Multiplying and Dividing Rational Expressions

Multiplying and Dividing Rational Expressions

Warm Up

Simplify each expression. Assume all variables are nonzero.

1. $x^5 \cdot x^2$ x^7

2. $y^3 \cdot y^3$ y^6

3. $\frac{x^6}{x^2}$ x^4

4. $\frac{y^2}{y^5}$ $\frac{1}{y^3}$

Factor each expression.

5. $x^2 - 2x - 8$ $(x - 4)(x + 2)$

6. $x^2 - 5x$ $x(x - 5)$

7. $x^5 - 9x^3$ $x^3(x - 3)(x + 3)$

Objectives

Simplify rational expressions.

Multiply and divide rational expressions.

Vocabulary

rational expression

Multiplying and Dividing Rational Expressions

A **rational expression** is a quotient of two polynomials. Other examples of rational expressions include the following:

$$\frac{x^2 - 4}{x + 2} \qquad \frac{10}{x^2 - 6} \qquad \frac{x + 3}{x - 7}$$

Multiplying and Dividing Rational Expressions

Because rational expressions are ratios of polynomials, you can simplify them the same way as you simplify fractions. Recall that to write a fraction in simplest form, you can divide out common factors in the numerator and denominator.

$$\frac{9}{24} = \frac{3 \cdot \cancel{3}}{8 \cdot \cancel{3}} = \frac{3}{8}$$

Caution!

When identifying values for which a rational expression is undefined, identify the values of the variable that make the original denominator equal to 0.

Example 1A: Simplifying Rational Expressions

Simplify. Identify any *x*-values for which the expression is undefined.

$$\frac{10x^8}{6x^4}$$

$$\frac{{}^{5}\cancel{10}x^{8-4}}{{}_{3}\cancel{6}} = \frac{5}{3}x^4$$

Quotient of Powers Property

The expression is undefined at $x = 0$ because this value of x makes $6x^4$ equal 0.

Example 1B: Simplifying Rational Expressions

Simplify. Identify any *x*-values for which the expression is undefined.

$$\frac{x^2 + x - 2}{x^2 + 2x - 3}$$

$$\frac{(x+2)\cancel{(x-1)}}{\cancel{(x-1)}(x+3)} = \frac{(x+2)}{(x+3)}$$

Factor; then divide out common factors.

The expression is undefined at $x = 1$ and $x = -3$ because these values of x make the factors $(x - 1)$ and $(x + 3)$ equal 0.

You Try! Example 1A

Simplify. Identify any *x*-values for which the expression is undefined.

$$\frac{16x^{11}}{8x^2}$$

$$\frac{{}^{2}\cancel{8}x^{11-2}}{{}_{1}\cancel{8}} = 2x^9$$

Quotient of Powers Property

The expression is undefined at $x = 0$ because this value of x makes $8x^2$ equal 0.

You Try! Example 1B

Simplify. Identify any *x*-values for which the expression is undefined.

$$\frac{3x + 4}{3x^2 + x - 4}$$

$$\frac{\cancel{(3x + 4)}}{\cancel{(3x + 4)}(x - 1)} = \frac{1}{(x - 1)}$$

Factor; then divide out common factors.

The expression is undefined at $x = 1$ and $x = -\frac{4}{3}$ because these values of x make the factors $(x - 1)$ and $(3x + 4)$ equal 0.

Example 1C

Simplify. Identify any *x*-values for which the expression is undefined.

$$\frac{6x^2 + 7x + 2}{6x^2 - 5x - 6}$$

$$\frac{(2x + 1)\cancel{(3x + 2)}}{\cancel{(3x + 2)}(2x - 3)} = \frac{(2x + 1)}{(2x - 3)}$$

Factor; then divide out common factors.

The expression is undefined at $x = -\frac{2}{3}$ and $x = \frac{3}{2}$ because these values of x make the factors $(3x + 2)$ and $(2x - 3)$ equal 0.

Multiplying and Dividing Rational Expressions

Example 2: Simplifying by Factoring by –1

Simplify $\dfrac{4x - x^2}{x^2 - 2x - 8}$. Identify any x values for which the expression is undefined.

$$\frac{-1(x^2 - 4x)}{x^2 - 2x - 8}$$

Factor out –1 in the numerator so that x^2 is positive, and reorder the terms.

$$\frac{-1(x)\cancel{(x - 4)}}{\cancel{(x - 4)}(x + 2)}$$

Factor the numerator and denominator. Divide out common factors.

$$\frac{-x}{(x + 2)}$$

Simplify.

The expression is undefined at $x = -2$ and $x = 4$.

You Try! Example 2A

Simplify $\dfrac{10 - 2x}{x - 5}$. Identify any x values for which the expression is undefined.

$\dfrac{-1(2x - 10)}{x - 5}$ *Factor out –1 in the numerator so that x is positive, and reorder the terms.*

$\dfrac{-1(2)\cancel{(x - 5)}}{\cancel{(x - 5)}}$ *Factor the numerator and denominator. Divide out common factors.*

$\dfrac{-2}{1}$ *Simplify.*

The expression is undefined at $x = 5$.

Multiplying and Dividing Rational Expressions

You can multiply rational expressions the same way that you multiply fractions.

Multiplying Rational Expressions
1. Factor all numerators and denominators completely.
2. Divide out common factors of the numerators and denominators.
3. Multiply numerators. Then multiply denominators.
4. Be sure the numerator and denominator have no common factors other than 1.

Example 3: Multiplying Rational Expressions

Multiply. Assume that all expressions are defined.

A. $\dfrac{3x^5y^3}{2x^3y^7} \cdot \dfrac{10x^3y^4}{9x^2y^5}$

$\dfrac{\cancel{3}x^{\cancel{5}^{3}}\cancel{y^3}}{\cancel{2}\cancel{x^3}\cancel{y^7}} \cdot {}^{5}\dfrac{\cancel{10}\cancel{x^3}\cancel{y^4}}{{}_{3}\cancel{9}\cancel{x^2}\cancel{y^5}}$

$\dfrac{5x^3}{3y^5}$

B. $\dfrac{x-3}{4x+20} \cdot \dfrac{x+5}{x^2-9}$

$\dfrac{\cancel{x-3}}{4\cancel{(x+5)}} \cdot \dfrac{\cancel{x+5}}{\cancel{(x-3)}(x+3)}$

$\dfrac{1}{4(x+3)}$

Multiplying and Dividing Rational Expressions

You Try! Example 3

Multiply. Assume that all expressions are defined.

C. $\dfrac{x}{15} \cdot \dfrac{x^7}{2x} \cdot \dfrac{20}{x^4}$

$\dfrac{x}{\cancel{15}_{3}} \cdot \dfrac{\cancel{x^7}^{\,2}}{\cancel{2x}} \cdot \dfrac{\cancel{20}^{\,2}}{\cancel{x^4}}$

$\dfrac{2x^3}{3}$

D. $\dfrac{10x - 40}{x^2 - 6x + 8} \cdot \dfrac{x + 3}{5x + 15}$

$\dfrac{^{2}\cancel{10(x - 4)}}{\cancel{(x - 4)}(x - 2)} \cdot \dfrac{\cancel{x + 3}}{5\cancel{(x + 3)}}$

$\dfrac{2}{(x - 2)}$

Multiplying and Dividing Rational Expressions

You can also divide rational expressions. Recall that to divide by a fraction, you multiply by its reciprocal.

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{\cancel{2}} \bullet \frac{\cancel{4}^{2}}{3} = \frac{2}{3}$$

Example 4A: Dividing Rational Expressions

Divide. Assume that all expressions are defined.

$$\frac{5x^4}{8x^2y^2} \div \frac{15}{8y^5}$$

$$\frac{5x^4}{8x^2y^2} \cdot \frac{8y^5}{15}$$

Rewrite as multiplication by the reciprocal.

$$\frac{\cancel{5}x^{\cancel{4}^{2}}}{\cancel{8}\cancel{x^2}\cancel{y^2}} \cdot \frac{\cancel{8}y^{\cancel{5}^{3}}}{\cancel{15}_{3}}$$

$$\frac{x^2y^3}{3}$$

Multiplying and Dividing Rational Expressions

Example 4B: Dividing Rational Expressions

Divide. Assume that all expressions are defined.

$$\frac{x^4 - 9x^2}{x^2 - 4x + 3} \div \frac{x^4 + 2x^3 - 8x^2}{x^2 - 16}$$

$$\frac{x^4 - 9x^2}{x^2 - 4x + 3} \cdot \frac{x^2 - 16}{x^4 + 2x^3 - 8x^2}$$

$$\frac{x^2(x^2 - 9)}{x^2 - 4x + 3} \cdot \frac{x^2 - 16}{x^2(x^2 + 2x - 8)}$$

$$\frac{\cancel{x^2}\cancel{(x - 3)}(x + 3)}{\cancel{(x - 3)}(x - 1)} \cdot \frac{\cancel{(x + 4)}(x - 4)}{\cancel{x^2}(x - 2)\cancel{(x + 4)}}$$

$$\frac{(x + 3)(x - 4)}{(x - 1)(x - 2)}$$

Rewrite as multiplication by the reciprocal.

You Try! Example 4A

Divide. Assume that all expressions are defined.

$$\frac{x^2}{4} \div \frac{x^4y}{12y^2}$$

$$\frac{x^2}{4} \cdot \frac{12y^2}{x^4y}$$

Rewrite as multiplication by the reciprocal.

$$\frac{\cancel{x^2}}{\cancel{4}} \cdot \frac{{}^{3}\cancel{12}y^{2/1}}{x^{\cancel{4}\,{}^{2}}\cancel{y}}$$

$$\frac{3y}{x^2}$$

Multiplying and Dividing Rational Expressions

You Try! Example 4B

Divide. Assume that all expressions are defined.

$$\frac{2x^2 - 7x - 4}{x^2 - 9} \div \frac{4x^2 - 1}{8x^2 - 28x + 12}$$

$$\frac{2x^2 - 7x - 4}{x^2 - 9} \cdot \frac{8x^2 - 28x + 12}{4x^2 - 1}$$

$$\frac{(2x + 1)(x - 4)}{(x + 3)(x - 3)} \cdot \frac{4(2x^2 - 7x + 3)}{(2x + 1)(2x - 1)}$$

$$\frac{\cancel{(2x + 1)}(x - 4)}{(x + 3)\cancel{(x - 3)}} \cdot \frac{4\cancel{(2x - 1)}\cancel{(x - 3)}}{\cancel{(2x + 1)}\cancel{(2x - 1)}}$$

$$\frac{4(x - 4)}{(x + 3)}$$

Note Card Check: Part I

Simplify. Identify any *x*-values for which the expression is undefined.

1. $\dfrac{x^2 - 6x + 5}{x^2 - 3x - 10}$

$\dfrac{x - 1}{x + 2}$ $x \neq -2, 5$

2. $\dfrac{6x - x^2}{x^2 - 7x + 6}$

$\dfrac{-x}{x - 1}$ $x \neq 1, 6$

Note Card Check: Part II

Multiply or divide. Assume that all expressions are defined.

3. $\dfrac{x+1}{3x+6} \cdot \dfrac{6x+12}{x^2-1}$

$\dfrac{2}{x-1}$

4. $\dfrac{x^2+4x+3}{x^2-4} \div \dfrac{x^2+2x-3}{x^2-6x+8}$

$\dfrac{(x+1)(x-4)}{(x+2)(x-1)}$

Homework

TEXTBOOK : pg. 190-191

#3-7 ODD, 8-14 ALL

19 & 21, 28-31 ALL