

Similarity

Warm Up

1. If $\triangle QRS \cong \triangle ZYX$, identify the pairs of congruent angles and the pairs of congruent sides.

$$\angle Q \cong \angle Z; \angle R \cong \angle Y; \angle S \cong \angle X;$$
$$\overline{QR} \cong \overline{ZY}; \overline{RS} \cong \overline{YX}; \overline{QS} \cong \overline{ZX}$$

Solve each proportion.

2. $\frac{2}{x-3} = \frac{8}{3x-3}$

$$x = 9$$

3. $\frac{x-6}{42} = \frac{2x-14}{77}$

$$x = 18$$

Objectives

Identify similar polygons.

Apply properties of similar polygons to solve problems.

Vocabulary

similar

similar polygons

similarity ratio

Figures that are **similar** ($\sim$) have the same shape but not necessarily the same size.

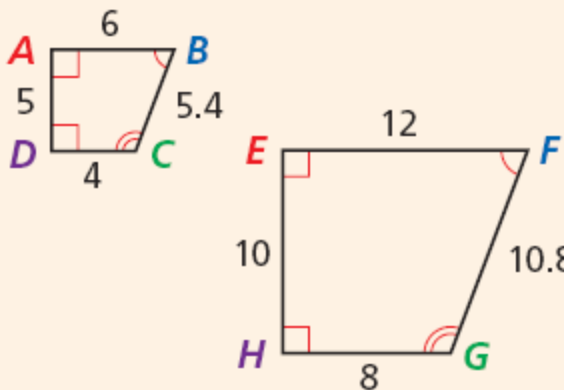


$\triangle 1$ is similar to $\triangle 2$ ($\triangle 1 \sim \triangle 2$).



$\triangle 1$ is not similar to $\triangle 3$ ($\triangle 1 \not\sim \triangle 3$).

Similar Polygons

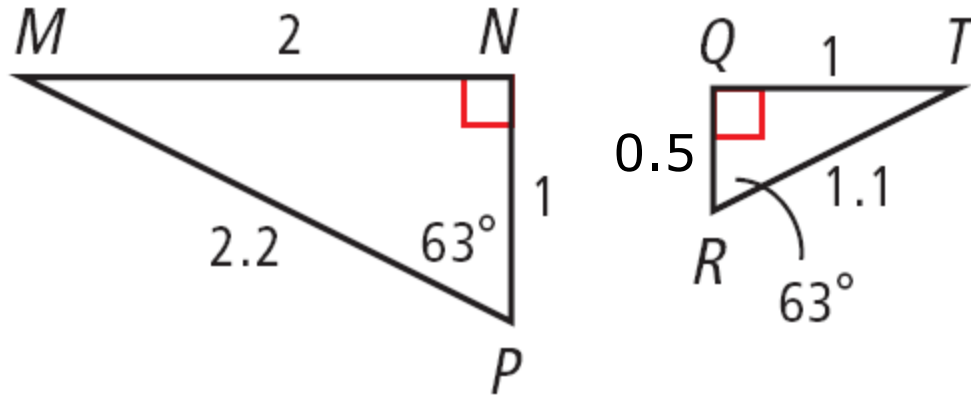
DEFINITION	DIAGRAM	STATEMENTS
Two polygons are similar polygons if and only if their corresponding angles are congruent and their corresponding side lengths are proportional.	 $ABCD \sim EFGH$	$\begin{aligned} \angle A &\cong \angle E \\ \angle B &\cong \angle F \\ \angle C &\cong \angle G \\ \angle D &\cong \angle H \\ \frac{AB}{EF} &= \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE} = \frac{1}{2} \end{aligned}$

“Similar Triangles.”

<http://www.mathopenref.com/similartriangles.html>

Example 1: Describing Similar Polygons

Identify the pairs of congruent angles and corresponding sides.



$\angle N \cong \angle Q$ and $\angle P \cong \angle R$.

By the Third Angles Theorem, $\angle M \cong \angle T$.

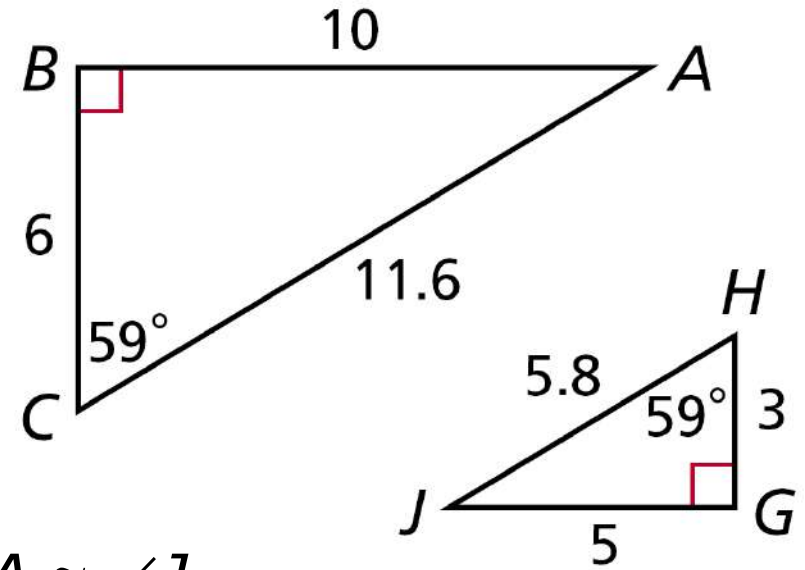
$$\frac{MP}{TR} = \frac{2.2}{1.1} = 2$$

$$\frac{MN}{TQ} = \frac{2}{1} = 2$$

$$\frac{NP}{QR} = \frac{1}{0.5} = 2$$

Check It Out! Example 1

Identify the pairs of congruent angles and corresponding sides.



$\angle B \cong \angle G$ and $\angle C \cong \angle H$.

By the Third Angles Theorem, $\angle A \cong \angle J$.

$$\frac{AB}{JG} = \frac{10}{5} = 2$$

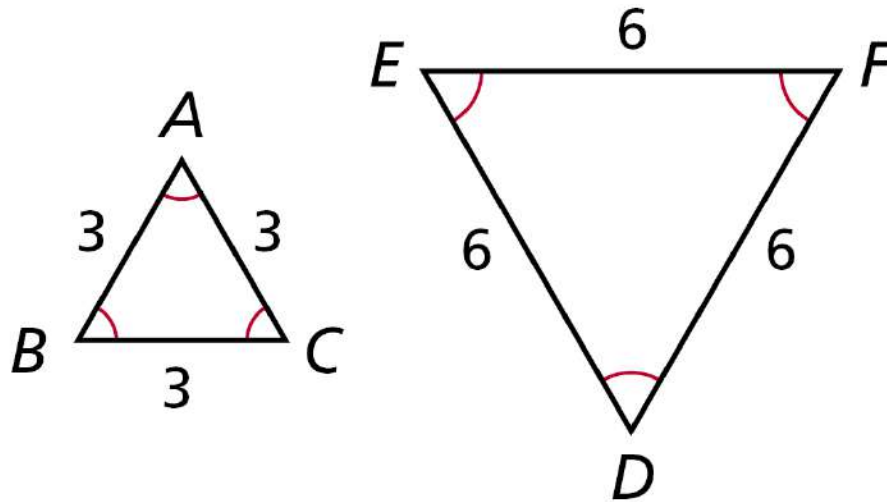
$$\frac{BC}{GH} = \frac{6}{3} = 2$$

$$\frac{AC}{JH} = \frac{11.6}{5.8} = 2$$

A **similarity ratio** is the ratio of the lengths of the corresponding sides of two similar polygons.

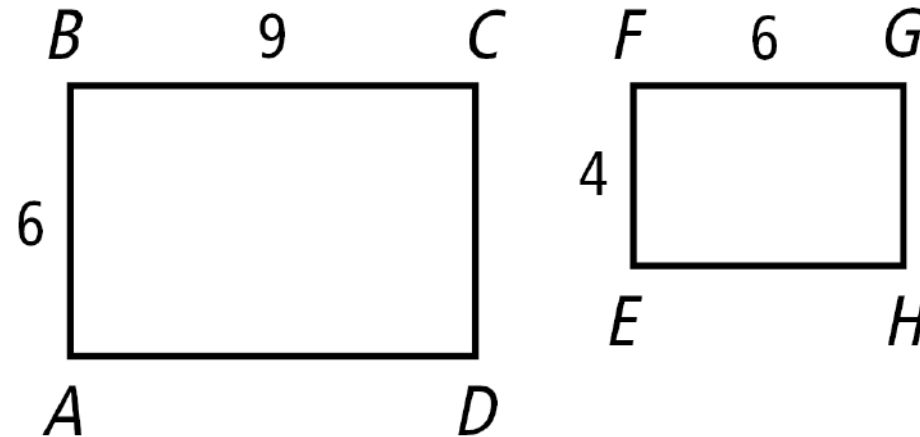
The similarity ratio of $\triangle ABC$ to $\triangle DEF$ is $\frac{3}{6}$, or $\frac{1}{2}$.

The similarity ratio of $\triangle DEF$ to $\triangle ABC$ is $\frac{6}{3}$, or 2.



Example 2A: Identifying Similar Polygons

**Determine whether the polygons are similar.
If so, write the similarity ratio and a
similarity statement.**



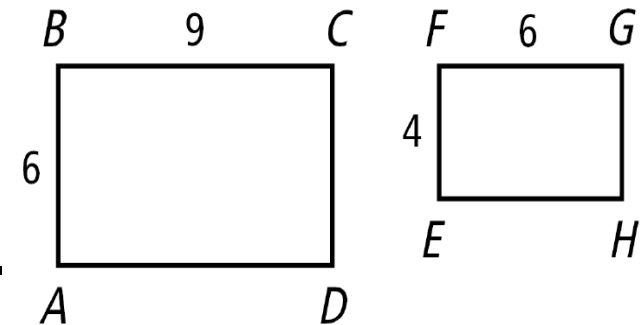
rectangles $ABCD$ and $EFGH$

Example 2A Continued

Step 1 Identify pairs of congruent angles.

$$\angle A \cong \angle E, \angle B \cong \angle F, \\ \angle C \cong \angle G, \text{ and } \angle D \cong \angle H.$$

*All $\angle$ s of a rect. are rt. $\angle$ s
and are $\cong$.*



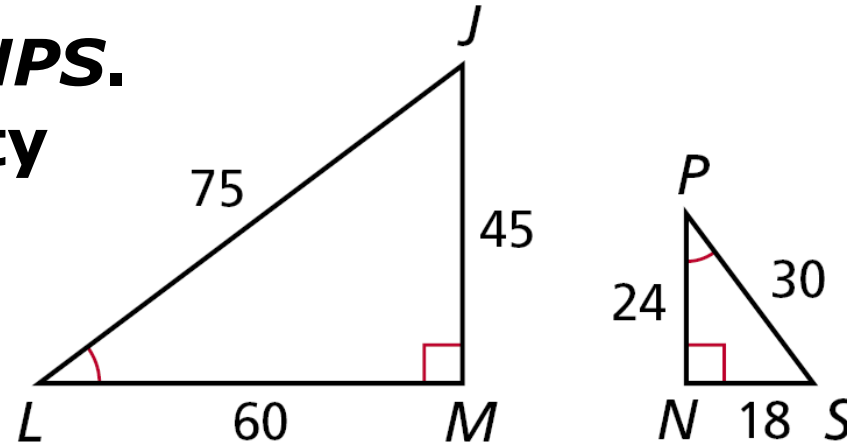
Step 2 Compare corresponding sides.

$$\frac{AB}{EF} = \frac{6}{4} = \frac{3}{2}, \frac{BC}{FG} = \frac{9}{6} = \frac{3}{2}, \frac{CD}{GH} = \frac{AB}{EF} = \frac{3}{2}, \text{ and } \frac{DA}{HE} = \frac{BC}{FG} = \frac{3}{2}.$$

Thus the similarity ratio is $\frac{3}{2}$, and rect. $ABCD \sim$ rect. $EFGH$.

Check It Out! Example 2

Determine if $\triangle JLM \sim \triangle NPS$.
If so, write the similarity
ratio and a similarity
statement.



Step 1 Identify pairs of congruent angles.

$$\angle N \cong \angle M, \angle L \cong \angle P, \angle S \cong \angle J$$

Check It Out! Example 2 Continued

Step 2 Compare corresponding sides.

$$\frac{LJ}{PS} = \frac{75}{30} = \frac{5}{2}, \frac{LM}{PN} = \frac{60}{24} = \frac{5}{2}, \frac{JM}{SN} = \frac{45}{18} = \frac{5}{2}$$

Thus the similarity ratio is $\frac{5}{2}$, and $\triangle LMJ \sim \triangle PNS$.

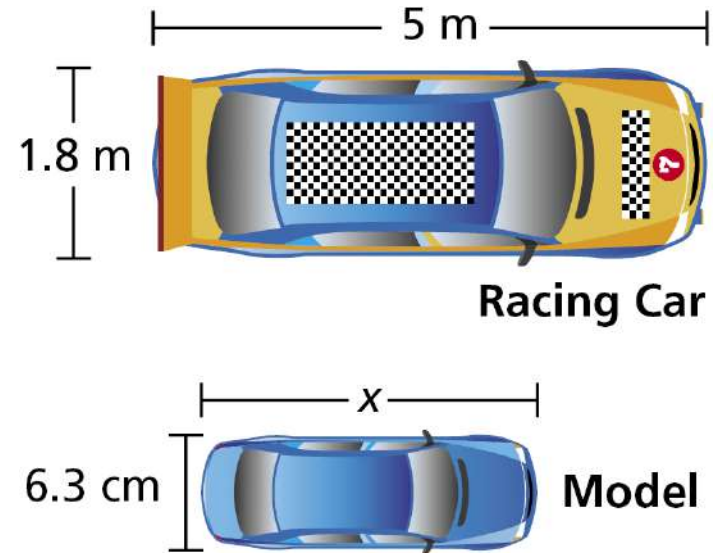
Helpful Hint

When you work with proportions, be sure the ratios compare corresponding measures.

Example 3: Hobby Application

Find the length of the model to the nearest tenth of a centimeter.

Let x be the length of the model in centimeters. The rectangular model of the racing car is similar to the rectangular racing car, so the corresponding lengths are proportional.



Example 3 Continued

$$\frac{\text{length of racing car}}{\text{length of model}} = \frac{\text{width of racing car}}{\text{width of model}}$$

$$\frac{5}{x} = \frac{1.8}{6.3}$$

$$5(6.3) = x(1.8) \quad \text{Cross Products Prop.}$$

$$31.5 = 1.8x \quad \text{Simplify.}$$

$$17.5 = x \quad \text{Divide both sides by 1.8.}$$

The length of the model is 17.5 centimeters.

Check It Out! Example 3

**A boxcar has the dimensions shown.
A model of the boxcar is 1.25 in. wide. Find
the length of the model to the nearest inch.**



$$\frac{\text{length of boxcar}}{\text{length of model}} = \frac{\text{width of boxcar}}{\text{width of model}}$$

$$\frac{36.25}{x} = \frac{9}{1.25}$$

Check It Out! Example 3 Continued

$$\frac{36.25}{x} = \frac{9}{1.25}$$

$$1.25(36.25) = x(9) \quad \text{Cross Products Prop.}$$

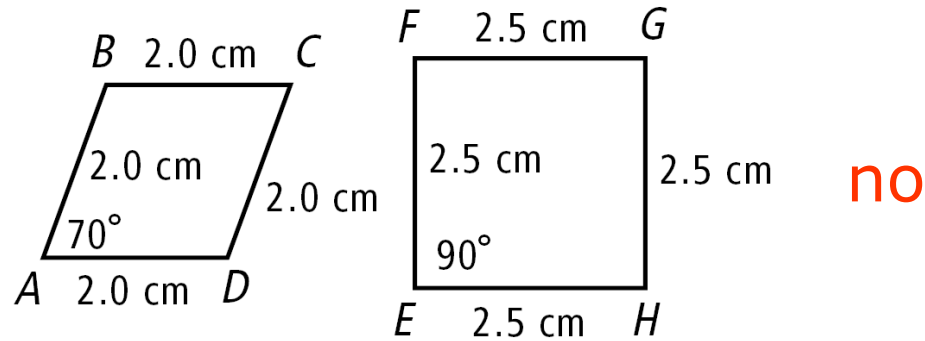
$$45.3 = 9x \quad \text{Simplify.}$$

$$5 \approx x \quad \text{Divide both sides by 9.}$$

The length of the model is approximately 5 inches.

Lesson Quiz: Part I

1. Determine whether the polygons are similar. If so, write the similarity ratio and a similarity statement.



2. The ratio of a model sailboat's dimensions to the actual boat's dimensions is $\frac{1}{30}$. If the length of the model is 10 inches, what is the length of the actual sailboat in feet?

25 ft

Lesson Quiz: Part II

- 3.** Tell whether the following statement is sometimes, always, or never true. Two equilateral triangles are similar.

Always