**AP Calculus First Semester REVIEW & CATCH UP**

**Name:**________________________________________________ **Period:** ______

***Show all work. Attach any additional paper used.***

**For #1-10, use the chart to the right of a differentiable function, p(x):**

| x | p(x) | $p'(x)$ |
|---|---|---|
| −3 | 5 | −2 |
| 2 | 3 | 6 |
| 4 | −1 | 7 |

1) Given $g(x) = x^2 p(x) + 2p(x)$, find $g'(x)$.

2) Now find $g'(2)$ in simplified form.

3) Given $L(x) = 8p(x) + \dfrac{x}{p(x)} + x - 6$, find $L'(x)$.

4) Now find $L'(4)$ in simplified form.

5) Find $g'(-3) + L'(2)$ in simplified form.

6) Estimate $p'(3)$.

7) If $Y(x) = p(2x)$, find $Y'(x)$.

8) Now find $Y'(2)$.

9) Find the local linearization of p(x) at x = −3.

10) Using your answer to #9, estimate p(−2.8).

**Find each derivative in simplified form. (don't forget chain when appropriate!!) ☺**

11) $y(x) = \dfrac{6x^6 - x^4 + 3x^2 - 9}{3x^2}$ **avoid quotient**

12) $g(x) = \frac{1}{2}x^4 - 3x^2 + x - 5$

13) $T(x) = 3x\cos 5x - 2\sin x + \sqrt[5]{x} - 5\cot 4x^2$

14) $f(x) = \dfrac{2x^2+1}{5x+3}$

15) $M(x) = \dfrac{6-x}{x^2-3} + 4x^2\sec x - 12$

16) $f(x) = x^3 \csc 8x - \tan x \cot x$ *think*

17) $g(\theta) = \dfrac{2\sin\theta + \cot\theta}{\cos\theta}$ *avoid quotient*

18) $y(x) = (8x^2 + 3)^5 (5x - 11)^8$

19) $j(x) = \dfrac{8x+2}{\sqrt{3x^4+1}}$

20) $T(x) = \csc^7(5x^2 + x)$

**Answer each.**

21) Choose True or False AND explain: A differentiable function is always continuous.

22) Choose True or False AND explain: A continuous function is always differentiable.

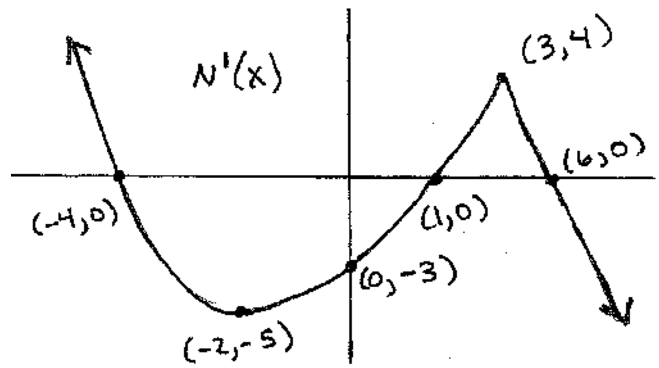
23) Find the equation of the tangent line to $h(x) = \sqrt{5x^2+6x-2}$ at x = 1.

24) Find the x-coordinates where $p(x) = 2x^3 - 9x^2 - 6x + 12$ has a slope of 3.

25) What is the instantaneous rate of change at x = 3 of the function $g$ given by $g(x) = \dfrac{x^2 - 2}{x - 1}$?

26) Find the average rate of change in g(x) in question #25 on [2, 5].

27) Using the graph of $N'(x)$, answer each question.




a) Give the x-coordinates where N(x) has horizontal tangents.

b) Give the equation of the tangent line to N(x) at $x = -2$ if $N(-2) = 6$.

c) Identify where $N''(x) = 0$. Explain.

d) Where is $N'(x)$ not differentiable? Explain.

e) Where is N(x) not differentiable? Explain.

28) A particle moves along the x-axis so that its position at time $t$ is given by $x(t) = t^2 - 6t + 5$. For what value of $t$ is the velocity of the particle zero?

29) If $h(x) = \tan 2x$, then $\dfrac{dh}{dx}$ is what at $x = \dfrac{\pi}{6}$? (simplified answer)

30) Find the local linearization of $y = x + \cos x$ at $x = \dfrac{\pi}{2}$.

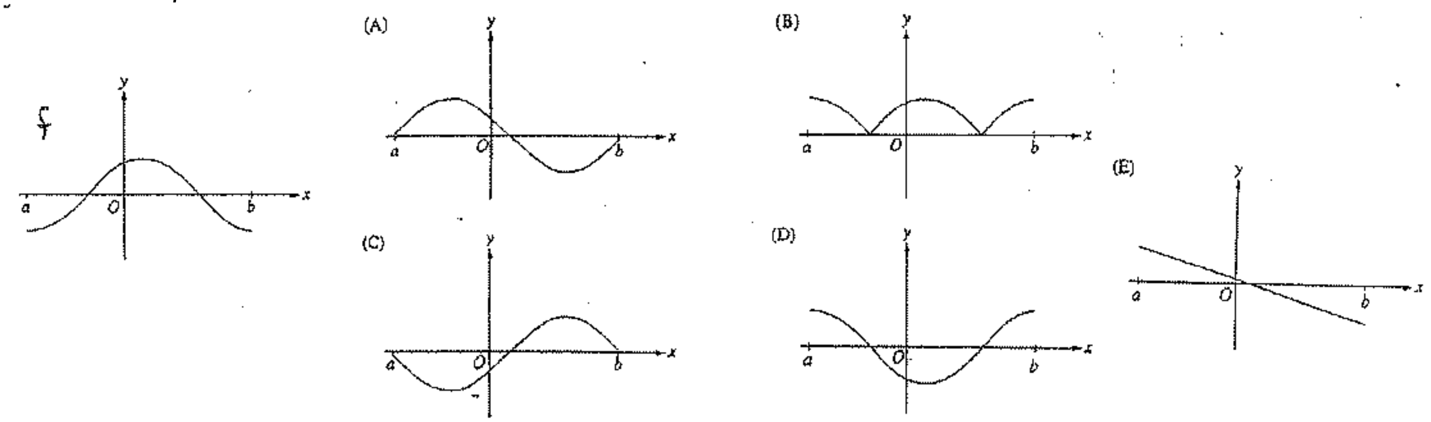
31) The function $f$ is continuous on the closed interval [0, 2] and has values that are given in the table. The equation $f(x) = \dfrac{1}{2}$ must have at least two solutions in the interval [0, 2] if $k$ = ____

| | | | |
|---|---|---|---|
| $x$ | 0 | 1 | 2 |
| $f(x)$ | 1 | $k$ | 2 |

*HINT: Sketch*

a) 0 b) ½ c) 1 d) 2 e) 3

32) The graph of $f$ is shown in the figure on the left. Which of the following could be the graph of the derivative of $f$?



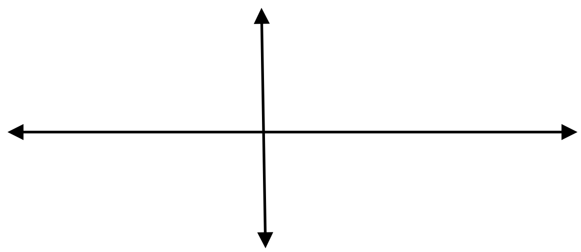

33) Sketch the graph of a function that meets the following criteria.

$\lim_{x->3} T(x)$ exists

$\lim_{x->3} T'(x)$ does not exist

$T'(x) > 0$ for $(-\infty, 5)$

$T'(x) < 0$ for $(5, \infty)$



34) This is an old free response question:

Let $f$ be the function defined by $f(x) = \begin{cases} 2x+1, & for\ x \leq 2 \\ \frac{1}{2}x^2 + k, & for\ \ x > 2 \end{cases}$

a) For what value of k will $f$ be continuous at x = 2? Justify your answer.

b) Using the value of k found in part (a), determine whether $f$ is differentiable at x = 2. Justify your answer.

c) Let k = 4. Determine whether $f$ is differentiable at x = 2. Justify your answer.

## AP Calculus First Semester REVIEW & CATCH UP….. **<u>**Continued**</u>**

35) This is an old free response question:

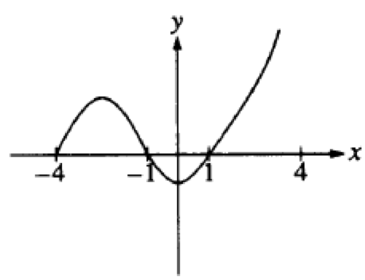
Let *g* be the function defined as follows: $g(x)=\begin{cases} |x-1|+2, & for\ x<1 \\ ax^2+bx, & for\ x\ge 1 \end{cases}$ where *a* and *b* are constants.

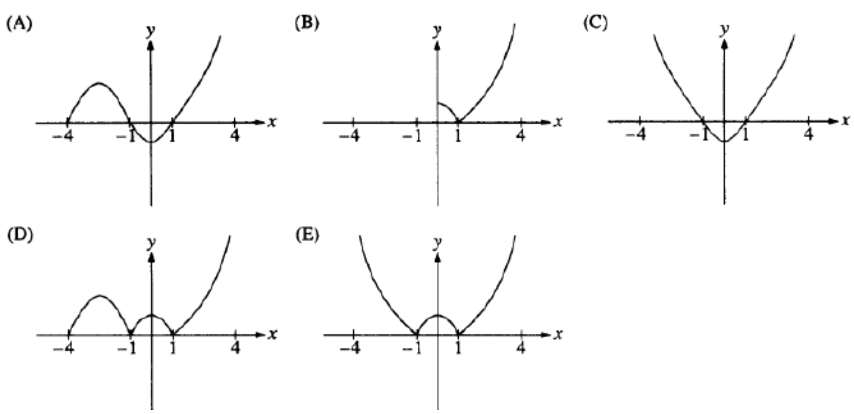
a) If a = 2 and b = 3, is *g* continuous for all x? Justify your answer.

b) <u>Describe</u> all values of *a* and *b* for which *g* is a continuous function. (hint: you will only be able to find the relationship between *a* and *b* for this one)

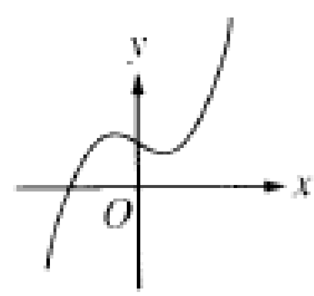
c) For what <u>values</u> of *a* and *b* is *g* both continuous and differentiable?

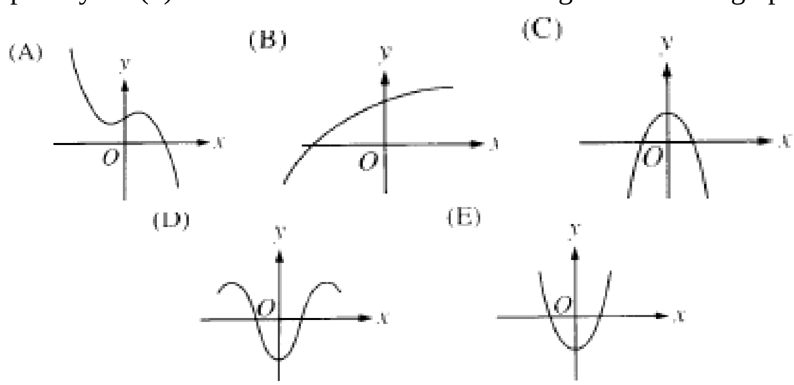
36) The graph of y = f(x) is shown in the figure. Which of the following could be the graph of y = f(|x|)?




37) The graph of y = h(x) is shown. Which of the following could be the graph of $h'(x)$



38) Identify the values for which g(x) is differentiable if g(x) = $\sqrt[7]{x^2}$ . (notice the function is continuous)

**For each piecewise, answer the questions.**

39) Consider $h(x) = \begin{cases} x^2 + 8 & if \quad x < 0 \\ 5\cos x + 3 & if \quad x > 0 \end{cases}$

a) Find $h'(x)$.

$h'(x) = \begin{cases} \\ \end{cases}$

b) Is h(x) continuous? Justify with calculus notation.

c) Is h(x) differentiable? Justify with calculus notation.

40) Consider $w(x) = \begin{cases} 5x - x^3 & if \quad x > 1 \\ x + 3 & if \quad x \le 1 \end{cases}$

a) Find $w'(x)$.

$w'(x) = \begin{cases} \\ \end{cases}$

b) Is w(x) continuous? Justify with calculus notation.

c) Is w(x) differentiable? Justify with calculus notation.

41) Consider $G(x) = \begin{cases} x^4 - 3x^2 + 5 & if \quad x < -1 \\ x^3 - x + 3 & if \quad x \ge -1 \end{cases}$

a) Find $G'(x)$.

$G'(x) = \begin{cases} \\ \end{cases}$

b) Is G(x) continuous? Justify with calculus notation.

c) Is G(x) differentiable? Justify with calculus notation.

42) Find $T'(x)$ if T(x) = 6 w(x) + f($x^3$) − y(5x) h(7$x^4$ − 3x + 2) + cot(9x) where y(x), w(x), and h(x) are all differentiable functions.