Warm Up

1. If $\triangle QRS \cong \triangle ZYX$, identify the pairs of congruent angles and the pairs of congruent sides.

$\angle Q \cong \angle Z$; $\angle R \cong \angle Y$; $\angle S \cong \angle X$;
$\overline{QR} \cong \overline{ZY}$; $\overline{RS} \cong \overline{YX}$; $\overline{QS} \cong \overline{ZX}$

Solve each proportion.

2. $\dfrac{2}{x-3} = \dfrac{8}{3x-3}$

$x = 9$

3. $\dfrac{x-6}{42} = \dfrac{2x-14}{77}$

$x = 18$

Objectives

Identify similar polygons.

Apply properties of similar polygons to solve problems.

Vocabulary

similar

similar polygons

similarity ratio

Figures that are **similar** (~) have the same shape but not necessarily the same size.
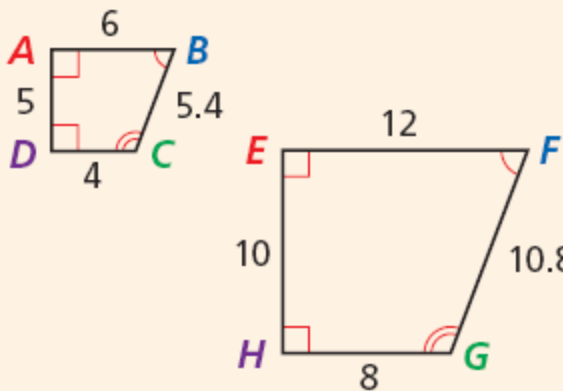

△1 is similar to △2(△1 ~ △2).

△1 is not similar to △3(△1 ≁ △3).

Similar Polygons

DEFINITION	DIAGRAM	STATEMENTS
Two polygons are **similar polygons** if and only if their corresponding angles are congruent and their corresponding side lengths are proportional.	$ABCD \sim EFGH$	$\angle A \cong \angle E$ $\angle B \cong \angle F$ $\angle C \cong \angle G$ $\angle D \cong \angle H$ $\dfrac{AB}{EF} = \dfrac{BC}{FG} = \dfrac{CD}{GH} = \dfrac{DA}{HE} = \dfrac{1}{2}$

Example 1: Describing Similar Polygons

Identify the pairs of congruent angles and corresponding sides.
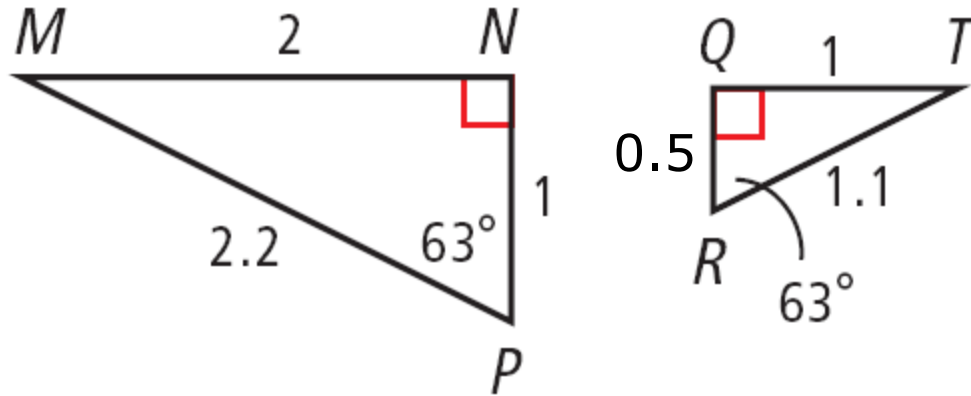
$\angle N \cong \angle Q$ and $\angle P \cong \angle R$.
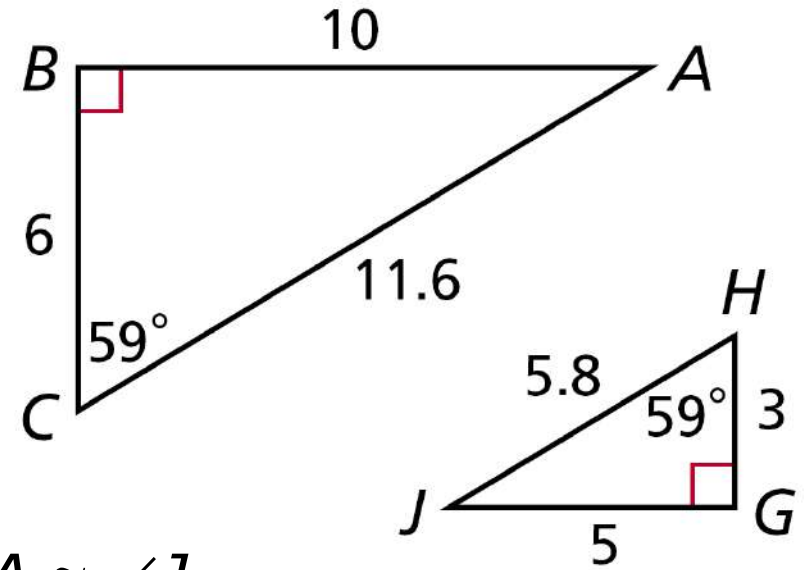By the Third Angles Theorem, $\angle M \cong \angle T$.

$$\frac{MP}{TR} = \frac{2.2}{1.1} = 2 \qquad \frac{MN}{TQ} = \frac{2}{1} = 2 \qquad \frac{NP}{QR} = \frac{1}{0.5} = 2$$

Identify the pairs of congruent angles and corresponding sides.

$\angle B \cong \angle G$ and $\angle C \cong \angle H$.
By the Third Angles Theorem, $\angle A \cong \angle J$.
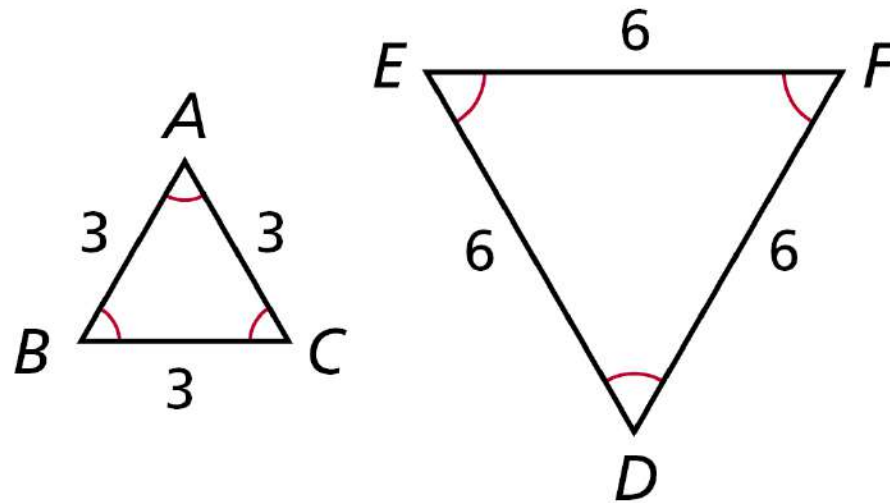
$$\frac{AB}{JG} = \frac{10}{5} = 2 \qquad \frac{BC}{GH} = \frac{6}{3} = 2 \qquad \frac{AC}{JH} = \frac{11.6}{5.8} = 2$$

A **similarity ratio** is the ratio of the lengths of the corresponding sides of two similar polygons.

The similarity ratio of $\triangle ABC$ to $\triangle DEF$ is $\frac{3}{6}$, or $\frac{1}{2}$.

The similarity ratio of $\triangle DEF$ to $\triangle ABC$ is $\frac{6}{3}$, or 2.
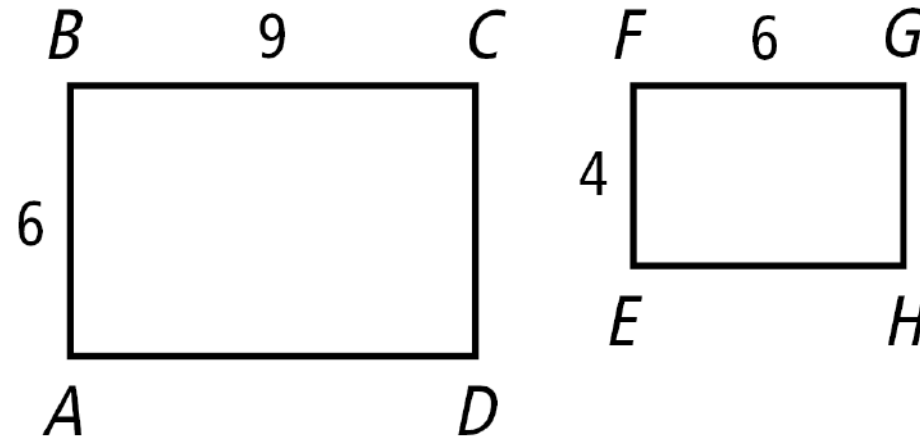
Writing Math

Writing a similarity statement is like writing a congruence statement—be sure to list corresponding vertices in the same order.

Example 2A: Identifying Similar Polygons

Determine whether the polygons are similar. If so, write the similarity ratio and a similarity statement.

rectangles *ABCD* and *EFGH*

Step 1 Identify pairs of congruent angles.

$\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, and $\angle D \cong \angle H$.

All $\angle$s of a rect. are rt. $\angle$s and are $\cong$.

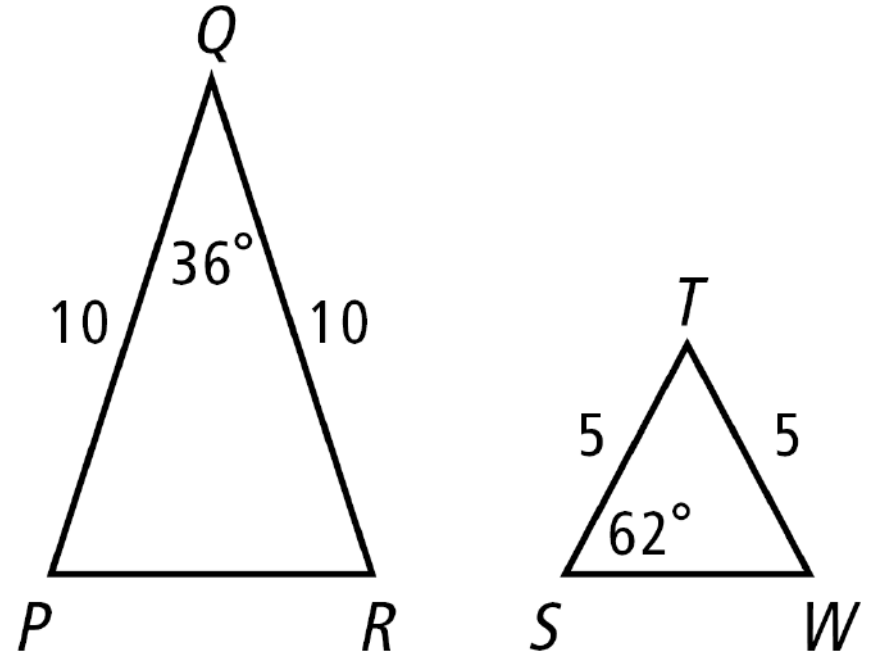**Step 2** Compare corresponding sides.

$$\frac{AB}{EF} = \frac{6}{4} = \frac{3}{2}, \frac{BC}{FG} = \frac{9}{6} = \frac{3}{2}, \frac{CD}{GH} = \frac{AB}{EF} = \frac{3}{2}, \text{ and } \frac{DA}{HE} = \frac{BC}{FG} = \frac{3}{2}.$$

Thus the similarity ratio is $\frac{3}{2}$, and rect. *ABCD* ~ rect. *EFGH*.

Example 2B: Identifying Similar Polygons

Determine whether the polygons are similar. If so, write the similarity ratio and a similarity statement.

$\triangle ABCD$ and $\triangle EFGH$

Example 2B Continued

Step 1 Identify pairs of congruent angles.

$\angle P \cong \angle R$ and $\angle S \cong \angle W$ *isos. Δ*

Step 2 Compare corresponding angles.
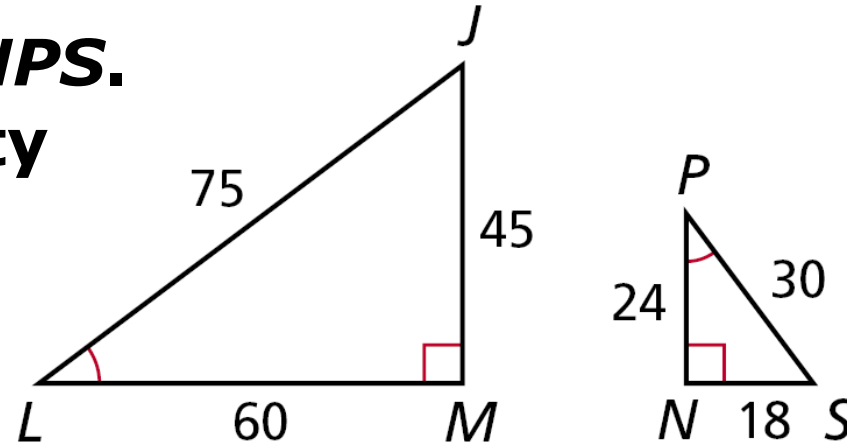
$$m\angle P = m\angle R = \frac{1}{2}\left(180° - 36°\right) = 72°$$

$$m\angle W = m\angle S = 62°$$

$$m\angle T = 180° - 2(62°) = 56°$$

Since no pairs of angles are congruent, the triangles are not similar.

Determine if △JLM ~ △NPS.
If so, write the similarity
ratio and a similarity
statement.

Step 1 Identify pairs of congruent angles.

∠N ≅ ∠M, ∠L ≅ ∠P, ∠S ≅ ∠J

Step 2 Compare corresponding sides.

$$\frac{LJ}{PS} = \frac{75}{30} = \frac{5}{2}, \frac{LM}{PN} = \frac{60}{24} = \frac{5}{2}, \frac{JM}{SN} = \frac{45}{18} = \frac{5}{2}$$

Thus the similarity ratio is $\frac{5}{2}$, and $\Delta LMJ \sim \Delta PNS$.

Helpful Hint

When you work with proportions, be sure the ratios compare corresponding measures.

Example 3: Hobby Application

Find the length of the model to the nearest tenth of a centimeter.

Let x be the length of the model in centimeters. The rectangular model of the racing car is similar to the rectangular racing car, so the corresponding lengths are proportional.
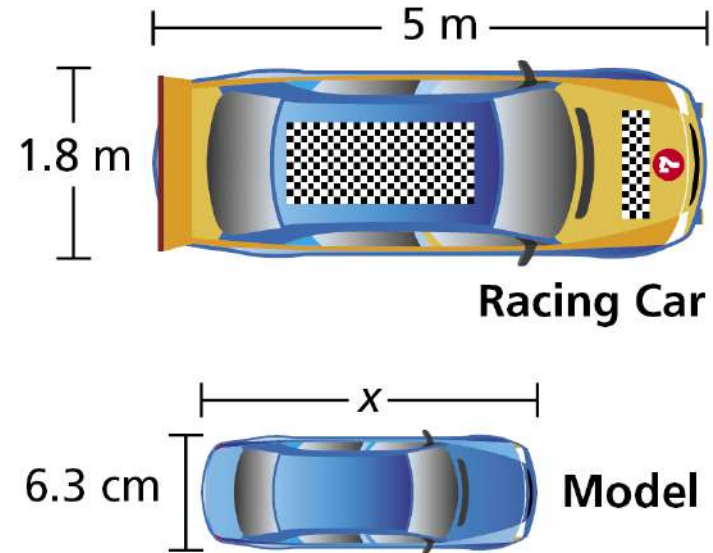
Racing Car

Model

Example 3 Continued

$$\frac{\text{length of racing car}}{\text{length of model}} = \frac{\text{width of racing car}}{\text{width of model}}$$

$$\frac{5}{x} = \frac{1.8}{6.3}$$

$5(6.3) = x(1.8)$ *Cross Products Prop.*

$31.5 = 1.8x$ *Simplify.*

$17.5 = x$ *Divide both sides by 1.8.*

The length of the model is 17.5 centimeters.

Check It Out! Example 3

A boxcar has the dimensions shown. A model of the boxcar is 1.25 in. wide. Find the length of the model to the nearest inch.

36.25 ft

x in.

9 ft | Boxcar |

Model | 1.25 in.

$$\frac{\text{length of boxcar}}{\text{length of model}} = \frac{\text{width of boxcar}}{\text{width of model}}$$

$$\frac{36.25}{x} = \frac{9}{1.25}$$

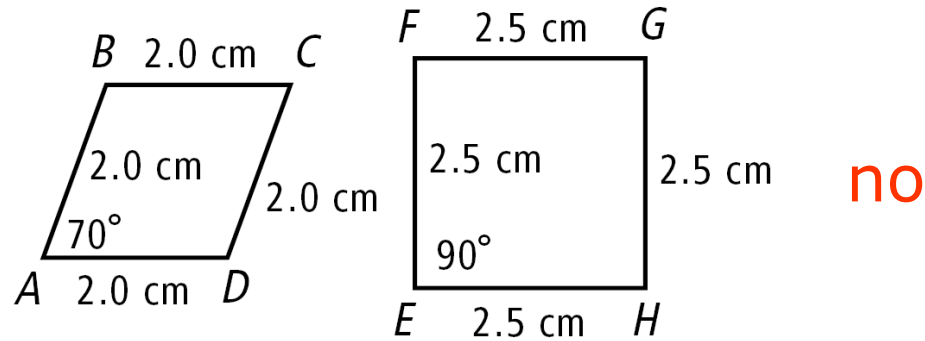$$\frac{36.25}{x} = \frac{9}{1.25}$$

$1.25(36.25) = x(9)$ *Cross Products Prop.*

$45.3 = 9x$ *Simplify.*

$5 \approx x$ *Divide both sides by 9.*

The length of the model is approximately 5 inches.

Lesson Quiz: Part I

1. Determine whether the polygons are similar. If so, write the similarity ratio and a similarity statement.

B 2.0 cm C

2.0 cm

2.0 cm

70°

A 2.0 cm D

F 2.5 cm G

2.5 cm

2.5 cm

90°

E 2.5 cm H

no

2. The ratio of a model sailboat's dimensions to the actual boat's dimensions is $\frac{1}{30}$. If the length of the model is 10 inches, what is the length of the actual sailboat in feet?

25 ft

3. Tell whether the following statement is sometimes, always, or never true. Two equilateral triangles are similar.

Always