Curriculum Framework for Mathematics

School: Sussex Preparatory Academy **Curricular Tool:** Holt McDougal *Algebra* **Course:** Algebra I

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Unit One: An Introduction To Algebra **Timeline : 2 weeks**			
Interpret the structure of expressions Interpret expressions that represent a quantity in terms of its context.* **CC.9-12.A.SSE.1** a. Interpret parts of an expression, such as terms, factors, and coefficients. **CC.9-12.A.SSE.1a** b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* **CC.9-12.A.SSE.1b**	Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.	Translate verbal expressions into mathematical expressions Write an expression containing identical factors as an expression using exponents Solve open sentences by performing arithmetic operations Recognize and use the properties of identity and equality Translate verbal expressions into equations and formulas Explore problem situations by asking and answering questions	**Informal:** Lesson Quiz Exit Tickets Journal Prompts Homework **Formal Assessment:** Unit Test Portfolio
Unit Two: Rational Numbers **Timeline: 2 weeks**			
Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **CC.9-12.A.CED.1** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **CC.9-12. A.REI.3**	An equation is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function.	Graphing/ Identifying Independent and Dependant variables Simplify expressions that contain rational numbers Multiply rational numbers Divide rational numbers	**Informal:** Lesson Quiz Exit Tickets Journal Prompts Homework **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. **CC.9-12.N.RN.3**	Numeric relationships can be symbolically represented as equations and inequalities and fluency in transforming these symbolic representations is a tool for graphing and solving problems. Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation. How properties of rational exponents, rational number, and irrational number are defined using characteristic patterns of equivalency and computation, to build a comprehensive knowledge of the structure and order of the real number system.	Define variables and write equations for verbal problems Write verbal problems for equations Solve problems involving direct variations Solve problems involving inverse variations	
Unit Three: Equations and Inequalities **Timeline: 3 weeks**			
Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* ***CC.9-12.*** **A-CED.1** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ***CC.9-12.*** **A-CED.2** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For*	An equation is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Numeric relationships can be symbolically represented as equations and inequalities and fluency in transforming these symbolic representations is a tool	Solve equations and inequalities by using addition Solve equations and inequalities by using subtraction Solve equations and inequalities by using multiplication. Solve equations and inequalities by using division. Solve problems by working	**Informal:** Lesson Quiz Exit Tickets Journal Prompts Homework **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
example, rearrange Ohm's law V = IR to highlight resistance R. ***CC.9-12.*** **A-CED.4** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **CC.9-12. A-REI.1.** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **CC.9-12. A-REI.3**	for graphing and solving problems. Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation.	backwards Solve equations and inequalities involving more than one operation Solve equations and inequalities with the variables on both sides Solve equations and inequalities containing grouping symbols Solve equations and inequalities s containing fractions or decimals	
Unit Four: Linear Functions **Timeline: 2 weeks**			
Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **CC.9-12. A-REI.6.** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **CC.9-12. F-IF.6**. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **CC.9-12. F-IF.7.** a. Graph linear and quadratic functions and show intercepts, maxima, and minima. Distinguish between situations that can be modeled with linear functions and with exponential functions. **CC.9-12. F-LE.1.** a. Prove that linear functions grow by equal	Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation. Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns.	Rate of Change in equations, tables, and graphs Graphing using slope intercept form Parallel and perpendicular lines Writing equations given various forms of information Scatter plot and Lines of Best Fit Solve problems involving uniform motion by using the formula $d = rt$	**Informal:** Lesson Quiz Exit Tickets Journal Prompts Homework **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **CC.9-12. F-LE.2.** Interpret the parameters in a linear or exponential function in terms of a context. **CC.9-12. F-LE.5.** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **CC.9-12. S-ID.6** c. Fit a linear function for a scatter plot that suggests a linear association. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **CC.9-12. S-ID.7.**	Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. When making statistical models, technology is valuable for varying assumptions, exploring consequences and comparing predictions with data. Causation implies correlation yet correlation does not imply causation.		
Unit Five: Polynomials **Timeline: 2 weeks**			
Add, subtract, and multiply polynomials. **CC.9-12. A.APR.1** Apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x-a is p(a), so p(a) = 0 if and only if (x-a) is a factor of p(x). **CC9-12 A.APR.2**	How to extend and apply the conceptual understanding of arithmetic structures and operation to polynomials. Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation	Solve problems by looking for a pattern Multiply monomials Simplify expressions involving powers of monomials Simplify expressions involving	**Informal:** Lesson Quiz Exit Tickets Journal Prompts Homework **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Identify zeros of polynomials when suitable factorizations are defined by the polynomial. **CC. 9-12.A.APR.3** Prove polynomial identities and use them to describe numerical relationships. **CC. 9-12.A.APR.4** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. **CC. 9-12.A.APR.6**		quotients of monomials Simplify expressions containing negative exponents Express numbers in scientific and decimal notation Find products and quotients of numbers expressed in scientific notation. Find the degree of a polynomial Arrange the terms of a polynomial so that the powers of a certain variable are in ascending or descending order Add and subtract polynomials Multiply a polynomial by a monomial Simplify expressions involving polynomials Multiply any two polynomials by using the distributive property Use the patterns for $(a+b)^2$, $(a-b)^2$, and $(a+b)(a-b)$	
Unit Six: Functions and Graphs **Timeline: 2 weeks**			
Knows that a graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **CC.9-12. A.REI.10** Determine an explicit expression, a recursive process, or steps for calculation from a context.**CC.9-12.**	Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation.	Identify the domain, range, and inverse of a relation Show relations as sets of ordered pairs and mappings Solve linear equations for a specific	**Informal:** Lesson Quiz Exit Tickets Journal Prompts Homework **Formal Assessment:**

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
F.BF.1a For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. **CC.9-12. F-IF.4.** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.***CC.9-12. F-IF.5.** a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	Understand how the concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical.	variable Solve linear equations for a given domain Determine whether a given relation is a function Calculate functional values for a given function Graph inequalities in the coordinate plane Write an equation to represent a relation, given a chart of values Solve problems by using bar graphs and line graphs	Unit Test Portfolio
Unit Seven: Graphing Linear Equations **Timeline: 3 weeks**			
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.** **CC.9-12.F.IF.5** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* **CC.9-12.F.IF.6**	Understand how the concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical.	Find the slope of a line, given the coordinates of two points on the line Write a linear equation in standard form given the coordinates of a point on the line and the slope of the line Write a linear equation in standard form given the coordinates of two points on the line Write an equation in slope-intercept	**Informal:** Lesson Quiz Exit Tickets Journal Prompts Homework **Formal Assessment:** Unit Test Portfolio

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Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **CC.9-12.F.IF.7** Graph linear and quadratic functions and show intercepts, maxima, and minima. **CC.9-12.F.IF.7a** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **CC.9-12.F.IF.7b** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **CC.9-12.F.IF.7c** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **CC.9-12.F.IF.7e** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **CC.9-12.F.IF.8**		from given the slope and y-intercept Determine the x- and y-intercept of a graph Determine the x- and y-intercepts of a graph Graph linear equations using the x- and y-intercepts or the slope and y-intercept Write a linear equation in slope-intercept form given the slope of a line and the coordinates of a point on the line Write a linear equation in slope-intercept form given the coordinates of two points on the line Write an equation of a line that passes through a given point and is parallel or perpendicular to the graph of a given equation Find the coordinates of the midpoint of a line segment in the coordinate plane given the coordinates of the endpoints Solve problems by using pictographs, circle graphs, and comparative graphs	

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Curriculum Framework for Mathematics

School: **Sussex Preparatory Academy** **Curricular Tool:** **Holt McDougal *Geometry*** **Course:** **Geometry**

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Unit One: Exploring Geometry **Timeline : 3 weeks**			
Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. **CC.9-12.G.CO.1** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). **CC.9-12.G.CO.2**	The concept of congruence and symmetry can be understood from the perspective of geometric transformation.	**Essential Questions:** How do points, lines, and rays differ? What definitions are used to classify angles? How can you describe the relationship between two lines? Why is Geometry important to us? **Learning Targets:** Begin to construct a geometry portfolio that will help to organize their work throughout the course. Understand and identify the undefined terms point, line and plane Define segment, ray, angle, collinear, intersect, intersection and coplanar Investigate postulates about points, lines and planes Construct a geometry ruler Define length, and congruent Identify and use the Segment Addition Postulate Measure angles with a protractor Identify and use the Angle Addition Postulate	**Informal:** Lesson Quiz Exit Tickets Journal Prompts Homework Math Connections Problems Chapter Review **Formal Assessment:** Chapter Test Chapter Project Cumulative Assessment Geometry Portfolio

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Use paper folding to construct perpendicular lines, parallel lines, segment bisectors and angle bisectors Define and make geometry conjectures Discover points of concurrency in triangles. Draw the inscribed and circumscribed circles of triangles. Identify and draw the three basic rigid transformations: translation, rotation, and reflection Review the algebraic concepts of coordinate plane, origin, x and y-coordinates, and ordered pair. Construct translations, reflections across axes, and rotations about the origin on a coordinate plane	
Unit Two: Parallels and Polygons **Timeline: 2 weeks**			
Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **CC.9-12.G.CO.9** Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* **CC.9-12.G.CO.10**	The concept of congruence and symmetry can be understood from the perspective of geometric transformation. Once the triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles,	**Essential Questions:** What similarities and differences exist between triangles? **Learning Targets:** Identify the properties of quadrilaterals and the relationships among properties Define transversal, alternate interior angles, alternate exterior angle, same-side interior angles and corresponding angles Identify and use the converse of the Corresponding	**Informal:** Lesson Quiz Exit Tickets Journal Prompts Homework Math Connections Problems Chapter Review **Formal Assessment:** Chapter Test Chapter Project Cumulative Assessment Geometry Portfolio

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* **CC.9-12.G.CO.11**	quadrilaterals, and other geometric figures Construction is another way to visualize and create a strategic pathway to proof.	Angles Postulate. Prove that lines are parallel by using theorems and postulates Identify and use the Parallel Postulate and the Triangle Sum Theorem Define interior and exterior angles of a polygon Develop and use formulas for the sums of the measures of interior and exterior angles of a polygon Define midsegment of a triangle and midsegment of a trapezoid. Develop and use formulas based on the properties of triangle and trapezoid midsegments Develop and use thermos about equal slopes and slopes of perpendicular lines Solve problems involving perpendicular and parallel lines in the coordinate plane by using appropriate theorems.	
Unit Three: Geometric Transformations and Triangle Congruence **Timeline: 3 weeks total**			
<u>Sub-Unit One:</u> Using Technology to Explore Geometric Transformations (DDOE Model Unit)			
Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). **CC.9-12.G.CO.2**	Objects in space can be oriented in an infinite number of ways. An object's location in space can be	<u>Essential Questions:</u> • How are objects transformed mathematically? • Why are objects transformed mathematically? • How can motion be described mathematically? • How can we use our knowledge of transformations to describe them mathematically?	**<u>Formative Assessment:</u>** Sketch and Describe Quizzes Student Self-Assessment **<u>Summative Assessment:</u>** Transformations Test

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. **CC.9-12.G.CO.3** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **CC.9-12.G.CO.4** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. **CC.9-12.G.CO.5** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **CC.9-12.G.CO.6** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **CC.9-12.G.CO.7** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **CC.9-12.G.CO.8**	described quantitatively. There are multiple ways to transform an object.	• Which transformations are commutative? Student Learning Targets: *Students will know…* • the differences between reflections, translations, rotations, and dilations. • which transformations create similar and/or congruent figures. *Students will be able to…* (21st century skills) • describe various transformations a shape has undergone. • develop their own designs and describe them using geometric transformations. • graph composite transformations on the Cartesian plane.	**Transfer Task:** Develop a repetitive design using composite transformations (of at least two types). Including a precise mathematical description, professional drawing on a Cartesian plane with the location of the pre-image clearly marked. If you choose to use graphing technology, be sure to format the diagram with axes on. For extra credit, create a model product for the client.
Sub-Unit Two: Properties of Polygons			
Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a*	Once the triangle congruence criteria	Develop and use the Isosceles Triangle Theorem	**Informal:** Lesson Quiz

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. **CC.9-12.G.CO.9** Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* **CC.9-12.G.CO.10** Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* **CC.9-12.G.CO.11**	(ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures	Prove quadrilateral conjectures by using triangle congruence postulates and theorems Develop conjectures about special quadrilaterals-parallelograms, rectangles, and rhombuses Construct congruent copies of segments, angles, and triangles. Construct an angle bisector Translate, rotate, and reflect figures by using a compass and straightedge Prove that translations , rotations, and reflections preserve congruence and other properties Use the Betweeness Postulate to establish the Triangle Inequality Theorem	Exit Tickets Journal Prompts Homework Math Connections Problems Chapter Review **Formal Assessment:** Chapter Test Chapter Project Cumulative Assessment Geometry Portfolio
Unit Four: Special Right Triangles **Timeline: 2 weeks**			
Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.* **CC.9-12.G.GPE.7** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **CC.9-12.G.SRT.6** Explain and use the relationship between the sine and cosine of complementary angles. **CC.9-12.G.SRT.7**	Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor"	Identify and use the Area of a Rectangle and the Sum of Areas Postulates Solve problems involving fixed perimeters and fixed areas Develop formulas for the areas of triangles, parallelograms, and trapezoids. Solve problems by using the formulas for the areas of triangles, parallelograms, and trapezoids	**Informal:** Lesson Quiz Exit Tickets Journal Prompts Homework Math Connections Problems Chapter Review **Formal Assessment:** Chapter Test Chapter Project

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.* **CC.9-12.G.SRT.8**	developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. The definition of trigonometric ratios is not only useful in solving right triangle problems but can also be applied to general triangles. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa	Identify and apply formulas for the circumference and area of a circle. Solve problems by using the formulas for the circumference and area of a circle. Identify and apply the Pythagorean theorem and its converse Solve problems by using the Pythagorean Theorem Identify and use the 45-45-90 Triangle Theorem and the 30-60-90 Triangle Theorem Identify and use the formulas for the area of a regular polygon Develop and apply the distance formula Use the distance formula to develop techniques for estimating the area under a curve. Develop coordinate proofs for the Triangle Midsegment Theorem, the diagonals of a parallelogram and the reflection point across the line y=x Use the concepts of coordinate proofs to solve problems on the coordinate plane. Develop and apply the basic formula for geometric probability	Cumulative Assessment Geometry Portfolio

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Unit Five: Surface Area and Volume **Timeline: 2 weeks**			
Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.* **CC.9-12.G.GMD.3** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* **CC.9-12.G.MG.1** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).* **CC.9-12.G.MG.2** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).* **CC.9-12.G.MG.3**	Perimeter, Area, and Volume of Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of shape (composing, decomposing or slicing) correspond to algebraic changes in their equations. Real-world situations are not organized and labeled for analysis; formulating flexible geometric models, representing such models, and analyzing them is a creative process. The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and	Explore the ratios of surface area to volume Develop the concepts of maximizing volume and minimizing surface area Define and use the formula for finding the surface area of a right prism. Define and use the formula for finding the volume of a right prism Use Cavalieri's Principle to develop the formula for the volume of a right or oblique prism Define and use the formula for the surface area of a regular pyramid Define and use the formula for the volume of a pyramid Define and use the formula for the surface area of a right cylinder Define and use the formula for the volume of a cylinder Define and use the formula for the surface area of a cone Define and use the formula for the volume of a cone Define and use the formula for the surface area of a sphere Define and use the formula for the volume of a sphere	**Informal:** Lesson Quiz Exit Tickets Journal Prompts Homework Math Connections Problems Chapter Review **Formal Assessment:** Chapter Test Chapter Project Cumulative Assessment Geometry Portfolio

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
	technical skills, and our ability to recognize significant variables and relationships among them.	Define various transformations in three-dimensional space Solve problems by using transformation in three-dimensional space	
Unit Six: Similar Shapes **Timeline: 2 weeks**			
Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **CC.9-12.G.SRT.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **CC.9-12.G.CO.6** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **CC.9-12.G.CO.7** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **CC.9-12.G.CO.8**	Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. Transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. The definition of trigonometric ratios is not only useful in solving right triangle problems but can also be applied to general triangles.	Constructing a dilation of a segment and a point by using a scale factor Construct a dilation of a closed plane figure Define similar polygons Use Properties of Proportions and scale factor to solve problems involving similar polygons Develop the AA Triangle Similarity Postulate and the SSS and SAS Triangle Similarity Theorems Develop and prove the Side-Splitting Theorem Use the Side-Splitting Theorem to solve problems Use the triangle similarity to measure distances indirectly Develop and use similarity theorems for altitudes and medians of triangles Develop and use ratios for areas of similar figures Develop and use ratios for volumes of similar solids Explore relationships between cross-sectional area, weight, and height.	**Informal:** Lesson Quiz Exit Tickets Journal Prompts Homework Math Connections Problems Chapter Review **Formal Assessment:** Chapter Test Chapter Project Cumulative Assessment Geometry Portfolio

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Unit Seven: Circles **Timeline: 2 weeks**			
Prove that all circles are similar. **CC.9-12.G.C.1** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **CC.9-12.G.C.2** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **CC.9-12.G.C.3** (+) Construct a tangent line from a point outside a given circle to the circle. **CC.9-12.G.C.4** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. **CC.9-12.G.C.5**	Properties of Circles can be described by theorems that integrate algebraic and geometric understanding, modeling, and proof. Properties of Circles can be used to derive an understanding of the radian measure of an angle.	Define a circle and its associated parts, and use them in constructions Define and use the degree measure of arcs Define and use the length measure of arcs Prove a theorem about chords and their intercepted arcs Define tangents and secants of circles Understand the relationship between tangents and certain radii of circles Understand the geometry of a radius perpendicular to a chord of a circle Define inscribed angle and intercepted arc Develop and use the Inscribed Angle Theorem and its corollaries Define angles formed by secants and tangents of circles Develop and use theorems about measures of arcs intercepted by these angles. Define special cases of segments related to circles, including secant-secant, secant-tangent, and chord-chord Develop and use theorems about measures of the segments	**Informal:** Lesson Quiz Exit Tickets Journal Prompts Homework Math Connections Problems Chapter Review **Formal Assessment:** Chapter Test Chapter Project Cumulative Assessment Geometry Portfolio

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		Develop and use the equation of a circle Adjust the equation for a circle to move the center in a coordinate plane	

Curriculum Framework for Mathematics

School: Sussex Preparatory Academy **Curricular Tool:** Holt McDougal *Algebra 2* **Course:** Algebra 2

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Unit One: Sampling and Reasoning **Timeline : 2 weeks**			
Solve quadratic equations with real coefficients that have complex solutions. **CC.9-12.N.CN.7** Solve quadratic equations in one variable. **CC.9-12.A.REI.4** a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **CC.9-12.A.REI.4a** b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **CC.9-12.A.REI.4b** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **CC.9-12.F.IF.7** a. Graph linear and quadratic functions and show intercepts, maxima, and minima. **CC.9-12.F.IF.7a** Choose and produce an equivalent form of an	How knowledge of number properties in the Real Number System can be use to develop and apply properties of the Complex Number System. Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure	Graph quadratic equations Implement the parent function for quadratic equations Determine the properties of the graph of $y = ax^2 + bx + c$ Determine minimum and maximum values Graph quadratic functions in vertex form or intercept form Create the graph of Vertex Form $y = a(x-h)^2 + k$ Determine the FOIL method Graph functions expressed symbolically and show key features of the graph Graph linear and quadratic functions Determine intercepts, maxima and minima Solve quadratic equations Factor a quadratic expression to reveal the zeros of the function it defines Determine special factoring patterns Implement the zero product property Use factoring to solve equations of the form $ax^2 + bx + c = 0$ Factor a quadratic expression to reveal the zeros of	**Informal Assessment:** Countdown to Mastery Exit Tickets Skill Practice Problem Solving problems Lesson Quiz (paper and online) Math Journal Chapter Summary Mixed Review **Formal Assessment:** Unit Test Portfolio

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expression to reveal and explain properties of the quantity represented by the expression.* **CC.9-12.A.SSE.3** a. Factor a quadratic expression to reveal the zeros of the function it defines. **CC.9-12.A.SSE.3a** Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real. **CC.9-12.N.CN.1** Solve quadratic equations in one variable. **CC.9-12.A.REI.4** a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **CC.9-12.A.REI.4a** b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*. **CC.9-12.A.REI.4b**		the function it defines Factor out monimals Solve multi-step problems Solve quadratic equations by finding square roots Solve quadratic equations by completing the square, the quadratic formula and factoring. Recognize when the quadratic formula gives complex solutions Rationalize denominators of fractions Perform operations with complex numbers Use the relation $i^2 = -1$ Implement the commutative, associative and distributive properties to add, subtract, and multiply complex numbers Determine the square root of a negative number Determine the sums and differences of complex numbers Plot complex numbers Determine the absolute values of complex numbers Solve quadratic equations by completing the square Make a perfect square trinomial Solve $ax^2 + bx + c = 0$ when $a = 1$ Solve $ax^2 + bx + c = 0$ when $a \neq 1$ Write a quadratic function in vertex form Find the maximum value of a quadratic function Solve quadratic equations using the quadratic	

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		formula Implement the discriminant Graph and solve quadratic inequalities Graph an quadratic inequality in two variables Solve a quadratic inequality using a table Solve a quadratic inequality by graphing Use a quadratic inequality as a model Solve a quadratic inequality algebraically	
Unit Two: Polynomials and Polynomial Functions **Timeline: 1week**			
Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **CC.9-12.F.IF.7** (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **CC.9-12.F.IF.7d** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **CC.9-12.F.IF.8**	Understand how the concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical.	Simplify expressions involving powers Evaluate numerical expressions Use scientific notation in real life Evaluate and graph other polynomial function Identify polynomial functions Add and subtract polynomials vertically and horizontally Multiply and divide polynomials vertically and horizontally Use special product patterns Find a common monomial factor Factor polynomials in quadratic form Solve a polynomial equation Factor a polynomial Use a polynomial model List possible rational zeros	**Informal Assessment:** Countdown to Mastery Exit Tickets Skill Practice Problem Solving problems Lesson Quiz (paper and online) Math Journal Chapter Summary Mixed Review **Formal Assessment:** Unit Test Portfolio

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		Solve a multi-step problem Find the number of solutions or zeros in a polynomial function Use zeros to write a polynomial function	
Unit Three: Rational Exponents and Radical Functions **Timeline: 1 week**			
Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **CC.9-12.F.IF.7** c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **CC.9-12.F.IF.7c** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **CC.9-12.F.IF.8** a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **CC.9-12.F.IF.8a** b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* **CC.9- 12.F.IF.8b** Explain how the definition of the meaning of rational exponents follows from extending the	Understand how the concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical. How properties of rational exponents, rational number, and irrational number are defined using characteristic patterns of equivalency and computation, to build a comprehensive knowledge of the structure and order of the real number system. Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation.	Find *n*th roots Evaluate expressions with rational exponents Approximate roots with a calculator Solve equations using *n*th roots Use n^{th} roots in problem solving Use properties of exponents Apply properties of exponents Use properties of radicals Write radicals in simplest form Add and subtract like radicals and roots Simplify expressions involving variables Write variable expressions in simplest form Add and subtract expressions involving variables Add and subtract functions Multiply and divide functions Solve a multi-step problem Find compositions of functions Solve a multi-step problem Find an inverse relation	**Informal Assessment:** Countdown to Mastery Exit Tickets Skill Practice Problem Solving problems Lesson Quiz (paper and online) Math Journal Chapter Summary Mixed Review **Formal Assessment:** Unit Test Portfolio

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properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.* **CC.9-12.N.RN.1** Rewrite expressions involving radicals and rational exponents using the properties of exponents. **CC.9-12.N.RN.2** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **CC.9-12.F.IF.7** a. Graph linear and quadratic functions and show intercepts, maxima, and minima. **CC.9-12.F.IF.7a** b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **CC.9-12.F.IF.7b** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **CC.9-12.A.REI.2**		Verify that functions are inverses Solve a multi-step problem using inverses Find the inverse of a power function Find the inverse of a cubic function Find the inverse of a power model Use an inverse power model to make a prediction Graph a square root function Graph a cube root function Solve a multi-step problem Graph a translated square root function Graph a translated cube root function Solve a radical equation Solve a radical equation given a function Solve an equation with a rational exponent Solve an equation with an extraneous solution Solve an equation with two radicals	
Unit Four: Exponential and Logarithmic Functions **Timeline: 1 week**			
Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential,	Understand how the concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical.	Graph $y = b^x$ for $b > 1$ Graph $y = ab^x$ for $b > 1$ Graph $y = ab^{x-h} + k$ for $b > 1$ Solve a multi-step problem using the exponential growth model Find the balance in an account	**Informal Assessment:** Countdown to Mastery Exit Tickets Skill Practice Problem Solving problems Lesson Quiz (paper and online) Math Journal Chapter Summary

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and logarithmic functions.* **CC.9-12.A.REI.11** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **CC.9-12.F.IF.7** e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **CC.9-12.F.IF.7e** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **CC.9-12.F.IF.8** b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* **CC.9-12.F.IF.8b** Distinguish between situations that can be modeled with linear functions and with exponential functions. **CC.9-12.F.LE.1** a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. **CC.9-12.F.LE.1a** b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **CC.9-12.F.LE.1b** c. Recognize situations in which a quantity grows	Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation. Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. When making statistical models, technology is valuable for varying assumptions, exploring consequences and comparing predictions with data.	Graph $y = b^x$ for $0 < b < 1$ Graph $y = ab^2$ for $0 < b < 1$ Graph $y = ab^{x-h} + k$ for $0 < b < 1$ Solve a multi-step problem using an exponential decay function Simplify natural base expressions Evaluate natural base expressions Graph natural base functions Solve a multi-step problem using a function involving *e* Model coninuously compounded interest Rewrite logarithmic equations Evaluate logarithms Evaluate common and natural logarithms Evaluate a logarithmic model Use inverse properties Find inverse functions Graph logarithmic functions Translate a logarithmic graph Use properties of logarithms Expand a logarithmic expression Use the change-of-base formula Use properties of logarithms in real life Solve by equating exponents Take a logarithm of each side of an equation Use an exponential model	Mixed Review **Formal Assessment:** Unit Test Portfolio

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or decays by a constant percent rate per unit interval relative to another. **CC.9-12.F.LE.1c** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **CC.9-12.F.LE.2** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. **CC.9-12.F.LE.4** Interpret the parameters in a linear or exponential function in terms of a context. **CC.9-12.F.LE.5** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **CC.9-12.S.ID.6** a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* **CC.9-12.S.ID.6a**	Causation implies correlation yet correlation does not imply causation.	Solve a logarithmic equation Exponent each side of an equation Use a logarithmic model Write an exponential function Find an exponential model Use exponential regression Write a power function Find a power model Use power regression	
Unit Five: Rational Functions **Timeline: 2 weeks**			
Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. **CC.9-12.A.APR.6**	How to extend and apply the conceptual understanding of arithmetic structures and operation to polynomials. Algebraic manipulations are governed by the properties of operations and exponents, and	Graph a rational function of the form y = a/x Graph a rational function of the form y = a/x-h + k Graphy a rational function of the form y = ax + b/ cx + d Solve a multi-step problem using simple and	**Informal Assessment:** Countdown to Mastery Exit Tickets Skill Practice Problem Solving problems Lesson Quiz (paper and online) Math Journal Chapter Summary

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(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. **CC.9-12.A.APR.7** Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f*(*x*), *f*(*kx*), and *f*(*x + k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* **CC.9-12.F.BF.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.* **CC.9-12.G.GMD.3** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* **CC.9-12.F.IF.6** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* **CC.9-12.F.IF.8b** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features*	the conventions of algebraic notation. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. Understand how the concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical. Perimeter, Area, and Volume of Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of shape (composing, decomposing or slicing) correspond to algebraic changes in their equations.	rational functions Graph a rational function (m < n) Graph a rational function (m = n) Graph a rational function (m > n) Solve a multi-step problem using rational functions Simplify a rational expression Solve a multi-step problem using surface area and volume formulas Add and subtract rational expressions Multiply a rational expression by a polynomial Multiply and divide rational expressions Divide a rational expression by a polynomial Add or subtract with like denominators Find a least common multiple (LCM) Add with unlike denominators Subtract with unlike denominators Simplify a complex fraction (Method 1) Simplify a complex fraction (Method 2) Solve a rational equation by cross multiplying Write and use a rational model	Mixed Review **Formal Assessment:** Unit Test Portfolio

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*include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** **CC.9-12.F.IF.4**		Solve a rational equation with two solutions Check for extraneous solutions Solve a rational equation given a function Sketch a graph given a verbal description Investigate average rate of change Compare functions in different representations using maximums and x-intercepts Identify even and odd functions	
Unit Six: Data Analysis and Statistics **Timeline: 1 week**			
(+) Use permutations and combinations to compute probabilities of compound events and solve problems. **CC.9-12.S.CP.9** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* **CC.9-12.A.CED.3** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1] **CC.9-12.A.APR.5** (+) Define a random variable for a quantity of	In a probability model, sample points represent outcomes and combine to make up events. The probabilities of the events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. How to extend and apply the conceptual understanding of arithmetic structures and	Find combinations Decide to multiply or add combinations Solve a multi-step problem using combinations Use Pascal's triangle Expand a power of a binomial sum Expand a power of a binomial difference Find a coefficient in an expansion Construct a probability distribution Interpret a probability distribution Construct a binomial distribution	**Informal Assessment:** Countdown to Mastery Exit Tickets Skill Practice Problem Solving problems Lesson Quiz (paper and online) Math Journal Chapter Summary Mixed Review **Formal Assessment:** Unit Test Portfolio

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interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. **CC.9-12.S.MD.1** (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. **CC.9-12.S.MD.2** (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each nd find the expected grade under various grading schemes.* **CC.9-12.S.MD.3**	operation to polynomials. Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. An equation is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Numeric relationships can be symbolically represented as equations and inequalities and fluency in transforming these symbolic representations is a tool for graphing and solving problems.	Interpret a binomial distribution Classify distributions as symmetric or skewed Find a normal probability Interpret normally distributed data Use a *z*-score and the standard normal table Classify samples Identify a biased sample Choose an unbiased sample Find a margin of error Identify and correct bias in survey questioning Identify experiments and observational studies Evaluate a published report Design an experiment or observational study	
Unit Seven: Sequences and Series **Timeline: 2 weeks**			
Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n□≥1.* **CC.9-12.F.IF.3** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **CC.9-12.F.LE.2**	Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. Understand how the concept of function can be used to interpret, analyze and model	Write terms of sequences Write rules for sequences Solve a multi-step problem using a sequence table Write series using summation notation Find the sum of a series Use a formula for a sum Identify arithmetic sequences	**Informal Assessment:** Countdown to Mastery Exit Tickets Skill Practice Problem Solving problems Lesson Quiz (paper and online) Math Journal Chapter Summary Mixed Review **Formal Assessment:**

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Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.* **CC.9-12.F.BF.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **CC.9-12.F.LE.2** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.** **CC.9-12.A.SSE.4** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n≥1.* **CC.9-12.F.IF.3** Write a function that describes a relationship between two quantities.* **CC.9-12.F.BF.1** Determine an explicit expression, a recursive process, or steps for calculation from a context. **CC.9-12.F.BF.1a** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.* **CC.9-12.F.BF.2**	functions that emerge from contexts including those contexts that are purely mathematical. Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.	Write a rule for the n^{th} term Write a rule given a term and common difference Write a rule given two terms Use an arithmetic sequence and series in real life Identify geometric sequences Write a rule for the n^{th} term Write a rule given a term and common ratio Write a rule given two terms Find the sum of a geometric series Use a geometric sequence and series in real life Find partial sums Find sums of infinite geometric series Use an infinite series as a model Evaluate recursive rules Write recursive rules Write recursive rules for special sequences Solve a multi-step problem using a recursive rule Iterate a function Translate from an explicit rule to a recursive rule	Unit Test Portfolio
Unit Eight: Quadratic Relations and Conic Solutions **Timeline: 1 week**			
Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.* **CC.9-**	Analytic geometry connects algebra and geometry, resulting	Classify a triangle using the distance formula	**Informal Assessment:** Countdown to Mastery Exit Tickets

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12.G.GPE.7 Derive the equation of a parabola given a focus and directrix. **CC.9-12.G.GPE.2** (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **CC.9-12.G.GPE.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* **CC.9-12.A.CED.3**	in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. An equation is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Numeric relationships can be symbolically represented as equations and inequalities and fluency in transforming these symbolic representations is a tool for graphing and solving problems	Find the midpoint of a line segment Find a perpendicular bisector Solve a multi-step problem using perpendicular bisectors Graph an equation of a parabola Write an equation of a parabola Solve a multi-step problem using a parabola Graph an equation of a circle Write an equation of a circle Graph an equation of an ellipse Write an equation given a vertex and a co-vertex Solve a multi-step problem using an ellipse equation Write an equation given a vertex and a focus Graph an equation of a hyperbola Write an equation of a hyperbola Solve a multi-problem using a hyperbola Graph the equation of a translated circle Graph the equation of a translated hyperbola Write an equation of a translated parabola Write an equation of a translated ellipse Identify symmetries of conic sections Solve a linear-quadratic system by graphing Solve a linear-quadratic system by substitution Solve a quadratic system by elimination Solve a real-life quadratic system	Skill Practice Problem Solving problems Lesson Quiz (paper and online) Math Journal Chapter Summary Mixed Review **Formal Assessment:** Unit Test Portfolio

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Unit Nine: Trigonometric Ratios and Functions **Timeline: 2weeks**			
Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **CC.9-12.F.TF.1** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **CC.9-12.F.TF.2** (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for □π/3,□ π/4 and□ π/6, and use the unit circle to express the values of sine, cosine, and tangent for □π-*x*,□ π+*x*, and 2π-*x* in terms of their values for *x*, where *x* is any real number. **CC.9-12.F.TF.3** (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). **CC.9-12.G.SRT.11** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **CC.9-12.G.CO.8** (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts;	The connection between extending the domain of trigonometric functions using the unit circle and graphing trigonometric functions in the Cartesian coordinate system to model periodic phenomena across the extended domain. That the graph of a function is a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties (amplitude, frequency, and midline). The concept of congruence and symmetry can be understood from the perspective of geometric transformation. Once the triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures Construction is another way to	Evaluate trigonometric functions Find an unknown side length of a right triangle Use a calculator to solve a right triangle Use indirect measurement Use and angle of elevation Draw angles in standard position Find co-terminal angles Convert between degrees and radians Solve a multi-step problem using angles and radians Evaluate trigonometric functions given a point Use the unit circle Find reference angles Use reference angles to evaluate functions Calculate horizontal distance traveled Model with a trigonometric function Evaluate inverse trigonometric functions Solve a trigonometric equation Write and solve a trigonometric equation Solve a triangle for the AAS or ASA case Solve SSA case with one solution Examine the SSA case with no solution	**Informal Assessment:** Countdown to Mastery Exit Tickets Skill Practice Problem Solving problems Lesson Quiz (paper and online) Math Journal Chapter Summary Mixed Review **Formal Assessment:** Unit Test Portfolio

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evaluate the solutions using technology, and interpret them in terms of the context.* **CC.9-12.F.TF.7** (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. **CC.9-12.F.TF.9**	visualize and create a strategic pathway to proof. Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. The definition of trigonometric ratios is not only useful in solving right triangle problems but can also be applied to general triangles.	Solve the SSA cased with two solutions Find the area of a triangle Solve a triangle for the SAS case Solve a triangle for the SSS case Use the law of cosines in real life Solve a multi-step problem using a triangle	
Unit Ten: Trigonometric Graphs, Identities and Equations **Timeline: 2 weeks**			
Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **CC.9-12.F.IF.7e** Solve quadratic equations in one variable. **CC.9-12.A.REI.4** a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same	Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation. The concept of congruence and symmetry can be understood from the	Graph sine and cosine functions Graph a cosine function Model with a sine function Graph a tangent function Graph a vertical translation Graph a horizontal translation Graph a model for circular motion	**Informal Assessment:** Countdown to Mastery Exit Tickets Skill Practice Problem Solving problems Lesson Quiz (paper and online) Math Journal Chapter Summary Mixed Review **Formal Assessment:**

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
solutions. Derive the quadratic formula from this form. **CC.9-12.A.REI.4a** b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **CC.9-12.A.REI.4b** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **CC.9-12.A.REI.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). **CC.9-12.G.CO.2**	perspective of geometric transformation. Construction is another way to visualize and create a strategic pathway to proof. Understand how the concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical.	Combine a translation and a reflection using sine Combine a translation and a reflection using tangent Model with a tangent function Find trigonometric values Simplify a trigonometric expression Verify a trigonometric identity Verify a real-life trigonometric identity Solve a trigonometric equation Solve a trigonometric equation in an interval Solve a real-life trigonometric equation Use the quadratic formula Solve an equation with an extraneous solution Evaluate a trigonometric expression Solve a trigonometric equation Evaluate trigonometric expressions Derive a trigonometric model Verify a trigonometric identity Solve a trigonometric equation	Unit Test Portfolio

Curriculum Framework for Pre-Calculus/Calculus

School: Sussex Preparatory Academy Curricular Tool: *Calculus I with Pre-Calculus* (Larson & Edwards) Grade: 12

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
Unit P: Prerequisites **Timeline : 10 days**			
Interpret expressions that represent a quantity in terms of its context.* **CC.9-12.A.SSE.1** a. Interpret parts of an expression, such as terms, factors, and coefficients. **CC.9-12.A.SSE.1a** b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* **CC.9-12.A.SSE.1b** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.* **CC.9-12.A.SSE.2** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* **CC.9-12.A.SSE.3** c. Factor a quadratic expression to reveal the zeros of the function it defines. **CC.9-12.A.SSE.3a** d. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. **CC.9-12.A.SSE.3b** e. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* **CC.9-12.A.SSE.3c** Understand that polynomials form a system analogous to the integers, namely, they are closed under the	Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. How to extend and apply the conceptual understanding of arithmetic structures and operation to polynomials. Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. An equation is a record of a computation with numbers, symbols that represent numbers, arithmetic	**Learning Targets:** Students will: • identify different types of equations • solve linear equations in one variable and equations that lead to linear equations • solve quadratic equations by factoring, extracting square roots, completing the square, and using the quadratic formula • solve polynomial equations of degree three or greater • solve equations using radicals • solve equations with absolute value • represent solutions of linear inequalities in one variable • represent use properties of inequalities to create equivalent inequalities and solve inequalities in one variable • solve inequalities involving absolute value • solve polynomial inequalities and rational inequalities	**Formative:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Summative Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **CC.9-12.A.APR.1** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **CC.9-12.A.APR.3** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. **CC.9-12.A.APR.6** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **CC.9-12.A.CED.1** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **CC.9-12.A.CED.2** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* **CC.9-12.A.CED.3** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.* **CC.9-12.A.CED.4** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the	operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Numeric relationships can be symbolically represented as equations and inequalities and fluency in transforming these symbolic representations is a tool for graphing and solving problems. Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation.	• plot points in the Cartesian plane • use the distance formula to find the distance between two points and use the midpoint formula to find the midpoint of a line segment • use a coordinate plane to model and solve real-life problems • sketch graphs of equations • find x- and y- intercepts of graphs of equations • use symmetry to sketch graphs of equations • find equations of and sketch graphs of circles • use graphs of equations in solving real-life problems • Students will use slope to graph linear equations in two variable • find the slope of a line given two points on the line • write linear equations in two variables • use slope to identify parallel and perpendicular lines • use slope and linear equations in two variables to model and solve real-life problems	

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
original equation has a solution. Construct a viable argument to justify a solution method. **CC.9-12.A.REI.1** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **CC.9-12.A.REI.3** Solve quadratic equations in one variable. **CC.9-12.A.REI.4** a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **CC.9-12.A.REI.4a** b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **CC.9-12.A.REI.4b** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **CC.9-12.A.REI.5** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **CC.9-12.A.REI.6** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$. **CC.9-12.A.REI.7** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **CC.9-**			

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
12.A.REI.10 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. **CC.9-12.G.CO.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **CC.9-12.G.GPE.1** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). **CC.9-12.G.GPE.5** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **CC.9-12.G.GPE.6**			
Unit One: Functions and their Graphs (sample unit attached) **Timeline : 12 days**			
Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. **CC.9-12.A.SSE.2** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **CC.9-12.A.APR.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. **CC.9-12.F.IF.1**	Mathematical data can be modeled with functions and their graphs. The concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the	Essential Questions: How are functions recognized, represented, and evaluated? How are graphs of functions sketched and analyze? How can graphs of functions be transformed? How can functions be combined? How to find inverse functions? How can functions be used to model mathematical data? Learning Targets: Students will: • determine whether relations between two	**Formative:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Summative Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **CC.9-12.F.IF.2** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** **CC.9-12.F.IF.4** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.** **CC.9-12.F.IF.5** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* **CC.9-12.F.IF.6** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **CC.9-12.F.IF.7** a. Graph linear and quadratic functions and show intercepts, maxima, and minima. **CC.9-12.F.IF.7a** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **CC.9-12.F.IF.8** a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **CC.9-12.F.IF.8a**	construction of mathematical models.	variables are functions. • use function notation and evaluate functions. • find the domains of functions. • use functions to model and solve real-life problems • use the Vertical Line Test for functions • find the zeros of functions. • determine intervals on which functions are increasing or decreasing and determine relative maximum and relative minimum values of functions. • identify and graph linear functions • identify and graph step and other piecewise-defined functions • identify even and odd functions. • recognize graphs of common functions. • use vertical and horizontal shifts to sketch graphs of functions. • use reflections to sketch graphs of functions. • use non-rigid transformations to sketch graphs of functions. • add, subtract, multiply and divide functions. • find the composition of one function with	

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **CC.9-12.F.IF.9** Write a function that describes a relationship between two quantities.* **CC.9-12.F.BF.1** a. Determine an explicit expression, a recursive process, or steps for calculation from a context. **CC.9-12.F.BF.1a** b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* **CC.9-12.F.BF.1b** c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.* **CC.9-12.F.BF.1c** Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* **CC.9-12.F.BF.3** Find inverse functions. **CC.9-12.F.BF.4** a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.* **CC.9-12.F.BF.4a**		another function. • use combinations and compositions of functions to model and solve real-life problems. • find inverse functions Formatively and verify that two functions are inverse functions of each other. • use graphs of functions to determine whether functions have inverse functions. • use graphs of functions to determine whether functions have inverse functions. • use the Horizontal Line Test to determine if functions are one-on-one. • find inverse functions analytically. • use mathematical models to approximate sets of data points. • use the *regression* feature of a graphing utility to find the equation of a least squares regression line. • write mathematical models for direct variation. • write mathematical models for direct variations as an *n*th power. • write mathematical models for inverse variation. • write mathematical models for joint variation.	

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
b. (+) Verify by composition that one function is the inverse of another. **CC.9-12.F.BF.4b** c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. **CC.9-12.F.BF.4c** d. (+) Produce an invertible function from a non-invertible function by restricting the domain. **CC.9-12.F.BF.4d**			
Unit Two: Polynomial and Rational Functions **Timeline: 12 days**			
Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. **CC.9-12.N.CN.1** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **CC.9-12.N.CN.2** (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **CC.9-12.N.CN.3** (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **CC.9-12.N.CN.4** (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.* **CC.9-12.N.CN.5** (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the	How knowledge of number properties in the Real Number System can be use to develop and apply properties of the Complex Number System. How to extend and apply the conceptual understanding of arithmetic structures and operation to polynomials. Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation. Understand how the concept of function can be used to	**Learning Targets:** Students will • analyze graphs of quadratic functions. • write quadratic functions in standard form and use the results to sketch graphs of quadratic functions. • find minimum and maximum values of quadratic functions in real-life applications. • use transformations to sketch graphs of polynomial functions. • use the Leading Coefficient Test to determine the end behavior of graphs of polynomial functions. • find and use zeros of polynomial functions as sketching aids. • divide polynomials using long division. • use synthetic division to divide polynomials by binomials of the form (x-k/).	**Formative:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Summative Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
midpoint of a segment as the average of the numbers at its endpoints. **CC.9-12.N.CN.6** Solve quadratic equations with real coefficients that have complex solutions. CC.9-12.N.CN.7 (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$. **CC.9-12.N.CN.8** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **CC.9-12.N.CN.9** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **CC.9-12.A.APR.1** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. **CC.9-12.A.APR.2** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **CC.9-12.A.APR.3** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.* **CC.9-12.A.APR.4** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. **CC.9-12.A.APR.6** (+) Understand that rational expressions form a system	interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical.	• use the Remainder Theorem and the Factor Theorem. • use polynomial division to answer questions about real-life problems. • use the imaginary unit i to write complex numbers. • add, subtract and multiply complex numbers. • use complex conjugates to write the quotient of two complex numbers in standard form. • find complex solutions of quadratic equations. • understand and use the Fundamental Theorem of Algebra. • find all the zeros of a polynomial function. • write a polynomial function with real coefficients, given its zeros. • find the domains of rational functions. • find the vertical and horizontal asymptotes of graphs of rational functions. • analyze and sketch graphs of rational functions. • sketch graphs of rational functions that have start asymptote.	

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. **CC.9-12.A.APR.7** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **CC.9-12.A.REI.2** Solve quadratic equations in one variable. **CC.9-12.A.REI.4** a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **CC.9-12.A.REI.4a** b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **CC.9-12.A.REI.4b** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **CC.9-12.F.IF.7** a. Graph linear and quadratic functions and show intercepts, maxima, and minima. **CC.9-12.F.IF.7a** b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **CC.9-12.F.IF.7b** c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **CC.9-12.F.IF.7c** d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **CC.9-**		• use rational functions to model and solve real-life problems.	

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
12.F.IF.7d e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **CC.9-12.F.IF.7e** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **CC.9-12.F.IF.8** a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **CC.9-12.F.IF.8a** b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* **CC.9-12.F.IF.8b**			
Unit Three: Limits and Their Properties **Timeline: 10 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Calculus, along with geometric and analytic information, can explain the observed local and global behavior of a function. Limits can be determined using algebra, graphs and/or tables of data. The concept of a limit is one of the foundations of Calculus. The limit of a function is the	**Learning Targets:** Students will • understand what calculus is and how it compares with pre-calculus. • understand that the tangent line problem is basic to calculus. • understand that the area problem is also basic to calculus. • estimate a limit using a numerical or graphical approach.	**Formative:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
	value approached by f(x) as x is approaching a given value or infinity.	• learn different ways that a limit can fail to exist. • study and use a formal definition of limit. • evaluate a limit using properties of limits. • develop and use a strategy for finding limits. • evaluate a limit using dividing out and rationalizing techniques. • evaluate a limit using the Squeeze Theorem. • determine continuity at a point and continuity on an open interval. • determine one-sided limits and continuity on a closed interval. • use properties of continuity. • understand and use the Intermediate Value Theorem. • determine infinite limits from the left and from the right. • find and sketch the vertical asymptotes of the graph of a function.	
Unit Four: Differentiation **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics.	The derivative is the instantaneous rate of change at	**Learning Targets:** Students will:	**Formative:** Lesson Exercises

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	a given point. Derivatives can be used to analyze curves. Derivatives can be used to model rates of change. Derivatives can be used in optimization problems.	• find the slope of the tangent line to a curve at a point. • use the limit definition to find the derivative of a function. • understand the relationship between differentiability and continuity. • find the derivative of a function using the Constant Rule. • find the derivative of a function using the Power Rule. • find the derivative of a function using the Constant Multiple Rule. • find the derivative of a function using the Sum and Difference Rules. • use derivatives to find rates of change. • find the derivative of a function using the Product Rule. • find the derivative of a function using the Quotient Rule. • find the higher-order derivative of a function. • find the derivative of a composite function using the Chain Rule. • find the derivative of a function using the General Power Rule.	Review Exercises Textbook Chapter Test Problem Solving problems **<u>Formal Assessment:</u>** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
		• simplify the derivative of a function using algebra. • distinguish between functions written in implicit form and explicit form. • use implicit differentiation to find the derivative of a function. • find a related rate. • use related rates to solve real-life problems	
Unit Five: Applications of Differentiation **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	The limit of a function is the value approached by f(x) as x is approaching a given value or infinity. The first derivative determines increasing or decreasing and the second derivative determines concavity.	**Learning Targets:** Students will • understand the definition of extrema of a function on an interval. • understand the definition of relative extrema of a function on an open interval. • find extrema on a closed interval. • understand and use Rolle's Theorem. • understand and use the Mean Value Theorem. • determine intervals on which a function is increasing or decreasing. • apply the First Derivative Test to find relative extrema of a function.	**Formative:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
		• determine intervals on which a function is concave upward or concave downward. • find any points of inflection of the graph of a function. • apply the Second Derivative Test to find relative extrema of a function. • determine (finite) limits at infinity. • determine the horizontal asymptotes, if any, of the graph of a function. • determine infinite limits at infinity. • analyze and sketch the graph of a function. • use calculus to solve applied minimum and maximum problems. • understand the concept of a tangent line approximation. • compare the value of the differential, *dy*, with the actual change in *y,* Δy. • estimate a propagated error using a differential. • find the differential of a function using differentiation formulas.	
Unit Six: Integration **Timeline: 14 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics.	Integration is a summation process.	**Learning Targets:** Students will	**Formative:** Lesson Exercises

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	The Fundamental Theorems of Calculus relate differentiation and integration as inverse functions. Antiderivatives follow directly from derivatives. Antiderivatives can be used to solve initial condition problems, including separable differential equations. There are several numerical techniques to approximate the definite integral.	• write the general solution of a differential equation. • use indefinite integral notation for antiderivatives. • use basic integration rules to find antiderivatives. • find a particular solution of a differential equation. • use sigma notation to write and evaluate a sum. • understand the concept of area. • use rectangles to approximate the area of a plane region. • find the area of a plane region using limits. • understand the definition of a Riemann sum. • evaluate a definite integral using limits. • evaluate a definite integral using properties of definite integrals. • evaluate a definite integral using the Fundamental Theorem of Calculus. • understand and use the Mean Value Theorem for Integrals. • find the average value of a function over a closed interval.	Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
		• understand and use the Second Fundamental Theorem of Calculus. • understand and use the Net Change Theorem. • use pattern recognition to find an indefinite integral. • use a change of variables to find an indefinite integral. • use the General Power Rule for Integration to find an indefinite integral. • use a change of variables to evaluate a definite integral. • evaluate a definite integral involving an even or odd function. • approximate a definite integral using the Trapezoidal Rule. • approximate a definite integral using Simpson's Rule. • analyze the approximate errors in the Trapezoidal Rule and Simpson's Rule.	
Unit Seven: Exponential and Logarithmic Functions **Timeline: 12 days**			
Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example,*	How properties of rational exponents, rational number, and irrational number are defined using characteristic	**Learning Targets:** Students will • recognize and evaluate exponential functions with base *a*.	**Formative:** Lesson Exercises Review Exercises Textbook Chapter Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. **CC.9-12.N.RN.1** Rewrite expressions involving radicals and rational exponents using the properties of exponents. **CC.9-12.N.RN.2** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. **CC.9-12.N.RN.3** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **CC.9-12.F.IF.7** e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **CC.9-12.F.IF.7e** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **CC.9-12.F.IF.8** b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. **CC.9-12.F.IF.8b** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **CC.9-12.F.IF.9**	patterns of equivalency and computation, to build a comprehensive knowledge of the structure and order of the real number system. Understand how the concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical. Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate.	• graph exponential functions. • recognize, evaluate and graph exponential functions with base *e*. • use exponential functions to model and solve real-life problems. • recognize and evaluate logarithmic functions with base *a*. • graph logarithmic functions. • recognize, evaluate and graph natural logarithmic functions. • use logarithmic functions to model and solve real-life problems. • use the change-of-base formula to rewrite and evaluate logarithmic expressions. • use properties of logarithms to evaluate or rewrite logarithmic expressions. • use properties of logarithms to expand or condense logarithmic expressions. • use logarithmic functions to model and solve real-life problems. • solve simple exponential and logarithmic equations. • solve more complicated exponential equations.	Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. **CC.9-12.F.BF.5** Distinguish between situations that can be modeled with linear functions and with exponential functions. **CC.9-12.F.LE.1** a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. **CC.9-12.F.LE.1a** b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **CC.9-12.F.LE.1b** c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **CC.9-12.F.LE.1c** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **CC.9-12.F.LE.2** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **CC.9-12.F.LE.3** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. **CC.9-12.F.LE.4** Interpret the parameters in a linear or exponential function in terms of a context. **CC.9-12.F.LE.5**		• solve more complicated logarithmic equations. • use exponential and logarithmic equations to model and solve real-life problems. • recognize the five most common types of models involving exponential and logarithmic functions. • use exponential growth and decay functions to model and solve real-life problems. • use Gaussian functions to model and solve real-life functions. • use logistic growth functions to model and solve real-life problems. • use logarithmic functions to model and solve real-life problems.	

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
Unit Eight: Exponential and Logarithmic Functions and Calculus **Timeline: 10 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Exponential and Logarithmic functions are inverse functions. The definition of the derivative and integral can be applied to logarithmic, exponential and transcendental functions.	**Learning Targets:** Students will • differentiate natural exponential functions. • integrate natural exponential functions. • find derivatives of functions involving the natural logarithmic function. • use logarithms as an aid in differentiating non logarithmic functions. • find derivatives of exponential and logarithmic functions in bases other than *e*. • use the Log Rule For Integration to integrate a rational function. • use separation of variables to solve a simple differential equation. • use exponential functions to model growth and decay in applied problems.	**Formative:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test
Unit Nine: Trigonometric Functions (sample unit attached) **Timeline: 12 days**			
Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **CC.9-12.F.TF.1** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **CC.9-12.F.TF.2**	Trigonometric Functions can be used to model and solve real-life mathematical problems. Understand how the concept of function can be used to interpret, analyze and model functions that emerge from	**Essential Questions:** How can angles be described, radian measures be used, and degree measures be used? How can trigonometric functions be evaluate using the unit circle? How can trigonometric functions be evaluated for acute angles and how can fundamental trigonometric identities be used?	**Formative:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Summative Assessment:**

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for □π/3,□ π/4 and□ π/6, and use the unit circle to express the values of sine, cosine, and tangent for □π-*x*,□ π+*x*, and 2π-*x* in terms of their values for *x*, where *x* is any real number. **CC.9-12.F.TF.3** (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **CC.9-12.F.TF.4** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* **CC.9-12.F.TF.5** (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **CC.9-12.F.TF.6** (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.* **CC.9-12.F.TF.7** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **CC.9-12.G.SRT.6** Explain and use the relationship between the sine and cosine of complementary angles. **CC.9-12.G.SRT.7** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.* **CC.9-12.G.SRT.8**	contexts including those contexts that are purely mathematical. Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. The definition of trigonometric ratios is not only useful in solving right triangle problems but can also be applied to general triangles.	How can reference angles be used to evaluate trigonometric functions of any angle? How can the graphs of sine and cosine be sketched? How can the graphs of tangent, cotangent, secant, and cosecant be sketched? How can inverse trigonometric functions be evaluated? How are real-life problems involving right triangles, directional bearings, and harmonic motion solved? **<u>Learning Targets:</u>** Students will: • describe angles. • use radian measure. • use degree measure. • use angles to model and solve real-life problems. • identify a unit circle and describe its relationship to real numbers. • evaluate trigonometric functions using the unit circle. • use the domain and period to evaluate sine and cosine functions. • use a calculator to evaluate trigonometric functions.	Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
		• evaluate trigonometric functions of acute angles. • use fundamental trigonometric identities. • use trigonometric functions to model and solve real-life problems. • evaluate trigonometric functions of any angle. • use reference angles to evaluate trigonometric functions. • sketch the graphs of basic sine and cosine functions. • use amplitude and period to help sketch the graphs of sine and cosine functions. • sketch translations of the graphs of sine and cosine functions. • use sine and cosine functions to model real-life data. • sketch the graphs of tangent functions. • sketch the graphs of cotangent functions. • sketch the graphs of secant and cosecant functions. • sketch the graphs of damped trigonometric functions. • evaluate and graph the inverse sine	

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
		function. • evaluate and graph the other inverse trigonometric functions. • evaluate and graph the compositions of trigonometric functions. • solve real-life problems involving right triangles. • solve real-life problems involving directional bearings. • solve real-life problems involving harmonic motion.	
Unit Ten: Analytic Trigonometry **Timeline: 10 days**			
Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. **CC.9-12.F.TF.8** (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. **CC.9-12.F.TF.9**	The connection between extending the domain of trigonometric functions using the unit circle and graphing trigonometric functions in the Cartesian coordinate system to model periodic phenomena across the extended domain. That the graph of a function is a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties (amplitude, frequency, and midline).	**Learning Targets:** Students will • recognize and write the fundamental trigonometric identities. • use the fundamental trigonometric identities to evaluate trigonometric functions, simplify trigonometric expressions, and rewrite trigonometric expressions. • verify trigonometric identities. • use standard algebraic techniques to solve trigonometric equations. • solve trigonometric equations of quadratic type.	**Formative:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Summative Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
		• solve trigonometric equations involving multiple angles. • use inverse trigonometric functions to solve trigonometric equations. • use sum and difference formulas to evaluate trigonometric functions, verify identities, and solve trigonometric equations. • use multiple-angle formulas to rewrite and evaluate trigonometric functions. • use power-reducing formulas to rewrite and evaluate trigonometric functions. • use half-angle formulas to rewrite and evaluate trigonometric functions. • use product-to-sum and sum-to-product formulas to rewrite and evaluate trigonometric functions. • use trigonometric formulas to rewrite real-life models.	
Unit Eleven: Trigonometric Functions and Calculus **Timeline: 10 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	Fundamental trigonometric identities can be used to simplify trigonometric expressions. Finding the derivative of a function may require the use of several rules, including rules for: sums, products,	**Learning Targets:** Students will • determine the limits of trigonometric functions. • find and use the derivatives of the sine and cosine functions. • find and use the derivatives of other	**Formative:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Summative Assessment:**

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	quotients, powers, exponentials, logarithms, trigonometric and inverse trigonometric functions, and the chain rule.	trigonometric functions. • apply the First Derivative Test to find the minima and maxima of a function. • integrate trigonometric functions using trigonometric identities and *u*-substitution. • use integrals to find the average value of a function. • differentiate an inverse trigonometric function. • review the basic differentiation rules for elementary functions. • integrate functions whose antiderivatives involve inverse trigonometric functions. • use the method of completing the square to integrate a function. • review the basic integration rules involving elementary functions. • develop properties of hyperbolic functions. • differentiate and integrate hyperbolic functions. • develop properties of inverse hyperbolic functions. • differentiate and integrate functions involving inverse hyperbolic functions.	Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
Unit Twelve: Topics in Analytic Geometry **Timeline: 12 days**			
(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **CC.9-12.G.GPE.3**	Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.	**Learning Targets:** Students will • recognize a conic as the intersection of a plane and double-napped cone. • write equations of parabolas in standard from and graph parabolas. • use the reflective property of parabolas to solve real-life problems. • write equations of ellipses in standard form and graph ellipses. • use implicit differentiation to find the slope of a line tangent to an ellipse. • use properties of ellipses to model and solve real-life problems. • find eccentricities of ellipses. • write equations of hyperbolas in standard form. • find asymptotes of and graph hyperbolas. • use implicit differentiation to find the slope of a line tangent to a hyperbola. • use properties of hyperbolas to solve real-life problems. • classify conics from their general equations.	**Formative:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Summative Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
		• evaluate sets of parametric equations for given values of the parameter. • sketch curves that are represented by sets of parametric equations. • rewrite sets of parametric equations as single rectangular equations by eliminating the parameter. • find sets of parametric equations for graphs. • find the slope of a tangent line to a curve given by a set of parametric equations. • understand the polar coordinate system. • rewrite rectangular coordinates and equations in polar form and vice versa. • find the slope of a tangent line to a polar graph. • graph polar equations by point plotting. • use symmetry, zeros and maximum *r*-values to sketch graphs of polar equations. • recognize special polar graphs. • define conic in terms of eccentricity. • write and graph equations of conic in polar form. • use equations of conic in polar form to model real-life problems.	

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
Unit Thirteen: Additional Topics in Trigonometry **Timeline: 12 days**			
(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lVert\boldsymbol{v}\rVert$, v). **CC.9-12.N.VM.1** (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **CC.9-12.N.VM.2** (+) Solve problems involving velocity and other quantities that can be represented by vectors. **CC.9-12.N.VM.3** (+) Add and subtract vectors. **CC.9-12.N.VM.4** a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **CC.9-12.N.VM.4a** b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **CC.9-12.N.VM.4b** c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. **CC.9-12.N.VM.4c** (+) Multiply a vector by a scalar. **CC.9-12.N.VM.5** a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. **CC.9-12.N.VM.5a** b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\lVert c\boldsymbol{v}\rVert = \lvert c\rvert v$. Compute the direction of $c\boldsymbol{v}$	Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. The definition of trigonometric ratios is not only useful in solving right triangle problems but can also be applied to general triangles.	**Learning Targets:** Students will • use the Law of Sines to solve oblique triangles (AAS, ASA, or SSA). • find the areas of oblique triangles. • use the Law of Sines to model and solve real-life problems. • use the Law of Cosines to solve oblique triangles (SSS or SAS). • use Heron's Area Formula to find the area of a triangle. • use the Law of Cosines to model and solve real-life problems. • represent vectors as directed line segments. • write the component forms of vectors. • perform basic vector operations and represent them graphically. • write vectors as linear combinations of unit vectors. • find the direction angles of vectors. • use vectors to model and solve real-life problems. • find the dot product of two vectors and use	**Formative:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
knowing that when $\|c\|v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$). **CC.9-12.N.VM.5b** (+) Prove the Laws of Sines and Cosines and use them to solve problems. **CC.9-12.G.SRT.10** (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). **CC.9-12.G.SRT.11**		the properties of the dot product. • find the angle between two vectors and determine whether two vectors are orthogonal. • write a vector as the sum of two vector components. • use vectors to find the work done by a force. • plot complex numbers in the complex plane and find absolute values or complex numbers. • write the trigonometric forms of complex numbers. • multiply and divide complex numbers written in trigonometric form. • use DeMoivre's Theorem to find powers of complex numbers. • find *n*th roots of complex numbers.	
Unit Fourteen: Systems of Equations and Matrices (WEB) **Timeline: 12 days**			
(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. **CC.9-12.N.VM.6** (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. **CC.9-12.N.VM.7**	Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation.	**Learning Targets:** Students will • write, graph, and solve systems of linear equations in two variable • write, graph, and solve multivariable linear systems	**Formative:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems Summative

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
(+) Add, subtract, and multiply matrices of appropriate dimensions. **CC.9-12.N.VM.8** (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **CC.9-12.N.VM.9** (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **CC.9-12.N.VM.10** (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. **CC.9-12.N.VM.11** (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. **CC.9-12.N.VM.12** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **CC.9-12.A.REI.5** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **CC.9-12.A.REI.6** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$. **CC.9-12.A.REI.7** (+) Represent a system of linear equations as a single matrix equation in a vector variable. **CC.9-12.A.REI.8**		• write, graph, and solve systems of inequalities • use matrices to model and solve systems of equations • operate on matrices • find and use the inverse of a square matrix • find and use the determinant of a square matrix • use Crammer's rule	<u>Assessment:</u> Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concepts	Student Learning Targets	Assessments
(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). **CC.9-12.A.REI.9** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **CC.9-12.A.REI.10** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* **CC.9-12.A.REI.11** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. **CC.9-12.A.REI.12**			

Curriculum Framework for Calculus BC

School: Sussex Preparatory Academy **Curricular Tool: *Calculus* 9th Ed. (Larson & Edwards)** **Grade: 12**

Standards Alignment	Unit Concept Big Ideas	Student Learning Targets	Assessments
Unit One: Limits and Their Properties **Timeline : 9 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Introduction/Preview of Calculus Finding limits Evaluating limits analytically Continuity and one-sided limits Infinite limits	**Learning Targets:** How Calculus Compares with precalculus How Tangent line and area problems are basic to Calculus How to find limits graphically and numerically upon existence How to evaluate limits analytically: using properties of limits, dividing out, rationalizing techniques, and the Squeeze Theorem How to determine continuity at a point and on an open interval, and how to determine one-sided limits Understand and use the Intermediate Value Theorem How to determine infinite limits and find vertical asymptotes	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concept Big Ideas	Student Learning Targets	Assessments
Unit Two: Differentiation **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	The derivative and the tangent line Basic differentiation rules and rates of change Product and quotient rules and high-order derivatives The chain rule Implicit differentiation Related rates	**Learning Targets:** How to find the derivative of a function using the limit definition and understand the relationship between differentiability and continuity How to find the derivative of a function using basic differentiation rules How to find the derivative of a function using the Product Rule and the Quotient Rule How to find the derivative of a function using the Chain Rule and the General Power Rule How to find the derivative of a function using implicit differentiation How to find a related rate	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test
Unit Three: Applications of Differentiation **Timeline: 14 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and	Extrema on an Interval Rolle's Theorem and the Mean Value Theorem Increasing and Decreasing Functions and the First Derivative test Concavity and the Second Derivative test	**Learning Targets:** How to use a derivative to locate the minimum and maximum values of a function on a closed interval How numerous results in this chapter depend on two important theorems called Rolle's Theorem and the Mean Value Theorem	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones

Standards Alignment	Unit Concept Big Ideas	Student Learning Targets	Assessments
persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Limits an Infinity A summary of the curve Sketching Optimization problems Newton's Method Differentials	How to use the first derivative to determine whether a function is increasing or decreasing How to use the second derivative to determine whether the graph of a function is concave upward or concave downward How to find horizontal asymptotes of the graph of a function How to graph functions using the techniques from Chapters P-3 How to solve optimization problems How to use approximation techniques to solve problems	Lesson quizzes Unit Test
Unit Four: Integration **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others.	Antiderivatives and Indefinite Integration Area Riemann Sums and Definite Integrals The Fundamental Theorem of Calculus Integration by Substitution Numerical Integration	**Learning Targets:** How to evaluate indefinite integrals using basic rules How to evaluate a sum and approximate the area of a plane region How to evaluate a definite integral using a limit How to evaluate a definite integral using Fundamental Theorem of Calculus	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concept Big Ideas	Student Learning Targets	Assessments
4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.		How to evaluate different types of definite and indefinite integrals using a variety of methods How to approximate a definite integral using the Trapezoidal Rule and Simpson's Rule	
Unit Five: Logarithmic, Exponential, and Other Transcendental Functions **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	The Natural Logarithmic Function: Differentiation and Integration Inverse Functions Exponential Functions: Differentiation and Integration Bases Other than e and Applications Inverse Trigonometric Functions: Differentiation and Integration Hyperbolic Functions	**Learning Targets:** The properties of the natural logarithmic function. How to find the derivative and antiderivative of the natural logarithmic function How to determine whether a function has an inverse function The properties of the natural exponential function. How to find the derivative and antiderivative of the natural exponential function The properties, derivatives, and antiderivatives of logarithmic and exponential functions that have bases other than e The properties of inverse trigonometric functions. How to find the derivatives and antiderivatives of inverse trigonometric functions The properties of hyperbolic functions. How to find the derivative	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concept Big Ideas	Student Learning Targets	Assessments
		and antiderivative of hyperbolic functions	
Unit Six: Differential Equations **Timeline: 9 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Slope fields and Euler's method Differential Equations: Growth and Decay Separation of Variables and the Logistics Equation First-Order Linear Differential Equations	**Learning Targets:** How to sketch a slope filed of differential equations and find a particular solution How to use an exponential function to model growth and decay How to use separation of variables to solve a differential equation How to solve a first-order linear differential equation and a Bernoulli differential equation	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test
Unit Seven: Applications of Integration **Timeline: 12 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices	Area of region Between Two Curves Volume: The Disk Method and Shell Method Arc Length and Surfaces of Revolution Work Moments, Centers of Mass, and	**Learning Targets:** How to use a definite integral to find the area of a region bounded by two curves How to find the volume of a solid of	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems

Standards Alignment	Unit Concept Big Ideas	Student Learning Targets	Assessments
contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Centroids Fluid pressure and force	revolution by the disk and shell method How to find the length of a curve and the surface area of a surface of revolution How to find the work done by a constant force and by variable force How to find centers of mass ands centriods How to find fluid pressure and fluid force	**Formal Assessment:** Capstones Lesson quizzes Unit Test
Unit Eight Integration Techniques, L'Hopital Rule, and Improper Integrals **Timeline: 15 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically.	Basic Integration Rules Integration by Parts Trigonometric Integrals Trigonometric Substitution Partial Fractions Integration by Tables and other Integration Techniques Indeterminate Forms and L'Hopital Rule Improper Integrals	**Learning Targets:** How to fit an integrand to one of the basic integration rules How to find an antiderivative using integration by parts How to evaluate trigonometric integrals How to use trigonometric substitution to evaluate an integral How to use partial fraction decomposition to integrate rational functions	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concept Big Ideas	Student Learning Targets	Assessments
6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.		How to evaluate an indefinite integral using a table of integrals and using reduction formulas How to apply L'Hopital Rule to evaluate a limit How to evaluate an improper integral	
Unit Nine: Infinite Series **Timeline: 15 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning. .	Sequences Series and Convergence The Integral Test and p-Series Comparison of Series Alternating series The Ratio and Root tests Taylor Polynomials and Approximations Power Series Representation of Functions by Power Series Taylor and Maclaurin Series	**Learning Targets:** How to determine whether a sequence converges or diverges How to determine whether an infinite series converges or diverges How to find Taylor and Maclaurin polynomial approximation of elementary functions How to find the radius and interval of convergence of a power series and to differentiate and integrate power series How to represent functions by power series How to find a Taylor and Maclaurin series for a function	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test
Unit Ten: Conics, Parametric Equations, and Polar Coordinates **Timeline: 9 days**			

Standards Alignment	Unit Concept Big Ideas	Student Learning Targets	Assessments
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Conics and Calculus Plane Curves and Parametric Equations Parametric Equations and Calculus Polar Coordinates and Polar Graphs Area and Arc Length in Polar Coordinates Polar Equations of Conics and Kepler's Laws	**Learning Targets:** How to analyze and write equations of a parabola, an ellipse, and a hyperbola How to sketch a curve represented by parametric equations How to use a set of parametric equations to find the slope of a tangent line to a curve and the arc length of a curve How to sketch the graph of an equation in polar form, find the slope of a tangent line to a polar graph, and identify special polar graphs How to find the area of a region bounded by a polar graph and find the arc length of a polar graph How to analyze and write a polar equation of a conic	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test
Unit Eleven Vectors and the Geometry of Space **Timeline: 9 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and	Vectors in the Plane Space Coordinates and Vectors in Space The Dot Product of Two Vectors The Cross Product of Two vectors in Space Lines and Planes in Space Surfaces in Space	**Learning Targets:** How to write vectors, perform basic vector operations, and represent vectors graphically How to plot points in a three-dimensional coordinate system and analyze vectors in space How to find the dot product of two vectors (in plane or in space)	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concept Big Ideas	Student Learning Targets	Assessments
quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Cylindrical and Spherical Coordinates	How to find the cross product of two vectors (in space) How to find equations of lines and planes in space, and how to sketch their graphs How to recognize and write equations of cylindrical and quadric surfaces and of surfaces of revolution How to use cylindrical and spherical coordinates to represent surfaces in space	
Unit Twelve: Vector-Valued Functions **Timeline: 7 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of	Vector-Valued Functions Differentiation and Integration of Vector-Valued Functions Velocity and Acceleration Tangent Vectors and Normal Vectors Arc Length and Curvature	**Learning Targets:** How to analyze and sketch a space curve represented by a vector-valued function. How apply the concepts of limits and continuity to vector-valued functions How to differentiate and integrate vector-valued functions How to describe the velocity and acceleration associated with a vector-value function How to use a vector-valued function to analyze projectile motion How to find tangent vectors and normal vectors	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concept Big Ideas	Student Learning Targets	Assessments
structure. 8. Look for and express regularity in repeated reasoning.		How to find the arc length and curvature of a curve	
Unit Thirteen: Functions of Several Variables **Timeline: 15 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Introduction to Functions of Several Variables Limits and Continuity Partial Derivatives Differentials Chain Rules for Functions of Several Variables Directional Derivatives and Gradients Tangent Planes and Normal Lines Extrema of Functions of Two Variable And their Applications Langrange Multipliers	**Learning Targets:** How to sketch a graph, level curves, and level surfaces How to find a limit and determine continuity How to find and use a partial derivative How to find and use a total differential and determine differentiability How to the Chain Rule and find a partial derivative implicitly How to find and use a directional derivative and a gradient How to find an equation of a tangent plane and an equation of a normal line to a surface How to find the angle of inclination of a plane How to find absolute and relative extrema How to solve an optimization problem including constrained	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concept Big Ideas	Student Learning Targets	Assessments
		optimization using Lagrange multiplier How to the method of least squares	
Unit Fourteen: Multiple Integrations **Timeline: 15 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Iterated Integrals and Area in the Plane Double Integrals and Volume Change of Variables: Polar Coordinates Center of Mass and Moments of Inertia Surface Area Triple Integrals and Applications Triple Integrals in Cylindrical and Spherical Coordinates Change of Variables: Jacobians	**Learning Targets:** How to evaluate an iterated integral and find the area of a plane region How to use a double integral to find the volume of a solid region How to write and evaluate double integrals in polar coordinates How to find the mass of a planar lamina, the center of mass of a planar lamina, and moments of inertia using double integrals How to use a double integral to find the area of a surface How to use a triple integral to find the volume, center of mass, and moments of inertia of a solid region How to write and evaluate triple integrals in cylindrical and spherical coordinates How to use a Jacobian to change variables in double integral	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test

Standards Alignment	Unit Concept Big Ideas	Student Learning Targets	Assessments
Unit Fifteen: Vector Analysis **Timeline: 15 days**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Vector Fields Line Integrals Conservative Vector Fields and Independence of Path Greens' Theorem Parametric Surfaces Divergence Theorem Stokes's Theorem	**Learning Targets:** How to sketch a vector field, determine whether a vector field is conservative, find a potential functions, find curl, and find divergence How to find a piecewise smooth parametrization, write and evaluate a line integral, and use Green's Theorem How to use the Fundamental Theorem of Line Integrals, independence of path, and conservation of energy How to sketch a parametric surface, find a set of parametric equations to represent a surface, find a normal vector, find a tangent plane, and find the area of a parametric surface How to evaluate a surface integral, determine the orientation of a surface, evaluate a flux integral, and use the Divergence Theorem How to use Stokes's Theorem to evaluate a line integral or a surface integral and how to use curl to analyze the motion of a rotating liquid	**Informal:** Lesson Exercises Review Exercises Textbook Chapter Test Problem Solving problems **Formal Assessment:** Capstones Lesson quizzes Unit Test

Curriculum Framework for Statistics

School: Sussex Preparatory Academy **Curricular Tool: *Elementary Statistics*** **Course: Statistics**

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
Unit One: Statistics **Timeline : 4 weeks**			
Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **CC.9-12.S.IC.1** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **CC.9-12.S.IC.3** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **CC.9-12.S.IC.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* **CC.9-12.S.CP.4**	Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. When making statistical models, technology is valuable for varying assumptions, exploring consequences and comparing predictions with data. Causation implies correlation yet correlation does not imply causation. In a probability model, sample points represent outcomes and combine to make up events. The probabilities of the events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities	Understand and be able to describe the difference between descriptive and inferential statistics Understand and be able to identify and interpret the relationships between sample and population. Know and be able to identify and describe the different types of variables Understand how conveniences and volunteer samples result in biased samples Understand the differences among and be able to identify experiments, observational studies, and judgment samples. Understand and be able to describe the single-stage sampling methods of "simple random sample" and "systematic sampling' Understand and be able to describe the multistage sampling methods of "stratified sampling" and "cluster sampling" Understand that variability is inherent in everything and in the sampling process	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
	relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.		
Unit Two: Descriptive Analysis and Presentation of Single-Variable Data **Timeline: 3 weeks**			
Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **CC.9-12.S.CP.1** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. **CC.9-12.S.CP.2** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. **CC.9-12.S.CP.3** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly*	The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. Collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments	Create and interpret graphical displays, including pie charts, bar graphs, Pareto diagrams, dotplots, and stem-and-leaf plots displays Understand and be able to describe the difference between grouped and ungrouped frequency distributions, frequency and relative frequency and cumulative relative frequency Identify and describe the parts of a frequency distribution; class boundaries, class width, and class midpoint Create and interpret frequency histograms, and relative frequency histograms Identify the shapes of distributions Compute, describe, and compare the four measures of central tendency: mean, median, mode and midrange. Understand the effect of outliers on each of the four measures of central tendency Compute, describe, compare, and interpret the two measures of position: quartiles, percentiles, and z-scores. Create and interpret boxplots	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. **CC.9-12.S.CP.4** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* **CC.9-12.S.CP.5**		Understand the empirical rule and Chebyshev's theorem and be able to assess a set of data's compliance to these rules Know when and when not to use certain statistics-graphic and numeric	
Unit Three: Linear Systems and Matrices **Timeline: 3 weeks**			
(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. **CC.9-12.N.VM.6** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **CC.9-12.A.REI.5** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **CC.9-12.A.REI.6** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$. **CC.9-12.A.REI.7** (+) Represent a system of linear equations as a single matrix equation in a vector variable.	Algebraic manipulations used to solve equations/systems are governed by the underlying properties and structure of number systems and the conventions of algebraic notation. Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. When making statistical models, technology is valuable for varying assumptions, exploring consequences and comparing predictions with data.	Understand and be able to present and describe data in the form of two qualitative variables, both in contingency table format and appropriate graphs Understand and be able to present and describe data in the form of one qualitative variable and one quantitative variable, in both table format and appropriate graphs. Understand and be able to present and describe the relationship between two quantitative variables using a scatter diagram Understand and be able to explain a linear relationship Compute, describe, and interpret a correlation coefficient Compute, describe, and interpret a line of best fit Define and understand the difference between correlation and causation	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
CC.9-12.A.REI.8 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). **CC.9-12.A.REI.9** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **CC.9-12.S.ID.7**	Causation implies correlation yet correlation does not imply causation.	Determine and explain possible luring variables and their effects on a linear relationship Understand and be able to explain the slope of the line of best fit with respect to the context it is presented in Understand and be able to explain the y-intercept of the line of best fit with respect to the context it is presented in Create a scatter diagram with the line of best fit drawn on it Compute prediction values based on the line of best fit Understand and be able to explain what predication values are. Understand that predictions should be made lonely for values within the sample domain and that caution must be exercised for values outside that domain.	
Unit Four: Probability **Timeline: 4 weeks**			
Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* **CC.9-12.S.IC.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to	The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.	Understand and be able to describe the basic concept of probability Understand and describe a simple event Understand and be able to describe the differences between empirical, theoretical, and subjective probabilities Compute and interpret relative frequencies Identify and describe a sample space for an	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
determine if they are independent. **CC.9-12.S.CP.2** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. **CC.9-12.S.CP.3** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* **CC.9-12.S.CP.4** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* **CC.9-12.S.CP.5**	Collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments In a probability model, sample points represent outcomes and combine to make up events. The probabilities of the events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. .	experiment Construct tables, tree diagrams, and/or Venn diagrams to aid in computing and interpreting probabilities Understand the properties of probability numbers: 1. 0≤ each *P*(A)≤ 1 2. ∑ *P*(A) = 1 Understand , describe, and use the law of large numbers to determine probabilities Understand, compute, and interpret odds of an event Understand that compound events involve the occurrence of more than one event Construct, describe, compute, and interpret a conditional probability Understand and be able to utilize the complement rule Compute probabilities of compound events using the addition rule. Compute probabilities of compound events using the multiplication rule Understand, describe, and determine mutually exclusive events Compute probabilities of compound events using the addition rule for mutually exclusive events Understand , describe, and determine independent events	

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
		Compute probabilities of compound events using the multiplication rule for independent events Recognize and compare the differences between mutually exclusive events and independent events	
Unit Five: Probability Distributions (Discrete Variables) **Timeline: 2 weeks**			
(+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* **CC.9-12.S.MD.3**	The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. Collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments	Understand that random variables is a numerical quantity whose value depends on the conditions and probabilities associated with an experiment Understand the difference between a discrete and a continuous random variable Be able to construct a discrete probability distribution based on an experiment or given function Understand the terms *mutually exclusive* and *all-inclusive* as they apply to the variable for probability distributions Understand the similarities and differences between frequency distributions and probability distributions Understand and be able to utilize the two main properties of probability distribution to verify compliance Understand that a probability distribution in a theoretical probability distribution and that the mean and standard deviation (μ and σ, respectively) are parameters. Compute, describe, and interpret the mean and standard deviation of a probability distribution	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
		Understand the key elements of a binomial experiment and be able to define *x* ,*n* ,*p* and *q*. Know and be able to calculate binomial probabilities using the binomial probability function Understand and be able to use Table 2 in Appendix B, Binomial Probabilities, to determine binomial probabilities. Compute, describe, and interpret the mean and standard deviation of a binomial probability distribution	
Unit Six: Normal Probability Distributions **Timeline: 3 weeks**			
Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **CC.9-12.S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **CC.9-12.S.ID.4**	Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. When making statistical models, technology is valuable for varying assumptions, exploring consequences and comparing predictions with data. Causation implies correlation yet correlation does not imply causation.	Understand the difference between a discrete and continuous random variable Understand the relationship between the empirical rule and the normal cure Understand that a normal curve is a bell-shaped curve, with total area under the curve equal to 1 Understand that the normal curve is symmetrical about the mean , with an area of 0.5000 on each side of the mean Be able to draw a normal curve, labeling the mean and various *z*-scores Understand and be able to use Table 3, Areas of the Standard Normal Distributions, in Appendix B Calculate probabilities for intervals defined on the standard normal distribution	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
		Determine *z*-values for corresponding intervals on the standard normal distribution. Compute, describe, and interpret a *z*-value for a data value from a normal distribution Compute *z*-scores and probabilities for applications of the normal distribution Draw, compute, and interpret *z* of alpha notation, $z(\alpha)$ Understand the key elements of a binomial experiment: *x, n, p, q.* Know its mean and standard deviation formulas Understand that the normal distribution can be used to calculate binomial probabilities, provided certain conditions are met Understand and be able to use the continuity correction factor when calculating *z*-scores. Compute *z*-scores and probabilities for normal approximation to the binomial	
Unit Seven: Sample Variability **Timeline: 2 week**			
Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **CC.9-12.S.IC.4**	The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.	Understand what a sampling distribution of a sample statistic is and that the distribution is obtained from repeated samples, all of the same size Be able to form a sampling distribution for a mean, median, or range based on a small, finite population Understand that a sampling distribution is a probability distribution for a sample statistic.	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
	Collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments.	Understand and be able to present and describe the sampling distribution of sample means and the central limit theorem Understand and be able to explain the relationship between the sampling distribution of sample means and the central limit theorem Determine and be able to explain the effect of sample size on the standard error of the mean Understand when and how the normal distribution can be used to find probabilities corresponding to sample means Compute, describe, and interpret *z*-scores corresponding to known values of $\bar{x}$ Compute *z*-scores and probabilities for applications of the sampling distribution of sample means	
Unit Eight: Introduction to Statistical Inferences **Timeline: 4 weeks**			
Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **CC.9-12.S.IC.1**	The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. Collecting data from a random	Understand the difference between descriptive statistics and inferential statistics. Understand that an unbiased statistic has a sampling distribution with a mean that is equal to the population parameter being estimated. **With respect to confidence intervals:** Understand that a confidence interval is an interval estimate of a population parameter, with a degree of certainty, used when the population parameter is unknown	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
	sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments.	Understand that a point estimate for a population parameter is the value of the corresponding sample statistic Understand that the level of confidence is the long-run proportion of the intervals, which will contain the true population parameters, based on repeated sampling Understand and be able to describe the key components for a confidence interval: point estimate, level of confidence, confidence coefficient, maximum error of estimate, lower confidence limit, and upper confidence limit	
Unit Nine: Applications of Chi-Square **Timeline : 3 week**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Chi-Square Statistic Inferences Concerning Multinomial Experiments Inferences Concerning Contingency Tables	Understand that enumerative data are data that can be counted and placed into categories Understand that the chi-square distributions will be used to test hypotheses involving enumerative data Understand the properties of the chi-square distribution and how series of distribution based on sample size (using degrees of freedom as the index) Understand the key elements of multinomial experiment and be able to define n, k, O_i, and P_i Know and be able to calculate $E = np$ Know and be able to calculate a chi-square statistic: $x^2 = \sum_{(\text{all cells})} (o - E)^2/E$ Know and be able to calculate the degrees of freedom for a multinomial experiment (df = k – 1)	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
		Perform, describe, and interpret a hypothesis test for a multinomial experiment using the chi-square distribution with the *p*-value approach and/or the classical approach Understand and know the definition of independence of two events Know and be able to calculate expected values using $E_{ij} = R_1 \bullet C_j/n$ Know and be able to calculate the degrees of freedom for a test of independence or homogeneity [df = (r – 1)(c – 1) Perform , describe, and interpret a hypothesis test for a test of independence or homogeneity using the chi-square distribution with the *p*-value approach and/or the classical approach Understand the differences and similarities between tests of independence and tests of homogeneity	
Unit Ten: Analysis of Variance **Timeline: 3 week**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.	Introduction to the Analysis of Variance Technique The Logic behind ANOVA Applications of Single-Factor ANOVA	Understand that analysis of variance technique (ANOVA) are use to test differences among more than two means Understand that ANOVA uses variances to complete the testing of several means Understand that the *F*-distribution is used to test the ratio of the variation between the means being tested to the variation within the samples being tested Understand that if the variation between the means	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.		is significantly more than the variation within the samples ,then the means are considered unequal Compute, describe, and interpret a hypothesis test for the differences among several means, using the *F*-distribution with the *p*-value approach and/or the classical approach	
Unit Eleven: Linear Correlation and Regression Analysis **Timeline: 3 weeks**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Linea Correlation Analysis Inferences about the Linear Correlation Coefficient Linear Regression Analysis Inferences Concerning the Slope of the Regression Line Confidence Intervals for Regression Understanding the Relationship between Correlation and Regression	Understand what bivariate data, independent variable, and dependent variable are Understand that the linear correlation coefficient, *r*, measures the strength of the linear relationship between two variables Understand that the centroid for bivariate data is $(\bar{x}, \bar{y})$. Understand that the centroid is used in the calculation of the correlation coefficient Understand that covariance is a measure of linear dependency but that it is affected by the spread of the data Understand that the correlation coefficient , *r*, standardizes covariance so that relative strengths can be compared Understand that the assumptions for inferences about the linear correlation coefficient are that the ordered pairs form a random sample and that the *y* values at each *x* have a normal distribution. Inferences will utilize the *t*-distribution using $(n - 2)$ degrees of freedom	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
		Compute, describe, and interpret a confidence interval for the population correlation coefficient, *p*, using Table 10 in Appendix B Perform, describe, and interpret a hypothesis test for the population correlation coefficient, *p*, using the *t*-distribution with the *p*-value approach and classical approach Understand that the significance of *r* does not imply a cause-and-effect relationship Understand that the estimate of the experimental error ,*e*, is the difference between the observed *y* and the predicted *y*, $(y - y)$, at a given value of *x* Understand that the variance about the line of best fit is the same as the variance of the error, *e* Understand that the line of best fit passes through the centroid Compute, describe, and interpret a confidence interval for population slope of the regression line, β_1, using the *t*-distribution Perform, describe, and interpret a confidence interval for population slope of the regression line, β_1, using the *t*-distribution with the *p*-value approach and classical approach Compute, describe, and interpret a confidence interval for the mean value of *y* for a particular *x*, $(\mu_{y/x0})$, using the *t*-distribution. Compute, describe and interpret a prediction interval for an individual value of *y* for a particular *x*, (y_{x0}), using the *t*-distribution Understand the difference between a confidence	

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
		interval and a prediction interval for a *y* value at a particular *x* value	
Unit Twelve: Elements of Nonparametric Statistics **Timeline: 2 weeks**			
This unit goes beyond the level of rigor detailed in the Common Core State Standards for Mathematics. However, in teaching this unit the teacher will employ the mathematics practices contained within the standards. 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Nonparametric Statistics The Sign Test The Mann-Whitney *U* Test The Runs Test Rank Correlation	Understand that parametric methods are statistical methods that assume that the parent population is approximately normal or that the central limit theorem gives (at least approximately) a normal distribution of a test statistic Understand that nonparametric methods (distribution –free methods) do not depend on the distribution of the population being sampled Understand that the power of a test (1- β) is its ability to reject a false null hypothesis Understand that efficiency of a nonparametric test takes into account the power of a test and the required sample size. Understand that the sign test is the nonparametric alternative to the *t*-test for one mean and the difference between two dependent means Compute, describe, and interpret a confidence interval for a population median using the sign test Perform, describe, and interpret a hypothesis test for a single median using the sign test with the *p*-value approach and classical approach Perform, describe, and interpret a hypothesis test for the median of paired differences using the sign test with the *p*-value approach and classical approach	**Informal Assessments:** Applied Examples Section Exercises Technology Instructions Lesson Quiz Math Journal **Formal Assessment:** Unit Test Portfolio

Standards Alignment	Unit Concept/Big Ideas	Essential Questions Student Learning Targets	Assessments
		Understand that the Mann-Whitney *U* test is the nonparametric alternative to the *t*-test for the difference between two independent means Perform, describe, and interpret a hypothesis test for the difference between two means using the Mann- Whitney *U* test with the *p*-value approach and classical approach Perform, describe, and interpret a hypothesis test for the difference between two means using the normal approximations to the Mann-Whitney *U* test with the *p*-value approach and classical approach Perform, describe, and interpret a hypothesis test for the randomness of data using the runs test with the *p*-value approach and classical approach Perform, describe, and interpret a hypothesis test for the randomness of data using normal approximation to the runs test with the *p*-value approach and classical approach Understand that the Spearman rank correlation coefficient is the nonparametric alternative to the Pearson linear correlation coefficient, *r*. Perform, describe, and interpret a hypothesis test for the significance of correlation between two variable s using the Spearman rank correlation coefficient with the *p*-value approach and classical approach	

Using Technology to Explore Geometric Transformations

Delaware Recommended Curriculum
Mathematics Unit Cover Page

Preface: This unit has been created as a model for teachers in their designing or redesigning of course curricula. It is by no means intended to be inclusive; rather it is meant to be a springboard for teacher thought and creativity. The information we have included represents one possibility for developing a unit based on the Delaware content standards and the Understanding by Design framework and philosophy.

Unit Title: Using Technology to Explore Geometric Transformations (GeoMaster, Cabri Jr., Geometer's Sketchpad, etc.) **Grade Level(s):** 10

Subject/Topic Area: Geometry Reasoning

Searchable Key Words: Similar, congruent, inductive reasoning, conjectures, deductive reasoning, transformations, technology, GeoMaster, Cabri, Sketchpad

Designed By: Pamela Mason, Leslie Williams
Delmar High School

Time Frame: 12 to 15 hours

Reviewed by:

Date: Spring 2006

SUMMARY OF PURPOSE: In this 10th grade Geometry unit, students will learn about Geometric Transformations and how they are used in today's workplace. They will be able to not only write mathematical descriptions of transformations but also perform these transformations by hand and on available technology. Students will use their inductive and deductive reasoning skills to explore the ideas of congruence and similarity. As a final project, students will create a design for one of three products.

Charter School Unit Modification

Guiding Questions

1. Why was this model unit of instruction selected as part of your schools' curricular submission?

   This unit is posted on the DDOE web site as an exemplary unit of instruction for math. It teaches fundamental information and strategies for geometric reasoning.

2. What modifications have been made to the model unit of instruction to meet the specific needs of the student population your school serves?

   No modifications will need to be made based on the student population.

3. What modifications have been made to the model unit of instruction that reflect the resources (human, time, building, technology etc.) available to your school?

   No modifications will need to be made based on the resources available. Students will have access to computers to complete the work in the unit.

4. Describe any other modifications that have been made to the model unit of instruction that will assist in the curricular review for your school.
   No other modification will need to be made to the unit.

Using Technology to Explore Geometric Transformations

Stage 1: Desired Results

Alignment to Common Core State Standards for Mathematics

Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). **CC.9-12.G.CO.2**

Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. **CC.9-12.G.CO.3**

Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **CC.9-12.G.CO.4**

Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. **CC.9-12.G.CO.5**

Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **CC.9-12.G.CO.6**

Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **CC.9-12.G.CO.7**

Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **CC.9-12.G.CO.8**

Big Idea

Objects in space can be oriented in an infinite number of ways.

Enduring Understandings

Students will understand that…

- An object's location in space can be described quantitatively.
- There are multiple ways to transform an object.

Essential Questions

- How are objects transformed mathematically?
- Why are objects transformed mathematically?
- How can motion be described mathematically?

Unit Question(s)

- How can we use our knowledge of transformations to describe them mathematically?
- Which transformations are commutative?

Targeted Knowledge & Skill

Students will know…

- the differences between reflections, translations, rotations, and dilations.
- which transformations create similar and/or congruent figures.

Students will be able to… (21st century skills)

- describe various transformations a shape has undergone.
- develop their own designs and describe them using geometric transformations.
- graph composite transformations on the Cartesian plane.

Stage 2: Assessment Evidence

Transfer Task

Performance Task

You are working for Dynamic Designs, Inc. The following accounts have requested new artwork. *Sonic Skateboard Co.* needs a new design for their deck, *Pineapple Computer* needs a new screensaver, and *Jet Tees* is developing a new line of graphic t-shirts. Since you are the senior designer, you may choose which design to develop.

Each company requires a repetitive design. You have chosen to use composite transformations (of at least two types). The company requires a precise mathematical description in order to program the design into their manufacturing software. Please include a complete professional drawing on a Cartesian plane with the location of the pre-image clearly marked. If you choose to use graphing technology, be sure to format the diagram with axes on. Your boss is offering you a bonus if you create a model product for the client.

Rubrics for Transfer Tasks

Performance Task

	4	3	2	1
Diagram	Graphs composite transformations of at least two types precisely on a Cartesian plane	Graphs composite transformations of at least two types on a Cartesian plane	Graphs composite transformations on a Cartesian plane	Attempts to graph composite transformations
Instructions	Clearly, coherently provides step-by-step instructions using sophisticated mathematical language	Provides step-by-step instructions using mathematical language	Provides step-by-step instructions	Instructions contain errors in basic understanding

Other Evidence (e.g., tests, quizzes, prompts, work samples, observations)
All copies can be found in Appendix A.

Sketch and Describe Quizzes

Quiz A
Quiz B
Quiz C

Transformations Test

Student Self-Assessment and Reflection

Pairs Communication Activity

Directions: Partners each create a pre-image and an image using a single transformation of their choice without showing their partner. Next each student will write directions for creating their pre-image and then transforming it. First student A will read their directions to student B while the student B sketches as instructed. Once complete they will compare sketches and discuss how to make the directions more clear if necessary. Then they will switch jobs with student B reading and student A sketching. Students should be given multiple opportunities including different partners to repeat the process to build confidence and precision. At the conclusion of this activity the following reflection may be used.

Reflection:

1. On which part of this activity were you most successful and why (giving or receiving directions)?
2. Which part was most difficult and why?
3. What types of strategies did you develop with your partner to write directions more clearly?

Stage 3: Learning Plan

Key learning events needed to achieve unit goals

Learning Activities: What learning experiences and instruction will enable students to achieve the desired results?

- **Calculator exploration of transformations using GeoMaster.**
- **Graphing transformations (single and composite) by hand on a Cartesian plane.**
- **Pairs Communication Activity.**

Group exploration using TI Navigator and Interwrite board.

Lesson 1

General Topics Introduction of formal mathematical terminology that will be used throughout the unit.
Transformation, translation, reflection, rotation, rigid, composition of transformations, line of reflection, point of rotation, horizontal and vertical shifts, dilation, scale factor

1. Use PowerPoint presentation (see sample PowerPoint in Appendix B) of people and places where knowledge of geometric transformations are applied to generate interest and discussion about transformations. Discuss with students the benefits of this unit. **W, H**
2. Brainstorm as a class current knowledge about geometric transformations and formalize the correct mathematical terminology. Students will be given a Vocabulary Template (see Appendix) to organize transformation terminology. Complete template on Interwrite Board with students and print for any special needs students. **E1, O, T**

Lesson 2

General Topics Reflection

1. Begin having students reflect basic polygons with paper and pencil. Make conjectures about what it's going to look like. **E1**
2. Ask students to discuss the ideas of congruence and similarity. Are all images created by reflections congruent? Are all images created by reflections similar? How could

various mathematical principles be used to prove that all reflections congruent? (distance formula, slope formula, SSS, SAS, ASA, etc.) **E1, R**

3. Provide a homework problem in which student will justify congruence of a reflected triangle. **E1**
4. Use direct instruction to show students how to reflect geometric shapes using a TI calculator with GeoMaster and Cabri Jr. (or using a computer with Geometer's Sketchpad software). **E1**
5. Allow students time to reflect polygons using any line, not just vertical and horizontal lines. Various student designs should be projected via TI Navigator (or computer projector) in order to encourage further exploration and generate possible conjectures for multiple reflections. **E1, T, R**
6. Informally assess student work via the TI Navigator and assist and encourage exploration as needed. **E1, E2**

Lesson 3

General Topics Rotations

1. As a warm-up begin class by polling students using the TI Navigator to informally assess reflection knowledge. **E2**
2. Using TI Navigator send students FLAGTURN program (see Appendix). Discuss what is occurring in the program. Allow students time to change the geometric shape being rotated by changing the coordinates in matrix B. **E1**
3. Use direct instruction to show students how to rotate geometric shapes using GeoMaster, Cabri Jr. or Geometer's Sketchpad, emphasizing the need to specify a center point and an angle of rotation. **E1**
4. Allow students time to rotate polygons using appropriate technology and explore the effects of rotations around various points with various angles of rotation. **E1**
5. Again discuss with students the ideas of similarity and congruence. When geometric shapes are rotated, is the image similar or congruent to the pre-image? How can this be proved using previous knowledge? **E1, R**
6. Provide a homework problem in which student will justify congruence of a rotated triangle. **E1**
7. Distribute graph paper, rulers, and protractors and discuss how to rotate by hand. Provide various examples of rotating segments and triangles 90 degrees, 180 degrees, 270 degrees, and 360 degrees. **E1**
8. Students should complete Quiz A (see Appendix).

Lesson 4

General Topics Translations

1. As a warm-up display the visual (see Appendix) and ask students to describe the transformations using correct mathematical terminology. **E2**
2. Discuss how to describe translations mathematically with the students. Again discuss with students the ideas of similarity and congruence. When geometric shapes are translated, is the image similar or congruent to the pre-image? How can this be proved

using previous knowledge? **E1, R**

3. Provide examples and practice problems of translating triangles. **E1**
4. Have students complete the Rigid Transformation Organizer (see Appendix) individually. Discuss results as a class using the Interwrite Board. Print for special needs students. **T, O**
5. Students complete the Pairs Communication Activity (see Stage 2: Student Self-Assessment and Reflection). **E2**
6. Provide a homework problem in which student will justify congruence of a translated parallelogram. **E1**
7. Students should complete Quiz B (see Appendix).

Lesson 5

General Topics Dilations

1. Discuss the need for dilations with scale drawings in construction, photo enlargement, camera operations, zoom feature on calculator, nesting cups, etc. **W**
2. Allow students time to dilate polygons using appropriate technology and explore the effects of various scale factors. **E1**
3. Again discuss with students the ideas of similarity and congruence. When geometric shapes are dilated, is the image similar or congruent to the pre-image? How can this be proved using previous knowledge? (AA, SAS, SSS) **E1, R**
4. Provide a homework problem in which student will justify similarity of a dilated triangle. **E1**
5. Students should complete Quiz C (see Appendix).

Lesson 6

General Topics Review and Assess

1. Students will complete the Pairs Communication Activity using different student groupings than in Lesson 4. (see Stage 2:Student Self-Assessment and Reflection). **E2**
2. Students should complete the Unit Test (see Appendix).
3. Students will work with a partner to correct any errors and misunderstandings from the Unit Test. Students will write two paragraphs each addressing each category from the Unit Test Reflection Checklist (see Appendix). One paragraph will highlight their strengths on the test. In the second paragraph students will explain any misunderstandings from the test including test corrections. **R, E2, O**
4. Students should complete the Performance Assessment.

The acronym WHERETO summarizes key elements to consider when designing an effective and engaging learning plan.

W – Help the students know Where the unit is going and What is expected? Help the teachers know Where the students are coming from (prior knowledge, interests)
H – Hook all students and Hold their interest?
E – Equip students, help them Experience the key ideas and Explore the issues?
R – Provide opportunities to Rethink and Revise their understandings and work?
E – Allow students to Evaluate their work and its implications?
T – Be Tailored (personalized) to the different needs, interests, and abilities of learners?
O – Be Organized to maximize initial and sustained engagement as well as effective learning?

Code

IP = international education perspective
IL = information literacy
WR = Workplace readiness/21st century skills
FA = a formative assessment, used to check for understanding

Resources & Teaching Tips

- **What resources best enrich or support this unit?**

Internet

http://www.shodor.org/interactivate/activities/transform/index.html
This site can be used by students and teachers to transform geometric rigid shapes on the coordinate plane.

http://education.ti.com/educationportal/sites/US/productDetail/us_ti_navigator.html
This part of the Texas Instruments website for educators discusses the TI Navigator.

http://education.ti.com/educationportal/activityexchange/activity_list.do?cid=us
This part of the Texas Instruments website for educators includes classroom activities using their technology.

http://www.calcomp.com/interwriteschoolpad.htm
This site shows how to use the Interwrite SchoolPad in the classroom, complete with video and ordering details.

http://education.ti.com/educationportal/sites/US/productDetail/us_geomaster_83_84.html
This site is where GeoMaster may be downloaded for free, complete with users guide.

Media

PowerPoint

Print

Coxford, Arthur F., Fey, James T., Hirsch, Christian R., Schoen, Harold L., Burrill, Gail, Hart, Eric W., & Watkins, Ann E. (1997). *Contemporary Mathematics in Context Course II*. Chicago, IL: Everyday Learning.

- **What tips to teachers of the unit can you offer about likely rough spots/students misunderstandings and performance weaknesses, and how to troubleshoot those issues?**

Accommodation/Differentiation ideas and tips

The time period needed to complete this unit will vary depending upon student achievement levels and class grouping.

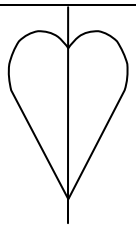
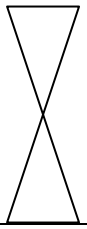
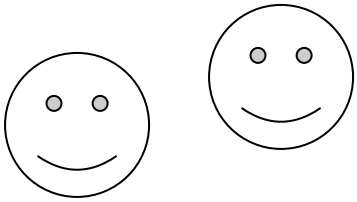
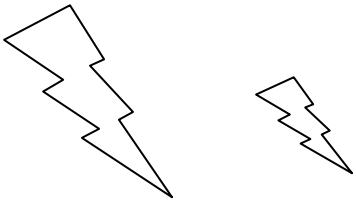
Appendix A

Sample Resources

Using Technology to Explore Geometric Transformations
Vocabulary Template

Transformation	Related Terms	Sketch

Example of Completed Vocabulary Template

Transformation	Related Terms	Sketch
Reflection (flip across a line)	Line of reflection Pre-image and image Rigid	
Rotation (turn about a point in a specific direction)	Point of rotation Degrees Clockwise or counterclockwise Rigid	
Translation (shifted copy)	Vertical and horizontal shift Rigid	
Dilation (reduction or enlargement)	Scale factor Not rigid	

Flagturn Program

```
ClrDraw
For(N,1,8,1)
[A]*[B]→[B]
Line([B](1,1),[B](2,1),[B](1,2),[B](2,2))
Line([B](1,2),[B](2,2),[B](1,3),[B](2,3))
Line([B](1,3),[B](2,3),[B](1,4),[B](2,4))
For(K,1,25,1)
End
ClrDraw
End
Stop
```

Pre-load matrices A and B as follows:

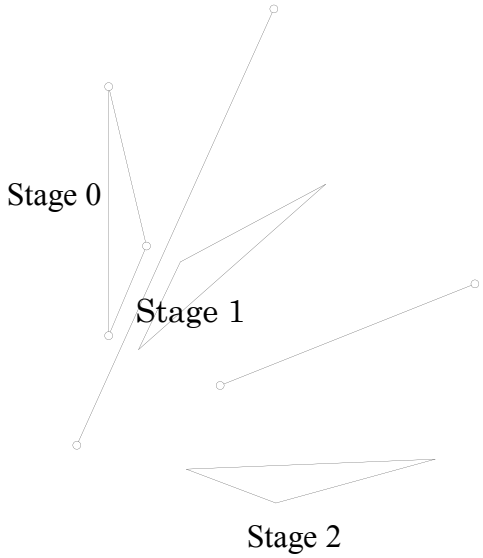
$$A = \begin{pmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 & 3 & 0 \\ 0 & 8 & 6 & 4 \end{pmatrix}$$

Reproduced from:
Coxford, Arthur F., Fey, James T., Hirsch, Christian R., Schoen, Harold L., Burrill, Gail, Hart, Eric W., & Watkins, Ann E. (1997). *Contemporary Mathematics in Context Course II*. Chicago, IL: Everyday Learning. (Page 153)

Using Technology to Explore Geometric Transformations

Quiz A

Use the following diagram to answer the questions below. You may use a protractor and ruler as needed.



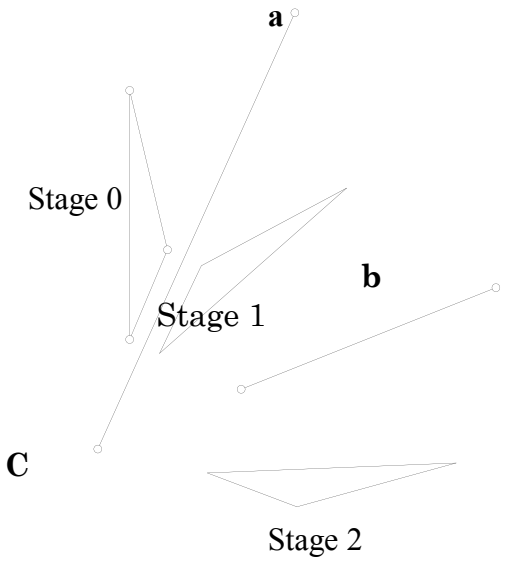
1. Describe the transformation from the pre-image at stage 0 to the image at stage 1.

1. Describe the transformation from the image at stage 1 to the image at stage 2.

2. Describe in a single transformation how the pre-image can be formed into the image at stage 2.

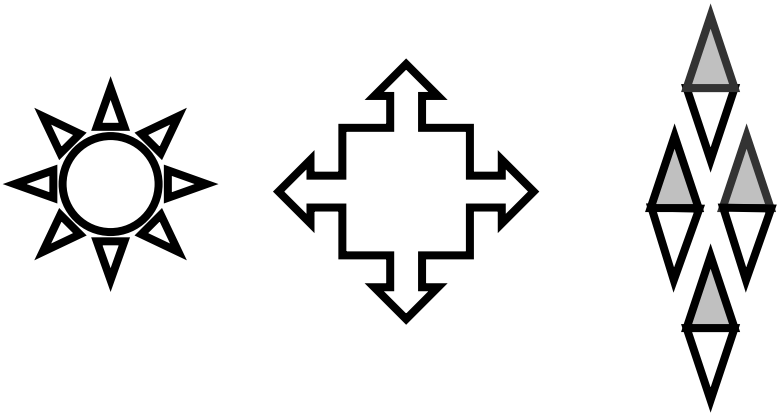
Using Technology to Explore Geometric Transformations

Key to Quiz A



1. reflection across line a (see above)
2. reflection across line b (see above)
3. 80 – 90 degree clockwise rotation about point C (see above)

Lesson 4 Warm-Up Visual



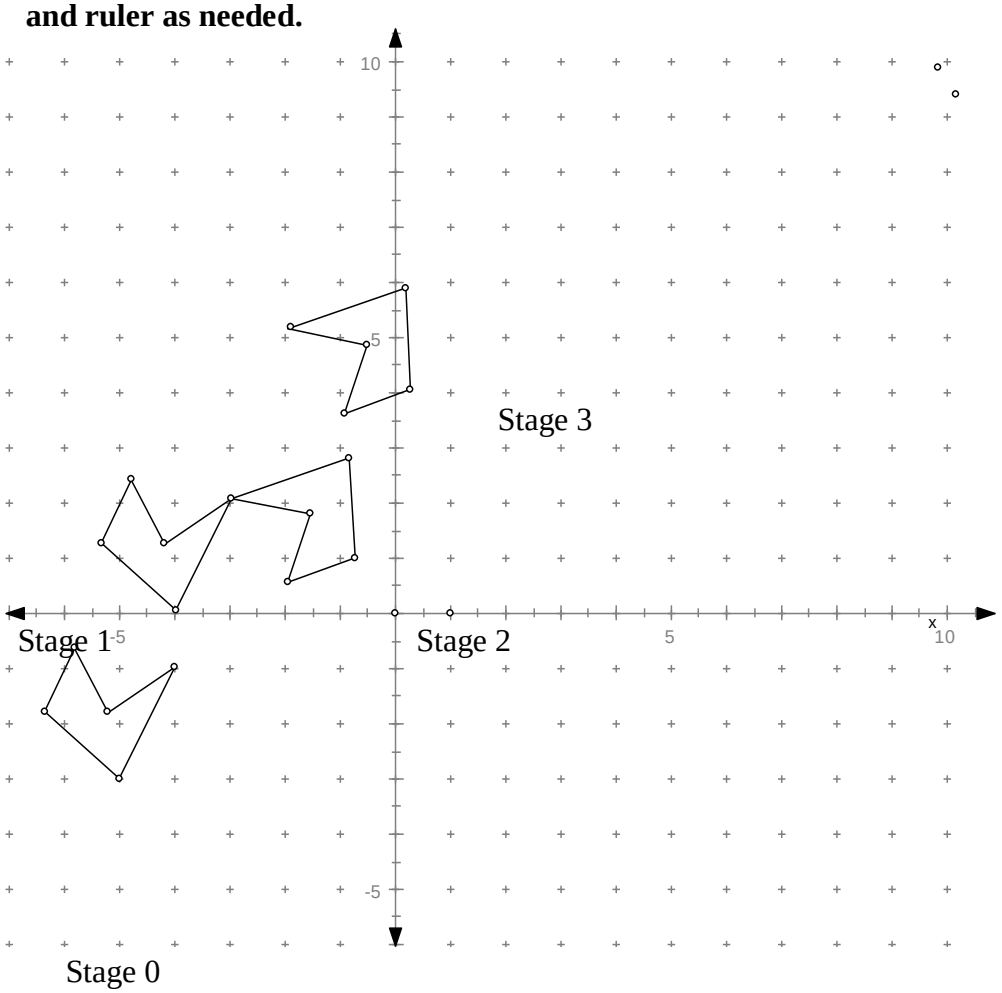
Rigid Transformation Organizer

Reflections	
x-axis	$(x, y) \rightarrow (\quad , \quad)$
y-axis	$(x, y) \rightarrow (\quad , \quad)$
$y = x$	$(x, y) \rightarrow (\quad , \quad)$
$y = -x$	$(x, y) \rightarrow (\quad , \quad)$
Rotations Counterclockwise	
90° about the origin	$(x, y) \rightarrow (\quad , \quad)$
180° about the origin	$(x, y) \rightarrow (\quad , \quad)$
270° about the origin	$(x, y) \rightarrow (\quad , \quad)$
Translations	
horizontal translation: h vertical translation: k	$(x, y) \rightarrow (\quad , \quad)$

Using Technology to Explore Geometric Transformations

Quiz B

Use the following diagram to answer the questions below. You may use a protractor and ruler as needed.



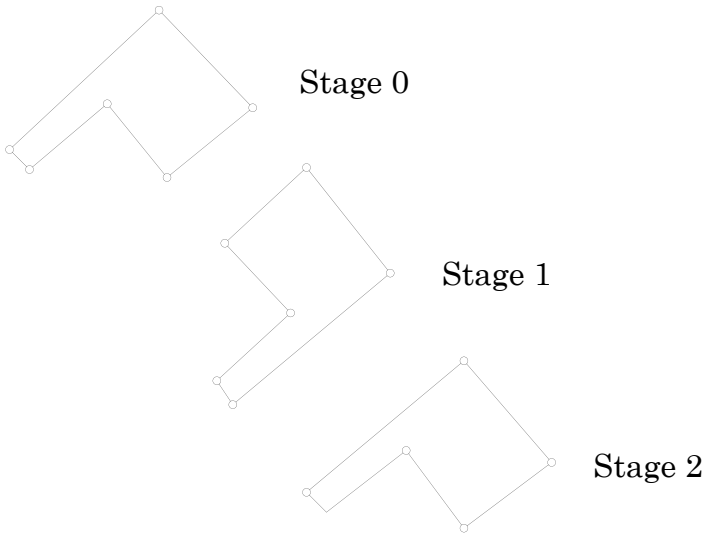
1. Describe the transformation from the pre-image at stage 0 to the image at stage 1.

2. Describe the transformation from the image at stage 1 to the image at stage 2.

3. Sketch stage 4 and stage 5 on the Cartesian plane above.

Using Technology to Explore Geometric Transformations

Use the following diagram to answer the questions below.



4. Describe the transformation from the pre-image at stage 0 to the image at stage 1.

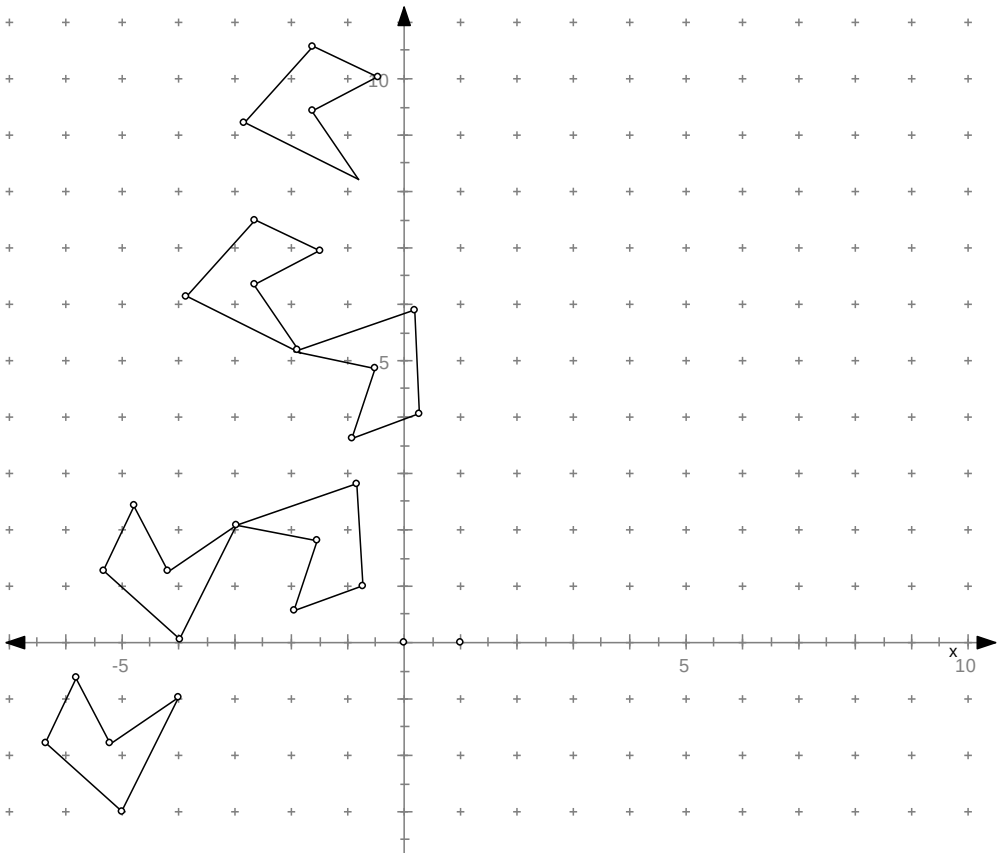
5. Describe the transformation from the image at stage 1 to the image at stage 2.

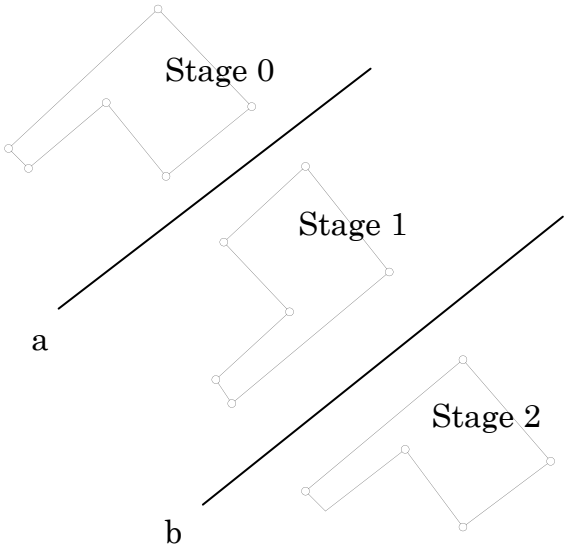
6. Describe in a single transformation how the pre-image can be formed into the image at stage 2.

Using Technology to Explore Geometric Transformations

Key to Quiz B

1. translation (x+1, y+3) or equivalent expression
2. rotation of 135º counterclockwise about (-3,2) – tall tip of pentagon



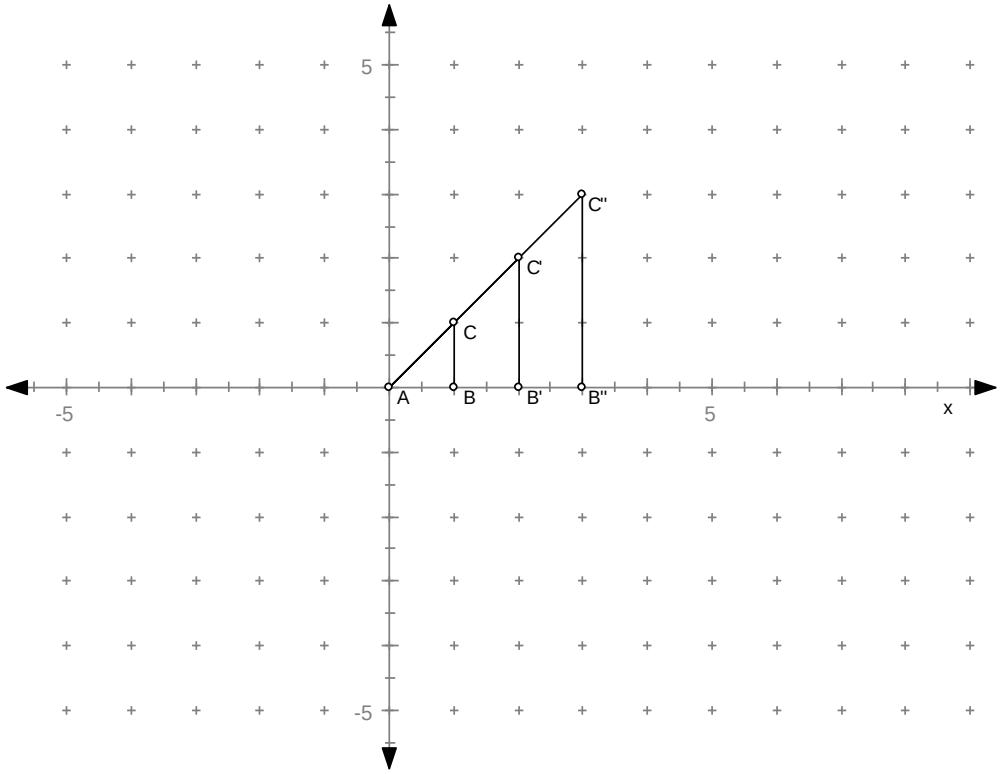


1. reflection across line a, (see above)
2. reflection across line b, (see above)
3. translation down and to the right, twice the distance between lines a and b

Using Technology to Explore Geometric Transformations

Quiz C

Use the following diagram to answer the questions below. You may use a protractor and ruler as needed.



1. Describe the transformation from the pre-image ΔABC to image ΔA'B'C'.

2. Describe the transformation from the image ΔA'B'C' to the image at ΔA"B"C".

3. Describe in a single transformation how the pre-image ΔABC can be formed into the image ΔA"B"C".

Using Technology to Explore Geometric Transformations

Key to Quiz C

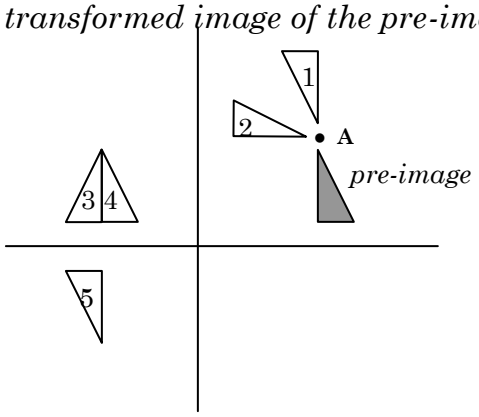
1. dilation scale factor 2
2. dilation scale factor 1.5
3. dilation scale factor 3

Using Technology to Explore Geometric Transformations

Transformations Unit Test

1. *MATCHING – Write the number of the transformed image of the pre-image, that best matches the description.*

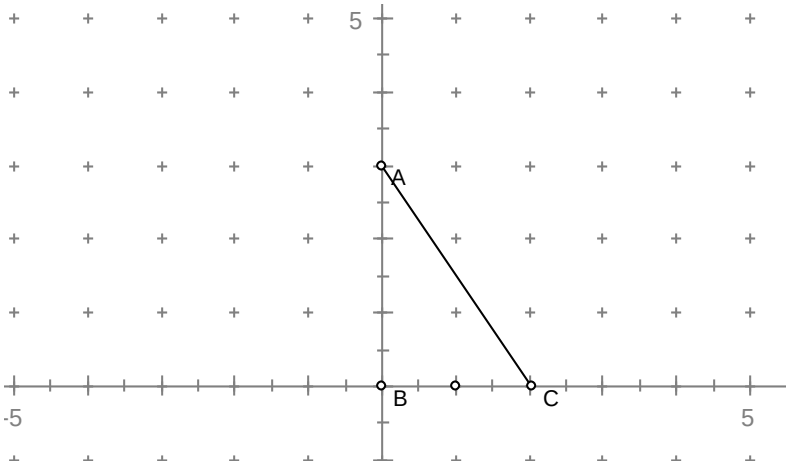
_____a. translation
_____b. 180° rotation about the origin
_____c. 180° rotation about point A
_____d. reflection across the y-axis
_____e. reflection across y = x



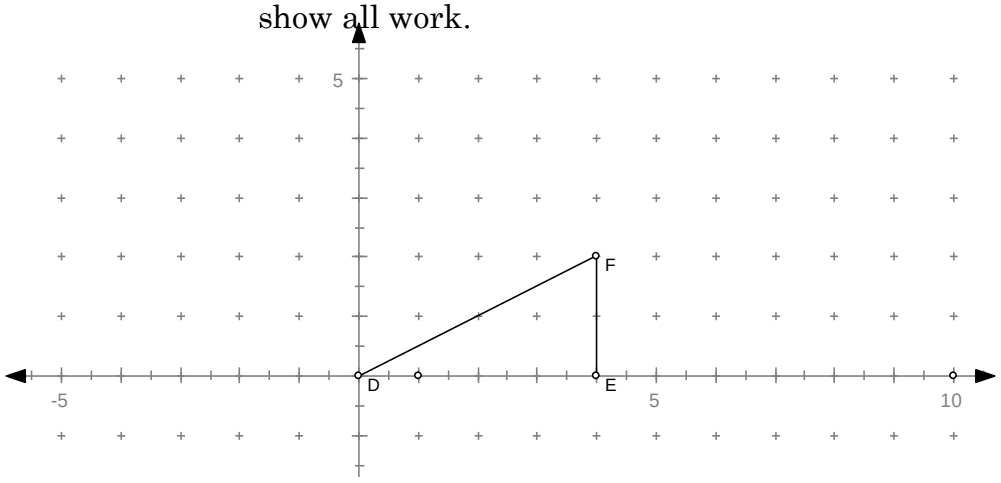
2. *Describe what happens to the x & y coordinates when a pre-image is*
   a. rotated 180°.

   b. reflected across the x-axis.

3. *Transform and Justify*
   a. Reflect triangle ABC across the y-axis.
   b. Justify that the image is congruent to the pre-image. Explain and show all work.



c. Dilate triangle DEF by a scale factor of ½ .
d. Justify that the image is similar the pre-image. Explain and show all work.



4. *Commutativity*
   a. Draw one example, complete with explanation, of a composition of geometric transformations that is commutative.

   b. Draw one example, complete with explanation, of a composition of geometric transformations that is NOT commutative.

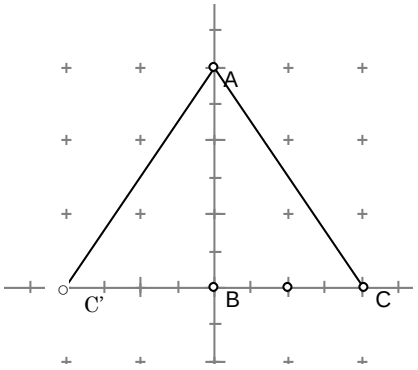
Using Technology to Explore Geometric Transformations

ANSWER KEY for Transformations Test

1. a. 4 c. 1 e. 2
   b. 5 d. 3

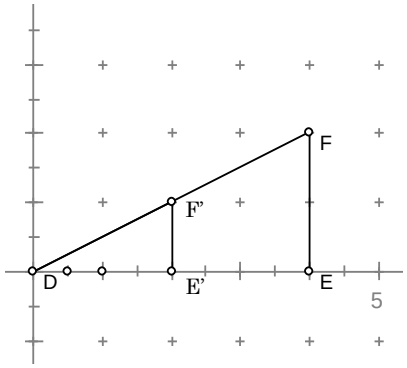
2. a. (-x,-y) or equivalent expression
   b. (x, -y) or equivalent expression

3. a.



b. AB = 3; BC = 2; AC = $\sqrt{13}$;
A'B' = 3; B'C' = 2; A'C' = $\sqrt{13}$
Since three pairs of corresponding sides are congruent, the triangles (the pre-image and image) are congruent by the SSS Theorem.

c.



d. Since EF and DE are perpendicular, angle E is a right angle. Likewise, since E'F' and D'E' are perpendicular, angle F'E'D' is a right angle. Since both are right angles, angle E and angle F'E'D' are congruent. Also, angle FDE is congruent to angle F'D'E'. Therefore, the triangles (the pre-image and image) are similar by the AA Theorem.

4.

	2	1
Diagram	Contains clear composite transformation which is (or is not) commutative	Contains composite transformation which is (or is not) commutative
Explanation	Clear explanation using sophisticated mathematical knowledge	Explanation contains a minor error

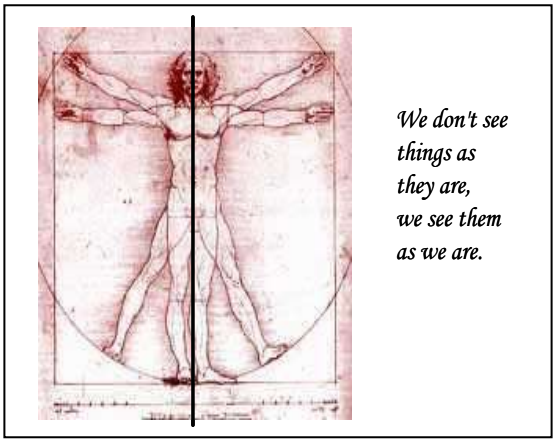
Unit Test Reflection Checklist

	Fully	Partially	Not At All
I explained at least 3 strengths that I demonstrated on this test.			
I explained all major misunderstandings I had when I took this test.			
I corrected all errors on my test.			

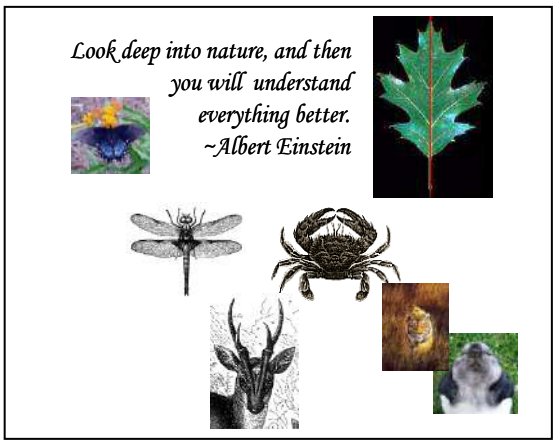
Appendix B

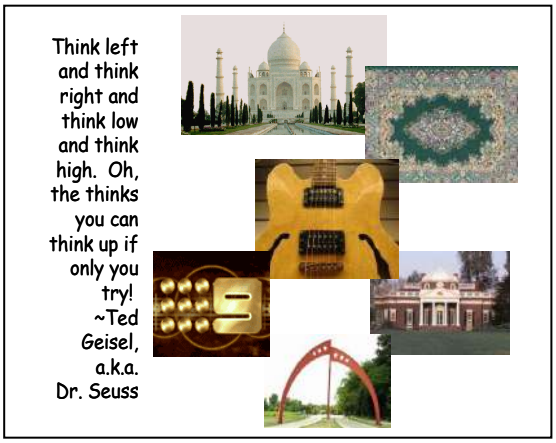
Sample PowerPoint Presentation

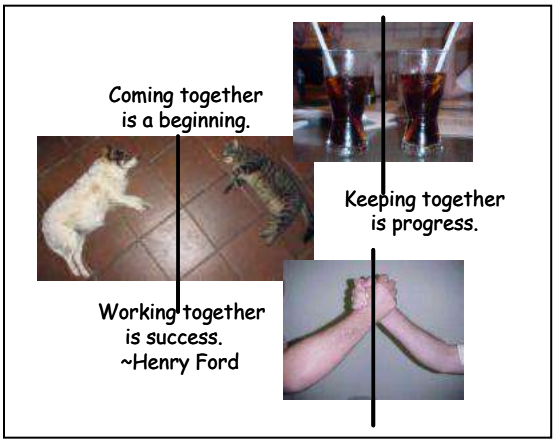

Transformations
In Our World


We don't see
things as
they are,
we see them
as we are.


Do you see
what I see?


Look deep into nature, and then
you will understand
everything better.
~Albert Einstein


Think left
and think
right and
think low
and think
high. Oh,
the thinks
you can
think up if
only you
try!
~Ted
Geisel,
a.k.a.
Dr. Seuss

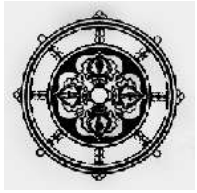

Coming together
is a beginning.
Keeping together
is progress.
Working together
is success.
~Henry Ford

Using Technology to Explore Geometric Transformations

Clocks slay time...
time is dead as long as it is being clicked off by little wheels;
only when the clock stops does time come to life.
~William Faulkner

People from a planet without flowers would think we must be mad with joy the whole time to have such things about us.

Art picks up where nature ends.

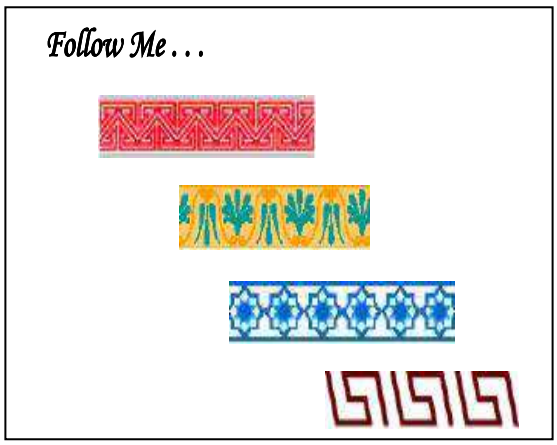


Form depends on function.

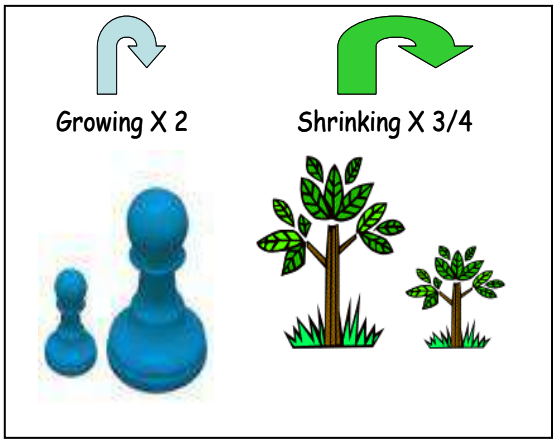
If you are seeking creative ideas, go out walking. Angels whisper to a man when he goes for a walk.

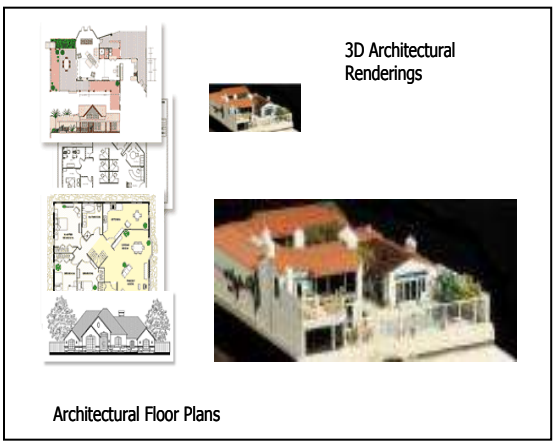
Lack of activity destroys the good condition of every human being, while movement and methodical physical exercise save it and preserve it.
~Plato

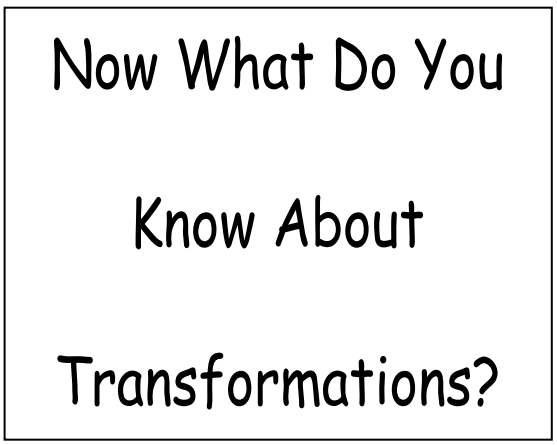

Though we travel the world
over to find the beautiful,
we must carry it with us
or we find it not.
~Ralph Waldo Emerson


Follow Me . . .


Babies
are such
a nice
way to
start
people.


Growing X 2
Shrinking X 3/4


3D Architectural
Renderings
Architectural Floor Plans


Now What Do You
Know About
Transformations?

Using Technology to Explore Geometric Transformations

Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title: Trigonometric Functions

Designed by: Christine Bichler and Takashi Rhoulac
For: Innovative Schools

Content Area: Calculus with Pre-Calculus
Grade Level(s): 9-12

Summary of Unit: In this Calculus with Pre-Calculus unit, students will learn about Trigonometric functions. Students will use Trigonometric functions to find relationships between sides and angles of triangles. Students will be able to model quantities that are periodic. Trigonometric Functions will be used to model real-life quantities.

Stage 1 – Desired Results

What students will know, do, and understand

Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **CC.9-12.F.TF.1**

Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **CC.9-12.F.TF.2**

(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. **CC.9-12.F.TF.3**

(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **CC.9-12.F.TF.4**

Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* **CC.9-12.F.TF.5**

(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **CC.9-12.F.TF.6**

(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.* **CC.9-12.F.TF.7**

Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **CC.9-12.G.SRT.6**

Explain and use the relationship between the sine and cosine of complementary angles. **CC.9-12.G.SRT.7**

Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.* **CC.9-12.G.SRT.8**

Big Idea(s)

Understand how the concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades.

These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definition of trigonometric ratios is not only useful in solving right triangle problems but can also be applied to general triangles.

Unit Enduring Understanding(s)

Students will understand that……..

- Trigonometric Functions can be used to model and solve real-life mathematical problems

Unit Essential Questions(s)

How can angles be described, radian measures be used, and degree measures be used?

How can trigonometric functions be evaluate using the unit circle?

How can trigonometric functions be evaluated for acute angles and how can fundamental trigonometric identities be used?

How can reference angles be used to evaluate trigonometric functions of any angle?

How can the graphs of sine and cosine be sketched?

How can the graphs of tangent, cotangent, secant, and cosecant be sketched?

How can inverse trigonometric functions be evaluated?

How are real-life problems involving right triangles, directional bearings, and harmonic motion solved?

Knowledge and Skills

Students will know…

- how to describe angles.
- how to use radian measure.
- how to use degree measure.
- how to identify a unit circle and describe its relationship to real numbers.
- how to use the domain and period to evaluate sine and cosine functions.
- how to use a calculator to evaluate trigonometric functions.
- how to evaluate trigonometric functions of acute angles.
- how to use fundamental trigonometric identities.
- how to evaluate trigonometric functions of any angle.
- how to use reference angles to evaluate trigonometric functions.
- how to use amplitude and period to help sketch the graphs of sine and cosine functions.

- how to translations of the graphs of sine and cosine functions.

Students will be able to…

- use angles to model and solve real-life problems.
- evaluate trigonometric functions using the unit circle.
- use sine and cosine functions to model real-life data.
- use trigonometric functions to model and solve real-life problems.
- evaluate and graph the inverse sine function, the other inverse trigonometric functions, and the compositions of trigonometric functions
- sketch the graphs of sine and cosine functions, tangent functions, cotangent functions, secant and cosecant functions, and damped trigonometric functions.
- solve real-life problems involving right triangles.
- solve real-life problems involving directional bearings.
- solve real-life problems involving harmonic motion

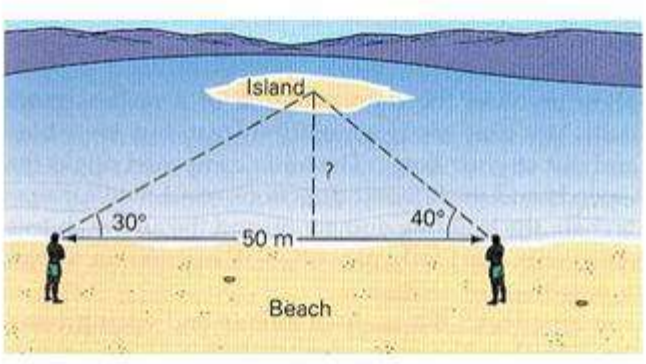
Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

Suggested Performance/Transfer Task(s)

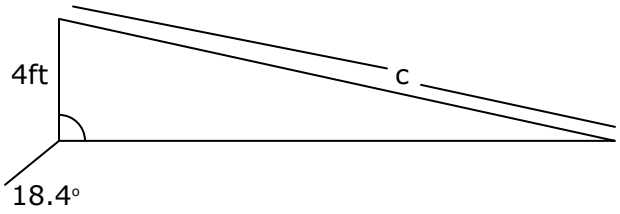
- Performance/Transfer tasks as evidence of student proficiency

1.



Juan is standing due west of Charlie. They are 50 meters apart. An island is spotted and the bearings from Charlie and Juan are N40°W and N30°E respectfully. Find the distance ? of the island from the line formed between Juan and Charlie. Show your work.

2. Find the length of the skateboard ramp c, shown below.



3. In right triangle trigonometry, explain why $\sin 30^\circ = \frac{1}{2}$ regardless of the size of the triangle.

Rubric(s)

- Scoring guide to evaluate performance/transfer tasks used as evidence of student proficiency

	3	2	1	0
	Student provide a correct answer with work to support the answer	Students have made minor mathematical errors	Attempt was made but their were major mathematical flaws	No attempt was made

Other Evidence

- Varied evidence that checks for understanding (e.g., tests, quizzes, prompts, student work samples, observations and supplements the evidence provided by the task)

Capstones
Lesson quizzes
Unit Test

Student Self-Assessment and Reflection

- Opportunities for self-monitoring learning (e.g., reflection journals, learning logs, pre- and post-tests, self-editing – based on ongoing formative assessments)

Lesson Exercises
Review Exercises
Writing about Concepts
Textbook Chapter Test
Problem Solving problems

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

- Instructional activities and learning experiences needed to align with Stage 1 and Stage 2 expectations

Include these instructional elements when designing an effective and engaging learning plan for ALL students:

*Align with expectations of Stage 1 and Stage 2

*Scaffold in order to acquire information, construct meaning, and practice transfer of understanding

*Include a wide range of research-based, effective, and engaging strategies

*Differentiate and personalize content, process, and product for diverse learners

*Provide ongoing opportunities for self-monitoring and self-evaluation

- Identify the basic ratios for the six trigonometric functions

- Identify the graphs of the six trigonometric functions
- Geometer's Sketchpad
- Texas Instrument Graphing Calculator

Lesson 1

General Topics: Radian and Degree Measure

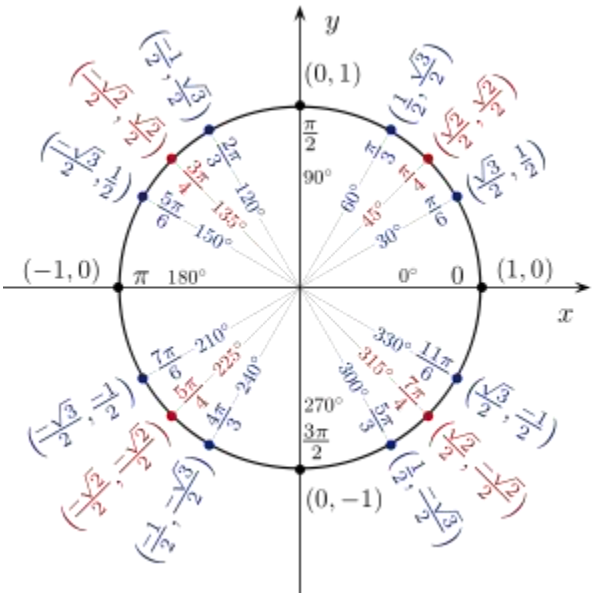
1. Discuss and review/ define the following vocabulary terms as encountered within the context of the lesson:
2. *trigonometry, angle, initial side, terminal side, vertex, standard position, positive angles, negative angles, $\theta(theta), \alpha\ (alpha), \beta\ (beta), \pi\ (pi)$coterminal, measure of an angle (acute, obtuse), radian measures, central angles, complementary, supplementary, degrees, arc length (S = 2πr)*. **E1, R**
3. Use direct instruction to model for students how to find the measure of an angle and how to convert the measure to either radian to degree measure depending upon the initial measure. Students should be shown how to derive the common angle/ radian measures $\frac{\pi}{6}\ (30), \frac{\pi}{4}, (45)\frac{\pi}{3}(60), \frac{\pi}{2}\ (90), \pi\ (180), 2\pi(360)$. Students should also be shown how to find the coterminal angle. Demonstrate on the graphing calculator how to convert between radian and degree measures. **E1**
4. Students will work with a partner and sketch diagrams using Geometer's Sketch Pad. Students should draw angles and identify the initial and terminal sides as well as classify them as acute or obtuse and positive or negative. Their partner should draw the corresponding coterminal angle. **E2**
5. Have students work in pairs to complete even exercise problems. Discuss results whole class using the Interactive write Board to draw figures. Print for special needs students. **T, O**
6. Students complete writing about concepts and capstone problems (Student Self-Assessment and Reflection). **E2**
7. For homework, students should complete the fill in the blanks section problems 1-10 to review vocabulary terms. Additional odd problems should be assigned to provide students with an independent opportunity. A real-world problem should also be assigned from problems 104 – 109 to allow students the opportunity to make practical application. **E1**
8. Students should complete quiz lesson 9:1

Lesson 2

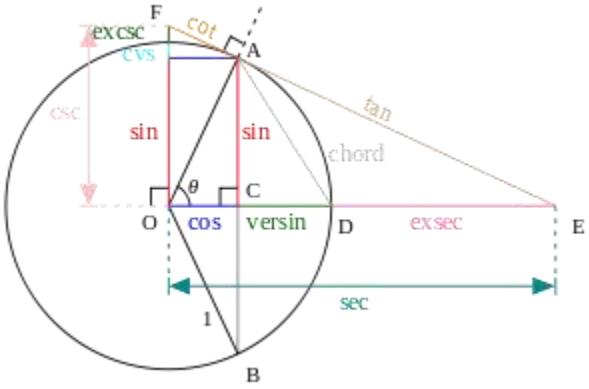
General Topics: Trigonometric Functions: The Unit Circle

1. Discuss and review/ define the following vocabulary terms as encountered within the context of the lesson:
   unit circle, trigonometric functions – sine, cosine, tangent, cotangent, secant, cosecant and (their abbreviations) sin, cos, tan, cot, sec, sec. **E1, R**
2. Direct instruction should be provided to explain how the formula for the unit circle: $x^2 + y^2 = 1$ is derived. Students should be given a copy of the unit circle and label corresponding information as it is presented. Students should also be shown how the points (x,y) on the unit circle are determined. **E1**

3. Provide handouts with practice unit circle problems where the students identify the point (x,y) for the corresponding real number. The diagram below should be shared with students. **E1**



4. Direct instruction should be provided to introduce the six trigonometric functions and corresponding points on the unit circle. Similar to the figure below.



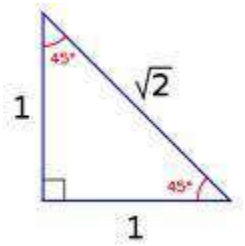
5. Have students work in pairs to complete problems 28 – 34 even. Discuss results as a class using the Interactive write Board. Print for special needs students. **T, O**
6. Students complete writing about concepts and capstone problems (Student Self-Assessment and Reflection). **E2**
7. For homework students should complete problems 5 – 8 and 27 – 33 odd. **E1**
8. Students should complete quiz lesson 9:2

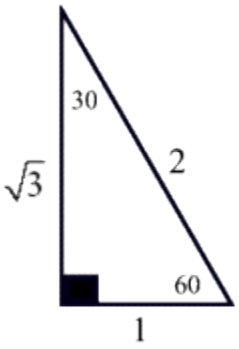
Lesson 3

General Topics: Right Triangle Trigonometry

1. Discuss and review/ define the following vocabulary terms as encountered within the context of the lesson:
   Hypotenuse, opposite side, adjacent side and (their abbreviations) opp, adj, hyp **E1, R**
2. Have students complete problems 1 – 4 to review vocabulary terms.

3. Provide direct instruction and to explain the special right triangles 30-60-90 and 45-45-90. Have a handout with the triangles drawn and have students complete angle measures and side lengths to use as a reference. **E1**





4. Have students complete problems 5 – 8 with a partner to ensure they are able to find the values of the six trigonometric functions. Discuss results as a class using the Interactive write Board. Print for special needs students. **T, O**
5. Students complete writing about concepts and capstone problems (Student Self-Assessment and Reflection). **E2**
6. For homework, have students complete problems 9 – 29 odd. Students should also complete problems 71 – 79 for a real world application of trigonometry. **E1**
7. Students should complete quiz lesson 9:3

Lesson 4

General Topics: Trigonometric Functions of Any Angles

1. Discuss and review/ define the following vocabulary terms as encountered within the context of the lesson:
   Quadrant, reference angle **E1, R**
2. Provide direct instruction on how to define the trigonometric functions of any angle using the formula $r = \sqrt{x^2 + y^2} \neq 0$ where (x,y) is a point on the terminal side. **E1, R**
3. Have students complete examples and practice problems identifying trigonometric functions using any angles and evaluate them. **E1**
4. Have students work in pairs to complete even exercise problems. Discuss results as a class using the Interactive write Board. Print for special needs students. **T, O**
5. Students complete writing about concepts and capstone problems (Student Self-Assessment and Reflection). **E2**
6. For homework, have students complete problems 1 – 17 odd in addition to problem 100 to provide a real word context of finding the trigonometric values of any angle. **E1**
7. Students should complete quiz lesson 9:4

Lesson 5

General Topics: Graphs of Sine and Cosine Functions

1. Discuss and review/ define the following vocabulary terms as encountered within the context of the lesson:

Sine curve, one cycle, key points in one period, amplitude, reflection, phase shift, cosine graph, horizontal translation, vertical translation **E1, R**

2. Provide direct instruction to students as to how the graphs of Sine and Cosine functions are created for one cycle. Use a graphing calculator overhead to project the images and allow the students to see the differences in the graphs **E1, R**
3. Have students construct and label the key points using graph paper. **E1**
4. After students construct their own graphs, show students how to graph the sine and cosine functions using their graphing calculators and allow students to check their work. **E1**
5. Provide examples and practice problems using graphs of sine and cosine functions **E1**
6. Have students work in pairs to complete problems using Geometer's Sketch Pad. Review whole class. **T, O**
7. Students complete writing about concepts and capstone problems (Student Self-Assessment and Reflection). **E2**
8. For homework, have students complete problems 1 – 10 all. Students should also complete problems 95 – 97 for real world application. **E1**
9. Students should complete quiz lesson 9:5

Lesson 6

General Topics: Graphs of Other Trigonometric Functions

1. Discuss and review/ define the following vocabulary terms as encountered within the context of the lesson:
   domain, symmetry, asymptotes, reciprocal **E1, R**
2. Discuss whole class the graphs of tangent, cotangent, secant, cosecant trigonometric functions. Use a graphing calculator overhead to project the images and allow the students to see the differences in the graphs. **E1, R**
3. Provide examples and practice problems using graphs of tangent, cotangent, secant, cosecant trigonometric functions. **E1**
4. Have students work in pairs to complete even exercise problems. Use a graphing calculator overhead to project the images. **T, O**
5. Students complete writing about concepts and capstone problems (Student Self-Assessment and Reflection). **E2**
6. For homework, have students complete problems 1 – 14 all and then have students use their graphing calculators and sketch the responses indicated in problems 15 – 21. **E1**
7. Students should complete quiz lesson 9:6

Lesson 7

General Topics: Inverse Trigonometric Functions

1. Discuss and define inverse trigonometric functions – *acrsin, arccos, arctan, domain, range*. **E1, R**
2. Provide direct instruction and examples problems using inverse trigonometric functions. A graphing calculator should be used to show students how to obtain exact values. Use an overhead projector to display each graph and its inverse. Allow students to compare the differences in the graphs. **E1**

3. Have students work in pairs to complete problems using Geometer's Sketchpad. Discuss results as a class. Print for special needs students. **T, O**
4. Students complete writing about concepts and capstone problems (Student Self-Assessment and Reflection). **E2**
5. For homework, have students complete problems 1 – 11 odd and 23 – 35 odd. Students should also complete problems 110 and 111 to provide a real world context. **E1**
6. Students should complete quiz lesson 9:7

Lesson 8

General Topics: Applications and Models

1. Have students brainstorm how trigonometric functions can model real-life problems. **E1, R**
2. Have students work with a partner to complete various problems 30 – 60. Discuss the results whole class. **E1**
3. Students complete writing about concepts and capstone problems (Student Self-Assessment and Reflection). **E2**
4. For homework, have students complete additional real world problems in the areas that are interest to them. **E1**
5. Students should complete quiz lesson 9:8

Lesson Review and P.S. Problem Solving

General Topics: Review and problem solving

1. Students will work in pairs to complete review excises and problem solving set at the end of the unit. E2
2. Students should complete the Unit Test.
3. Students will work with a partner to make corrections on their unit test **E2**

The acronym WHERETO summarizes key elements to consider when designing an effective and engaging learning plan.

W – Help the students know Where the unit is going and What is expected? Help the teachers know Where the students are coming from (prior knowledge, interests)

H – Hook all students and Hold their interest?

E – Equip students, help them Experience the key ideas and Explore the issues?

R – Provide opportunities to Rethink and Revise their understandings and work?

E – Allow students to Evaluate their work and its implications?

T – Be Tailored (personalized) to the different needs, interests, and abilities of learners?

O – Be Organized to maximize initial and sustained engagement as well as effective learning?

Code

IP = international education perspective

IL = information literacy

WR = Workplace readiness/21st century skills

FA = a formative assessment, used to check for understanding

Resources and Teaching Tips

- A variety of resources are included (texts, print, media, web links)
- Help in identifying and correcting student misunderstandings and weaknesses

Print

Larson, Ron, Edwards, Bruce, & Falvo, David (2012). *Calculus with Precalculus: a one year course.* Boston, MA: Brooks/Cole.

Web Links

http://www.cengagebrian.com
http://www.CalcChat.com
http://www.mathgraphs.com

Media

Power Lecture through CengageBrain.com

Differentiation

- Stage 2 and 3 allow students to demonstrate understanding with choices, options, and/or variety in the products and performances without compromising the expectations of the Content Standards.
- Instruction is varied to address differences in readiness, interest, and/or learning profiles.
- Accommodations and differentiation strategies are incorporated in the design of Stage 2 and 3.

The time period needed to complete this unit will vary depending upon student achievement levels and class grouping.

Provide additional examples and use additional exercise problems to provide additional support when needed.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.
- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning –** the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

Universal Design for Learning – using multiply representations of a function: equations, tables, and graphs

21st Century Learning – Student solve application, real-world problems, and conceptual problems throughout the practice exercise and in the problem solving section.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- **8th Grade Technology Literacy** - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century(SETDA, 2003).

TI – Calculator applications will be integrated throughout the book

Content Connections

Content Standards integrated within instructional strategies

Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title: Functions and Their Graphs

Designed by: Christine Bichler and Takashi Rhoulac
District: Innovative Schools

Content Area: Calculus with Pre-Calculus
Grade Level(s): 9-12

Summary of Unit: In this Calculus with Pre-Calculus unit, students will learn about Functions and their graphs. Students will learn how functions can be used to model and solve real-world problems fitting mathematical data to functions. Students will be able to write and evaluate a function as well as combinations of functions. Students will sketch, analyze, and transform graphs of functions.

Stage 1 – Desired Results

What students will know, do, and understand

Common Core Standards

- Include those addressed in Stage 3 and assessed in Stage 2.

Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.* **CC.9-12.A.SSE.2**

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **CC.9-12.A.APR.1**

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*). **CC.9-12.F.IF.1**

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **CC.9-12.F.IF.2**

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** **CC.9-12.F.IF.4**

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.** **CC.9-12.F.IF.5**

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* **CC.9-12.F.IF.6**

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* **CC.9-12.F.IF.7**

a. Graph linear and quadratic functions and show intercepts, maxima, and minima. **CC.9-12.F.IF.7a**

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **CC.9-12.F.IF.8**

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **CC.9-12.F.IF.8a**

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **CC.9-12.F.IF.9**

Write a function that describes a relationship between two quantities.* **CC.9-12.F.BF.1**

a. Determine an explicit expression, a recursive process, or steps for calculation from a context. **CC.9-12.F.BF.1a**

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* **CC.9-12.F.BF.1b**

c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.* **CC.9-12.F.BF.1c**

Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f*(*x*), *f*(*kx*), and *f*(*x + k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* **CC.9-12.F.BF.3**

Find inverse functions. **CC.9-12.F.BF.4**

a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x-1) for $x \neq 1$.* **CC.9-12.F.BF.4a**

b. (+) Verify by composition that one function is the inverse of another. **CC.9-12.F.BF.4b**

c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. **CC.9-12.F.BF.4c**

d. (+) Produce an invertible function from a non-invertible function by restricting the domain. **CC.9-12.F.BF.4d**

Big Idea(s)

- Transferable core concepts, principles, theories, and processes from the Content Standards

Mathematical data can be modeled with functions and their graphs.

Unit Enduring Understanding(s)

- Full-sentence, important statements or generalizations that specify what students should understand from the Big Ideas (s) and/or Content Standards and that are transferable to new situations

Students will understand that……..

- Mathematical data can be modeled with functions and their graphs.
- The concept of function can be used to interpret, analyze and model functions that emerge from contexts including those contexts that are purely mathematical.
- Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

Unit Essential Questions(s)

- Open-ended questions designed to guide student inquiry and learning

How are functions recognized, represented, and evaluated?

How are graphs of functions sketched and analyze?

How can graphs of functions be transformed?

How can functions be combined?

How to find inverse functions?

How can functions be used to model mathematical data?

Knowledge and Skills

- Needed to meet Content Standards addressed in Stage 3 and assessed in Stage 2

Students will know…

- whether relations between two variables are functions.
- how to use function notation and evaluate functions.
- how to find the domains of functions.
- how use the Vertical Line Test for functions
- how to find the zeros of functions.
- how to determine intervals on which functions are increasing or decreasing and determine relative maximum and relative minimum values of functions.
- how to identify and graph linear functions
- how to identify and graph step and other piecewise-defined functions
- how to identify even and odd functions.
- how to recognize graphs of common functions.
- vertical and horizontal shifts effects on sketching graphs of functions.
- how reflections cab be used to sketch graphs of functions.
- how nonrigid transformations can be used to sketch graphs of functions.
- how to add, subtract, multiply and divide functions.
- how to find the composition of one function with another function.
- how to find inverse functions informally and verify that two functions are inverse functions of each other.
- how to use graphs of functions to determine whether functions have inverse functions.
- how to use graphs of functions to determine whether functions have inverse functions.
- how to use the Horizontal Line Test to determine if functions are one-on-one.
- how to find inverse functions analytically.
- how to use the *regression* feature of a graphing utility to find the equation of a least squares regression line.

Students will be able to…

- use functions to model and solve real-life problems
- use combinations and compositions of functions to model and solve real-life problems.

- use mathematical models to approximate sets of data points.
- write mathematical models for direct variation.
- write mathematical models for direct variations as an *n*th power.
- write mathematical models for inverse variation.
- write mathematical models for joint variation

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

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Suggested Performance/Transfer Task(s)

- Performance/Transfer tasks as evidence of student proficiency

1. The weekly cost C of producing x units in a manufacturing process is given by $C(x) = 60x + 750$

The number of units x produced in t hours is $x(t) = 50t$

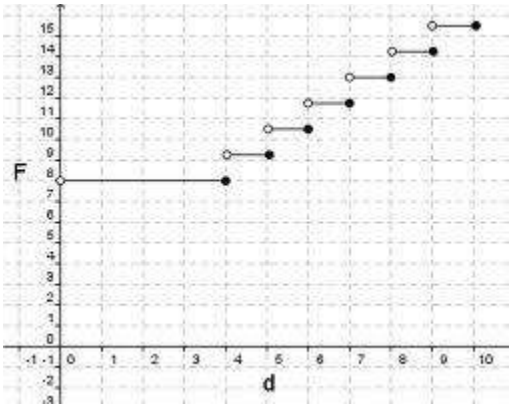
a) Find and interpret $C(x(t))$
b) Find the number of units produced in 4 hours. Show your calculations
c) Produce a graph of the cost as a function of time. Use your graph to estimate the time that must elapse until the cost increases to $15,000.

Rubric:

- Scoring guide to evaluate performance/transfer tasks used as evidence of student proficiency

	3	2	1	0
Part a			Correctly write the compound function	Incorrectly writes the compounded function
Part b		Student used the function from part a and showed their calculators with the correct answer	Student made a minor in their calculations	Calculations are completely wrong
Part c	Correctly graphed the function with labels and correctly estimated the solution	Correctly graphed the function with labels and did not estimate the solution correctly Correctly graphed the function but did not label the graph and did estimate the solution correctly	Made minor errors in graphing and estimated the solution correctly Graphed but did not provide labels and did not estimate the solution	Show little to no attempt at graphing the function and estimating the solution

2. Determine whether the following diagram is a function. Give proof to verify your answer.



3. An auditorium has 1200 seats. For the past several shows, the auditorium has been filled to capacity. Tickets currently cost $5 and the owner wants to increase the ticket prices. She estimates that for each $.50 increase in price, 100 fewer people will attend. What ticket price will maximize the profit?

Rubric for problems 2 and 3:

- Scoring guide to evaluate performance/transfer tasks used as evidence of student proficiency

	3	2	1	0
	Student provide a correct answer with work to support the answer	Students have made minor mathematical errors	Attempt was made but their were major mathematical flaws	No attempt was made

Other Evidence

- Varied evidence that checks for understanding (e.g., tests, quizzes, prompts, student work samples, observations and supplements the evidence provided by the task)

Capstones
Lesson quizzes
Unit Test

Student Self-Assessment and Reflection

- Opportunities for self-monitoring learning (e.g., reflection journals, learning logs, pre- and post-tests, self-editing – based on ongoing formative assessments)

Lesson Exercises
Review Exercises
Writing about Concepts
Textbook Chapter Test
Problem Solving problems

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

- Instructional activities and learning experiences needed to align with Stage 1 and Stage 2 expectations

Include these instructional elements when designing an effective and engaging learning plan for ALL students:

*Align with expectations of Stage 1 and Stage 2

*Scaffold in order to acquire information, construct meaning, and practice transfer of understanding

*Include a wide range of research-based, effective, and engaging strategies

*Differentiate and personalize content, process, and product for diverse learners

*Provide ongoing opportunities for self-monitoring and self-evaluation

Lesson 1

General Topics: Functions

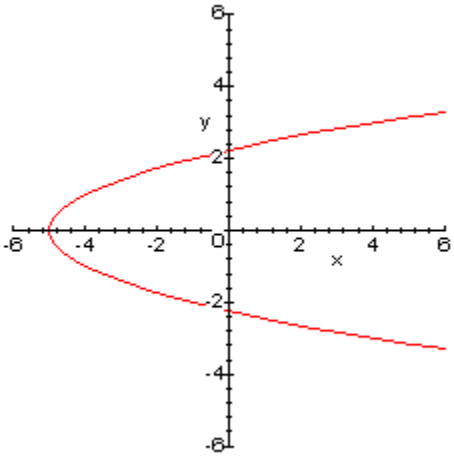
1. Discuss and review/ define the following vocabulary terms when applicable within the lesson:
   function, characteristics of a function, domain, range, relation, independent variable, dependent variable, function notation, piecewise-defined function, implied domain. **E1, R**
2. Provide direct instruction along with example problems showing students how to identify functions, represent functions verbally, numerically and analytically. Students must also know what a piecewise function is. **E1**
3. Have students work in pairs to complete even exercise problems. Discuss results as a class using the Interactive write Board. Print for special needs students. **T, O**
4. Students will complete Writing about Concepts and Capstone problems. (Student Self-Assessment and Reflection). **E2**
5. For homework, students will complete vocabulary review and problems in which student will practice with functions. **E1**
6. Students should complete quiz lesson 1:1

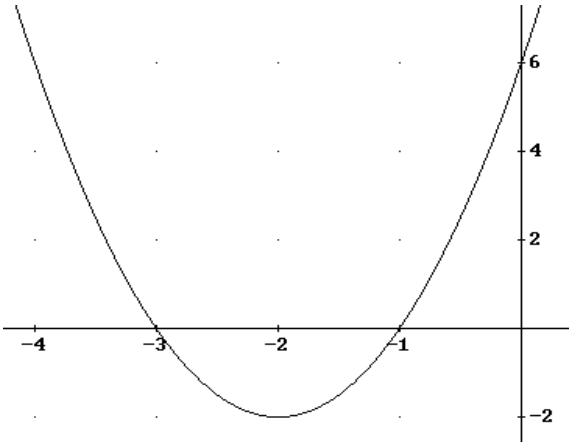
Lesson 2

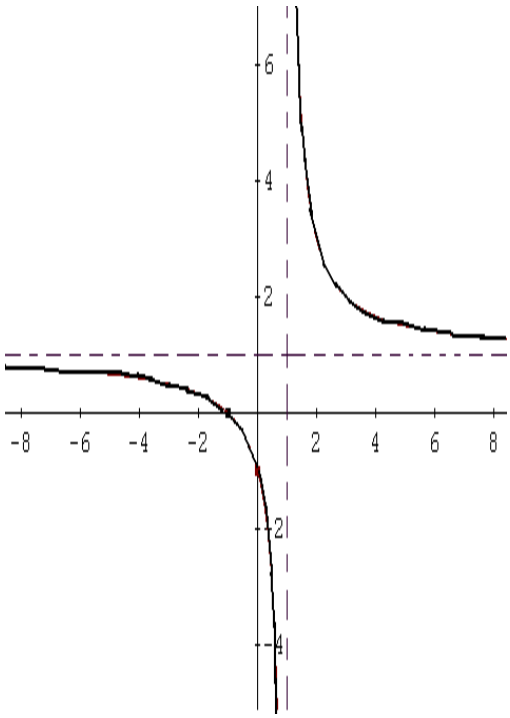
General Topics: Analyzing Graphs of Functions

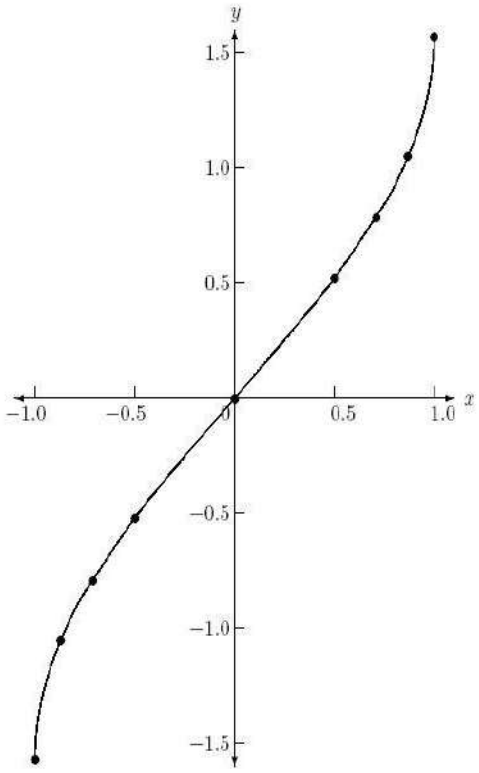
1. Discuss and review/ define the following vocabulary terms when applicable within the lesson:
   graph of a function, vertical line test, zeros of a function, increasing, decreasing, constant as it relates to functions, relative minimum, relative maximum. **E1, R**

2. Provide direct instruction to show students how to:
    - use the vertical line test to determine a function
    - find the zeros of a function
    - determine whether a function is increasing, decreasing or constant
    - graph a piecewise function
    - identify even and odd functions **E1**
3. Have students work in pairs to complete exercise problems. Discuss results as a class using the Interactive write Board. Print for special needs students. **T, O**
4. Also have students sort diagrams in piles of functions and not functions. Some example problems are below. Have students use the vertical line test to determine if the graph is a function. If so, students should be able to find the zeros of a function, determine whether a function is increasing, decreasing or constant and identify even and odd functions. **T, O**









5. Students will complete Writing about Concepts and Capstone problems in their Math Journals (Student Self-Assessment and Reflection). **E2**
6. For homework, students will complete the vocabulary review and other example problems that will allow them to work with functions analytically and graphically. **E1**
7. Students should complete quiz lesson 1:2

Lesson 3

General Topics: Transformations of Functions

1. Discuss and review/ define the following vocabulary terms when applicable within the lesson:
   vertical and horizontal shifts, reflections, rigid transformations, nonrigid transformations, vertical stretch and shrink, horizontal stretch and shrink. **E1, R**
2. Provide direct instruction using examples to show students how to perform rigid and nonrigid transformations. **E1**
3. Have students work in pairs to with Geometer's Sketchpad to experiment with various shifts, reflections, stretches and shrinkage. Have students identify/ sketch the original function and explain what transformation took place. **T, O**
4. Students will complete Writing about Concepts and Capstone problems in their Math Journals (Student Self-Assessment and Reflection). **E2**
5. Provide a homework problem in which student review vocabulary as well as practice rigid and nonrigid transformations. **E1**
6. Students should complete quiz lesson 1:3

Lesson 4

General Topics: Combinations of Functions

1. Discuss and review/ define the following vocabulary terms when applicable within the lesson:
   Sum, difference, product and quotient of functions, composition of a function **E1, R**
2. Provide direct and instruction to model and explain the arithmetic compositions of a function as well as composite functions. **E1**
3. Have students work in pairs to complete exercise problems. Discuss results as a class using the Interactive write Board. Print for special needs students. **T, O**
4. Students will complete Writing about Concepts and Capstone problems in their Math Journals (Student Self-Assessment and Reflection). **E2**
5. For homework, students should complete the vocabulary review along with problems that allow students practice with composite functions. **E1**
6. Students should complete quiz lesson 1:4

Lesson 5

General Topics: Inverse Functions

1. Discuss and review/ define the following vocabulary terms when applicable within the lesson:
   inverse functions, f^{-1}, horizontal line test, one-to-one functions E1**, R**
2. Provide examples and direct instruction to help students understand how to obtain the inverse function and use the horizontal line test. Students should also be able to determine if a function is one-to-one. **E1**
3. Have students work in pairs to complete exercise problems. Discuss results as a class using the Interactive write Board. Print for special needs students. **T, O**
4. Students will complete Writing about Concepts and Capstone problems in their Math Journals (Student Self-Assessment and Reflection). **E2**
5. Provide a homework problem in which student will practice with functions. **E1**
6. Students should complete quiz lesson 1:5

Lesson 6

General Topics: Mathematical Modeling and Variation

1. Have students brainstorm how functions can be used in real-life problems. **E1, R**
2. Have students work in pairs to complete exercise problems. Have students complete problems from several different disciplines to ensure an greater understanding of functions in real word situations. Discuss results as a class. **T, O**
3. Students will complete Writing about Concepts and Capstone problems in their Math Journals (Student Self-Assessment and Reflection). **E2**
4. Provide a homework in which student will gain additional practice with functions in the real world. **E1**
5. Students should complete quiz lesson 1:6

Lesson Review and P.S. Problem Solving

General Topics: Review and problem solving

1. Students will work in pairs to complete review excises and problem solving set at the end of the unit. E2
2. Students should complete the Unit Test.
3. Students will work with a partner to make corrections on their unit test **E2**

The acronym WHERETO summarizes key elements to consider when designing an effective and engaging learning plan.

W – Help the students know Where the unit is going and What is expected? Help the teachers know Where the students are coming from (prior knowledge, interests)

H – Hook all students and Hold their interest?

E – Equip students, help them Experience the key ideas and Explore the issues?

R – Provide opportunities to Rethink and Revise their understandings and work?

E – Allow students to Evaluate their work and its implications?

T – Be Tailored (personalized) to the different needs, interests, and abilities of learners?

O – Be Organized to maximize initial and sustained engagement as well as effective learning?

Code

IP = international education perspective

IL = information literacy

WR = Workplace readiness/21st century skills

FA = a formative assessment, used to check for understanding

Resources and Teaching Tips

- A variety of resources are included (texts, print, media, web links)
- Help in identifying and correcting student misunderstandings and weaknesses

Print

Larson, Ron, Edwards, Bruce, & Falvo, David (2012). *Calculus with Precalculus: a one year course*. Boston, MA: Brooks/Cole.

Web Links

http://www.cengagebrian.com
http://www.CalcChat.com
http://www.mathgraphs.com

Media

Power Lecture through CengageBrain.com

Differentiation

- Stage 2 and 3 allow students to demonstrate understanding with choices, options, and/or variety in the products and performances without compromising the expectations of the Content Standards.
- Instruction is varied to address differences in readiness, interest, and/or learning profiles.
- Accommodations and differentiation strategies are incorporated in the design of Stage 2 and 3.

The time period needed to complete this unit will vary depending upon student achievement levels and class grouping.

Provide additional examples and use additional exercise problems to provide additional support when needed.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.
- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning –** the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

Universal Design for Learning – using multiply representations of a function: equations, tables, and graphs

21st Century Learning – Student solve application, real-world problems, and conceptual problems throughout the practice exercise and in the problem solving section.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- **8th Grade Technology Literacy** - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century(SETDA, 2003).

TI – Calculator applications will be integrated throughout the book

Content Connections

Content Standards integrated within instructional strategies