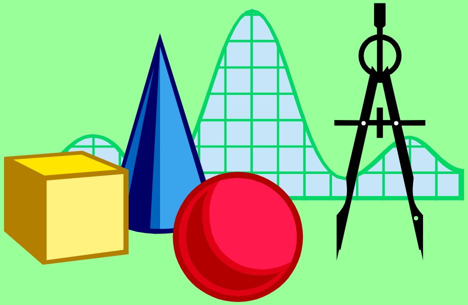
Section 4.7

Sequences
&
Series



Sequences – sets of numbers

Notation:

$a_n = \left(\text{represents the formula for finding terms}\right)$

$n = \text{term number}$

a_4 is the notation for the 4$^{\text{th}}$ term

a_{32} is the notation for the 32$^{\text{nd}}$ term

Examples:

If $a_n = 2n - 3,$ find the first 5 terms.

If $\boldsymbol{a_n = -3n + 1}$, find the 20th term.

Ex.1

Find the first four terms of the sequence

$$a_n = 3n - 2$$

$$a_1 = 3(1) - 2 = 1$$ **First term**

$$a_2 = 4$$ **Second term**

$$a_3 = 7$$ **Third term**

$$a_4 = 10$$ **Fourth term**

Writing Rules for Sequences

We can calculate as many terms as we want as long as we know the rule or equation for a_n.

Example:

3, 5, 7, 9, ___ , ___,……. ______ .

$a_n = 2n + 1$

Arithmetic Sequences

- When you want to find a large sequence, this process is long and there is great room for error.
- To find the 20^{th}, 45^{th}, etc. term use the following formula:

$$\mathbf{a_n = a_1 + (n - 1)d}$$

Arithmetic Sequences

$$a_n = a_1 + (n - 1)d$$

Where:

a_1 is the first number in the sequence

n is the number of the term you are looking for

d is the common difference

a_n is the value of the term you are looking for

Arithmetic Sequences

- Find the 15th term of the sequence:

34, 23, 12,…

Using the formula $\mathbf{a_n = a_1 + (n - 1)d}$,

$\mathbf{a_1 = 34}$

$\mathbf{d = -11}$

$\mathbf{n = 15}$

$\mathbf{a_n = 34 + (n-1)(-11) = -11n + 45}$

$\mathbf{a_{15} = -11(15) + 45}$

$\mathbf{a_{15} = -120}$

Geometric Sequences

- Again, use a formula to find large numbers.
- $$a_n = a_1 \bullet (r)^{n-1}$$

Geometric Sequences

- **Find the 10^{th} term of the sequence :**

4,8,16,…

$a_n = a_1 \cdot (r)^{n-1}$

- **$a_1 = 4$**
- **$r = 2$**
- **$n = 10$**

Geometric Sequences

$$a_n = a_1 \bullet (r)^{n-1}$$

$$a_{10} = 4 \bullet (2)^{10-1}$$

$$a_{10} = 4 \bullet (2)^{9}$$

$$a_{10} = 4 \bullet 512$$

$$a_{10} = 2048$$

Series – the sum of a certain number of terms of a sequence

Sigma Notation: $\displaystyle\sum_{i=1}^{n} a_i$

Stop

Formula

Start

"Add up the terms in the sequence beginning at term number 1 and going through term number "n".

Examples

1. $\sum_{i=1}^{4} -5i = -5(1) + -5(2) + -5(3) + -5(4) = -50$

2. $\sum_{i=1}^{5} \frac{1}{2} i + 1 = \frac{25}{2}$

3. $\sum_{i=3}^{7} i = 25$

4. $\sum_{i=1}^{6} 3$

SUMMATION NOTATION

Sum of the terms of a finite sequence

Upper limit of summation

(Ending point)

$$\sum_{i=1}^{n} a_i =$$

Lower limit of summation

(Starting point)

Ex 7b

$$\sum_{k=3}^{6}(1+k^2) = (1+3^2)+(1+4^2)+(1+5^2)+(1+6^2)$$

$$= (10)+(17)+(26)+(37)$$

$$= 90$$