10, the LCD of the fractions.*

$$10 \left( \frac{2x}{5} \right) - 10 \left( \frac{1}{2} \right) = 10(5)$$

*Distribute 10 on the left side.*

$$4x - 5 = 50$$

$$\begin{array}{r} + 5 \quad + 5 \\ \hline \end{array}$$

$$4x = 55$$

*Simplify.*

*Since 5 is subtracted from 4x, add 5 to both sides to undo the subtraction.*

## Check It Out! Example 2a

**Solve**  $\frac{2x}{5} - \frac{1}{2} = 5$ .

**Method 2** Multiply by the LCD to clear the fractions.

$$\frac{4x}{4} = \frac{55}{4}$$

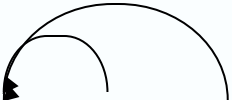
$$x = \frac{55}{4}$$

*Simplify. Since 4 is multiplied by x, divide both sides by 4 to undo the multiplication.*

## Check It Out! Example 2b

**Solve**  $\frac{3}{4}u + \frac{1}{2} = \frac{7}{8}$ .

**Method 2** Multiply by the LCD to clear the fractions.


$$8 \left( \frac{3}{4}u + \frac{1}{2} \right) = 8 \left( \frac{7}{8} \right)$$

*Multiply both sides by 8, the LCD of the fractions.*

$$8 \left( \frac{3}{4}u \right) + 8 \left( \frac{1}{2} \right) = 8 \left( \frac{7}{8} \right)$$

*Distribute 8 on the left side.*

$$6u + 4 = 7$$

$$\begin{array}{r} 6u + 4 = 7 \\ -4 \quad -4 \\ \hline 6u = 3 \end{array}$$

*Simplify.*

*Since 4 is added to 6u, subtract 4 from both sides to undo the addition.*



## Check It Out! Example 2b Continued

**Solve**  $\frac{3}{4}u + \frac{1}{2} = \frac{7}{8}$ .

**Method 2** Multiply by the LCD to clear the fractions.

$$\frac{6u}{6} = \frac{3}{6}$$

$$u = \frac{1}{2}$$

*Since  $u$  is multiplied by 6, divide both sides by 6 to undo the multiplication.*

Equations that are more complicated may have to be simplified before they can be solved. You may have to use the Distributive Property or combine like terms before you begin using inverse operations.

## Example 3A: Simplifying Before Solving Equations

**Solve  $8x - 21 + 5x = -15$ .**

$$8x - 21 - 5x = -15$$

$$8x - 5x - 21 = -15 \quad \text{Use the Commutative Property of Addition.}$$

$$3x - 21 = -15 \quad \text{Combine like terms.}$$

$$\begin{array}{r} +21 \quad +21 \\ \hline 3x = 6 \end{array} \quad \text{Since 21 is subtracted from } 3x, \text{ add 21 to both sides to undo the subtraction.}$$

$$\frac{3x}{3} = \frac{6}{3} \quad \text{Since } x \text{ is multiplied by 3, divide both sides by 3 to undo the multiplication.}$$

$$x = 2$$

## Example 3B: Simplifying Before Solving Equations

**Solve  $10y - (4y + 8) = -20$**

$$10y + (-1)(4y + 8) = -20$$

*Write subtraction as addition of the opposite.*

$$10y + (-1)(4y) + (-1)(8) = -20$$

*Distribute  $-1$  on the left side.*

$$10y - 4y - 8 = -20$$

*Simplify.*

$$6y - 8 = -20$$

*Combine like terms.*

$$\begin{array}{r} + 8 \quad + 8 \\ \hline \end{array}$$

*Since 8 is subtracted from  $6y$ , add 8 to both sides to undo the subtraction.*

$$6y = -12$$

$$\begin{array}{r} 6y = -12 \\ \hline 6 \quad 6 \end{array}$$

*Since  $y$  is multiplied by 6, divide both sides by 6 to undo the multiplication.*

$$y = -2$$

## Check It Out! Example 3a

**Solve  $2a + 3 - 8a = 8$ .**

$$2a + 3 - 8a = 8$$

$$2a - 8a + 3 = 8$$

$$-6a + 3 = 8$$

$$\underline{-3 \quad -3}$$

$$-6a = 5$$

$$\frac{-6a}{-6} = \frac{5}{-6}$$

$$a = -\frac{5}{6}$$

*Use the Commutative Property of Addition.*


*Combine like terms.*

*Since 3 is added to  $-6a$ , subtract 3 from both sides to undo the addition.*

*Since  $a$  is multiplied by  $-6$ , divide both sides by  $-6$  to undo the multiplication.*

## Check It Out! Example 3b

Solve  $-2(3 - d) = 4$


$$-2(3 - d) = 4$$

$$(-2)(3) + (-2)(-d) = 4$$

$$-6 + 2d = 4$$

$$-6 + 2d = 4$$

$$\begin{array}{r} + 6 \quad \quad + 6 \\ \hline \end{array}$$

$$2d = 10$$

$$\begin{array}{r} 2d = 10 \\ \hline 2 \quad \quad 2 \end{array}$$

$$d = 5$$

*Distribute  $-2$  on the left side.*

*Simplify.*

*Add 6 to both sides.*

*Since  $d$  is multiplied by 2,  
divide both sides by 2 to  
undo the multiplication.*

## Check It Out! Example 3c

$$\text{Solve } 4(x - 2) + 2x = 40$$

$$4(x - 2) + 2x = 40$$

$$(4)(x) + (4)(-2) + 2x = 40$$

$$4x - 8 + 2x = 40$$

$$4x + 2x - 8 = 40$$

$$6x - 8 = 40$$

$$\begin{array}{r} + 8 \quad + 8 \\ \hline 6x = 48 \end{array}$$

$$\begin{array}{r} 6x = 48 \\ \hline 6 \quad 6 \\ x = 8 \end{array}$$

*Distribute 4 on the left side.*

*Simplify.*

*Commutative Property of Addition.*

*Combine like terms.*

*Since 8 is subtracted from  $6x$ , add 8 to both sides to undo the subtraction.*

*Since  $x$  is multiplied by 6, divide both sides by 6 to undo the multiplication.*

# Solving Equations with Variables on Both Sides

To solve an equation with variables on both sides, use inverse operations to "collect" variable terms on one side of the equation.

## **Helpful Hint**

Equations are often easier to solve when the variable has a positive coefficient. Keep this in mind when deciding on which side to "collect" variable terms.



## Example 4: Solving Equations with Variables on Both Sides

**Solve  $7n - 2 = 5n + 6$ .**

$$\begin{array}{r} 7n - 2 = 5n + 6 \\ \underline{-5n} \quad \underline{-5n} \end{array}$$

*To collect the variable terms on one side, subtract  $5n$  from both sides.*

$$\begin{array}{r} 2n - 2 = 6 \\ \underline{+ 2} \quad \underline{+ 2} \\ 2n = 8 \end{array}$$

$$\frac{2n}{2} = \frac{8}{2}$$

$$n = 4$$

*Since  $n$  is multiplied by 2, divide both sides by 2 to undo the multiplication.*

## Check It Out! Example 4a

**Solve  $4b + 2 = 3b$ .**

$$4b + 2 = 3b$$

$$\underline{-3b} \quad \underline{-3b}$$

$$b + 2 = 0$$

$$\underline{-2} \quad \underline{-2}$$

$$b = -2$$

*To collect the variable terms on one side, subtract  $3b$  from both sides.*

## Check It Out! Example 4b

**Solve  $0.5 + 0.3y = 0.7y - 0.3$ .**

$$\begin{array}{rcl} 0.5 + 0.3y & = & 0.7y - 0.3 \\ \underline{-0.3y} & \underline{-0.3y} & \\ 0.5 & = & 0.4y - 0.3 \\ \underline{+0.3} & \underline{+0.3} & \\ 0.8 & = & 0.4y \end{array}$$

$$\frac{0.8}{0.4} = \frac{0.4y}{0.4}$$

$$2 = y$$

*To collect the variable terms on one side, subtract  $0.3y$  from both sides.*

*Since  $0.3$  is subtracted from  $0.4y$ , add  $0.3$  to both sides to undo the subtraction.*

*Since  $y$  is multiplied by  $0.4$ , divide both sides by  $0.4$  to undo the multiplication.*

To solve more complicated equations, you may need to first simplify by using the Distributive Property or combining like terms.

## Example 5A: Simplifying Each Side Before Solving Equations

**Solve  $4 - 6a + 4a = -1 - 5(7 - 2a)$ .**

$$4 - 6a + 4a = -1 - 5(7 - 2a) \quad \text{Distribute } -5 \text{ to the expression in parentheses.}$$
$$4 - 6a + 4a = -1 - 5(7) - 5(-2a)$$

$$4 - 6a + 4a = -1 - 35 + 10a$$

$$4 - 2a = -36 + 10a$$

$$\begin{array}{r} +36 \\ \hline \end{array} \quad \begin{array}{r} +36 \\ \hline \end{array}$$

$$40 - 2a = 10a$$

$$\begin{array}{r} + 2a \\ \hline \end{array}$$

$$40 = 12a$$

*Combine like terms.*

*Since  $-36$  is added to  $10a$ , add  $36$  to both sides.*

*To collect the variable terms on one side, add  $2a$  to both sides.*

## Example 5A Continued

**Solve  $4 - 6a + 4a = -1 - 5(7 - 2a)$ .**

$$40 = 12a$$


$$\frac{40}{12} = \frac{12a}{12}$$

$$\frac{10}{3} = a$$

*Since  $a$  is multiplied by 12,  
divide both sides by 12.*

## Check It Out! Example 5a

**Solve**  $\frac{1}{2}(b+6) = \frac{3}{2}b - 1.$


$$\frac{1}{2}(b+6) = \frac{3}{2}b - 1$$

$$\frac{1}{2}b + 3 = \frac{3}{2}b - 1$$

$$\underline{-\frac{1}{2}b} \quad \underline{-\frac{1}{2}b}$$

$$3 = b - 1$$

$$\underline{+1} \quad \underline{+1}$$

$$4 = b$$

Distribute  $\frac{1}{2}$  to the expression in parentheses.

To collect the variable terms on one side, subtract  $\frac{1}{2}b$  from both sides.

Since 1 is subtracted from  $b$ , add 1 to both sides.

**Check It Out! Example 5b**

**Solve  $3x + 15 - 9 = 2(x + 2)$ .**

$3x + 15 - 9 = 2(x + 2)$  *Distribute 2 to the expression in parentheses.*

$3x + 15 - 9 = 2(x) + 2(2)$

$3x + 15 - 9 = 2x + 4$

$3x + 6 = 2x + 4$  *Combine like terms.*

$-2x$	$-2x$
<hr/>	<hr/>
$x + 6 =$	$4$
$- 6$	$- 6$
<hr/>	<hr/>
$x = -2$	

*To collect the variable terms on one side, subtract 2x from both sides.*

*Since 6 is added to x, subtract 6 from both sides to undo the addition.*



An **identity** is an equation that is true for all values of the variable. An equation that is an identity has infinitely many solutions.

Some equations are always false. These equations have no solutions.

# Identities and False Equations

<b>WORDS</b>	<b>Identity</b> When solving an equation, if you get an equation that is always true, the original equation is an identity, and it has infinitely many solutions.
<b>NUMBERS</b>	$2 + 1 = 2 + 1$ $3 = 3 \checkmark$
<b>ALGEBRA</b>	$2 + x = 2 + x$ $\begin{array}{r} -x \\ \hline 2 = 2 \checkmark \end{array}$

# Identities and False Equations

WORDS	<p>False Equations</p> <p>When solving an equation, if you get a false equation, the original equation has no solutions.</p>
NUMBERS	$1 = 1 + 2$ $1 = 3 \text{ ✗}$
ALGEBRA	$x = x + 3$ $\begin{array}{r} -x \quad -x \\ 0 = 3 \text{ ✗} \end{array}$ <div style="text-align: center;"><div style="display: inline-block; width: 100px; border-bottom: 1px solid red; margin: 0 10px;"></div><div style="display: inline-block; width: 100px; border-bottom: 1px solid red; margin: 0 10px;"></div></div>

## Example 6A: Infinitely Many Solutions or No Solutions

**Solve  $10 - 5x + 1 = 7x + 11 - 12x$ .**

$$10 - 5x + 1 = 7x + 11 - 12x$$

$$10 - 5x + 1 = 7x + 11 - 12x \quad \text{Identify like terms.}$$

$$11 - 5x = 11 - 5x \quad \text{Combine like terms on the left and the right.}$$

$$\begin{array}{r} + 5x \\ \hline 11 \end{array} \quad \begin{array}{r} + 5x \\ \hline 11 \end{array} \quad \text{Add } 5x \text{ to both sides.}$$

$$11 = 11 \checkmark \quad \text{True statement.}$$

The equation  $10 - 5x + 1 = 7x + 11 - 12x$  is an identity. All values of  $x$  will make the equation true. All real numbers are solutions.

## Example 6B: Infinitely Many Solutions or No Solutions

**Solve  $12x - 3 + x = 5x - 4 + 8x$ .**

$$12x - 3 + x = 5x - 4 + 8x$$

$$12x - 3 + x = 5x - 4 + 8x \quad \text{Identify like terms.}$$

$$13x - 3 = 13x - 4 \quad \text{Combine like terms on the left and the right.}$$

$$\begin{array}{r} -13x \\ \hline \end{array} \quad \begin{array}{r} -13x \\ \hline \end{array} \quad \text{Subtract } 13x \text{ from both sides.}$$

$$-3 = -4 \quad \text{False statement.}$$

The equation  $12x - 3 + x = 5x - 4 + 8x$  is a false equation. There is no value of  $x$  that will make the equation true. There are no solutions.

## Check It Out! Example 6a

**Solve  $4y + 7 - y = 10 + 3y$ .**

$$4y + 7 - y = 10 + 3y$$

$$4y + 7 - y = 10 + 3y \quad \text{Identify like terms.}$$

$$3y + 7 = 3y + 10 \quad \text{Combine like terms on the left and the right.}$$

$$\begin{array}{r} -3y \quad \quad -3y \\ \hline 7 = 10 \end{array} \quad \text{Subtract } 3y \text{ from both sides.}$$

$$7 = 10 \quad \text{False statement.}$$

The equation  $4y + 7 - y = 10 + 3y$  is a false equation. There is no value of  $y$  that will make the equation true. There are no solutions.

## Check It Out! Example 6b

**Solve  $2c + 7 + c = -14 + 3c + 21$ .**

$$2c + 7 + c = -14 + 3c + 21$$

$$2c + 7 + c = -14 + 3c + 21 \text{ Identify like terms.}$$

$$3c + 7 = 3c + 7 \quad \text{Combine like terms on the left and the right.}$$

$$\begin{array}{r} -3c \\ \hline \end{array} \quad \begin{array}{r} -3c \\ \hline \end{array} \quad \text{Subtract } 3c \text{ both sides.}$$

$$7 = 7 \quad \checkmark \text{ True statement.}$$

The equation  $2c + 7 + c = -14 + 3c + 21$  is an identity. All values of  $c$  will make the equation true. All real numbers are solutions.