Curriculum Framework for Mathematics

School: Reach Academy for Girls   **Curricular Tool: Singapore Math**   **Grade: K**   **Teacher: Dunlap/Keane/Luce**

Standards Alignment	Unit Concepts Enduring Understandings	Essential Questions Student Learning Targets	Assessments
Unit One: Shapes and Attributes **Timeline: Approximately 30 school days**			
K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.*	All shapes can be sorted according to their attributes. Using the correct words about shapes and positions helps us communicate mathematically.	**Essential Questions:** What strategies help us identify and discuss shapes around us? Why is it important to use the correct words when we communicate mathematically?	Formative Assessments: Math journals Class work Classroom discussions Group work checklist
K.G.2 Correctly name shapes regardless of their orientations or overall size.	Shapes can be modeled in different materials.	Why are only certain attributes used to identify shapes?	Summative Assessments: Attributes Assessment Shapes Rubric
K.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").	Shapes can be joined to create larger shapes.	How are two- and three-dimensional shapes related?	
K.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).		In what ways can shapes in the real world be modeled? What strategies can be used to make larger shapes from smaller shapes?	
K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.		**Learning Targets:** I can identify, compare, locate and describe plane shapes and solid figures.	
K.G.6 Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"		I can analyze, create and compose shapes I can identify and discuss shapes. I can create shapes with different materials.	

Standards	Enduring Understandings	Essential Questions / Learning Targets	Assessments
		I can use smaller shapes to build larger shapes.	
Unit Two: Counting and Comparing Numbers (1-100) **Timeline: Approximately 50 days**			
K.CC.1 Count to 100 by ones and by tens. **K.CC.2** Count forward beginning from a given number within the known sequence (instead of having to begin at 1). **K.CC.3** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). **K.CC.4** Understand the relationship between numbers and quantities; connect counting to cardinality. **K.CC.4 a.** When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. **K.CC.4 b.** Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. **K.CC.4 c.** Understand that each successive number name refers to a quantity that is one larger. **K.CC.5** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.	Our number system is made up of ones, tens and hundreds. Counting strategies help determine how many objects are in a group. Each object counted corresponds to only one number in the counting sequence and vice versa Spoken and written numbers tell us how many objects there are in a group. Numbers can be compared to other numbers. Quantities can be represented in pictures, words and numerals.	**Essential Questions:** Why is it important to know the correct way to count objects? What strategies can we use to help us count objects in a group? How do pictures, words and numerals help us communicate mathematically about groups of objects? Why do we need to be able to read number words? What strategies can we use to compare two numbers? **Learning Targets:** I can count and name numbers. I can write numbers from 0 to 20. I can count to tell the number of objects. I can count out up to 20 objects. I can touch and say the names of numbers in order I can tell if the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using comparison strategies I can compare two numbers	**Formative Assessments:** Unit Pre-test Math journals Classroom discussion Class work Group work checklist Counting Assignment **Summative Assessments:** Unit Post Test Number Relationship Rubric Counting and Comparing Assessment How many in the bag? Assessment Ordinal Number Rubric

Standards	Enduring Understandings	Essential Questions / Learning Targets	Assessments
	between 1 and 10 as written numerals.		**Formative Assessments:** Unit Pre-test Math journals Class work Classroom discussions Group work checklist Measurement Assignment **Summative Assessments:** Unit Post-Test Measurement Rubric Analyzing and Comparing Assessment Classifying Rubric
K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1] **K.CC.7** Compare two numbers between 1 and 10 presented as written numerals. **K.OA.4** For any number from 1 to 9, find the number that makes 10 when added to he given number, e.g., but using objects or drawings, and record the answer with a drawing or equation.			
Unit Three: Measuring and Analyzing Data/Problem Solving **Timeline: 60 Days**			
K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. **K.MD.2** Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. **K.MD.3** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. **K.OA.1** Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	Objects can be sorted by their attributes. Attributes help to classify and compare objects. Using strategies can help solve problems	**Essential Questions:** Why are attributes used to describe objects? How can attributes be used to classify objects? How can strategies help solve problems? Why is it useful to classify objects? How do our words help us communicate mathematically? **Learning Targets:** I can name different attributes of an object. I can compare two objects based on one attribute. I can sort objects based on attributes. I can use strategies to help solve	

		addition and subtraction problems.	
Unit Four: Investigating Addition and Subtraction **Timeline: Approximately 35 Days**			
K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $18 = 10 + 8$)*; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. **K.OA.5** Fluently add and subtract within 5. **K.OA.4** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. **K.OA.3** Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$). **K.OA.2** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. **K.OA.1** Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	Working with the numbers 11-19 can help us learn more about place value. Understanding place value relationships helps solve problems. Using different strategies can help solve addition and subtraction problems.	**Essential Questions:** How does understanding place value help solve problems? What strategies can we use to help us solve addition and subtraction problems? How can using objects be helpful when solving problems? How do we communicate addition and subtraction problems as mathematicians? **Learning Targets:** I can take numbers apart and put numbers back together. I can use objects to help me with addition and subtraction word problems. I can use objects to show the quantity of a number. I can add and subtract to 20. I can draw number problems. I can write a number sentence for an addition or subtraction problem.	**Formative Assessment:** Teacher/Student conferencing Government Pre-Assessment Class work Classroom discussions Plants Pre Assessment Plant graphic organizer **Summative Assessment:** Post Assessment 10s and 1s Rubric Number Sentence Rubric

Understanding by Design Template
Mathematics - Kindergarten
Title: Counting and Comparing Numbers to 10 - Time Frame: 30 days
Developed by Reach Academy for Girls
Teachers: Dunlap - Luce - Keane

Unit Summary:
This unit represents the portion of Counting and Comparing numbers that happens in October-December of the school year. Additional work on Counting and Comparing Numbers will happen in the late winter and spring.

Students learn that numbers, including written forms, are used to communicate quantities. Each spoken number corresponds to exactly one item when counting, and strategies can make the counting process more efficient. These numbers help us to solve problems, such as determining how many in a set, counting out a specific number of items and comparing quantities. Understanding counting strategies is integral to the work of this unit, and also to building place value concepts in future units. While this unit focuses on values to 10, it lays important groundwork for the standards that include counting to 100, writing numbers to 20, and counting to 20.

Stage 1 – Desired Outcomes

Common Core State Standards
K.CC.1 Count to 100 by ones and by tens. *This unit focuses first on counting to 10 by ones.*
K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). *This unit focuses first on numerals to 10.*
K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.
K.CC.4 a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
K.CC.4 b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
K.CC.4 c. Understand that each successive number name refers to a quantity that is one larger.
K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. *This unit focuses on counting to 10 in scattered configurations, and counting to 10 objects.*

This unit also incorporates Mathematical Processes 1-8

Big Ideas:
- Numbers have specific names, place values and a specific counting sequence
- Counting tells us the number of objects in a group
- Quantities can be represented in pictures, words and numerals
- Numbers can be compared to one another

Understandings:
- Our number system is made up of ones, tens and hundreds
- Each object counted corresponds to only one number in the counting sequence and vice versa
- Counting strategies help determine how many objects are in a group
- Spoken and written numbers tell us how many objects there are in a group
- Numbers can be compared to other numbers

Essential Questions:
- Why is it important to know the correct way to count objects?
- What strategies can we use to help us count objects in a group?
- How do pictures, words and numerals help us communicate mathematically about groups of objects?
- What strategies can we use to compare two numbers?

Students will Know:
- The counting sequence and how to name numbers
- How to write numerals from 0 to 20
- Correct counting sequence with 1:1 correspondence to tell the number of object in s group, up to 20 objects
- The relationship between spoken names and written numerals
- Strategies to compare two groups of objects
- Strategies to compare two numbers between 1 and 10 as written numerals
- Strategies for determining how far a number is from 10

Stage 2 – Assessment Evidence

Performance Task: How Many are in The Bag? [1]

Students count objects in three different bags.
Responses are recorded as pictures and numerals
Students determine a number that is greater than the number of objects in the bag, and one number that is less than the number of objects in the bag, and to explain their reasoning.
Number lines and number boxes should be available for students if necessary.

Preparation:
Label three bags A, B, and C. Fill each with a different quantity of small objects (counting bears, buttons, cubes), 0-10, for each student. In order to avoid confusion, make sure each bag has different items inside.

Task Directions:
Instruct students to look in each bag, count the number of objects and record it on their recording sheet:

Bag	Quantity	Number Word	Picture	A Number that is GREATER than this quantity	A number that is LESS than this quantity
A					
B					
C					

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[1] Adapted from Georgia Department of Education:
http://www.gcssk12.net/fullpanel/uploads/files/kindergarten-math-unit-one-00001-00001.pdf

For Bag C, explain how you know your number is GREATER than the quantity in the bag:

For Bag C, explain how you know your number is LESS than the quantity in the bag:

Formative Assessment Questions:
- What are some of the different ways you can represent numbers?
- What strategy are you using to count your objects?
- How do you know your answers are correct?
- If you had 1 more in your bag, how many would you have? If you had 1 less?

Differentiation:
> **Intervention:**
> - Allow students to use ten frames, number boxes or number lines to assist in counting the objects.
> - Allow students to use a spelling list for the number words (this may be posted in the classroom as well)
> - Limit quantities in the bags to 5 objects for students who need additional time and practice to build to 10; reassess later.
>
> **Extension:**
> - Ask students to determine how far the quantity in each bag is from 10.
> - Provide students with bags that have 11-19 items and have them complete the same activity.

Scoring Rubric:
4 - Exceeds
> Student has:
> - All responses correct on the recording sheet
> - Strong reasoning for selecting the greater than and less than numbers
> - Clear written explanation that does not require additional inferencing on the part of the scorer

3 – Meets
> Student has:
> - 1-3 errors on the recording sheet (these should not be in the same row or column)
> - Strong reasoning for selecting the greater than and less than numbers
> - Written explanation that is appropriate, but may require some inferencing on the part of the scorer

2 – Nearing
> Student has:
> - 4-6 errors on the recording sheet
> - Inaccurate reasoning for selecting greater than and less than numbers
> - Written explanation requires additional probing for clarity

1 – Below
> Student has:
> - More than 6 errors on the recording sheet
> - Inaccurate or missing reasoning for selecting greater than and less than numbers
> - Written explanation is incoherent or not present

Other Evidence:
- Observations of Learning Center activities
- Classroom discussions
- Small group work observations
- Class work
- Homework
- Popsicle stick game
- Unit 3 review, page 84-85
- Activity book pages 16-25

- Lesson 3.14: what strategies do students use to count?
- Singapore assessments
- Notes on activities/discussions
- Number relationship rubric
- Ordinal numbers rubric
- Student self-assessment and reflection in Math Journals

Stage 3 – Learning Plan

Lesson 1: Counting to 5
- Students will complete Unit 2 in the textbook. This lesson develops understanding of counting quantities up to 5, and begins to lay the foundation for the concept that arrangement of objects does not affect the quantity of objects. Knowing this, students also begin to learn what "shapes" different numbers of objects can take.
  - 2.1: Counting to 3
  - 2.2: Counting to 4
  - 2.3: Counting to 5
  - 2.4: Making arrangements of objects
  - Assessment: Given at least 5 objects, have students count out 4, 2, 1, 3, and 5 objects
  - 2.5: Representing counting data on a picture graph
  - 2.6: Representing counting data on a picture graph
    - *Optional Support:* Students can complete activity book p. 8, where students count bags of wool, then cut an paste the corresponding quantity of bags for each person onto the chart.
  - 2.7: Identifying small numbers without counting
  - 2.8: Representing 1 in writing and pictures
  - 2.9: Representing 2 in writing and pictures
  - 2.10: Representing 3 in writing and pictures
  - 2.11: Representing 4 in writing and pictures
    - *Optional Support:* Students can complete activity book p. 9-10, where students count the number of objects in each group and write the numerals 1-5.
  - 2.12: Representing 5 in writing and pictures
    - *Optional Support:* Students can complete activity book p. 11-12, where students count objects in a picture and write the numerals 1-5.
  - 2.13: Counting and representing up to 5
    - *Optional Support:* Students can complete activity book p. 13, where students are asked to cut out corresponding numbers and paste them on a chart. Then they practice writing numerals 1-5.
  - 2.14: Counting data from a picture graph

Lesson 2: Counting to 10
- Students will complete textbook lessons 3.1 through 3.9, which each focus on counting one more to ten.
  - This lesson solidifies the fact that arrangement does not affect quantity when objects are counted. Each lesson includes pictures of objects to count in which objects are organized in different ways. For example, on a 3 x 3 number grid, Lesson 3.2 demonstrates for students that there are multiple ways to show the quantity 6 by coloring six different boxes within the grid.
  - Each lesson also continues to reinforce previously learned counting sequences and practice written numerals.
    - 3.1: Counting to 8
    - 3.2: Representing 6 in writing and pictures
    - 3.3: Representing 7 in writing and pictures
    - 3.4: Representing 8 in writing and pictures
      - *Optional Support:*
        - Students complete activity book p. 16, where students must cut out the

corresponding quantity of carrots and paste them next to the given numerals, 4-8.

- 3.5: Counting to interpret data from a picture graph
- 3.6: Counting to 8 and representing numbers to 8
- 3.7: Counting to 10
- 3.8: Representing 9 in writing and pictures
- 3.9: Representing 10 in writing and pictures
    - *Optional Supports:*
        - Students complete activity book p. 17, where students match corresponding quantities of different objects. (Students match the picture of five rabbits to the picture of five stones.)
        - Students complete activity book p. 18, where students match the numerals 5-10 with corresponding pictures of the objects, with objects neatly lined up.
        - Students complete activity book p. 19, where students match the numerals 5-10 with corresponding pictures of the objects, with the objects in disarray.
        - Students complete activity book p. 20-21, where students count the number of objects in a picture 0-10 and write the numeral.
- Assessment: Play popsicle stick game with small groups:
    - With 4-5 players, give each 10 popsicle sticks
    - Explain to students that you are going to flash a numeral card and you want them to pick up the corresponding number of ice cream sticks.
    - When students are demonstrating proficiency, change roles. You lay out the ice cream sticks and they need to select the appropriate numeral cards.

Lesson 3: Using the number box/number line to count to 10
- Using cardstock numbers 0-10, have students come to the front of the room and line up, putting the numbers in order
- On their desks, have students label a number box with the numerals 0-10
- Complete Lesson 3.10, where students work on matching objects in pictures to boxes on the number box as a strategy for counting accurately. In this lesson, students are still expected to draw lines to show the correspondence of each object to its space in the number box. Students should still count orally once they have matched objects to the number boxes, to reinforce the counting sequence.
- During the course of the lesson, students should be asked which numbers are greater than the selected quantity, and which are less than. Encourage discussion around this concept.
- Once students have mastered the use of the number box, shift the picture of the number box into a number line for Lesson 3.11. Students will still match each item to a number on the number line, and should still count orally once they have matched the objects to the number line to reinforce the counting sequence.

Lesson 4: The importance of zero
- Lessons 3.12 and 3.13 expose students to the concept of zero representing an empty set.
    - *Optional Support:* Students complete Activity Book p. 22-23, in which students count the numbers of carrots, including numbers 0-5.
- Practice the song: *Five Currant Buns*
    - 5 currant buns in a baker's shop, (hold up five fingers)
      Round and fat with sugar on the top.
      Along came a boy with a penny one day,
      Bought a currant bun and took it away.

      4 currant buns in a bakers shop… (hold up four fingers)

      1 currant bun in a baker's shop, (hold up one finger)
      Round and fat with sugar on the top.

Along came a boy with a penny one day,
Bought a currant bun and then, there were none! (hold up fist)

- Bowling: give each group of 4-5 students 10 water bottles and a small ball. Have students count the bottles as they stand them up, starting with zero. Then, as students bowl, they should tell how many bottles were knocked down and how many remain standing. Reinforce the concepts of "greater than" and "less than" during the game: with each turn taken, the number of bottles standing should be less than the previous number.

Lesson 5: Putting it all together: counting pictures and objects
- Using the materials from Lesson 3.14, students should count the objects in each picture and write the corresponding numeral. Students will have access to number boxes and manipulatives; this is a great opportunity to assess students' comfort level with counting.
- *Optional Support:* Students complete activity book p. 24-25, which provides students another opportunity to count objects in a picture and write the corresponding numerals, 2, 3, 7, and 10.
- Assessment: Give students the review activity on pages 84-85. Students are first asked to count the number of each fruit from a picture, with responses of 0, 8, 9, 10 objects. Then, students are asked to count shapes on cards, with responses of 4, 6, 8, 7, 9, 10. Have students work together to find a number that is greater than each and less than each of these responses given. Note which methods students are using to count the objects.

Resources and Teaching Tools:
- *Singapore Math* Text Books and Teacher Guides
- *Singapore Math* Activity Guide

Differentiation:
Students are given the opportunity to work on the lessons at a rate appropriate to their level, and with whatever manipulatives they choose. Manipulatives are available to students throughout the unit. Additional counting strategies will be introduced on an as-needed basis.

Students can be placed into small groups based on their preferred counting strategies, to build proficiency. A jigsaw activity can be done where students are assigned to mixed strategy groups and all solve the same problem using different methods.

Optional support activities from the coordinating activity book are included with the lessons to help students who need additional support and, in some cases, provide extensions for students.

The teachers' edition provides additional support and activities for students who need more exposure and practice. Manipulatives should be available to all students throughout the unit. Among these activities are:
- Use play dough and counters, such as buttons. Have student make the correct numeral with the play cough, then decorate it with the corresponding number of counters.
- Working in partners, have one student select a quantity of objects to put in a cup. The partner must count the objects from the cup.

Design Principles for Unit Development:
- Universal Design for Learning: students have the opportunity to interact with the content and demonstrate understanding in various formats. Each lesson includes visual and hands-on methods for instruction and assessment.

Technology Integration:
- While technology is not specifically addressed within the lesson, the PBS kids website provides numerous games that can be used to reinforce counting concepts during the morning seatwork time or at home: http://pbskids.org/games/counting.html

Content Connections:
- These counting skills can be applied to all content areas by asking students to count: objects, items in pictures, steps down the hallway, etc.
- Science:
    - Counting leaves when sorting them by various characteristics
    - Making and interpreting graphs
- Social Studies:
    - Counting out linking cubes to demonstrate various elevations
- Literature counting connections:
  - *The Very Hungry Caterpillar*, by Eric Carle
  - *The Tortoise and the Hare*
  - *Ten Little Ladybugs,* by Melanie Gerth
  - *How Many Snails*, by Paul Giganti, Jr.

Curriculum Framework for Mathematics

School: Reach Academy for Girls   **Curricular Tool: Singapore Math**   **Grade: 1**   **Teacher: Mongillo/Wahl/Giamonti**

Standards Alignment	Unit Concept Enduring Understandings	Essential Questions Student Learning Targets	Assessments
Unit One: Number Sense/Number Bonds and Number Stories **Timeline: Approximately 60 school days**			
1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **1.OA.2** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **1.OA.3** Apply properties of operations as strategies to add and subtract. **1.OA.4** Understand subtraction as an unknown-addend problem. **1.OA.5** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). **1.OA.6** Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).	Addition and subtraction are related to each other. Numbers can be composed and decomposed. The relationship between addition and subtraction can help solve problems. Multiple strategies can be used to represent and solve problems. In an equation, the equal sign means that the parts on each side of the symbol have the same value.	**Essential Questions:** What strategies can be used to count larger numbers? How do numbers represent and define value? What are the relationships between numbers? How do addition and subtraction relate to each other? What strategies can I use to solve addition and subtraction problems? How do I know which operation to use to solve a problem? How do I determine which computational strategy to use? What are number bonds and how are they useful for solving problems? What strategies can be used to solve problems? How can creating graphs be useful in problem solving? **Learning Targets:** I can read, write and count to 120	Formative Assessments: Math journals • Observations of small group work • Group work checklist • Class work • Homework • Class discussions • Centers work Summative Assessments: • Math Rubric • Number Bond Assessment • Number Bond Rubric • Singapore Assessments

1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.	I know that a bundle of ten ones is called a ten.
1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating to three whole numbers.	I can identify how many tens are in a 2-digit number. I can identify how many ones are in a 2-digit number.
1.NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	I can identify the number of tens and ones in a numbers that are multiples of ten.
1.NBT.2.a Understand that the two digits of a two-digit number represent amounts of tens and ones.	I can write any 2-digit number in expanded form.
1.NBT.2.b The numbers from 11 to 19 are composed of ten and one, two, three, four, five, six, seven, eight, or nine ones.	I can compare numbers using tens and ones.
1.NBT.2.c The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	I can compare two 2-digit numbers to determine if a number is equal using the tens and ones.
1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	I can use the symbols >, <, and = to compare two 2-digit numbers.
1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	I can add a 2-digit number and a 1-digit number. I can add a 2-digit number and a multiple of ten. I can explain what strategy I used to solve my problem. I can determine when to regroup in an addition problem. I can explain the steps I used to solve my problem.

1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

| I can add two 2-digit numbers without regrouping. |
| I can add two 2-digit numbers with regrouping. |

1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

| I can find 10 more than a number without having to count. |
| I can find 10 less than a number without having to count. |
| I can explain how to find 10 more than a number. |
| I can explain how to find 10 less than a number. |
| I can subtract a multiple of 10 from another multiple of 10. (10-90) |
| I can explain the strategy I use to solve my problem. |
| I have multiple strategies that help me solve problems. |

Unit Two: Comparing and Measuring Length/Time and Shapes
Timeline: Approximately 30 days

Standards	Essential Understandings	Essential Questions	Assessments
1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.	Objects can be compared through indirect measurement.	**Essential Questions:** What strategies can we use to measure and compare the lengths of objects?	**Formative Assessments:** Unit Pre-test, Math journals, Group work checklist, Classroom discussion, Class work, Homework, Counting Assignment, Centers work
1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.	Measurement requires specific strategies and precision.	What strategies help us tell time quickly?	
1.MD.3 Tell and write time in hours and half-hours using analog and digital clocks.	Telling time is an important real-world skill.	Why is it important to organize and understand information?	**Summative Assessments:** Unit Post Test, Number Relationship Rubric
1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer	Being able to organize, represent and interpret data helps us understand the world around us.	Why do only some attributes matter when naming a shape?	

Standards	Enduring Understandings	Essential Questions / Learning Targets	Resources / Assessments
questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. **1.G.1.** Distinguish between defining attributes (e.g., triangles are closed and three sided) versus non defining attributes (e.g., color, orientation, overall size) ; build and draw shapes to possess defining attributes. **1.G.2.** Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half circles, and quarter, circles) or three dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. **1.G.3.** Partition circles and rectangles into two and four equal shares, describe the shares using the words halves fourths and quarters and use the phrases half of fourth of and quarter of… Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares	Certain attributes matter when defining a shape. Smaller shapes and solids can be combined to create larger shapes or solids. Rectangles and circles can be divided into two and four equal shares. The more shares, the smaller each share.	What strategies can we use to combine smaller shapes or solids into larger ones? How does dividing circles and rectangles into pieces help us understand equal shares? How does the number of equal shares affect the size of each piece? **Learning Targets:** I can order objects by length. I can measure an object using nonstandard units. I can tell time to the hour using a digital clock. I can tell time to the half-hour using a digital clock. I can tell time to the hour using an analog clock. I can tell time to the half-hour using an analog clock. I can write the time in hours. I can write the time in half-hours. I can organize data with up to three categories. I can use a graph to ask and answer questions. I can distinguish between attributes that define a shape and attributes that do not define a shape.	Counting and Comparing Assessment Ordinal Number Rubric Singapore Assessments

I can use attributes to build shapes.

I can use attributes to draw shapes.

I can build a new shape using two 2-dimensional shapes. (rectangle, square, trapezoid, triangle, ½ circle, ¼ circle)

I can build a new shape using two 3-dimensional shapes. (cube, right rectangular prism, right circular cone, right circular cylinder)

I can take a shape I have made from two shapes and change it to make a new shape.

I can divide into equal shares

I can divide objects into fourths and quarters

I can describe the whole as the number of parts needed to make the whole.

Unit Three: Addition/Subtraction to 100/Graphing/Time
Timeline: 60 Days

1.OA.3 Apply properties of operations as strategies to add and subtract.	There are certain rules that apply to addition and subtraction.	**Essential Questions:** How can addition and subtraction properties help us solve problems?	**Formative Assessments:** Unit Pre-test Math journals Group work checklist Class work Class discussions Homework Measurement Assignment Math Learning Stations
1.NBT.2.c The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	Some strategies for addition and subtraction are faster than other strategies.	Why do we use symbols to show comparisons?	**Summative Assessments:** Unit Post-Test Measurement Rubric Analyzing and Comparing Assessment Performance Assessment Rubric Singapore Assessments
1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	Each numeral in a number represents a different place value.	Why is place value so important in addition and subtraction?	
1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	Understanding the rules of addition and subtraction can help solve problems. Organizing, representing and interpreting data can help solve problems.	Why is it important that the rules of addition and subtraction are followed? How can we use graphs to help us understand data and solve problems? **Learning Targets:** I can use strategies to add and subtract. I can count by 10's.	
1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.		I can say 7 tens equals 70 I can compare numbers with the symbols >, =, and <. I can add to 100, using a two-digit number and a one-digit number, and adding a two-digit number	
1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.		I can use concrete models or drawings to show addition and subtraction	
1.MD.3 Tell and write time in hours and half-		I can use math strategies to a write and explain how I solved a	

hours using analog and digital clocks.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

problem.
I can add two-digit numbers with and without regrouping
I can find 10 more or less than a number without having to count.
I can explain how to find 10 less than a number.
I can organize, represent, and interpret data with up to three categories.
I can tell and write time in hours and half-hours using analog and digital clocks.
I can explain the strategy I use to solve my problem.
I can use addition and subtraction within 20 to solve word problems.

Unit Four: Addition and Subtraction Revisited
Timeline: 30 days

	Essential Questions:	**Formative Assessments:**
1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **1.NBT.1** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **1.NBT.6** Subtract multiples of 10 in the range	Understanding the relationship between addition and subtraction helps solve problems. Understanding place value and numbers helps compare numbers and solve addition and subtraction problems. Objects can be compared through indirect measurement.	What strategies can help solve addition and subtraction problems? Why is place value important when comparing numbers? Why is measurement important in our everyday life? What strategies can be used to measure weight using everyday objects?	Teacher/Student conferencing Government Pre-Assessment Plants Pre Assessment Plant graphic organizer Class discussions Small group work Centers work Homework **Summative Assessment:** Post Assessment Number Position and Relationship Rubric

10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **1.MD.1** Order three objects by length; compare the lengths of two objects indirectly by using a third object. **1.NBT.3** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. **1.NBT.2.a 10** can be thought of as a bundle of ten ones - called a "ten." **1.NBT.1** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	How can objects be compared, ordered and sorted by weight? **Learning Targets:** I can take apart and put numbers back together I can use arrays to describe groups or sets I can use objects to help me with addition and subtraction word problems I can compare the weight of one object to another I can use objects to show the quantity and position of a number I can add and subtract to 100 I can draw number problems I can write and read numbers to 120	Multiples Assessment Ordinal Number Rubric Hundreds Chart Rubric Singapore Assessments

Understanding by Design Template

Mathematics – First Grade
Title: Addition, Subtraction, and Data - Time Frame: 25 days
Developed by Reach Academy for Girls
Teachers: Mongillo - Wahl – Giamonti

Summary:
This 30-day unit is part of the Number Sense/Number Bonds and Number Stories unit. The Number Sense/Number Bonds and Number Stories unit is completed later in the year.

In this unit, students will develop skills for adding and subtracting with two-digit numbers. Students continue to build an understanding of place value, and apply this understanding to both addition and subtraction problems. Using number stories, students build the connections between the parts and the whole. Students also spend considerable time working with the concepts of commutativity in addition and the inverse relationship between addition and subtraction. Finally, students

Stage 1 - Desired Outcomes

CC.1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

CC.1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

CC.1.OA.3 Apply properties of operations as strategies to add and subtract.   Examples: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known. (Commutative property of addition.) To add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$. (Associative property of addition.)

CC.1.OA.4 Understand subtraction as an unknown-addend problem. For example, subtract $10 - 8$ by finding the number that make 10 when added to 8.

CC.1.OA.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

CC.1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).

CC.1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $5 = \_\_ - 3$, $6 + 6 = \_\_$.

CC.1.NBT.2.a Understand that the two digits of a two-digit number represent amounts of tens and ones.

CC.1.NBT.2.b The numbers from 11 to 19 are composed of ten and one, two, three, four, five, six, seven, eight, or nine ones.

CC.1.NBT.2.c The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

CC.1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

CC.1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

This unit also incorporates Mathematical Processes 1-8

Big Ideas:
- Understanding place value helps make addition and subtraction with larger quantities efficient.
- Addition and subtraction are inverse operations.
- Addition and subtraction follow different rules.
- Because each problem we encounter is different, it is useful to have several strategies for adding and subtracting efficiently.

Understandings:
- Place value is critical to addition and subtraction.
- Addition and subtraction are related to each other.
- Numbers can be composed and decomposed.
- The relationship between addition and subtraction can help solve problems.
- Multiple strategies can be used to represent and solve addition and subtraction problems.
- Some strategies may be more efficient than other strategies.
- In an equation, the equal sign means that the parts on each side of the symbol have the same value.

Essential Questions:
- How do numbers represent and define value?
- What are the relationships between numbers?
- How do addition and subtraction relate to each other?
- How do I know which operation to use to solve a problem?
- How do I determine which computational strategy to use?
- What strategies can I use to add or subtract numbers?
- What are number bonds and how are they useful for solving problems?
- What strategies can be used to solve problems?

Students will Know:
- addition and subtraction are used in everyday situations.
- in addition, the order of the quantities does not matter.
- in subtraction, the order of the quantities matters.
- addition and subtraction are inverse operations
- skip counting is adding the same number over and over again.
- decomposing numbers maintains the same quantity, just in different group sizes
- several strategies for solving addition and subtraction problems.
- when adding a two digit number, you must add the ones places together and the tens places together.

Stage 2 - Assessment Evidence

Performance Task: Max's Numbers[1]

Students will be given the following assessment:

> Max has three number cards:
>
> | 3 | 1 | 7 |
>
> What is the largest two-digit number he can create?

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[1] Adapted from New York City Department of Education:
http://schools.nyc.gov/NR/rdonlyres/B8F6F552-ED31-498A-A1B6-4AA86018FE5D/0/NYCDOEG1MathNinasNumbers_Final.pdf

_____ _____

How do you know?
Use pictures, numbers and words to explain your thinking.

Using these same cards, what is the smallest two-digit number he can create?

_____ _____

How do you know?
Use pictures, numbers and words to explain your thinking.

Add your largest and smallest numbers together:

_____ _____ + _____ _____ =

Explain your thinking:

Differentiation:
   Intervention:
   - Students can be given a set of Max's cards so they can manipulate the numbers when looking for the greatest and least possible values.
   - Students should have access to multiple manipulatives, including number lines and hundreds charts.
   - Number values can be modified to preclude regrouping for this assessment.
   - Students can be offered the opportunity to respond in other, non-written formats.
   Extension:
   - Students can be given number values that add to more than 100, requiring regrouping in both tens and hundreds place value positions.
   - Students can be asked, "What number can you make that is closest to ___?"

Scoring Rubric:

Students receive the following points:

Response:	Points:
Greatest number = 73	1

Explanation for greatest number:
    1. Student provides and accurate explanation for reasoning, specifically mentioning place value concepts.
    2. Written response is clear and does not require additional probing from the teacher.
	2
	(1 each)

Smallest number = 13	1

Explanation for smallest number:
    1. Student provides and accurate explanation for reasoning, specifically mentioning place value concepts.
    2. Written response is clear and does not require additional probing from the teacher.
	2
	(1 each)

Sum: 73+13 = 86 * *If either of the first two values is incorrect, student should still receive credit if a correct addition sentence is included based on the initial error made.*	1

Explanation for sum:
    1. Student provides and accurate explanation for reasoning, specifically mentioning place value concepts.
    2. Written response is clear and does not require additional probing from the teacher.
	2

Total Possible Points:	9

4 – Exceeds – 9 points earned
3 – Meets – 7 or 8 points earned
2 – Nearing – 4, 5 or 6 points earned
1 – Below – 1-3 points earned

Other Evidence:
- Observations of Learning Center activities
- Observations during small group work
- Notes on activities/discussions
- Math Journal
- Class work
- Group work
- Homework assignments
- Singapore assessments
- Pre- and posttests

Stage 3 Learning Plan: Framework for the unit:

Lesson 1: Counting to 100
- In 18.1a, students work on using bundles to represent groups of tens, and practice counting groups of ten up to 100. Students work with bundles, dot cards, base 10 blocks and hundreds charts to make connections. At this stage, students are working only with whole tens, no ones.
- In 18.1b, students are given groups of items that total less than 100 (using some ones) and are expected to count them. During this lesson, students also discuss methods for counting the items – either grouping by tens or counting individually by ones. The goal is to have students work to make groups of ten to count the objects. Students are asked to share their quantities, stating the tens and ones separately, then giving the total quantity. This reinforces place value concepts.
- In 18.1c, students work on combining dot cards with groups of tens and a group of ones to create a two-digit number. For example, 60 and 4 = 64 = sixty-four

Lesson 2: One More, One Less, Ten More, Ten Less
- In 18.3a, students work with manipulatives again to build understanding of what happens when we add one or ten, or take away one or ten. Students discuss strategies for answering these problems, and use equations to represent their work.

Lesson 3: Comparing and Ordering Numbers
- In 18.3b, students develop strategies to use place value to compare two two-digit numbers. Students spend time building proficiency with the concepts of "greater" and "smaller"
- In 18.3c, students continue working with comparing numbers, and are asked to put four numbers in order based on their value.
- Play game: *Count on and Count Back.* Students work on a hundreds grid. They roll number cubes to represent addition and subtraction of tens and ones, and must move according to the values rolled. During the game, students are expected to state the movements in terms of tens and ones.

Lesson 4: Addition
- In 18.5a, students begin by working with two-digit plus one-digit addition without regrouping. Students use base ten blocks and number bonds to help reinforce the importance of place value when adding numbers.
- In 18.5b, students continue two-digit plus one digit addition problems, but move into regrouping by using the strategy of "making ten."
- In 18.5c, students shift their place value focus, and add two-digit numbers with a multiple of 10.
- In 18.5c, students are given the task of adding any two two-digit numbers, using manipulatives as needed.
- In 18.5e and 18.5f, students are introduced to the use of algorithms to add without regrouping.

Lesson 5: Subtraction
- In 18.6a, students begin by subtracting a one-digit number from a two-digit number without regrouping. During this lesson, students interact with base ten blocks, number bonds and the "counting back" strategy.
- In 18.6b, students use the benchmark ten to work to build concepts for subtracting a one-digit number from a two-digit number with regrouping. Students continue the use of multiple representations to reinforce place value concepts.
- In 18.6c, students subtract groups of ten from two-digit numbers

Differentiation:
Students are given the opportunity to work on the lessons at a pace appropriate to their level, and with whatever manipulatives they choose. Manipulatives are available to students throughout the unit. Additional strategies for addition and subtraction may be introduced on an as-needed basis.

Small group instruction will focus on playing games to reinforce skills, and extension games will also be available to students who demonstrate proficiency.

The teachers' edition provides many "practice" and "extra practice" opportunities for students.

Students should be given the opportunity to demonstrate understanding in a variety of ways during the instructional unit: spoken responses, computer-based, SMART board, iPad, etc.

Resources and Teaching Tools:
- *Singapore Math* text books and teachers' guide
- *Singapore Math* workbook

Design Principles for Unit Development:
- Universal Design for Learning: students are given various formats to interact with content. Students spend time with visuals, manipulatives, speaking and writing in each lesson. These are all used as formative assessment techniques.
- 21st Century Learning: Higher-order thinking skills are incorporated into each lesson within the unit, and students are given frequent opportunities to explain their own and consider others' work. Students are

expected to apply their knowledge in a novel situation in the Performance Task.

Technology Integration:
- Interactive base ten blocks: http://www.fuelthebrain.com/Interactives/app.php?ID=201
- Various place value, addition, and subtraction games, including number bonds: http://resources.woodlands-junior.kent.sch.uk/maths/interactive/

Content Connections:
- These skills can be applied to all other content areas when there are quantities that can be compared and combined:
- Specific literature connections:
  - *The King's Commissioners* by Marilyn Burns
  - *Twelve Ways to get to Eleven* by Eve Merriam

Curriculum Framework for Mathematics

School: Reach Academy for Girls   **Curricular Tool: Singapore Math**   **Grade: 2**   **Teacher: Gimbel/Laughman/Wood**

Unit One: Numbers to 1000/Place Value/Comparing Numbers/ Addition and Subtraction with Regrouping
Timeline: Approximately 40 school days

Standards Alignment	Unit Concept Enduring Understandings	Essential Questions Student Learning Targets	Assessments
2.OA.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	There are many ways to represent addition and subtraction problems up to 100. In three digit numbers, the digits represent hundreds, tens and ones.	**Essential Questions:** How does place value help with addition and subtraction problems? How can you use place value to model and write numbers to 100? What strategies help us when comparing numbers?	**Formative Assessments:** Math journals Classroom observations Class work Group work checklist Pattern Assignment **Summative Assessments:** Math Rubric Number Bond Assessment Solving for Unknown Rubric
2.OA.2. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.	There are many strategies that help us count efficiently. Understanding place value helps us add and subtract numbers efficiently.	Why is it useful to know whether a number is odd or even? How can models and mental math help with addition of two-digit numbers?	
2.NBT.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.	Addition and subtraction of larger numbers follows the same pattern as addition and subtraction of smaller numbers.	Why is 'making ten' such a useful strategy for addition? What strategies model and record adding one and two-digit numbers to two-digit numbers with and without regrouping?	
2.NBT.2. Count within 1000; skip-count by 5s, 10s, and 100s.			
2.NBT.3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.		What strategies help us add three or four two-digit numbers with and without regrouping?	
2.NBT.4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.		What strategy do you use to select a method for adding numbers? **Learning Targets:**	
2.NBT.5. Fluently add and subtract within 100 using strategies based on place value,			

properties of operations, and/or the relationship between addition and subtraction.	I can write an equation to express an even number as a sum of two equal addends.
2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations.	I can use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns.
2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it are necessary to compose or decompose tens or hundreds.	I can write an equation to express the total as a sum of equal addends. I can demonstrate that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. I can demonstrate how 100 can be thought of as a bundle of ten tens — called a "hundred." I can demonstrate how the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2.NBT.8. Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.	I can count within 1000. I can skip-count by 5s, 10s, and 100s. I can read and write numbers to 1000.
2.NBT.9. Explain why addition and subtraction strategies work, using place value and the properties of operations.	I can show numbers to 1000 using base-ten numerals. I can read and write number names to 1000.

I can show numbers to 1000 in expanded form.

I can compare two three-digit numbers using >, =, and < symbols.

I can fluently add and subtract within 100 using varying strategies.

I can add up to four two-digit numbers using varying strategies.

I can add and subtract within 1000, using concrete models or drawings and strategies.

I can relate concrete and mental strategies used to add and subtract within 1000 to a written method.

I can demonstrate that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

I can mentally add and subtract 10 or 100 to a given number 100-900.

I can explain why addition and subtraction strategies work, using drawings or objects.

Unit Two: Comparing and Measuring Length/Time and Money
Timeline: Approximately 60 days

Standards Alignment	Unit Concept Enduring Understandings	Essential Questions Student Learning Targets	Assessments
CC.2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	One object can have many different measurements, depending upon the measuring unit selected.	**Essential Questions** What strategies do you used to select a measurement tool?	**Formative Assessments:** Unit Pre-test Math journals Classroom discussions Class work Homework Group work checklist Counting Assignment Measurement Learning Stations
CC.2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	Measurement requires precision. Estimation can help determine whether an answer is reasonable. Any given unit of measurement is always exactly the same, independent of measurement tool.	Why is measurement important in everyday life? Why are there different measurements for the same object? Why do we need to recognize numbers through 1000?	
CC.2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.	(One inch is always the same, regardless of which standard unit ruler is used)	How can you use measurement to solve problems?	**Summative Assessments:** Unit Post Test Number Relationship Rubric Counting and Comparing Assessment Measurement Tools Assessment Measurement Rubric
CC.2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	Attributes measured in the same units can be added and subtracted.	How and when is estimation a useful tool?	
CC.2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	Clocks have standard units, just like other measurement tools. Precision is important when telling time. Money has standard units, just like other measurement tools.	**Learning Targets:** I can order objects by length. I can use one object to help me describe the length of other objects. I can measure an object using nonstandard units.	
CC.2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	Understanding and solving problems involving money is an important life skill.	I can recognize the hour hand. I can recognize the minute hand.	
CC.2.MD.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ (dollars) and ¢ (cents) symbols appropriately.	Graphs are a useful method for organizing data for solving problems.	I can tell time to the hour using a digital clock. I can tell time to the half-hour using a digital clock.	
CC.2.MD.9 Generate measurement data		I can tell time to the hour using an analog	

by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. **CC.2.MD.10** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.	clock. I can tell time to the half-hour using an analog clock. I can write the time in hours. I can write the time in half-hours. I can organize data with up to three categories. I can interpret a graph by asking questions about the data. I can interpret a graph by answering questions about the data. I can interpret a graph by comparing how many more are in one category than another. I can interpret a graph by comparing how many less are in one category than another I can distinguish between attributes that define the shape and attributes that do not define the shape. I can use attributes to build shapes. I can use attributes to draw shapes. I can build a new shape using two 2-dimensional shapes. (rectangle, square, trapezoid, triangle, ½ circle, ¼ circle) I can build a new shape using two 3-dimensional shapes. (cube, right rectangular prism, right circular cone, right circular cylinder)

	I can take a shape I have made from two shapes and change it to make a new shape. I can divide into equal shares I can divide objects into fourths and quarters I can describe the whole as the number of parts needed to make the whole.

Standards Alignment	Unit Concept Enduring Understandings	Essential Questions Student Learning Targets	Assessments
Unit Three: Fractions/Geometry/Graphing/Time Timeline: 60 Days			
CC.2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (Sizes are compared directly or visually, not compared by measuring.)

CC.2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

CC.2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

CC.2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

CC.2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. | Using mathematical language helps us discuss and compare attributes of different shapes.

Using same-size squares to cover rectangles can help us discuss the size of different rectangles.

Equal shares have to have equal value, and do not have to be the same shape.

Graphs are a useful method for organizing data for solving problems.

Clocks can be used to help us understand equal shares.

Precision is important when telling time. | **Essential Questions:**
How do we describe geometric figures?

Where can we find geometric figures in the world around us?

How does mathematical language help us communicate with each other about shapes?

Why are fractions important in everyday life?

What strategies can be used to help us understand fractions, parts and wholes?

Why are parts and wholes important in fractions?

How can we divide shapes into fractional parts?

How does dividing rectangles into rows and columns help us understand and solve multiplication problems?

What strategies can we use to compare fractions?

How do different graphs help us organize data and solve problems?

What strategies help us tell time to the nearest five minutes, using a.m. and p.m.?

Learning Targets:
I use mathematical language when discussing and comparing shapes.

I can recognize and draw shapes having | **Formative Assessments:**
Unit Pre-test
Math journals
Classroom discussions
Class work
Homework
Group work checklist
Graphing Assignment
Fraction Matching
Geometry Assignment
Math Learning Stations
Teacher Conference

Summative Assessments:
Unit Post-Test
Graphing Rubric
Geometry Rubric
Analyzing and Comparing Assessment
Fraction Checklist
Performance   Assessment
Rubric |

specified attributes, such as a given number of angles or a given number of equal faces.

I can identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

I can partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

I can partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths.

I can recognize that equal shares of identical wholes need not have the same shape.

I can draw a picture graph to represent a data set with up to four categories.

I can draw a bar graph to represent a data set with up to four categories.

I can solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

I can generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object.

I can show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

| | I can solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ (dollars) and ¢ (cents) symbols appropriately.

I can tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. | |

Standards Alignment	Unit Concept Enduring Understandings	Essential Questions Student Learning Targets	Assessments
Unit Four: Solidifying Place Value Concepts **Timeline: Approximately 35 Days**			
CC.2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	Using rectangular arrays can help us begin to understand multiplication concepts. There are many ways to represent addition and subtraction problems up to 100. In three digit numbers, the digits represent hundreds, tens and ones.	**Essential Questions:** How is addition related to multiplication? How can rectangular arrays help us understand multiplication? How can we develop and use number sense to investigate characteristics of numbers in a variety of forms and models of operation? How do operations affect numbers? How can we use information from a problem to choose an operation?	**Formative Assessments:** Teacher/Student conferencing Pre-Assessment Classroom discussions Class work Homework Repeated Pre Assessment Arrays graphic organizer **Summative Assessments:** Post Assessment Number Position Addition and Subtraction Rubric Multiples Assessment Ordinal Number Rubric Hundreds Chart Rubric
CC.2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.			
CC.2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	There are many strategies that help us count efficiently.		
CC.2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.	Understanding place value helps us add and subtract numbers efficiently. Addition and subtraction of larger numbers follows the same pattern as addition and subtraction of smaller numbers.	**Learning Targets:** I can compare two three-digit numbers using $>$, $=$, and $<$ symbols.	
CC.2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)		I can skip-count by 5s, 10s, and 100s.	
CC.2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it are necessary to compose or decompose tens or hundreds.		I can demonstrate that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. I can partition a rectangle into rows and columns of same-size squares and count to find the total number of them. I can explain why addition and subtraction strategies work, using	

	drawings or objects. I can add and subtract within 1000, using concrete models or drawings and strategies. I can relate concrete and mental strategies used to add and subtract within 1000 to a written method. I can demonstrate that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Understanding by Design Template

Mathematics Grade 2
Title: Place Value - Time Frame: 4 to 6 weeks
Developed by Reach Academy for Girls

Unit Summary:
In first grade, students worked solidifying addition and subtraction concepts to 20, while counting to 100. In this unit, students will use knowledge about the number system to develop an understanding of larger numbers, up to 1000. In working toward 1000, students are able to use patterns they already know about working with ones and tens to build knowledge about working with hundreds and thousands.

Stage 1 - Desired Outcomes

- **2.OA.1**. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- **2.OA.2**. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.
- **2.NBT.1**. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.
- **2.NBT.2.** Count within 1000; skip-count by 5s, 10s, and 100s.
- **2.NBT.3.** Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
- **2.NBT.4.** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
- **2.NBT.5.** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
- **2.NBT.6.** Add up to four two-digit numbers using strategies based on place value and properties of operations.
- **2.NBT.7.** Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it are necessary to compose or decompose tens or hundreds.
- **2.NBT.8.** Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.
- **2.NBT.9.** Explain why addition and subtraction strategies work, using place value and the properties of operations.

This unit also includes Mathematical Practice standards 1-8.

Big Ideas:
- Regardless of value, all whole numbers follow predictable patterns in addition and subtraction
- With each move left in a number, the value of the digit increases 10 times.
- Counting strategies can be expanded to add and subtract large numbers.

Understandings:
- There are many ways to represent addition and subtraction problems up to 100.
- In three digit numbers, the digits represent hundreds, tens and ones.
- There are many strategies that help us count efficiently.
- Understanding place value helps us add and subtract numbers efficiently.
- Addition and subtraction of larger numbers follows the same pattern as addition and subtraction of smaller numbers.

Essential Questions
- How does place value help with addition and subtraction problems?
- How can you use place value to model and write numbers to 100?
- What strategies help us when comparing numbers?
- Why is it useful to know whether a number is odd or even?
- How can models and mental math help with addition of two-digit numbers?
- Why is 'making ten' such a useful strategy for addition
- What strategies model and record adding one and two-digit numbers to two-digit numbers with and without regrouping
- What strategies help us add three or four two-digit numbers with and without regrouping?
- What strategy do you use to select a method for adding numbers?

Learning Targets
- I can write an equation to express an even number as a sum of two equal addends.
  I can use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns.
  I can write an equation to express the total as a sum of equal addends.
- I can demonstrate that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.
- I can demonstrate how 100 can be thought of as a bundle of ten tens — called a "hundred."
- I can demonstrate how the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
- I can count within 1000.
- I can skip-count by 5s, 10s, and 100s.
- I can read and write numbers to 1000.
- I can show numbers to 1000 using base-ten numerals.
- I can read and write number names to 1000.
- I can show numbers to 1000 in expanded form.
- I can compare two three-digit numbers using >, =, and < symbols.
- I can fluently add and subtract within 100 using varying strategies.
- I can add up to four two-digit numbers using varying strategies.
- I can add and subtract within 1000, using concrete models or drawings and strategies.
- I can relate concrete and mental strategies used to add and subtract within 1000 to a written method.
- I can demonstrate that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
- I can mentally add and subtract 10 or 100 to a given number 100-900.
- I can explain why addition and subtraction strategies work, using drawings or objects.

Stage 2 - Assessment Evidence

Performance Task:

MATERIALS
- One die per pair of students
- Recording sheet

GROUPING
Partners

DIRECTIONS
Students will order digits to create the highest or lowest possible number. Students will use previous experiences with place value to predict the place a number should be written on the recording sheet in order to create the highest or lowest number.

Students should be given a recording sheet that is divided into columns and rows. The number of columns will determine how large the number should be. In this lesson, there should be four columns to represent thousands, hundreds, tens, and ones. The number of rows will determine how many rounds the students should play.

The goal is to create the highest or lowest number for each row. The student partners will decide before playing whether they are looking for the highest or lowest number per round. The first student rolls the die, decides where the digit should be written on their recording sheet and describes it to their partner. "I rolled a 4, I'm going to put it in the tens place to make 40."
The second student does the same. The game continues in this way until both students have made a complete number. The person with the highest (or lowest) number will draw a star next to their number.

Example:

Thousands	Hundreds	Tens	Ones	Expanded Notation	Total
2	4	6	2	2000 + 400 + 60 + 2	2,462

Questions for Teacher Reflection
- As students create the numbers do they always choose the smallest number for the ones place? Are they able to explain why they are doing this?
- Can the students explain why they are putting any number in a particular place?
- Can the students explain why they would not put a number in a particular place?
- Can the students describe/express the number they have made in words?

Place Value Breakdown

Thousands 1,000	Hundreds 100	Tens 10	Ones 1	Expanded Notation _ _ _ _ + _ _ _ + _ _ + _ _	Total

1. What is the largest number you made? _____ How do you know ?

   _____

   _____

2. What is the smallest number you made? _____ How do you know ?

   _____ _____

3. Using your smallest number, what number adds to it to equal 1000? _____

   Write an equation for your work:

   _____

4. What does "place value" mean to you? _____ _____

   _____

   _____

Rubric:
Scoring Rubric:

Students receive the following points:

Response:	Points:
Greatest number is correctly identified	1

Explanation for greatest number:
  1. Student provides and accurate explanation for reasoning, specifically mentioning place value concepts.
  2. Written response is clear and does not require additional probing from the teacher.
	2
	(1 each)

Smallest number is correctly identified	1

Explanation for smallest number:
  1. Student provides and accurate explanation for reasoning, specifically mentioning place value concepts.
  2. Written response is clear and does not require additional probing from the teacher.
	2
	(1 each)

Addition to 1000:
  1. Student provides a correct response based upon their smallest number.
  2. Student shows a correct equation for their problem (student can use addition or subtraction to solve the problem)
	2

Explanation for place value:
  1. Student provides and accurate explanation for reasoning, specifically mentioning place value concepts.
  2. Written response is clear and does not require additional probing from the teacher.
	2

Total Possible Points:	10

4 – Exceeds – 9-10 points earned
3 – Meets - 7- 8 points earned
2 – Nearing – 4-6 points earned
1 – Below – 1-3 points earned

Other Evidence:

Formative Assessments:
    Math journals
    Group work checklist
    Class discussions
    Class work
    Homework
    Base Ten Pictures
    Place Value Show Down

Summative Assessments:
    Math Rubric
    Number Bond Assessment
    Solving for Unknown Rubric

Lesson Plan: Framework for the unit

Lesson 1: Counting On, Counting Backwards

- In 1.1c, students will use the "counting on or "counting backwards" strategies to engage with addition and subtraction of numbers up to 100.
- In 1.2a, students use base 10 blocks to review place values up to 999. This lesson provides a foundation for students as they move into expanding place values later in the unit.
- In 1.3a, students work with place value discs to compare and order numbers up to 1000. Students learn the importance of place value when comparing two numbers. They also write inequality statements for these comparisons.
- In 1.3b, students continue to work with counting on and counting backwards, looking at patterns in the answers as they do so. This again brings focus to place value concepts.

Lesson 2: Adding and Subtracting within 1000,without Regrouping

- In 2.2a, students work with adding a two-digit number
- In 2.2b, students add a three-digit number
- In 2.3a, students subtract two-digit numbers
- In 2.3b, students subtract three-digit numbers to solve problems
- In 2.3c, students practice these skills

Lesson 3: Adding with Regrouping

- In 2.4a, students mentally add ones or tens to a three digit number
- In 2.4b, Student add with renaming in the ones place
- In 2.4c, students add with renaming in the tens place
- In 2.4d, students add with renaming in both the tens and the ones places
- In 2.4e, students add three numbers within 1000
- In 2.4f, students practice these skills

Lesson 4: Subtracting with Regrouping

- In 2.5a, students mentally subtract ones or tens from a three digit number
- In 2.5b, students subtract with renaming in the tens place
- In 2.5c, students subtract wih regrouping in the hundreds place
- In 2.5d, students subtract with regrouping in both the tens and hundreds places
- In 2.5f and 2.5g, students practice all the addition and subtraction skills in the unit.

Games to Reinforce Learning:

Where am I on the Number Line?

In this game students will be reviewing counting up and counting back to get an answer. As the students play the game they will also see where a number lives on a number line and their relative position to each other (who its neighbor's are.) Being able to locate a number on the number line (the relative position of numerals) is essential to developing solid number sense. It will also help the student understand its value in relation to other numbers.

Part I
Introduce the game with the whole class before assigning partners to play. Using adding machine tape, create a 0 -50 number line. Use this number line to introduce "Where Am I on the Number Line?" Each time the spinner is spun, a student will move a paper clip or clothespin the appropriate number of spaces either up or down the number line. Have students state whether their new position is "greater than" or "less than" their previous position. Have students give a number sentence that matches with their move. Example: The player's clothespin is on 23 and he spins a -6. He will move the clothespin back and tell the class, "23 – 6 = 17." Students will begin on "5" and move forward or back accordingly. If they spin a number that is more than they can subtract they lose that turn. **When this happens make sure to discuss the fact that there ARE numbers on the other side of zero, negative numbers, but for now we are only working with/talking about the positive numbers.**

Student Directions
- Each player puts a paper clip or clothespin on 5.
- Place a transparent spinner on the game spinner. (Use A and then B)
- Player A spins the spinner, adds or subtracts that number to 5 and places the paper clip on that answer.

- Player B spins the spinner and moves as above.
- Player A spins the spinner, adds or subtracts the number to where his/her paper clip is, then moves the paper clip to the new answer.
- Player B does the same.
- The game continues until one of the players reaches or passes 50 on the number line.
- The first player to reach or pass 50 wins the game.

Where Am I On the Number Line?
Students will cut these apart and glue together to make a 0 -50 number line.

0	1	2	3	4	5	6	7	8	9	
10	11	12	13	14	15	16	17	18	19	
20	21	22	23	24	25	26	27	28	29	
30	31	32	33	34	35	36	37	38	39	
40	41	42	43	44	45	46	47	48	49	50

Variation: As the year progresses, change the number lines to show counts by 5's, 10's or 100's. Using dice, each roll of the dice has to be changed into the corresponding multiple of that number. Example: If a student rolls a 3 on a 10's number line that roll will represent 30.

Questions for Teacher Reflection
- Are students able to articulate the number sentence each time?
- Do students understand which direction to move the clothespin marker when they roll a number that calls for subtraction?
- Can students tell where a number lives on the number line and who its neighbors are?

What's My Number?

Students will use multiple representations of numbers to connect symbols to quantities.
   a. Represent numbers using a variety of models, diagrams, and number sentences).
   b. Understand the relative magnitudes of numbers using 10 as a unit, 100 as a unit, or 1000 as a unit.

MATERIALS

- Math Journals to record/explain concepts (optional)
- Base 10 manipulatives, as needed

GROUPING

Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I

The students will choose any two-digit number. After thinking of a way to describe their number, students will give hints to another classmate so they can try to figure out the number.

This activity helps students build flexibility using language and equivalent representations of numbers. Base ten manipulatives should be available if students desire to use them to visualize the numbers. Students could also draw pictures to help them make up their clues to find the mystery number. Children may begin with very simple, straightforward clues about their number. But eventually they will start to try to make up more difficult clues by combining the amount of ones, tens, or hundreds.

Part II

Create own game set with clues. Each child could create 5 or more cards and then place these cards in a center or for calendar review.

Examples of clues:
- I have a 4 in my tens place and a 2 in my ones place. Who am I?
- I have 1 more ten than the number 14 and 3 ones. Who am I?
- I am 35. I have 25 ones. How many tens do I have?
- CHALLENGE – I am 1 ten 5 hundreds, and 29 ones. Who am I?

Questions for Teacher Reflection
- Are students able to think of a number without writing it down?
- Can they describe their number in terms of place value and then communicate that information to a classmate or the teacher?
- Can they describe the number in more than one way?

Differentiation:

Students are given the opportunity to work on the lessons at a pace appropriate to their level, and with whatever manipulatives they choose. Manipulatives are available to students throughout the unit. Additional strategies for addition and subtraction may be introduced on an as-needed basis.

Small group instruction will focus on playing games to reinforce skills, and extension games will also be available to students who demonstrate proficiency.

The teachers' edition provides many "practice" and "extra practice" opportunities for students.

Students should be given the opportunity to demonstrate understanding in a variety of ways during the instructional unit: spoken responses, computer-based, SMART board, iPad, etc.

Resources and Teaching Tools:
- *Singapore Math* text books and teachers' guide
- *Singapore Math* workbook

Design Principles for Unit Development:
- Universal Design for Learning: students are given various formats to interact with content. Students spend time with visuals, manipulatives, speaking and writing in each lesson. These are all used as formative assessment techniques.
- 21st Century Learning: Higher-order thinking skills are incorporated into each lesson within the unit, and students are given frequent opportunities to explain their own and consider others' work. Students are expected to apply their knowledge in a novel situation in the Performance Task.

Technology Integration:

- Interactive base ten blocks: http://www.fuelthebrain.com/Interactives/app.php?ID=201
- Various place value, addition, and subtraction games, including number bonds: http://resources.woodlands-junior.kent.sch.uk/maths/interactive/

Content Connections:

- These skills can be applied to all other content areas when there are large quantities that can be compared and combined.

Curriculum Framework for Mathematics

School: Reach Academy for Girls **Curricular Tool: Singapore Math** **Grade: 3** **Teacher: Gimbel/Laughman/Wood**

Unit One: Numbers to 1000/Place Value/Numbers to 10,000, Addition/Subtraction
Timeline: Approximately 40 school days

Standards Alignment	Unit Concept Enduring Understandings	Essential Questions Student Learning Targets	Assessments
3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explains them using properties of operations. 3.NBT.2 Fluently adds and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (A range of algorithms may be used.)	Numbers can help me solve problems The base ten number system is based on combinations of ones, tens and hundreds Ordinal numbers can help me describe positions Numbers can be expressed in multiple forms Logical reasoning can help me solve mathematical and real-world problems Mathematicians fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	**Essential Questions:** How do numbers represent and define value? How are place value patterns repeated in large numbers? When should I use mental math? How do I use ordinal numbers in everyday life? How can I represent expanded notation? How do I know which operation to use to solve a problem? How do I determine which computational strategy to use? How do I assess the reasonableness of answers using mental computation and estimation strategies? **Learning Targets:** I can solve two-step word problems using all four operations. I can represent addition and subtraction word problems using equations.	**Formative Assessments:** • Math journals • Group work checklist • Class discussions • Class work • Homework • Addition Assignment • Subtraction Assignment **Summative Assessments:** • Math Rubric • Identifying Number Patterns • Number Bond Assessment • Number Bond Rubric • Unit Assessment

I can use mental computation strategies to see if my answer is reasonable.

I can explain the arithmetic patterns in addition and subtraction.

I can use strategies to add and subtract up to 1000.

Formative Assessments:
Unit Pre-test
Math journals
Class discussions
Class work
Homework
Group work checklist
Counting Assignment
Multiplication Task Cards

Summative Assessments:
Unit Post Test
Number Relationship Rubric
Counting and Comparing Assessment
Ordinal Number Rubric
Multiplication Scrapbook

Unit Two: Addition Ones, Tens, Hundreds, and Thousands (Ten Thousands)/ Multiplying Ones, Tens, and Hundreds/Dividing
Timeline: Approximately 40 days

Standards	Essential Understandings	Essential Questions
3.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each.	Understanding number relationships will helps determine the appropriate strategy to use in solving a problem	**Essential Questions:** How can mathematical situations and problems be represented?
3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.	Sums and differences of two and three digit numbers can be Estimated	How do operations relate to one another? How can I use what I know about hundreds, tens and ones to add and subtract greater numbers?
3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Mathematical properties govern addition and multiplication Multiples can be used to solve problems Skip counting can help me determine multiples of 2,5,10	What strategies do I use to find the sums or differences of two and three digit numbers? How are multiplication and addition related?
3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers.	Multiplication and division share a relationship	What are the mathematical operations that govern addition and multiplication? How do I use them?
3.OA.5 Apply properties of operations as strategies to multiply and divide.	Numbers can be divided with a number of strategies	How can multiples be used to solve a problem?
3.OA.6 Understand division as an unknown-factor problem.		How can numbers be broken down into smallest factors?
3.OA.7 Fluently multiply and divide within		

E-47

| 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explains them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. | **Learning Targets:**
I can model multiplication facts by forming groups to represent the factors.

I can model division facts by separating objects into equal shares.

I can solve multiplication and division word problems by using equations. (within 10)

I can determine the unknown whole number in multiplication and division problems.

I can determine the unknown whole number in a division problem.

I can apply commutative property to solve multiplication problems.

I can apply associative property to solve multiplication problems.

I can apply distributive property to solve multiplication problems.

I can solve a division problem by using multiplication. |

Standards Alignment	Unit Concept Enduring Understandings	Essential Questions Student Learning Targets	Assessments
Unit 3: Probability/Fractions **Timeline: 60 Days**			**Formative Assessments:** Unit Pre-test
3.NF.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)	Logical reasoning can be applied to mathematical problems	**Essential Questions:** How do I compare and read data? How can I draw a picture to solve an addition problem?	Math journals Class discussions Class work Homework Group work checklist Measurement Assignment Math Learning Stations Singapore Math assessments Observations Practice through games
	Fractions can be used in everyday life		
3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.	Fractions can be represented by models, drawings, and constructs	How does the type of data influence the type of graph?	
	Fractions can represent events or data	What is a fraction? How can I use fractions in real life?	**Summative Assessment:** Unit Post-Test Measurement Rubric Analyzing and Comparing Assessment Performance Task
3.NF.2 a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.	Fractions represent numbers on the number line	How are whole numbers and fractions related?	
	Fractions can be compared	How can I tell a fraction name indicates the number of equal parts in the whole?	
3.NF.2 b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)	The likelihood of an event can be described or written as a fraction	How does changing the size of the whole affect the value of a fraction? How can modeling show how fractional parts can be combined or separated?	
3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)		How does using a number line help me understand the value of fractions? What strategies can I use to place fractions on a number line?	
3.NF.3 a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.		**Learning Targets:** I know that a fraction means to	
3.NF.3 b. Recognize and generate simple			

equivalent fractions (e.g., $1/2 = 2/4$, $4/6 = 2/3$), Explain why the fractions are equivalent, e.g., by using a visual fraction model.	divide a whole object into equal size parts.
3.NF.3 c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3=3/1$; recognize that $6/1=6$; locate $4/4$ and 1 at the same point of a number line diagram.	I know that the numerator of the fraction tells how many of the equal size parts there are, and the denominator tells me how many parts are in one whole. I can show where fractions are on the number line, including fractions that are larger than 1.
3.NF.3 d. Compare two fractions with the same numerator or the same denominator by reasoning about their sizes. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, $<$ and justify the conclusions, e.g., by using a visual fraction model.	I can tell if two fractions are equivalent or which fraction is larger or smaller. I can use the symbols $<$ and $>$ to compare fractions. I can write a whole number as a fraction with a denominator of 1.

Standards Alignment	Unit Concept Enduring Understandings	Essential Questions Student Learning Targets	Assessments
Unit Four: Measuring Volume and Capacity Geometry/Order of Operations **Timeline: Approximately 35 Days**			
3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	Measurement can be represented in many different forms Measurements can be illustrated, described and extended with standard and non-standard units Attributes of shapes can be described, sorted and graphed Shapes can be transformed	**Essential Questions:** How do I use measurement in my life? What tools can I use to measure? How can I communicate measurement? How can I transform shapes? How can I demonstrate symmetry among shapes?	**Formative Assessments:** Teacher/Student conferencing Government Pre-Assessment Class work Class discussions Homework Plants Pre Assessment Graphic organizer Group work Teacher guided math conference **Summative Assessments:** Post Assessment Pattern Tessellations Rubric Mathematic Pattern Assessment Oder of Operations Rubric Elapsed Time Rubric
3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.	2 and 3 dimensional shapes can be represented by models and Constructs Shapes can be congruent Shapes can be manipulated to demonstrate symmetry	How can geometric ideas and relationships help me solve problems with numbers and measurement? How can I display my understanding of geometric term? Where can we find geometric figures in the world around us?	
3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.		**Learning Targets:** I can solve two-step word problems using addition, subtraction, multiplication and division.	
3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems		I can write an equation to show what I do to solve the problem. I can estimate the answer. I can find patterns in the addition and multiplication table and explain how they work.	

(problems involving notions of "times as much.")	
3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.	I can measure and estimate volume of liquids in liters and mass of objects in grams and kilograms.

I can tell and write time to the nearest minute. I can solve word problems involving intervals of time.

I can draw a bar graph or a picture graph and answer questions about the data on the graph. |
| **3.MD.8** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different area or with the same area and different perimeter. | I can draw a line plot from data I measure using a ruler marked in inches, halves and fourths of an inch.

I can find the area of a rectangle or square by measuring it with unit squares. |
| **3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. | I can find the area of a rectangle by multiplying the length time's width. I can explain why this works.

I can find the area of figures that are made up of rectangles. |
3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.	
3.MD.5 a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.	I can find the perimeter of a shape made up of straight sides, or work backward to find the length of a missing side.
3.MD.5 b. A plane figure, which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.	I can draw rectangles, squares, and rhombuses and I can explain how they are alike and different. (Note: the terms "parallel" and "perpendicular" are not introduced until 4th grade.)
3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units).	

	I can divide a shape into parts with equal areas and label each part with the correct fraction.
3.MD.7 Relate area to the operations of multiplication and addition.	
3.MD.7 a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.	
3.MD.7 b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.	
3.MD.7 c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.	
3.MD.7 d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	
3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describes the area of each part is 1/4 of the area of the shape.	

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title: Fractions

Designed by: Emilie Saintus

District: Developed by Thomas Edison Charter School, used with permission by the Reach Academy for girls

Content Area: Mathematics

Grade Level(s):   Grade Three

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Summary of Unit

This unit on fractions is part of a larger unit which teachers fractions and probability. This unit will review the uses of fractions and fractional notation and help students develop a solid understanding of equivalent fractions –a key element in comparing fractions and, later in calculating with fractions. Students will also learn strategies to place fractions on a number line. The unit involves whole group, small group, partner, hands on activities and individual learning. It is the eighth unit of the school year, and at this point in the year students should have a basic understanding of fractions.

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Stage 1 – Desired Results
What students will know, do, and understand

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Delaware Content Standards

CC3NF1:Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size 1/*b*.

CC3NF2:Understand a fraction as a number on the number line; represent fractions on a number line diagram.

CC3NF2a: Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

CC3NF2b: Represent a fraction *a/b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size*ba/b* and that its endpoint locates the number *a/b* on the number line.

CC3NF3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

CC3NF3a: Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

CC3NF3b: Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

CC3NF3c: Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

CC3NF3d: Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual

Big Idea(s)

This unit has six areas of focus:

- To explore fractional and spatial relationships
- To introduce the number line for fractions
- To Find equivalent fractions
- To compare factions using region models
- To name quantities greater than 1with fractions and mixed numbers
- To solve number stories involving fractions

Unit Enduring Understanding(s)

- Fractions can be represented by models and drawings
- Fractions can represent events or data
- Fractions represent numbers on the number line
- Fractions can be used in everyday life
- Equivalent fractions are fractions with the same value, but different denominators.

Unit Essential Questions(s)

- What is a fraction?
- How can I use fractions in real life?
- How are whole numbers and fractions related?
- How can I tell a fraction name indicates the number of equal parts in the whole?
- How does changing the size of the whole affect the value of a fraction?
- How can modeling show how fractional parts can be combined or separated?
- How does using a number line help me understand the value of fractions?

Knowledge and Skills

Students will know…

- The uses of fractions and of fraction notation
- How to identify equivalent fractions

Students will be able to…

- Students will be able to isolate and plot fractions on number lines
- Students will construct and deconstruct models and shapes to represent whole figures, shapes, and fractions
- Students will explain why fractions are equivalent and recognize simple equivalent fractions
- Students will express whole numbers as fractions

- Students will compare two fractions

Suggested Performance/Transfer Task(s)

Now that you have learned about fractions you are going to be in charge of teaching the rest of the class about a certain fraction by setting up an exhibit in the fraction museum. Your teacher will give you one fraction and you will be responsible for showing it in 3 different ways.

The first way will be finding items in the room to show the fraction, the same way you did with your group. For example, if your fraction was 2/3 you could find 3 pencils in the classroom and then separate 2 of them, showing 2 out of 3.

The second way will be taking an index card, which is a rectangle, and coloring it or cutting it up to show your fraction.

Finally, you will be taking a number line of 0 through 1, and plotting your fraction on it. The number line will have ½ shown to help you figure out where your fraction will go.

When you have finished all 3 tasks set up your desk like an exhibit in the fraction museum, showing your fraction in all 3 ways for your classmates to view.

When everyone is ready your class will be able to walk around and visit the fraction museum.

- Before starting, set parameters as to which items in the classroom can be used for them to show their fraction as part of a set.
- Give each student a fraction to 'be in charge' of.
- Students will have done these activities as a group already, but be available to assist students with each of the 3 tasks
- Help the students to set up their fraction museum exhibit on their desks, and allow students to walk around and see everyone's fractions

Rubric(s)

Rubric for small group work (evaluate each group)

	1	2	3
Definitions and explanations of terms	The group shows little knowledge of fraction terms and explanations	The group shows some understanding of fraction terms and explanations	The group shows thorough understanding of fraction terms and explanations
Ability to make fraction from items in the room	The group was unable to take classroom items to form a set and show a fraction	The group understood how to take classroom items to form a set and show a fraction, however they were unable to correctly a show/identify the fraction	The group was able to take classroom items and show/identify a fraction in the set
Place Fraction on the Smartboard	The group was unable to correctly place their fraction on the number line and was unable to correctly draw the fraction.	The group was able to correctly draw the fraction; however they were unable to appropriately place it on the number line.	The group was able to correctly draw the fraction and appropriately place it on the number line.

Rubric for Fraction Museum (individual)

	1	2	3
Showing fraction as part of a set	The student was unable to take classroom items to form a set and show a fraction	The student understood how to take classroom items to form a set and show a fraction, however they were unable to correctly a show/identify the fraction	The student was able to take classroom items and correctly show/identify a fraction in the set
Ability to make fraction from rectangle	The student was unable to correctly show their fraction using a rectangle	The student was able to correctly show their fraction using a rectangle	The student was able to correctly show their fraction using their rectangle; in addition they were able to show an equivalent fraction using the rectangle
Place fraction on number line	The student was unable to correctly place their fraction on the number line	The student was not able to correctly place their fraction on the number line; however, they were able to do so with help from the teacher	The student was able to correctly place their fraction on a number line

Other Evidence

- Singapore Math assessments
- Cumulative assessment
- Math boxes
- Mental journal
- Observations
- Practice through games
- Verbal and written expression through journal and discussions

Student Self-Assessment and Reflection

- Exit Slips to reflect on personal learning
- Self-Correcting
- Student discovery of ways to use fractions in everyday scenarios

Stage 3 – Learning Plan
(Design learning activities to align with Stage 1 and Stage 2 expectations)

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Key learning events needed to achieve unit goals

Introduce Fractions:

Have students complete p. 85-87, questions 1-4 which reviews the uses of fractions to express parts of a whole. As a whole group discuss the students' answers of these questions.

Ask students to 'think-pair-share' their own definition of the word equal. Use food to explain equal to students. Tell them you are going to give 'student A' a Snickers bar. Next tell them you are going to give 'student B' a piece of a Snickers bar. Ask them why 'student B' will not be happy- Because s/he did not get the same size of Snickers- they were not equal pieces.

Ask the students to again do a 'think-pair-share', but this time about the word whole. After discussing the word as a whole, use an orange to demonstrate whole. Have the orange peeled, but not sectioned. Show the class the orange, and section it in front of them. Explain that while the orange is now in pieces, it is still the whole orange, and demonstrate that by putting it back together.

Continue using the orange and introduce that fractions contain two parts- the numerator and the denominator. Use the orange to show the denominator-with the orange back together- and then split it up to demonstrate numerator. "I am holding 2 out of 8 pieces of this orange." Do 4 or 5 examples of this using the orange.

Using the Smartboard model fractions using a rectangle, or slate. Show fractions out of 2, 3, and 4.

Divide the slate into 2 equal parts, but shade both of them. Ask students to brainstorm which fraction this would be. Go on to demonstrate that a fraction that has the same numerator and denominator is a 'whole.' If students are struggling with this concept use the orange again.

Have students complete the remaining problems on p.90. Additional practice can be found on p. 94-95 in the students workbook.

Review Fraction Names for Parts of a Set:

As a whole group ask 5 children to lend their journals to make a set of books. Show the pile of 5 books. The 5 books are a whole set or ONE. Hold up several books and children to name fractional part of the set, always remembering to include the denominator. "I have 3 out of 5 books, the fraction is 3/5" Continue this exercise until students show understanding.

As small groups invite students to come up with their own fractions from a set, using things from the classroom. Give a set amount of time and rules as to what items in the classroom can be used (pencils, markers, books, ect.) Have the groups share their fractions from a set with the rest of the class.

Have students complete p. 102-104 in their textbook. Additional practice can be found on p. 115 in the student workbook.

<u>Investigating Equivalent Fractions:</u>

Review the idea of fractions. On the board draw 2 rectangles. Split one rectangle into 2 and shade one side. Write ½ next to it. Next, split the second rectangle into 4 sections and shade 2 of them. Write 2/4 next to this rectangle. Ask students to look at these fractions and think of similarities and differences.

Introduce the idea of equivalent fractions. Demonstrate by using different examples of ½.

Now, give each student a piece of paper. Have them fold it into two equal pieces. Have them shade one of the two parts. Then fold again, demonstrating that ½ is equivalent to 2/4, and continue to 4/8. Have students brainstorm other equivalent fractions using white boards.

Students should complete p. 92-94 in their student textbook. Additional practice can be found in the workbook on p. 100-107.

Review with students that these fractions are called equivalent fractions. Equivalent fractions name the same fractional part of ONE.

<u>Fractions Greater than one:</u>

Ask the students who likes pizza! Call on one student and ask them how many pizzas they usually order at their house. Tell students at your house you usually order 3 whole pizzas. Draw 3 circles on the board and then 'slice' them into 8 slices each.

Demonstrate the fractions for the 3 pizzas. There are 3 whole pizzas, or each one is 8/8. All together they are 24/8.

Ask a few students if they would like a 'slice of pizza' and tell them you've eaten ½ of one of the pizza

Ask students to write down how many whole pizzas there are, and then come up with a fraction (4/8) for the leftover pizza. Explain that this is also ½. Introduce mixed numbers 2 ½ is a mixed number because it has a whole number and a fraction!

Use the pizza to demonstrate a few more mixed numbers, until students really show understanding.

Teacher writes the following on the board. Students should write a response in their math journals

Emily has 3 apples. She cut one in half and ate one of the halves. How many apples where left? (2 ½ .) Then she cut each of the other whole apples in half. She gave all of the half apples to her friends. How many half apples did she give away? (five half apples) Write 2 ½ and 5/2 on the board. Do these numbers 2 ½ and 5/2 name equivalent amount of apples?

**Differentiation- When reviewing and practicing equivalent fractions, you may want to have students who are struggling with the concept only do ½ or 1 whole; students who are succeeding may move onto harder fractions. Also, the 'math message' may be used as a challenge question

<u>Placing Fractions on a number line:</u>

Use the Smartboard to display a number line between 0 and 1. Give each small group of students an index card with a fraction on it (1/2, 1/3, ¼, 1/5, 2/3, 2/4, etc.) As a group the students should decide where on the number line the index card will go. Teach the students to draw a rectangle and color their fraction so they are able to compare it to fractions with other denominators.

Have the ½ group go first, and help students arrange their index cards on the number line. Have students share their strategies out loud. Record strategies for reference later.

If necessary, this activity can be done in small groups for further reinforcement.

<div align="center">**Resources and Teaching Tips**</div>

Resources

Quizzes, tests, worksheets, Smartboard slides, Internet resources

Textbook materials (i.e. Math Student Journal, Math student skill links, Math students Homelinks)

Teaching Tips

Students may benefit from seeing a teacher model most activities prior to engaging in them independently.

Teacher should watch for students who have difficulty remembering which part of a fraction is the numerator and which part is the denominator

Students have a folder of challenging and engaging activities to use if they complete their work early.

Differentiation

Every lesson summary includes a list of Key Concepts and Skills addressed in the lesson. This list highlights the range of mathematics in each lesson so that teachers can better use the materials to meet students' needs. The Key Concepts and Skills are linked to the Grade-Level Goals and Program Goals and thus clarify how lesson activities connect to and support *Everyday Mathematics* long-range goals.

Each lesson provides point-of-use ways to modify activities. These suggestions are called "Adjusting the Activity." If children are having difficulty with a certain activity or need to be challenged a bit more, teachers might find one of these modification suggestions helpful.

At the beginning of each unit a "Multiage Classroom" chart can be found. Companion lessons from previous and future grade levels are given. If students have not had previous experience with *Everyday Mathematics*, teachers might find this information especially useful.

Lessons have to be reorganized to include optional differentiation activities. Each lesson contains one or more of the following types of activities:

- o Readiness
- o Enrichment
- o Extra Practice, and
- o English Language Learner Support.

Students are grouped according to ability levels for centers. Assignments vary to challenge each ability level. Individualized attention is provided to scaffold each group. Groups are frequently assessed to allow fluid movement between groups as students require more and less support.

Students learn in a variety of ways. In this unit students are able to express their knowledge through oral communication (whole group and partner work). Students are given the opportunity to work with partners to gain a better understanding of the material. Student created notebooks are a key resource for students to use when practicing math skills as they create them. Students create visual aids showing their knowledge of place value. Working in small groups, as well as teacher led groups, allows students to work at a pace and style that is conducive to their learning.

Lessons involve movement and transition to ensure that kinesthetic learners are engaged and involved.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.

Universal Design for Learning is embedded within this unit as students use different means to learn about and present their knowledge of topics. Students are encouraged to work in different environments (i.e. individually, small group, whole class). They are also encouraged to demonstrate their knowledge through written and oral formats.

Technology Integration
The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

8th Grade Technology Literacy - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century(SETDA, 2003).

Students use technology through Smartboard slides to further understand the concepts of fractions, fractions notation and equivalent fractions. They also use computers during centers.

Journal pages, Math Masters, Minute Math, etc. can be done on a (whole) class basis at times. To be able to project this on a SMARTboard and use the journal as a learning tool can be quite beneficial

Curriculum Framework for Mathematics

School: Reach Academy for Girls   **Curricular Tool: Singapore Math**   **Grade: 4**   **Teacher: Gimbel/Laughman/Wood**

Unit One: Multi-step Equations/Factors /Multiples
Timeline: Approximately 40 school days

Standards Alignment	Unit Concept Enduring Understandings	Essential Questions Student Learning Targets	Assessments
4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	The value of a number is determined by the place of its digits A number can be written using its name, standard, or expanded form	**Essential Questions:** How does understanding place value help me solve problems? Why is number sense the foundation for all mathematics?	**Formative Assessments:** • Math journals • Class work • Class discussions • Homework • Group work checklist • Pattern Assignment
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	Place value can be extended beyond hundreds Whole numbers can be rounded	What makes a computation strategy effective and efficient? When should an exact number be used? When should an estimate be used?	**Summative Assessments:** • Math Rubric • Number Multi-step equation Assessment • Multiples Rubric • Singapore Assessments • Performance Task
4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Numbers can be rounded and ordered Multi-digit whole numbers signify base ten numerals Multi-digit whole numbers can be compared Numbers can be represented in multiple ways	In a multi-digit whole number, what place value does each digit represent? How can I compare multi-digit numbers? How can I round multi-digit whole numbers?	
4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	Addition and multiplication share mathematical properties Multiples can be used to solve Problems Multiplication can be used to assist in solving everyday problems	How can numbers be represented? How do we solve multi-step word problems? How can multiples be used to solve problems?	
4.NBT.1 Recognize that in a multi-digit whole		How do I use prime factors to	

E-63

Standards		
number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)	Division shares mathematical properties with addition, subtraction and multiplication	solve problems?

What strategies assist me in solving multiplication problems? |
| 4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place. | Divisors and dividends share Relationships | How does my knowledge about multiplication help me solve other problems? |
| 4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. A range of algorithms may be used.) | Division can be used to assist in solving everyday problems | How do I know if a number is divisible by 2, 3, 5 and 10?

How can I represent divisibility?

How can I determine the GCF?

How do I identify the LCM? |
| 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. | | **Learning Targets:**
I can solve multiplication problems and explain whether my answers make sense or not.

I can solve multi-step word problems. |
| 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. | | I can find factors and multiples within the 10x10 multiplication table. I can tell if a number is prime or composite.

I can create a pattern of numbers or shapes if I'm given the rule.

I understand the concept of place value.

I know strategies that help me determine divisibility.

I can determine GCF and LCM.

I can round numbers to any given |

Standards	Concepts/Understandings	Essential Questions	Assessments
	I can add and subtract any multi-digit numbers fluently. I can multiply a four digit number by a one digit number and I can multiply two two-digit numbers. I can explain why. I can divide a four digit number by a one digit number and explain why. *(place.)*		**Formative Assessments:** Unit Pre-test, Math journals, Class work, Class discussions, Homework, Group work checklist, Fraction Assignments
Unit Two: Equivalent Fractions/Fraction of a Set **Timeline: Approximately 50 days** 4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.) 4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. 4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)	Relationships exist among fractions, decimals and percentages Fractions demonstrate numbers, decimals, measurements and percentages Fractions can be expressed with improper and proper representation Percentages represent values and can be compared with whole numbers, data sets and information	**Essential Questions:** How can we be sure that we have equal parts? Why is it important to divide into equal parts? How is half represented? How are shapes divided into halves and fourths? What is an improper fraction and how can it be represented? What is a mixed number and how can it be represented? How can I demonstrate the connection among fractions, decimals and percentages? What is the relationship between multiplication and division of fractions? How can models be used to	**Summative Assessments:** Unit Post Test Comparing Fractions Rubric Problem Based Fraction Assessment Halves/fourths Rubric Singapore Assessments

Standards	Essential Questions / Learning Targets
4.NF.3 a. Understand addition and subtraction of fractions as joining and separating parts of fractions referring to the same whole.	compute fractions with like and unlike denominators?

How many ways can we use models to compare and determine equivalent fractions?

When is it appropriate to use fractions/decimal fractions?

Learning Targets:
I can understand fractions, model fractions, and understand how two different fractions are equivalent. |
| 4.NF.3 b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

Examples: $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2\,1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$. | I can represent fractions.

I can explain how to represent improper fractions and mixed numbers. |
4.NF.3c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.	I can compare and order fractions.
4.NF.3d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	I can determine which operation (addition or subtraction) to use when solving word problems involving fractions with like denominators.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)	I can solve word problems that involve multiplying a fraction by a whole number.
4.NF.4a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.	I can determine which operation (addition or subtraction) to use when solving word problems involving fractions with unlike denominators (tenths and hundredths).
4.NF.4b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to	I can convert fractions into

	decimals and decimals into fractions (tenths and hundredths). I can work flexibly with fractions, decimals and percents. I can compare two decimals up to the hundredths place.
multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.) 4.NF.4 c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? 4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100 and add $3/10 + 4/100 = 34/100$. (Addition and subtraction with unlike denominators in general is not a requirement at this grade.) 4.NF.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. 4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	

Unit 3: Probability/Geometry
Timeline: 40 Days

Standards	Concepts	Essential Questions:	Learning Targets:	Assessments
4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement	Solve problems with ratios and Proportions	How is the probability of an event determined and described?		**Formative Assessments:** Unit Pre-test Math journals Class work Class discussions Homework Group work checklist Measurement Assignment Math Learning Stations
4.MD.5.a An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.	Shade, construct, deconstruct Ratios Identify proportions Solve percent problems Utilize "shortcut" strategies	How can estimation be used to identify proportions? How can I choose the appropriate mathematical operation to solve problems?		
4.MD.5.b An angle that turns through n one-degree angles is said to have an angle measure of n degrees.	2 and 3 dimensional shapes can be represented by models and constructs	How can I represent a shape as a 2 or 3 dimensional figure?		**Summative Assessments:** Unit Post-Test Rubric Analyzing and Comparing Checklist Singapore Assessments
4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	Geometric shapes can be described, classified and compared according to their characteristics	How can I record characteristics of geometric shapes? What is the relationship among geometric shapes or patterns? How can I identify symmetry?		
4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	Geometric patterns can be recognized, described, extended and created based on attributes and numbers Symmetry represents balanced and repeated patterns found in geometric shapes and in nature	How can we quantify an event or data without whole numbers? **Learning Targets:** Given an event, I can determine the probability of it happening. I can identify proportional relationships and use models to represent them.		
4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.			I can draw and identify points, lines, line segments, rays, angles (right, acute, obtuse), perpendicular and parallel lines and two-dimensional figures.	
4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right			I can classify two dimensional shapes based on their line	

triangles as a category, and identify right triangles.

segments and angles.

I can recognize lines of symmetry in 2 dimensional figures.

I can classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines.

I can classify two-dimensional figures or the presence or absence of angles of a specified size.

I can recognize right triangles as a category, and identify right triangles.

I can draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines.

I can identify point, lines and line segments in two-dimensional figures.

Formative Assessments:
Teacher/Student conferencing
Class work
Class discussions
Homework
Math journals
Government Pre-Assessment
Plants Pre Assessment
Plant graphic organizer

Summative Assessments:

Unit Four: Measurement and Data
Timeline: Approximately 35 Days

4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.	Mathematicians know relative sizes of measurement units within one system of units The four operations are used to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money. Representation of measurement
4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money,	**Essential Questions:** Why is measurement important in our everyday life? How can objects be compared, ordered and sorted by weight? What is a unit of measure? How does a measurement instrument work?

Standards			Post Assessment
including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	quantities using diagrams such as number line diagrams that feature a measurement scale.	What happens to the measurement when we change units?	Number Position and Relationship Rubric Multiples Assessment Ordinal Number Rubric Hundreds Chart Rubric
	The possible results of an experiment can be described and graphed	How can I determine which graph or table is appropriate to use in representing data?	
4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.	A difference can be determined between predicted and actual outcomes	How can I represent and organize data?	
4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.	Data can be represented in various forms Data can be interpreted	How can I interpret data? How can I solve problems involving measurements?	
4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.		**Learning Targets:** I can identify units of measurement, convert larger units of measurement into smaller units of measurement I can create a conversion table showing equivalent units of measurement. I can compare objects based on measurement attributes. I can use multiple units of measurement. I can recognize an angle, define a circular arc and recognize the relationship between an angle and a circle. I can measure an angle and draw an angle of a certain degree. I can show how angles can be	

broken down into smaller angles, how smaller angles can be put together to form one large angle.

I can create and use graphs and tables to organize data so that I can use it to solve problems.

I can solve addition and subtraction problems to find unknown angles.

I can determine which operation to use when solving word problems involving measurement.

I can use the formulas for area and perimeter to solve problems.

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title: **Place Value and Notation**

Designed by: **Rebecca Warren and Davina Roach**

District: **Developed by Thomas Edison Charter School, used with permission by the Reach Academy for Girls**

Content Area: **Math**

Grade Level(s):   **4ᵗʰ grade**

_____

Summary of Unit

In this unit, students will learn about whole numbers and their place value. Students will understand the place and value of numbers up to one million. They will be able to round any number within a given whole number up to one million. Students will be able to express numbers in base-ten numerals, number names, and expanded form. During this unit, students will use collaboration and individual exploration to guide their discovery of each standard. This unit will be the start of the math curriculum. This unit is imperative to starting off beginning routines and basic understanding of numbers and their values.

Stage 1 – Desired Results
What students will know, do, and understand

_____

Delaware Content Standards

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* **CC.4.NBT.1**

2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **CC.4.NBT.2**

3. Use place value understanding to round multi-digit whole numbers to any place. **CC.4.NBT.3**

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm. **CC.4.NBT.4**

Big Idea(s)

Generalize place value understanding for multi-digit whole numbers.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Unit Enduring Understanding(s)

The value of a number is determined by the place of its digits

A number can be written using its name, standard, or expanded form

Place value can be extended beyond hundreds

Whole numbers can be rounded

Numbers can be rounded and ordered

Multi-digit whole numbers signify base ten numerals

Multi-digit whole numbers can be compared

Numbers can be represented in multiple ways

Unit Essential Questions(s)

How does understanding place value help me solve problems?

In a multi-digit whole number, what place values do each digit represent?

How can I compare multi-digit numbers?

How can I round multi-digit whole numbers?

How can numbers be represented?

Knowledge and Skills
Students will know…

Vocabulary for base-ten numerals.
Vocabulary for number names.
Place and Value for numbers up to one million.
Standard algorithm for adding and subtracting multi-digit whole numbers.
Comparison symbols (<= >)
How to round whole numbers at any place value in a number up to one million.

Students will be able to…

Express the places of numbers up to one million and the value of each place.
Express that each place increases by ten times the prior place moving to the left.
Write numbers in base-ten numerals, number names, and expanded form less than or equal to one million.
Compare numbers less than or equal to one million based on place and value with use of comparison symbols.
Round whole numbers less than or equal to one million.
Fluently add and subtract multi-digit whole numbers using standard   algorithm.

_____

Suggested Performance/Transfer Task(s)
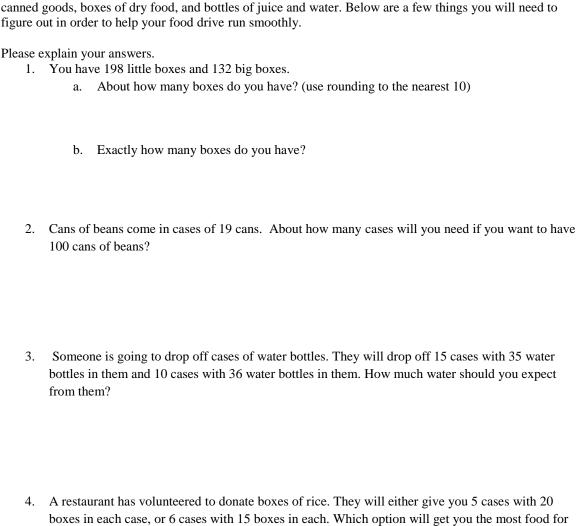
Your class is going to be running a food drive for less fortunate families. You are going to be collecting canned goods, boxes of dry food, and bottles of juice and water. Below are a few things you will need to figure out in order to help your food drive run smoothly.

Please explain your answers.
1. You have 198 little boxes and 132 big boxes.
   a. About how many boxes do you have? (use rounding to the nearest 10)

   b. Exactly how many boxes do you have?

2. Cans of beans come in cases of 19 cans. About how many cases will you need if you want to have 100 cans of beans?

3. Someone is going to drop off cases of water bottles. They will drop off 15 cases with 35 water bottles in them and 10 cases with 36 water bottles in them. How much water should you expect from them?

4. A restaurant has volunteered to donate boxes of rice. They will either give you 5 cases with 20 boxes in each case, or 6 cases with 15 boxes in each. Which option will get you the most food for your food drive?

Rubric for Performance Task (by Caroline Strunk)

	4	3	2	1
Calculations	All calculations are correct	Most calculations are correct with one exception	Attempts calculations but makes more than one mistake	Few calculations are correct
Explanation	Explanations are clear, concise and use mathematical language	Explanations are clear and concise	Attempts to explain but explanations are incomplete	The student's explanations contain conceptual misunderstandings

Rubric for assessing overall unit skills

	4	3	2	1
I can recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.	Understand differences and similarities in place value with no errors	Understand differences and similarities in place value with few errors	Understand differences and similarities in place value with multiple errors	Understand differences and similarities in place value with assistance
I can read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form.	Able to write number in word form, expanded, and represent base ten with no errors	Able to write number in word form, expanded, and represent base ten with few errors	Able to write number in word form, expanded, and represent base ten with multiple errors	Writes number in word, expanded, and represents base ten with assistance
I can fluently add and subtract multi-digit whole numbers.	Add and subtract with accurate calculations	Add and subtract with few errors	Add and subtract with multiple errors	Add and subtract with assistance
I can use place value understanding to round multi-digit whole numbers to any place.	Able to round numbers to any place value with no errors	Able to round numbers to any place value with few errors	Able to round numbers to any place value with multiple errors	Able to round numbers to any place value with assistance
I can compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of the comparisons.	Able to use comparison symbols to represent differences in numbers with no errors.	Able to use comparison symbols to represent differences in numbers with few errors	Able to use comparison symbols to represent differences in numbers with multiple errors	Able to use comparison symbols to represent differences in numbers with assistance

Student Created Math Journals	3	2	1
Attractiveness of Notebook	Notebook pages are neat and in order according to the table of contents.	Notebook pages placed in notebook according to the table of contents; pages are fairly neat and in some order	Notebook pages are missing and not in order. Assistance required.
Content of Notebook	Notebook pages are completed with no errors	Notebook pages are completed with few errors.	Notebook pages are completed with assistance.

Group Project	4	3	2	1
Attractiveness	Game board and pieces are neat with no spelling or grammatical errors.	Game board and pieces are neat with few spelling or grammatical errors.	Game board and pieces are all accounted for with multiple spelling and grammatical mistakes.	Game board and pieces are missing and frequented with mistakes.
Cooperative Work	The group worked well together with all members contributing significant amounts of quality work.	The group generally worked well together with all members contributing some quality work.	The group worked fairly well together with all members contributing some work.	The group often did not work well together and the game appeared to be the work of only 1-2 students in the group.
Accuracy of Content	All information cards made for the game are accurate with self-checking answers	All information cards made for the game are closely accurate with self-checking answers; few errors.	All information cards made for the game are mostly accurate with self-checking answers; multiple errors	Several information cards made for the game are not accurate.
Creativity	The group put a lot of thought into making the game interesting and fun to play.	The group put thought into making the game interesting and fun to play.	The group tried to make the game interesting and fun, but some of the things made it harder to understand/enjoy the game.	Little thought was put into making the game interesting or fun.

Team Work Checklist

My partner and me Evaluation	Check for completion and notes if needed
I shared ideas and listened to others ideas	
I worked on my part of the project to the best of my ability	
My partner shared ideas and listened to my ideas	
My partner worked on their part of the project to the best of their ability	

Peer Evaluation: Game Board	3	2	1
Accuracy of Game board and pieces	The game was easy to use and the question/answer cards were correct	The game was some what easy to use and most of the question/answer cards were correct	The game was difficult to understand and the question/answer cards were incorrect.

Other Evidence
*Class discussions
*Class work
*Homework
*Conference with small groups

Student Self-Assessment and Reflection
 *Pre/Post Quiz given to track progress
* Student Checklist with examples
* Math Journal
*Exit Slips for lesson reflection
*Review games

Stage 3 – Learning Plan
(Design learning activities to align with Stage 1 and Stage 2 expectations)

_____

Key learning events needed to achieve unit goals

Before the unit can begin, students must become familiar with their student reference books and other math manipulatives (student math journals, base ten blocks). As a whole group, teacher guides students in an introductory lesson of what they know about place and value of a number.

7-10 days: Students will use their background knowledge of base ten blocks. Students work as a whole group, identify base ten block values (creating an anchor chart that shows the blocks and their values). As students begin to discover the value of the blocks they will work with partners to create numbers using the base ten blocks. Using a given number, students will create the numbers using base ten blocks. Students develop a sense of place and value as they build number knowledge. Students begin answer questions about place and value (How many _____(hundreds) are in 300 ones?) Understanding that each place value to the right is increased by ten times. Students also explore the relationship between similarities and differences of place and value of numbers (ex: How is the number 4 in 456 similar and different to the number 4 in 546?) As a whole group, students work together to create a place value chart. Also, recognizing that numbers are written in different forms (expanded form, written form and digits). Students are engaged in practicing math skills individually and with a partner. Students build their math notebooks from the daily skills introduced. As students have begun to build their knowledge and skills of numbers place and value they will work with a partner to create a number gallery poster. Posters show their knowledge of a created number and the way numbers can be displayed (number, base ten blocks, word form, expanded form, and place value on a chart).

With a better understanding of numbers and their place and value, students' progress to comparing numbers using place and value represented by comparison symbols   (>=<) . Working as a whole group to determine ways or rules to follow when comparing numbers. Students work with partners to continue to practice comparing numbers.

Students have had a great deal of practice with comparing numbers and looking at place and value of a number to determine differences and similarities. The next skill student's focus on is rounding numbers to a given place value up to the millions. Students are familiar with the necessary knowledge of determining place and value. Working as a whole group, students use their background knowledge of rounding numbers to create an anchor chart of rules to follow when rounding.

Each day student's reflect on the previous day with a warm up and at the end of each lesson with an exit slip.

To review the different skills learned, students will work in small groups and participate in self-correcting review games. Students will also work on their checklists to determine their own progress.

4-5 days: Building basic addition facts will help students to build a better understanding of adding and subtracting multi-digit whole numbers. To start this lesson, students work in their student notebooks to create ways to make different numbers (ex: ways to make 15, 14, 10, 9, 6, etc.). Students also create a list of

doubles to reference. As a whole group, create an anchor chart that helps students remember the process for adding multi-digit whole numbers (with and without regrouping). Practice adding multi-digit whole numbers fluently. Students recognize the opposite of addition is subtraction. Creating an anchor chart that models the process of subtracting with and without borrowing. Students model their understanding through individual and partner practice.

4 days: Students begin their group project and review of skills to prepare for the summative assessment.

Create a board game that uses place value, rounding, adding/subtracting numbers, and expanded word form.

Our classroom has a problem. We don't have any fun math board games! I am looking for your help to put together a board game that uses our recently acquired math knowledge!

Your job is to create instructions for the board game, design the board, player pieces, and card questions and answers.

Students will give a brief introduction of their game to the whole class. Students will be given a chance to play each groups game and perform a peer evaluation about their thoughts on the groups game.

Final assessment for learned skills will be determined on the summative assessment.

Resources and Teaching Tips

Teacher Worksheets and review cards:
http://havefunteaching.com/worksheets/math-worksheets/place-value-worksheets/

Games for practice: http://gamequarium.com/placevalue.html

Differentiation
- Students learn in a variety of ways. In this unit students are able to express their knowledge through oral communication (whole group and partner work). Students are given the opportunity to work with partners to gain a better understanding of the material. Student created notebooks are a key resource for students to use when practicing math skills as they create them. Students create visual aids showing their knowledge of place and value. Students also learn ways to use the internet to help them research ways to practice skills and reinforce learning. Working in small groups, as well as teacher led groups, allows students to work at a pace and style that is conducive to their learning.

- Teacher and student created anchor charts help students reference learned material when applying their knowledge.
- Visual displays, hands-on interaction with manipulatives, review games, skill practice, and partner work add a variety of learning methods for all students.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

Universal Design for Learning - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.

(Briefly explain how design principle(s) are embedded within the unit design.)

Universal Design for learning plays an important part in this unit. Students are actively engaged

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- **8th Grade Technology Literacy** - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century(SETDA, 2003).

* Students work individually and with partners to research the latest math resources on the Internet. They also use the Internet as a resource for researching different types of game boards.

Content Connections

Content Standards integrated within instructional strategies

Students engage in social responsibility when they work with partners and small groups to accomplish a similar goal.

Students use reading research to help them plan their own game board. Also, using decision making skills to help them decide on their own design.

Students also use written knowledge to create instructions for their games.

Student Checklist 4 NBT 1-4

_____I can recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

For example:

How is the number 5 in 456 similar and different to the number 5 in 546?
How many tens in 70 ones?_____ How many thousands in 8,000 ones?_____

_____I can read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form.

 For example:

Number	Word Form	Expanded Form
56,789		
1,293,673		
345,953		

_____ I can compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of the comparisons.

For example:

Compare using > = < 567_____765	23,421_____24,421
876,543_____876,345	12,345_____12,346

_____I can use place value understanding to round multi-digit whole numbers to any place.

For example:

Round to the nearest tens place: 345_____ 923_____	Round to the nearest millions: 2,345,766_____ 5,799,345_____

_____I can fluently add and subtract multi-digit whole numbers.

For example:

Solve: 234 + 876_____	43,654 - 40,768_____

8,345 +765_____	983 – 345_____

Name that Place and Value Game
Instructions: 2 players. Taking turns, player 1 reads the question (may show the front of the card) while player 2 answers the question. If the question is answered correctly, the player may keep the card.

What is the place and value of the underlined number? 4,5<u>6</u>7,203	Place: Ten Thousands Value: 60,000
What is the place and value of the underlined number? 210,<u>8</u>97	Place: Hundreds Value: 800
What is the place and value of the underlined number? 98,6<u>7</u>9	Place: Tens Value: 70
What is the place and value of the underlined number? <u>2</u>,345,005	Place: Millions Value: 2,000,000
What is the place and value of the	Place: Hundred Thousands

underlined number? <u>7</u>88,009	Value: 700,000
What is the place and value of the underlined number? 4<u>9</u>,234	Place: Thousands Value: 9,000

Rounding
Instructions: 2 Players. Taking turns, player 1 reads the question (may show the front of the card) while player 2 answers the question. If the question is answered correctly, the player may keep the card.

Round the underlined number to the nearest <u>tens</u> 45,678	45, 6<u>8</u>0
Round the underlined number to the nearest <u>hundreds</u> 12,537	12,<u>5</u>00
Round the underlined number to the nearest <u>thousands</u> 134,877	13<u>5</u>,000

Round the underlined number to the nearest <u>millions</u> 3,456,<u>3</u>45	<u>3</u>,000,000
Round the underlined number to the nearest <u>hundred thousands</u> 987,533	<u>1</u>,000,000

Comparison Symbols
Instructions: 2 players. Taking turns, player 1 reads the question (may show the front of the card) while player 2 answers the question. If the question is answered correctly, the player may keep the card.

Compare the two numbers using >, =, < 567,543 _____ 568,543	56<u>7</u>,543 is less than (<) 56<u>8</u>,543
Compare the two numbers using >, =, < 21,223 _____ 20,344	2<u>1</u>,223 is greater than (>) 2<u>0</u>,344
Compare the two numbers using >, =, < 1,234,224 _____ 1,234,345	1,234,<u>2</u>24 is less than (<) 1,234,<u>3</u>45

Compare the two numbers using >, =, < 798 _____ 800	<u>7</u>98 is less than (<) <u>8</u>00
Compare the two numbers using >, =, < 45,567 _____ 45,567	45,567 is equal to (=) 45,567

4 NBT Pre/Post Quiz

1. How is the number 4 in 456 similar and different to the number 4 in 546?	2. Solve: _____hundreds = 600 ones _____tens = 50 ones _____thousands= 4,000 ones
3. Write the following in words: 543 5,876	4. Write the following in expanded form: 234 _____ 8,765 _____

45,889 607,803	134,567 _____

5. Round to the nearest tens place:	6. Round to the nearest thousands place:
567_____	323,445_____
67,996_____	986,443_____
453_____	214,005_____
7. Solve: 234 + 765_____	8. Solve: 723-544_____

9. The school is collecting books the library. We collected 34,689 books last week and 23,456 books this week. How many books have we collected total?

Answer:_____
_____

10. Emily is closing her shoe store and has to sell all of her 3,421 pairs of shoes. She sold 1,256 pairs as of today. How many shoes does she have left to sell?

Answer:_____
_____

11. Compare the numbers using < = >

234 _____ 324

14,678_____ 41,678

3,459_____ 4,359

967_____ 976

12. How much is your number worth?

2,378,439

The value of
2:_____

The place of
2:_____

The value of
7:_____

The place of
7:_____

The value of 4:_____

The place of 4:_____

Summative Assessment
4 NBT 1-4

1. write the **greatest** number using these five digits:

| 5 7 8 3 0 |

_____ _____ _____ _____ _____

2. Write the **least** number using these four digits:

| 4 9 2 4 |

_____ _____ _____ _____ _____

Explain why your answer is correct.

_____
_____
_____

_____
_____
_____

_____
_____
_____

3. Name the place and value of the underlined digit:

2<u>3</u>,451,980

Place:_____ Value:_____

5,4̲81

Place:_____ Value:_____

4. Write the number in expanded form:

68,876

_____

___

54,231

_____

___

5. Compare the numbers using > = <

456_____546   34,567_____35,435   987_____978

12,352_____123,520   2,312,453_____2,412,453

6. How is the number 5 in 7,056 similar and different to the number 5 in 7,564?

7. Round

56,684 to the nearest **tens** place_____

32,445 to the nearest **thousands** place_____

4,321,876 to the nearest **millions** place_____

8. Solve:

794 - 175_____	734 - 141_____
5,986 + 2,332_____	24,554 + 13,203_____

9. Jenny wrote a letter to her best friend that recently moved away. She counted the words in her letter and discovered she wrote 567 words. Jenny likes round numbers so she decided to round her number to the nearest hundreds. What will be her new number? And how many more words will she need to reach her new number? Show work.

Answer:_____

10. **1,220 students** in the school have **green** backpacks and **992 students** in the school have **blue** backpacks. Also, **574 students** have **red** backpacks. How many backpacks are there in all? Show work.

Answer:_____

Curriculum Framework for Mathematics

School: Reach Academy for Girls   Curricular Tool: Singapore Math   **Grade: 5**   **Teacher: Gimbel/Laughman/Wood**

Unit One: Operations and Algebraic Thinking
Timeline: Approximately 40 school days

Standards Alignment	Unit Concept Enduring Understandings	Essential Questions Student Learning Targets	Assessments
5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Place value: read, write, compare, and order whole numbers to millions and decimal numbers through the millionths place.	**Essential Questions:** What is algebra? Why is it important to follow an order of operations?	<u>Formative Assessments:</u> • Math journals • Group work checklist • Class work • Class discussions • Homework • Pattern Assignment
5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	Prime numbers up to 50 and composite numbers	How can I write an expression that demonstrates a situation or context?	<u>Summative Assessments:</u> • Math Rubric • Number Operations Assessment • Number Bond Rubric • Singapore Assessments
5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.	Prime factorization using both exponents and standard notations 2 digit by 2 digit multiplication Problems involving the division of two- and three- digit whole numbers by one- and two- digit whole numbers	How can I evaluate numerical expressions written using parenthesis, brackets, and braces? How can I write numerical expressions written using parenthesis, brackets, and braces? How can I interpret numerical expressions without evaluating them?	
5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.		**Learning Targets** I can make sense of problems and persevere in solving them.	
5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.		I can use appropriate tools strategically.	
5.NBT.3 Read, write, and compare decimals to thousandths.		I can look for and express regularity in repeated reasoning.	
5.NBT.3.a Read and write decimals to		I can make sense of problems and persevere in solving them.	

thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

5.NBT.3.b Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.

5.NBT.4 Use place value understanding to round decimals to any place.

5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

I can reason abstractly and quantitatively.

I can look for and make use of structure.

I can look for and express regularity in repeated reasoning.

Standards Alignment	Unit Concept Enduring Understandings	Essential Questions Student Learning Targets	Assessments
Unit Two: Number Operations and Fractions **Timeline: Approximately 40 days**			
5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.	Adding and subtracting fractions, mixed numbers, and decimals Reasonable estimates of fractions and decimal sums and differences	**Essential Questions:** How do you use equivalent fractions as a strategy to add and subtract fractions? How do we apply prior knowledge of multiplication and division to multiply and divide fractions?	**Formative Assessments:** Unit Pre-test Math journals Fraction Strips Number Lines Class work Class discussions Homework Hundred Chart Fraction Models
5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.	Terminating decimals in the form of fractions or mixed numbers Evaluating algebraic and/or multi-step numerical expressions involving decimals and fractions using order of operations	How are fractions related to division? How can you multiply fractions and whole numbers? How can you use compatible numbers to estimate with fractions and mixed numbers?	**Summative Assessment:** Unit Post Test Fraction Relationship Rubric Comparing Fraction Assessment Fraction Models Rubric Singapore Assessments
5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.		**Learning Targets** I can add fractions with unlike denominators using the strategy of equivalent fractions.	
5.NF.4a Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b.			
5.NF.4b Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.		I can subtract fractions with unlike denominators using the strategy of equivalent fractions. I can solve word problems involving the addition of fractions with unlike denominators.	
5.NF.5 Interpret multiplication as scaling (resizing) **5.NF.5a** Comparing the size of a product to the size of one factor on the basis of the size of		I can solve word problems involving the subtraction of fractions with unlike denominators.	

Standards	Concepts	Essential Questions	Assessments
the other factor, without performing the indicated multiplication. **5.NF.5.b** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1. **5.NF.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **5.MD.2** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.		I can use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. How does the coordinate system work? How do coordinate grids help you organize information? How can a line graph help us determine relationships between two numerical patterns? How can the coordinate system help you better understand other map systems?	

Unit 3: Measurement/Volume
Timeline: 40 Days

Standards	Concepts	Essential Questions	Assessments
5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. **5.MD.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **5.MD.3.b** A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units. **5.MD.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and	Linear and area measurements can be determined Real-world mathematical problems can be solved with measurement 2D shapes are measured with 2D units, 3D shapes are measured with 3D units Area and perimeter are both attribute of plane figures Equations can help me solve measurement problems	**Essential Questions:** How can we quantify an event or data without whole numbers? How do area and perimeter relate to volume of a given space/shape? What is volume? What strategies can I use to determine the volume of a given space or container? How is volume connected to multiplication and addition?	**Formative Assessments:** Unit Pre-test Math journals Class work Class discussions Homework Group work checklist Measurement Assignment Math Learning Stations **Summative Assessments:** Unit Post-Test Plot Rubric Fraction Measurement Assessment

Standards / Content	Enduring Understandings	Essential Questions / Learning Targets	Assessments
improvised units.			Performance Assessment Rubric Singapore Assessments
5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.	Volume is an attribute of solid Figures Volume can be measured	How can I use volume to solve real-world problems?	
5.MD.5.a Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.	The possible results of an experiment can be described & graphed Data can be represented and Interpreted	**Learning Targets:** I can build, draw, and work with prisms by a variety of views, and nets. I can describe and identify the five regular (Platonic) solids and their properties with respect to faces, shapes of faces, edges, and vertices.	
5.MD.5.b Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.	Data can be documented in appropriate units A difference can be determined between predicted and actual outcomes	I can solve problems that require calculating the area of triangles and parallelograms. I can decompose irregular shapes to find perimeter and area.	
5.MD.5.c Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.		I can identify a three-dimensional object from two-dimensional representations of that object and vice versa. I can solve problems involving surface area and volume of rectangular prisms. I can record measurements to a reasonable degree of accuracy using decimals and/or fractions.	

Unit Four: Geometry
Timeline: Approximately 35 Days

Standards / Content	Enduring Understandings	Essential Questions	Assessments
5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number	Geometric shapes share Attributes Geometric shapes can be described, classified & compared according to their characteristics	**Essential Questions:** How can I represent a shape as a 2 or 3 dimensional figure? How can I record characteristics of geometric shapes?	**Formative Assessments:** Teacher/Student conferencing Government Pre-Assessment Plants Pre Assessment Plant graphic organizer Class work Class discussions

indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). **5.G.2** Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **5.G.3** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. **5.G.4** Classify two-dimensional figures in a hierarchy based on properties.	Geometric patterns can be recognized, described, extended & created based on attributes & Numbers Geometric shapes can be Partitioned 2 and 3 dimensional shapes	What is the relationship among geometric shapes or patterns? **Learning Targets:** I can decompose irregular shapes to find perimeter and area. I can identify a three-dimensional object from two-dimensional representations of that object and vice versa. I can solve real world problems involving whole numbers by graphing points. I can sort two-dimensional figures based on properties of their sides and angles. I can use the coordinate system to help me better understand other map systems I can use a line graph help me determine relationships between two numerical patterns?	Homework Class journals **Summative Assessment:** Post Assessment Singapore Assessments

Unit Title: **Volume**
District: **The Reach Academy for Girls**
Content Area: **Mathematics**
Grade Level(s):   **Fifth Grade**
_____

Summary of Unit

This unit is a portion of the measurement unit for fifth graders. This is the first exposure students have to the concept of volume (other than liquid volume). The third grade standards have students working with concepts of area and perimeter. Now, those concepts are used as a foundation for volume. First, students build an understanding of a unit cube as a 1 x 1 x 1 cube with the unit determined by the measurement unit used. Students begin to understand that, when we place layers of unit cubes into 3-dimensional shapes, we can determine the volume of the shape. Finally, students connect the layers of cubes to the volume formula: *l* x *w* x *h,* and use this knowledge to solve real-world problems.

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Stage 1 – Desired Results
What students will know, do, and understand
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Delaware Content Standards

5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

5.MD.3.b A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

5.MD.5.a Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

5.MD.5.b Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

5.MD.5.c Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Big Idea(s)

- Volume is measured in 1 x 1 x 1 cubes, and those cubes can be of any unit length
- Using formulas allows us to solve problems efficiently
- The selected measurement unit must be appropriate for the task (e.g., volume cannot be measured in flat squares)

Unit Enduring Understanding(s)

- Real world mathematical problems can be solved with measurement
- Equations can help me solve measurement problems
- Volume is an attribute of solid Figures
- 2D shapes are measured with 2D units, 3D shapes are measured with 3D units
- Volume can be measured and counted

Unit Essential Questions(s)

- What is volume?
- What strategies can I use to determine the volume of a given space or container?
- How is volume connected to multiplication and addition?
- How can I use volume to solve real-world problems?

Knowledge and Skills

Students will know…

- A unit cube is a 1 x 1 x 1 cube, with all side lengths using the same measurement unit
- Stacking unit cubes in layers inside a solid will determine the volume
- l x w x h is a formula for calculating volume

Students will be able to…

- Define what a unit cube is, and give examples
- Differentiate between the use of squares (area) and cubes (volume)
- Count volume using manipulatives and/or visuals
- Calculate volume using the formula l x w x h
- Solve real-world problems involving volume situations

Suggested Performance/Transfer Task(s)

Building a Box[1]

You are designing a cabinet for the classroom. The cabinet must have 36 cubic feet of storage. What dimensions could the cabinet be?

1. Draw and label two possible designs.

2. How do you know each of these designs will work?

3. Explain which of your designs would be better for the classroom.

Formative Assessment Questions

• How could you determine which 3 numbers could be multiplied together to get 30?

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[1] Adapted from Georgia Department of Education:
https://www.georgiastandards.org/Common-Core/Common%20Core%20Frameworks/CCGPS_Math_5_Unit7FrameworkSE.pdf

• Is your answer reasonable? How do you know?
• What expression might you use to find volume?

Differentiation Opportunities

Extension:

- Have students find all possible combinations that could create a 48 cubic foot cabinet
- Have students design another box with a capacity of 36 cubic feet.

Intervention:

- Allow students to work with partners or in groups.
- Allow students to use calculators.
- Allow students to use cubes to build a model.

Rubric:

4	Student: • Provides two sketches and sets of dimensions that multiply to achieve a volume of 36 cubic feet • Provides a solid explanation for each set of dimensions, which may include a visual or formulaic explanation • Selects one design as "better" and is able to create a viable argument for that selection, related to the overall shape of the box based on the dimensions.
3	See 4 above, but student makes one of the following mistakes: • Provides only one set of dimensions that multiply to achieve a volume of 36 cubic feet or • Provides a solid explanation for only one set of dimensions, which may include a visual or formulaic explanation or • Selects one design as "better" but does not provide explanation.
2	Student: • Is able to give one set of dimensions • Provides one reasonable explanation for the set of dimensions given. • Gives some argument for question 3 based on the overall shape of the box based on the dimensions
1	Student: • Is able to give up to one set of dimensions that works • Does not provide any reasonable explanation for questions 2 and 3

Other Evidence

Formative Assessments:

Unit Pre-test
Math journals
Class work
Class discussions

Homework
Group work checklist
Measurement Assignment
Math Learning Stations

Summative Assessments:
Unit Post-Test
Singapore Assessment
Performance Task Rubric

Stage 3 – Learning Plan
(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

Lesson 1: Counting Volume
- Students begin the unit by learning about the concept of unit cubes. These are introduced to students, allowing them to interact with various unit cubes: cubic inch, cubic cm, cubic foot, etc. Larger unit cubes can be made from boxes or PVC pipe, if desired.
- Next, students are exposed to various shapes, including rectangular prisms and irregular shapes. All of these have images in which the cubes are outlines, and can at first be directly counted.
- Students are asked to count volumes where cubes are still shown on the object, but with some now hidden within the object. This encourages students to consider volume as "layers" of cubes.

Lesson 2: Calculating Volume
- Measurements are provided along with the cube drawings on the objects, and the cubes gradually disappear, leaving only measurements remaining for students.
- Students are then are expected to begin using the formula l x w x h to calculate other volumes.
- *Optional Support:* p. 37 in the student workbook
- Students are given problems in which they are given the volume and need to find a missing side length. This also reinforces the commutative property of multiplication.

Lesson 3: Using Volume to Solve Problems
- Students are exposed to a variety of real-world problems using volume. Initially, some support/hints are given, and these are weaned away.
- Students complete Practice B on p. 57 and items 19-20 on p. 60.
- *Optional supports:* p. 39-40 and 41-42 in the student workbook

Lesson 4: What's in the Box?

- Students should be broken into pairs or small groups. Each group is given a box (if possible, whole-number dimensions should be used). For groups who may be struggling, the box dimensions can be labeled. Other groups may be required to measure their box, as deemed appropriate.
- Groups will need to record the dimensions of the box and the total volume of the box in their notes.
- Groups then switch boxes, and repeat.
- Circulate to ask students probing questions:
  - Why can't we use square units to measure volume?
  - What information do you need to find the volume?
  - How do you know your answer makes sense?
  - What happens if you start over, turning the box on its side?
  - Which shape of box has the most cubes that aren't touching the outside faces?
  - Extension: Do you think you can figure out how much wrapping paper you would need to wrap your box (with no overlaps)? How?

Resources and Teaching Tips

Resources
- Singapore textbooks
- Unit cubes
- Various boxes (look for a variety of rectangular prism shapes - long/skinny, cube, etc.)
- Rulers/yard sticks/tape measures

Teaching Tips
- This unit provides an excellent connection between the Commutative property of multiplication and volume. Students should also be realizing that the order in which the measurements are multiplied does not affect the total volume calculation.

Differentiation
- Students will work with partners and groups to provide additional support while learning
- Struggling students will also be pared with a higher performing partner
- Teacher will spend time with small groups that require additional support
- Visual learners will benefit from the reading and writing from in the activities
- Struggling students can use cubes to build the initial boxes so they can count those cubes
- Extension: students can begin to consider how they might determine the surface area of a rectangular prism (by thinking about wrapping it).

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- Universal Design for Learning: students are given various formats to interact with content. Students spend time with visuals, manipulatives, speaking and writing in each lesson. These are all used as formative assessment techniques.
- 21st Century Learning: Higher-order thinking skills are incorporated into each lesson within the unit, and students are given frequent opportunities to explain their own and consider others' work. Students are expected to apply their knowledge in a novel situation in the Performance Task.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

8th Grade Technology Literacy - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century (SETDA, 2003).

Technology

In this unit, technology is primarily a support for students who need additional exposure to the content.

- Students can review volume concepts with an oniine tutorial: https://www.khanacademy.org/math/geometry/basic-geometry/volume-introduction-rectangular/v/how-we-measure-volume
- Students fill boxes with cubes as they investigate volume: http://illuminations.nctm.org/ActivityDetail.aspx?id=6
- Students experiment with constant volume and varying dimensions: http://illuminations.nctm.org/LessonDetail.aspx?ID=L240

Content Connections

Content Standards integrated within instructional strategies

Reading, vocabulary, and writing are embedded within all of the activities.

Curriculum Framework for Mathematics

School: Reach Academy for Girls   **Curricular Tool: SpringBoard**   **Grade: 6**   **Teacher:**

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Unit One: Number Concepts **Timeline : 6 weeks**			
Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express $36 + 8$ as $4 (9 + 2)$. **CC.6.NS.4**	Numbers can be represented in multiple ways. The same operations can be applied in problem situations that seem quite different from one another. Being able to compute fluently means making smart choices about which tools to use and when to use them. Knowing the reasonableness of an answer comes from using good number sense and estimation strategies.	**Essential Questions:** How can you use a prime factorization to find the greatest common factor of two or more numbers? Why can you use either a fraction or a decimal to name the same rational number? **Learning Targets:** I can define prime and composite numbers	**Formative Assessments:** Informal Observations Graded Homework Assignments Weekly Quizzes (called CFUs) Embedded Assessments Think-Pair-Share **Summative Assessments:** Unit Test Performance Tasks
Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **CC.6.NS.6**		I can use factors and multiples to derive divisibility rules I can draw factor trees in order to find the Prime Factorization of numbers	
Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. **CC.6.NS.6a**		I can use prime factors to find the GCF of two numbers	
Understand ordering and absolute value of rational numbers. **CC.6.NS.7**		I can use prime factors to find the LCM of two numbers	
Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right. **CC.6.NS.7a**		I can compare and order fractions with unlike denominators	
Write, interpret, and explain statements of order		I can compare and order mixed	

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
for rational numbers in real-world contexts. For example, write -3oC > -7oC to express the fact that -3oC is warmer than -7oC. **CC.6.NS.7b** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars. **CC.6.NS.7c** Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars. **CC.6.NS.7d**		numbers and improper fractions I can order integers and define absolute value numbers I can add and subtract negative numbers	
Unit Two: Operations with Numbers **Timeline: 6 weeks**			
Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? **CC.6.NS.1** Fluently divide multi-digit numbers using the standard algorithm. **CC.6.NS.2**	Numbers can be represented in multiple ways. The same operations can be applied in problem situations that seem quite different from one another. Being able to compute fluently means making smart choices about which tools to use and when to use them. Knowing the reasonableness of an answer comes from using good number sense and estimation strategies.	**Essential Questions:** How are operations with fractions similar to and different from operations with whole numbers? What kinds of numbers do you know about and how do mathematical properties apply to operations with different kinds of numbers? **Learning Targets:** I can add and subtract fractions with like and unlike denominators and use physical models to justify your answer I can round whole numbers and	**Formative Assessments:** Informal Observations Graded Homework Assignments Weekly Quizzes (called CFUs) Embedded Assessments Think-Pair-Share **Summative Assessments:** Unit Test Transfer Task

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		decimals to multiple place values	
Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. **CC.6.NS.3**		I can add and subtract mixed numbers with like and unlike denominators and use physical models to justify your answer	
Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **CC.6.NS.5**		I can multiply fractions by other fractions and mixed numbers using physical models, ratio/rate tables, and arrays	
		I can divide fractions by other fractions and mixed numbers using physical models, ratio/rate tables, and arrays	
		I can add, subtract and multiply decimals and apply it to real world situations	
		I can divide decimals by whole numbers and other decimals and apply it to real world situations	
Unit Three: Linear Patterns **Timeline: 5 weeks**			
Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **CC.6.NS.6**

Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs | Change is fundamental to understanding functions.

Numbers or objects that repeat in predictable ways can be described or generalized.

An operation can be "undone" by its inverse. | <u>**Essential Questions:**</u>
Why are tables, graphs, and equations useful for representing relationships?

How can you use equations to solve real-world problems?

<u>**Learning Targets:**</u> | <u>**Formative Assessments:**</u>
Informal Observations
Graded Homework Assignments
Weekly Quizzes (called CFUs)
Embedded Assessments
Think-Pair-Share

<u>**Summative Assessments:**</u>
Unit Test |

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
differ only by signs, the locations of the points are related by reflections across one or both axes. **CC.6.NS.6b**	Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.	I can describe and predict a linear pattern using tables and words	
Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **CC.6.NS.6c**		I can describe and predict a linear pattern using graphs	
		I can describe and predict a linear pattern using the expression y=ax	
Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **CC.6.NS.8**		I can graph linear equations and describe the relationship between the independent and dependent variables	
Write and evaluate numerical expressions involving whole-number exponents. **CC.6.EE.1**		I can write one-step equations and apply concepts to real world situations	
Write, read, and evaluate expressions in which letters stand for numbers. **CC.6.EE.2**		I can solve one-step equations and apply concepts to real world situations	
Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 - y. **CC.6.EE.2a**		I can write two-step equations and apply concepts to real world situations	
Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. **CC.6.EE.2b**		I can solve two-step equations and apply concepts to real world situations	
Evaluate expressions at specific values of their			

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole- number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s3$ and $A = 6 s2$ to find the volume and surface area of a cube with sides of length $s = 1/2$. **CC.6.EE.2c** Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3 (2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6 (4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$. **CC.6.EE.3** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for. **CC.6.EE.4** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **CC.6.EE.5** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending			

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
on the purpose at hand, any number in a specified set. **CC.6.EE.6** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. **CC.6.EE.7** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **CC.6.EE.8** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time. **CC.6.EE.9**			

Unit Four: Ratio and Rates
Timeline: 5 weeks

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C	Knowing the unit rate or value of an item allows for accurate comparison. Ratio compares two quantities	**Essential Questions:** Why are proportional relationships an important part of mathematics? How is percent related to fractions and decimals, and why is it such a	**Formative Assessments:** Informal Observations Graded Homework Assignments Weekly Quizzes (called CFUs) Embedded Assessments Think-Pair-Share

Standards Alignment	Unit Concept / Big Ideas	Essential Questions / Student Learning Targets	Assessments
received nearly three votes." **CC.6.RP.1** Understand the concept of a unit rate a/b associated with a ratio $a{:}b$ with $b \square \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* **CC.6.RP.2** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **CC.6.RP.3** Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **CC.6.RP.3a** Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? **CC.6.RP.3b** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **CC.6.RP.3c**	Proportions are two ratios that are equal to one another Proportions are used to calculate the quantities of other equal ratios	useful tool in everyday life? Why are proportional relationships an important part of mathematics? How is percent related to fractions and decimals, and why is it such a useful tool in everyday life? **Learning Targets:** I can write and compare ratios in different ways I can explore proportional relationships using graphs, and algebra I can use algorithms to solve proportions I can explore proportional relationships using graphs, and algebra I can use proportional reasoning to express rates I can identify a percent is a ratio out of 100 I can convert between percents and fractions I can convert and compare between	**Summative Assessments:** Unit Test Performance task Rubric

[1] Expectations for unit rates in this grade are limited to non-complex fractions.

Standards Alignment	Unit Concept Big Ideas	Essential Questions / Student Learning Targets	Assessments
Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **CC.6.RP.3d**		fractions, decimals, and percents I can use percent to find the part of a whole I can make good eating choices based on interpreting percentages on nutrition labels. I can find the percentage of a number to interpret %DV on nutrition labels.	
Unit Five: Geometry **Timeline: 6 weeks** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **CC.6.G.1** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\,w\,h$ and $V = b\,h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **CC.6.G.2** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context	Two- and three-dimensional objects can be described, classified, and analyzed by their attributes. An object in a plane or in space can be oriented in an infinite number of ways while maintaining its size or shape. An object's location on a plane or in space can be described quantitatively. Linear measure, area, and volume are fundamentally different but may be related to one another in ways that permit calculation of one given the other.	**Essential Questions:** What characteristics do various quadrilaterals share, and why is it possible to determine perimeter and area of quadrilaterals using related formulas? In what ways is symmetry important in real-world situations? **Learning Targets:** I can calculate the area and perimeter of rectangles I can calculate the areas and perimeters of circles and polygons I can calculate the area and perimeter of composite figures I can measure angles and sides to explore the properties of triangles	**Formative Assessments:** Informal Observations Graded Homework Assignments Weekly Quizzes (called CFUs) Embedded Assessments **Summative Assessments:** Unit Test Performance Assessment Rubrics

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
of solving real-world and mathematical problems. **CC.6.G.3** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. **CC.6.G.4**		I can measure angles and sides to explore the properties of quadrilaterals I can explore symmetry and transformations of polygons I can find the circumference of circles and explore finding its area.	
Unit Six: Data Analysis and Probability **Timeline: 5 weeks**			
Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages. **CC.6.SP.1** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **CC.6.SP.2** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **CC.6.SP.3** Display numerical data in plots on a number line, including dot plots, histograms, and box plots. **CC.6.SP.4** Summarize numerical data sets in relation to their context, such as by: **CC.6.SP.5**	The question to be answered determines the data to be collected and how best to collect it. Basic statistical techniques can be used to analyze data in the workplace. The probability of an event can be used to predict the probability of future events.	**Essential Questions:** How does understanding probability help you make decisions? Why is it important for you to understand how data is organized and presented in real-world situations? **Learning Targets:** I can define and predict the theoretical probability of future events I can use trials to define and predict the experimental probability of future events I can compare and contrast theoretical and experimental probabilities I can compare and contrast the likelihood of outcomes of different	**Formative Assessments:** Informal Observations Graded Homework Assignments Weekly Quizzes (called CFUs) Embedded Assessments **Summative Assessments:** Unit Test Performance task Rubrics

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Reporting the number of observations. **CC.6.SP.5a**		spinners	
Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. **CC.6.SP.5b**		I can quantify probability of outcomes in spinners and predict future outcomes	
		I can explore data using real life pie and bar graphs	
Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. **CC.6.SP.5c**		I can create bar graphs to interpret data	
		I can create circle graphs to interpret data	
Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. **CC.6.SP.5d**		I can use bar and circle graphs to make real-life decisions	
		I can define and distinguish between categorical and numerical variables	
		I can use mean, median, and mode to summarize categorical data	
		I can determine the effect of outliers on mean, medians, and modes	
		I can create dot plots to interpret and summarize data	
		I can create stem plot to interpret and summarize data	
		I can create box and whisker plot to interpret and summarize data	

Unit Title: **Ratio and Rates**
Designed by: **Rodolpho Loureiro, Stephanie Donofry[1]**
District: **Developed by Thomas Edison Charter School, used with permission by the Reach Academy for Girls**
Content Area: **Mathematics**
Grade Level(s):   **6th grade**

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Summary of Unit

Proportional understanding is essential to students because it teaches students to compare two quantities as it relates to another two quantities. Proportional analysis is essential to understanding relationships between very important concepts like fractions, decimals, and percent, as well as other concepts in geometry and algebra as well.

Stage 1 – Desired Results
What students will know, do, and understand

_____

Delaware Content Standards

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes." **CC.6.RP.1**

Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[2] **CC.6.RP.2**

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **CC.6.RP.3**

Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **CC.6.RP.3a**

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[1] (Adapted from http://dnet01.ode.state.oh.us/IMS.ItemDetails/LessonDetail. aspx?id=0907f84c80530b51)

[2] Expectations for unit rates in this grade are limited to non-complex fractions.

Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? **CC.6.RP.3b**

Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **CC.6.RP.3c**

Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **CC.6.RP.3d**

Big Idea(s)
The big idea of this unit is the comparison of two quantities to one another and in a form of a ratio and using it to calculate further ratios.

Unit Enduring Understanding(s)
Ratio compares two quantities

Proportions are two ratios that are equal to one another

Proportions are used to calculate the quantities of other equal ratios

Unit Essential Questions(s)
Why are proportional relationships an important part of mathematics?

How is percent related to fractions and decimals, and why is it such a useful tool in everyday life?

Why are proportional relationships an important part of mathematics?

How is percent related to fractions and decimals, and why is it such a useful tool in everyday life?

Knowledge and Skills

Students will know…
Vocabulary for speaking using ratios and rates

How to set up a ratio and a proportion

Set up a proportion to find the percent of a number

How to find the part of a whole of a number using percents

Students will be able…
To write and compare ratios in different ways

To explore the effects of additions to ratios using ratio tables

To explore proportional relationships using graphs, and algebra

To define and calculate unit rates and unit prices

To use algorithms to solve proportions

To define and calculate unit rates and unit prices

To define and calculate unit rates and unit prices

To use an algorithm to solve proportions

To use proportional reasoning to express rates

To use proportional reasoning to express rates and predict outcomes

To identify a percent is a ratio out of 100

To convert between percents and fractions

To convert between fractions, decimals, and percents

To convert and compare fractions, decimals, and percents

To demonstrate knowledge of concepts of a percent

To use percents to find the part of a whole

To make good eating choices based on interpreting percentages on nutrition labels.

To find the percentage of a number to interpret %DV on nutrition labels.

To find the percentage of a number to determine good eating choices.

Stage 2 – Assessment Evidence
Evidence that will be collected to determine whether or not Desired Results are achieved

Suggested Performance/Transfer Task(s)

Summer Job

You have just landed a summer job. You will paint houses Monday through Friday, 8 hours each day. You have to make many decisions in order to maximize your earnings,
- choose from two different pay options.
- decide how much paint to purchase.
- mix paints to get the right colors.

Make these important decisions.
1. The two different pay options are: either $62 per day or $304 per week. Which is the better deal for you? Use unit rates to explain your decision.
2. Using the option you chose in Question 1, determine how much money you will earn by working 4 weeks. Justify your answer and identify the number of hours you will work per week.
3. To get the right color to paint the house, you must mix 1 gallon of green paint with 3 gallons of white paint.
   a. Write a ratio in 3 different ways to show the relationship between green paint and white paint.
   b. How many gallons of paint will your mixture make?
4. You are told ahead of time that you will need to purchase about 12 gallons of paint in order to cover the entire house. Write a proportion to determine the amount of green and white paint you will need to purchase.
5. How many gallons of green paint would be needed if you have 10 gallons of white paint? Explain your reasoning.
6. How many gallons of white paint would you need to mix with 0.5 gallon of green paint? Explain your answer.

7. Suppose that you accidentally mixed 2 gallons of green paint with 3 gallons of white paint.
a. How would the color change? Would it be darker or lighter? Explain.
b. Without starting over, how could you fix your mistake to get the right color to paint the house?

A Special Birthday

Aiden's grandparents put $2,000 in a bank savings account when he was born. The bank account earns simple interest at a rate of 4.8% a year. They told Aiden that he could have the money on his sixteenth birthday. He plans to use the money to buy a used car.

1. How much money will be in the account when he turns sixteen?
2. Aiden found ads for used cars on the internet. Look at three ads that he found. The ads are from dealers in different states, so the sales tax rates are different.

Option 1	Option 2	Option 3
$4015	Originally $4350	18% off
15% off	Now $3915	You Save $750!
Sales Tax: 5.6%	Sales Tax: 7.25%	Sales Tax: 4.5%

a. Which offer has the best discount? Explain.
b. Calculate the total cost of each car including sales tax.

On his birthday, Aiden's family and relatives go out to dinner to celebrate. The three families decide to get separate checks.

3. Copy and complete each receipt. Show your work.

Aiden's Family

PASTA HOUSE
RECEIPT
March 12   6:30 PM

Food Total $72.50
Tax (6%)   _____
Subtotal   _____
Tip Amount (15%)   _____
Total Due   _____

Thank you! Please come again!

Aunt Elle's Family

PASTA HOUSE
RECEIPT
March 12   6:30 PM

Food Total $49.87
Tax (6%)   _____
Subtotal   _____
Tip Amount (%) $10.57
Total Due   _____

Thank you! Please come again!

Grandma & Grandpa

PASTA HOUSE
RECEIPT
March 12   6:30 PM

Food Total   _____
Tax (6%) $2.23
Subtotal   _____
Tip Amount (10%) $3.94
Total Due   _____

Thank you! Please come again!

4. Explain how each family could have used mental math to estimate the tip.

Rubric(s)

Rubric for "A Summer Job"

	Exemplary	Proficient	Emerging
Math Knowledge #2–6	Student accurately calculates pay (2) and finds the amounts of paint (3–6).	Student attempts at least 4 calculations and accurately determines at least 3 of these calculations.	Student attempts at least 3 calculations and accurately determines 2 of these calculations.
Problem Solving #1, 7a, 7b	Student correctly selects the better deal for pay using unit rates (1), determines how the paint color will change (7a), and finds a way to correct the mistake (7b).	Student correctly solves two of the three problems.	Student correctly solves at least one of the three problems.
Representation #3a, 4	Student writes three correct ratios (3a) and gives a correct proportion to determine paint quantities (4).	Student writes at least 2 correct ratios and sets up a proportion.	Student provides at least one correct ratio and sets up a proportion.
Communication #1, 5, 6, 7a	Student clearly communicates the better pay option (1), gives clear explanations of the methods used to find the amount of green paint (5) and white paint needed (6), and gives clear reasoning for whether the paint would be darker or lighter (7a).	Student clearly communicates logical reasoning for three of the items.	Student clearly communicates logical reasoning for two of the items.

Rubric for "A Special Birthday"

	Exemplary	Proficient	Emerging
Math Knowledge #1, 2b, 3, 4	Student accurately determines the amount of money in the account (1), correctly calculates the cost of all three cars (2b), correctly completes all three receipts (3), and provides three logical estimates for tips (4). Student makes no more than one minor error in calculations.	Student completely and correctly solves three of the problems. OR The student solves all the problems using correct methods, but the calculations may contain minor errors.	Student completely and correctly solves at least two of the problems.
Problem Solving #2a	Student accurately determines which of the three offers gives the best discount, providing appropriate supporting calculations (2a).	Student accurately determines two of the three discounts but makes an incorrect conclusion. OR The best discount may be chosen, but the supporting calculations provided are incomplete.	Student is able to determine the best discount without any appropriate supporting calculations. OR Student logically determines the best discount based on incorrect work.
Communication #2a, 4	Student clearly explains his or her reasoning for the discount chosen (2a) and for each of the three mental math strategies used (4).	Student gives explanations for the discount and the three mental math strategies. At most one explanation may be incomplete or unclear.	In providing the explanations for the discount and 3 mental math strategies, at least two of the four must be clear and complete.

Other Evidence

Group Activities focusing on practicing skills
Stations Review
Jeopardy! Review Game
Weekly quizzes
Exit Tickets
Worksheets

Student Self-Assessment and Reflection

Unit Test Analysis – Students self-correct own wrong answers and provide clear explanation of why they got the answer incorrectly

Unit Test Retakes – After student completes the Unit Test Analysis, student then verbally explain misunderstandings prior to retaking the Assessment

Task Reflections – Students reflect upon performance and habits that led to such outcome in the task.

Stage 3 – Learning Plan
(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

<u>Introduction to Ratios:</u>
- Ask students to describe situations in which they might want to compare the quantity of one thing to the quantity of another.
  - For example, how heavy one person's book bag is compared to another person's bag.
  - Record student ideas on chart paper.
  - Students typically generate ideas such as the number of boys compared to the number of girls in a class, the number of teachers to number of students in a grade level, number of desks to number of students.
  - Encourage students to think outside the classroom by comparing the number of pets to people, 1 table in the cafeteria to the number of seats at the table, and other such ideas.
- Select items from the student-generated comparisons and have the class discuss different ways of writing those comparisons.
  - For example, if comparing heights between two people, one might write: her height is 54 inches and his height is 53 inches, 54 inches to 53 inches, 54:53, or many other representations, such as a table or chart.
  - Introduce the common notation using a colon (54:53) and the terminology *ratio* (a comparison of two quantities).
- Place students in pairs and provide each pair with a bag of cubes.
  - Ask the pairs to sort the cubes by color, count and record how many of each color and how many cubes were in the bag. All groups have the same configuration of cubes.
- Conduct a class discussion about the contents of the bags.
  - Ask students to describe the contents of the bags. Expect responses that indicate the bags contain cubes, the cubes are different colors, there are three red cubes, four yellow cubes, and two green cubes in each bag.
  - Ask several different students to describe the number of red cubes in relation to the whole bag. Listen to how students phrase their responses.

Write a statement on the board to summarize the individual students' descriptions. For example, there are three red cubes out of the nine total cubes. Repeat this with the other colors.

- Ask the pairs to write down all the comparisons they can make for the cubes in the bag. Share the lists with the class. Include all comparisons, including the reverse order (e.g., one blue cube and two white cubes, and two white cubes and one blue cube).
- Ask students to suggest shorter ways to write some of the comparisons. Use student suggestions and encourage the use of common mathematical forms e.g., three red to 12 total, 3 red:12 total, and 3:12 if students do not suggest it. Avoid using fractions to represent the ratios in this lesson unless students suggest it.

- Combine pairs to form small groups. Ask the groups to complete the table below assuming that the later rows of the table are found by combining bags.

Number of Red Cubes	Number of Yellow Cubes	Number of Green Cubes	Total Number of Cubes
3	4		12
6	8		
			36

- Have groups fill in the table. Remind them that they will be asked to explain their strategies.
  - Ask groups to share the strategies they used to complete parts of the table and complete the table.

Number of Red Cubes	Number of Yellow Cubes	Number of Green Cubes	Total Number of Cubes
3	4	2	12
6	8	4	24
9	12	6	36

- Remind the class that a ratio's "job" is to describe a relationship between quantities. Facilitate a discussion to introduce equal ratios.Ask questions such as;
  - When we went from single bags to combined bags, what happens to the relationship between the number of yellow cubes and the number of red cubes? (Relationship stayed the same: still four yellow for every three red.)
  - If we combined six bags would there still be more yellow than red? Why? If necessary, tell the class, that a 4:3 ratio can be thought of as saying for every four yellow cubes there are three red cubes. Then, ask if this holds true for the combined bags.

- Have students group the cubes of the combined bags to show that for every four yellow cubes there are three red cubes.
- Explain that since the single bags and the combined bags have the same relationship, the ratios are said to be equal. Therefore, the ratio 4:3 is equal to the ratio 8:6. The
relationship between the number of yellow cubes to red cubes in the single and combined bags did not change.
- Tell students that when the bags were created, the ratio of yellow to green cubes was intended to be 2:1 and now the students have determined that the ratio is 4:2 for each original group.
- Ask students to suggest methods of determining if a mistake has been made, or if these are equal ratios. Allow students to make suggestions and to critique each other suggestions. Typically, students suggest some form of building up a set. They start with 2 yellow cubes and 1 green cube and then add 2 more yellow and 1 more green, making the ratio 4:2. Allow the class to reach agreement on the method they use.
- Ask students to work in small groups to find other ratios equal to the yellow: green ratio of 4:2. Encourage them to use strategies the class has already explored. Allow groups a few minutes in small groups, and then have groups share their ideas and approaches with the class.
- Ask the class if the ratio of yellow to green can be described as 6:3. Have students work in small groups again to investigate the idea. Initially, many groups may say it does not because they cannot build get that ratio by starting with 4 yellow cubes and 2 green cubes and then adding on those amounts again.
  - Ask these groups to explain why 4:2 was the same as 2:1, and to use that thinking to get to 6:3. Also these groups could build up tables to see that the amounts work out to the same number of cubes at some point. In the tables below, both ratios reach 24 yellow and 12 green at some point.

Number of Yellow Cubes	Number of Green Cubes
4	2
8	4
12	6
16	8
20	10
24	12

Number of Yellow Cubes	Number of Green Cubes
6	3
12	6
18	9
24	12

- Ask students if the ratio of yellow to red cubes can be described as 3:2. Have student's think-pair-share about this question. (No, the ratio of yellow to red if 4:3 which cannot be represented as 3:2.) To see why 4:3 cannot be represented as 3:2, suggest that students build up tables for these two ratios. 4:3 will be seen to be equivalent to 8:6, and 3:2 is equivalent to 9:6. Students should be able to see intuitively that 8:6 cannot possibly be the same ratio as 9:6.)

- Use ratios to represent students in the classroom. Do the following as a think-pair-share activity. Have students work alone to answer the first question, and then share thoughts with a partner. Finally, discuss the responses and strategies as a class.
  - Find the ratio of female students to the total students in the room. Write the ratio in several ways on the board. For example: 12 girls to 26 students, 12 girls:26 students, 12:26.
  - Find the ratio of male students to the total students. Write two different ratios to compare the number of male students to the total students. For example, 14:26 or 7:13.
  - Find the ratio of female students to male students. Explain how this ratio is different from girls to total, or boys to total.

Further Understanding
- Discuss ratios and the meaning with students.
  - Pose situations such as:
    - In the winter when people wear mittens, what is the ratio of mittens to a person? (2:1)
    - Have 4 boys and 6 girls stand at the front of the class. Ask students to identify the ratio of boys to girls (4:6), girls to boys (6:4). Ask students if there are other ways these students could be grouped that would visually demonstrate the same ratio. The students can be separated into two groups, each containing 2 boys and 3 girls, so the ratio could also be 2:3. This should be done as an intuitive separation of groups, and should not be approached in this lesson using formal algorithms for finding equivalent ratios.
    - What is the ratio of feet to inches? (1:12); What is the ratio of inches to feet? (12:1)
  - Ask students what is meant when a teacher or tutor provides one-to-one help. Ask students to identify the ratio of students to teachers in those settings. (1:1)
    - If the Cavaliers are playing the Pistons and are using a one-on-one defense, what does that mean? What is the ratio of Cavalier players to Piston players on the court in that setting? (1:1)
    - How is one computer to three students different from three computers to one student? Describe both ratios and tell how they are different.
- Select students to write the ratios on the board. Accept answers written in words or using the short notation, such as 2:1.
- Ask students to find another ratio equivalent to the given ratio. For example, if the answer was 2:1, another ratio would be 14:7. Do not provide students with a process or algorithm for finding equivalent ratios. This will occur later in this part of the lesson.
- At this point in the lesson, asking students to describe a systematic way for finding equivalent ratios would be appropriate if so desired.

Practicing Ratios
- Ask students to complete this sentence:

If two friends have one cookie, then the ratio of friends to cookies can be described as 1 to _____. The answer is $1:\frac{1}{2}$ because one friend can have half a cookie. Ask students: how else might we describe the ratio of friends to cookies? (2:1, because there are two friends and one cookie.) So the ratios 2:1 and $1:\frac{1}{2}$ are equal ratios because they are describing the same scenario in two different ways.

- Provide students with the following situation:

A pancake mix combines eggs, milk, and mix in the following ratio: 1 egg to $\frac{3}{4}$ cup milk to 1 cup mix. One batch serves 4 people.

Have students work in small groups to answer the following questions.

- What is the ratio of people to eggs if one batch is made? (4:1)
- What is the ratio of eggs to cups of mix if two batches are made? (2:2)
- What ratio could be described by 1:4? (eggs to people, or cups of mix to people)
- Sam made pancakes, but he used a ratio of two eggs to eight people. Is his ratio correct? What did he do? (The ratio is correct. Think of Sam making two batches.)
- If making two batches, what is the ratio of eggs to milk? ($1:\frac{3}{4}$ or $2:1\frac{1}{2}$)
- Why could one egg to one cup mix still be used to describe the ratio of eggs to mix if five batches of pancakes are made?

Resources and Teaching Tips

Resources
- SpringBoard Textbook
- Prentice Hall – Mathematics Textbook and Student Workbook
- Teacher Created Powerpoints
- Teacher Created activities, quizzes, and supplementary worksheets
- Nutrition Labels

Teaching Tips
- Students may have trouble remembering the three different ways of writing ratios
- Students will likely confuse where numbers are placed within the proportion
- Students would benefit with a strong model of how to write and label proportions
- Students will have trouble finding certain information pieces on nutritional labels
- Students may forget how to properly label numbers with appropriate units

- Using fractions to represent ratios is discouraged throughout this entire lesson. An incomplete understanding of fractions combined with an incomplete understanding of ratios can facilitate the development of misconceptions that are difficult to correct.
- Prior to the lesson, create bags of color cubes. Place three red cubes, four yellow cubes, and two green cubes in each bag. Prepare one bag for each pair of students.

Differentiation
- Students will be divided into leveled groups when performing inquiry-based lessons with more scaffold provided to such groups.
- Students will be able to partake in different roles within group presentations, according to their skills of their choice (writing, presenting, time-keeping)
- Students may work in pairs during certain activities for additional support or stretch of understanding
- Vocabulary will be presented at a slower pace for struggling students

Design Principles for Unit Development
At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.

  Students will be encouraged explore the nutrition labels of foods of different countries when researching nutrition facts**.**

- **21st Century Learning –** the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

  Students will be learning using a guided release model, an inquiry-based model of learning, and present materials by applying what they have learned in previous activities into a new format. They will also use physical activities to demonstrate understanding of rates

Technology Integration
The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- **8th Grade Technology Literacy** - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century(SETDA, 2003).

Students will present information on nutrition labels through Microsoft PowerPoint.

One aspect of quality in computer monitors and television screens is measured by using pixels per square inch. Have students explore the ratios for different computer monitors or televisions and the total number of pixels. Explore the relationship between cost and ratio of pixels.

Content Connections
Content Standards integrated within instructional strategies

Physical Education – Students will use jumping jacks to determine unit rates

Health – Students will read and interpret nutritional labels to make better food choices

Science – Students will learn the difference between calories, fats, and carbohydrates when reading nutritional labels.

Curriculum Framework for Mathematics

School: Reach Academy for Girls   **Curricular Tool: Spring Board**   **Grade: 7**   **Teacher:**

Unit One: Introduction to Rational Numbers & Adding/Subtracting Integers
Timeline : 3 weeks

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **CC.7.NS.1** Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. **CC.7.NS.1a** Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. **CC.7.NS.1b** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **CC.7.NS.1c** Solve real-world and mathematical problems involving the four operations with rational numbers.[1] **CC.7.NS.3**	Rational numbers are a subset of the number system including & beyond whole numbers Understanding and interpreting rules for adding and subtracting integers is necessary to solve real-world problems Number lines and diagrams are important tools to help understand operations with rational numbers	**Essential Questions:** <u>Essential Questions:</u> What is the relationship between subsets of rational numbers? Why is it important to understand properties and operations involving addition and subtraction of rational numbers? How can number lines and diagrams be used to interpret solutions of real-world problems? **Learning Targets:** <u>Learning Targets:</u> I can identify and describe relationship between subsets of rational numbers I can compare and order integers on the number line I can find absolute value and explain what it means I can find opposite of an integer and explain what opposite means I can understand why the sum of	**Formative Assessments:** <u>Formative Assessments:</u> Informal Classroom Observation Individual Whiteboards Classroom Discussion Think, Pair, Share "Quickwrite" Daily Exit Ticket Weekly Quiz **Summative Assessments:** <u>Summative Assessments:</u> Unit Test Performance Task

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		opposites is zero	
		I can add and subtract integers on a number line	
		I can solve multi-step real-world problems involving addition and subtraction of integers	

Unit Two: Multiplying & Dividing Integers, Computation with other Rational Numbers
Timeline: 5 weeks

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **CC.7.NS.2**	Understanding and interpreting rules for multiplying and dividing integers is necessary to solve real-world problems	**Essential Questions:** Why is it important to understand properties and operations involving integers and rational numbers?	**Formative Assessments:** Informal Classroom Observation Individual Whiteboards Classroom Discussion Think, Pair, Share "Quickwrite" Daily Exit Ticket Weekly Quiz Embedded Assessment
Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. **CC.7.NS.2a**	Operating with rational numbers such as fractions, decimals and percents is necessary to solve real-world problems	How can number lines and diagrams be used to interpret solutions of real-world problems?	**Summative Assessments:** Unit Test Performance Task
Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real- world contexts. **CC.7.NS.2b**	Number lines and diagrams are important tools to help understand operations with rational numbers	**Learning Targets:** I can multiply and divide integers	
		I can compute fluently with decimals	
Apply properties of operations as strategies to add and subtract rational numbers. **CC.7.NS.1d**		I can compute fluently with fractions and mixed numbers	
Apply properties of operations as strategies to multiply and divide rational numbers. **CC.7.NS.2c**		I can understand meaning of a percent	
		I can convert fluently between fractions, decimals & percents	
		I can apply knowledge of computation with rational numbers	

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **CC.7.NS.2d** Solve real-world and mathematical problems involving the four operations with rational numbers.[2] **CC.7.NS.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* **CC.7.RP.3**		by calculating discounts, markups, percent of increase, percent of decrease, simple interest & compound interest	

Unit Three: Equations, Inequalities & Linear Relationships
Timeline: 6 weeks

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **CC.7.EE.1** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."* **CC.7.EE.2** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of	Real-life and mathematical problems can be solved using algebraic expressions and equations Expressions can be manipulated to suit a particular purpose such as solving problems efficiently	**Essential Questions:** Why is it important to understand how to solve linear equations and inequalities? How can algebraic expressions and equations help us to solve real-world problems? Why is it important to be able to manipulate expressions? **Learning Targets:** I can define variable I can write an equation to represent a table of values I can define coefficient and	**Formative Assessment:** Informal Classroom Observation Individual Whiteboards Classroom Discussion Think, Pair, Share "Quickwrite" Daily Exit Ticket Weekly Quiz Embedded Assessment **Summative Assessments:** Unit Test Performance Task

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
answers using mental computation and estimation strategies. **CC.7.EE.3**		understand its meaning in an equation	
Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **CC.7.EE.4**		I can graph and interpret the graphs of linear equations	
		I can find slope of a line	
Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. **CC.7.EE.4a**		I can solve a two-step equation	
		I can combine like terms	
		I can solve an equation with variables on both sides	
Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. **CC.7.EE.4b**		I can solve an equation using the distributive property	
		I can write an solve a multi-step equation in order to solve a real-world problem	
Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. **CC.7.RP.2c**		I can define inequality and write inequalities to represent real world problems	
		I can find and graph solution to an inequality	
		I can write, solve and graph situations involving direct variation	

Unit Four: Two-Dimensional Geometry, Proportions & Scale
Timeline: 6 weeks

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step	Relationships between geometric figures are useful for	**Essential Questions:** Why is it important to understand	**Formative Assessment:** Informal Classroom Observation

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
problem to write and solve simple equations for an unknown angle in a figure. **CC.7.G.5** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **CC.7.G.2** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.* **CC.7.RP.1** Recognize and represent proportional relationships between quantities. **CC.7.RP.2** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. **CC.7.RP.2a** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **CC.7.RP.2b** Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate. **CC.7.RP.2d** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and	building new knowledge and solving real world and mathematical problems accurately. Extend an understanding of ratios to develop a deeper understanding of proportionality building the knowledge and skill levels needed to solve single- and multi-step problems	properties of angles and figures to solve problems? How can an understanding of ratios help build an understanding of proportional relationships? How can identifying proportional relationships help to solve problems involving scale drawings? **Learning Targets:** I can identify and calculate complementary, supplementary, vertical and adjacent angles I can solve for an unknown angle in a figure I can identify and define different triangles and polygons I can construct and describe triangles given the angle measures I can write ratio I can identify and solve problems involving proportional relationships I can graph proportional relationships in a coordinate grid I can calculate unit rate	Individual Whiteboards Classroom Discussion Think, Pair, Share "Quickwrite" Daily Exit Ticket Weekly Quiz Embedded Assessment **Summative Assessments:** Unit Test Performance Task

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
reproducing a scale drawing at a different scale. **CC.7.G.1** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **CC.7.G.4**		I can identify scale factor and use scale factor to draw scale models	
Unit Five: Three-Dimensional Geometry **Timeline: 4 weeks**			
Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **CC.7.G.6** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. **CC.7.G.3**	Investigate the relationship between two-dimensional and three-dimensional figures in order to integrate these concepts for problem-solving Use knowledge of 2-dimensional figures in order to measure the surface area and volume of three-dimensional figures	**Essential Questions:** Why is it important to be able to relate two-dimensional drawings with three-dimensional figures? How can surface area and volume be used to find answers to real-world problems? **Learning Targets:** I can draw two-dimensional representations of three-dimensional figures I can draw and describe plane sections of right rectangular prisms and right rectangular pyramids I can find area of polygons I can find surface area and volume of rectangular prisms I can find surface area and volume of cubes I can find surface area and volume	**Formative Assessment:** Informal Classroom Observation Individual Whiteboards Classroom Discussion Think, Pair, Share "Quickwrite" Daily Exit Ticket Weekly Quiz Embedded Assessment **Summative Assessments:** Unit Test Performance Task

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
		of triangular prisms I can find surface area and volume of cylinders I can find surface area and volume of pyramids I can find surface area and volume of compound solids	
Unit Six: Statistics **Timeline: 5 weeks** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. **CC.7.SP.1** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* **CC.7.SP.2** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean*	Formulating questions, designing studies, and collecting data about a population allows us to make inferences and compare data. It is important to use random, representative sampling methods in order to obtain accurate information on a population so that reasonable inferences and predictions can be drawn	**Essential Questions:** Why is it important to understand different sampling methods? Why is it important to be able to represent data using graphs and measures of central tendency/variability? **Learning Targets:** I can calculate measures of central tendency I can calculate measures of variability I can use measures of central tendency and variability to draw conclusions about a data set I can construct data displays including dot plots, histograms and box plots to compare two data sets	**Formative Assessment:** Informal Classroom Observation Individual Whiteboards Classroom Discussion Think, Pair, Share "Quickwrite" Daily Exit Ticket Weekly Quiz Embedded Assessment **Summative Assessments:** Unit Test Unit Project

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. **CC.7.SP.3** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* **CC.7.SP.4**		I can identify various methods of sampling and explain the advantages and disadvantages of each method I can use data from a random sample to draw inferences about the sample I can generate multiple samples and explain how a prediction made from multiple samples is different from a prediction made from a single sample	
Unit Seven: Probability **Timeline: 6 weeks**			
Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **CC.7.SP.5** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* **CC.7.SP.6** Develop a probability model and use it to find probabilities of events. Compare probabilities	Probabilities are fractions derived from modeling real world experiments and simulations of chance It is important to compare probability models with the results of actual probability simulations because often there are discrepancies between models and simulations	**Essential Questions:** How can probability models be used to predict the outcome of future events? How are theoretical and experimental probabilities related? **Learning Targets:** I can construct and describe sample space or probability model for a probability experiment I can make predictions based on a probability model I can determine probability for independent and dependent events I can analyze probabilities	**Formative Assessments:** Informal Classroom Observation Individual Whiteboards Classroom Discussion Think, Pair, Share "Quickwrite" Daily Exit Ticket Weekly Quiz Embedded Assessment **Summative Assessments:** Unit Test

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. **CC.7.SP.7**		determined from a model to observed probability frequencies and analyze possible sources of discrepancy	
Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* **CC.7.SP.7a**		I can calculate compound probability using sample spaces and probability models	
Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? **CC.7.SP.7b**		I can calculate probability of compound events by creating and conducting a simulation	
Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. **CC.7.SP.8**			
Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. **CC.7.SP.8a**			
Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. **CC.7.SP.8b**			
Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to*			

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood? **CC.7.SP.8c**			

Unit Title:	Introducing Rational Numbers & Adding/Subtracting Integers
Designed by:	Caroline Strunk
District:	Thomas Edison Charter School, used with permission by the Reach Academy for Girls
Content Area:	Mathematics
Grade Level(s):	7th Grade

_____

Summary of Unit

In this Unit students will learn about subsets of rational numbers, focusing especially on integers. Students will build upon their previous understanding of operations and negative numbers to learn how to add and subtract integers. Students will utilize number lines, diagrams, and real-world contexts to ensure a meaningful understanding of adding and subtracting integers as well as the relationship between the two operations.

Stage 1 – Desired Results
What students will know, do, and understand

_____

Delaware Content Standards

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **CC.7.NS.1**

Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. **CC.7.NS.1a**

Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. **CC.7.NS.1b**

Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **CC.7.NS.1c**

Solve real-world and mathematical problems involving the four operations with rational numbers. **CC.7.NS.3**

Big Idea(s)

Rational numbers are a subset of the number system including & beyond whole numbers

Understanding and interpreting rules for adding and subtracting integers is necessary to solve real-world problems

Number lines and diagrams are important tools to help understand operations with rational numbers

Unit Enduring Understanding(s)

Students will finish this unit with an understanding of the relationship between subsets of rational numbers and will be able to add and subtract fluently with positive and negative numbers.

Students will also have a meaningful understanding of the relationship between addition and subtraction of integers. When faced with a real-world problem that requires adding and subtracting positive and negative numbers, students will be able to apply their skills to successfully solve the problem.

Unit Essential Questions(s)

Why is it important to understand properties and operations involving addition and subtraction of rational numbers?

How can number lines and diagrams be used to interpret solutions of real-world problems?

What is the relationship between subsets of rational numbers?

Knowledge and Skills

Students will know…

The following vocabulary:
- Rational numbers
- Counting numbers
- Integers
- Whole Numbers
- Opposites
- Absolute Value

Students will be able to…
- Identify and describe relationship between subsets of rational numbers
- Compare and order integers on the number line
- Find opposite of an integer and explain what opposite means
- Understand why the sum of opposites is zero
- Find absolute value of a number and explain what it means
- Add and subtract integers on a number line
- Solve multi-step, real-world problems involving addition and subtraction of integers

Stage 2 – Assessment Evidence
Evidence that will be collected to determine whether or not Desired Results are achieved

_____

Suggested Performance/Transfer Task(s)

Overview:
Weather reporters throughout the world are in desperate need of your help. The computer program typically used to determine high and low temperatures and to calculate temperature ranges has been destroyed by a computer virus. Because weather reporters have become so reliant on technology, they no longer remember how to make the calculations necessary to determine the temperatures—especially in cold regions. Their public is waiting, they want information now. The weather reporters need you to research and calculate the temperatures for

them. They would also be very grateful if you would explain how you arrived at your answers so they are not helpless should a similar situation happen again in the future.

Requirements:
1. Using the Internet, find the record high and low temperatures for January in Wisconsin or another state with extreme winter temperatures.
2. Identify the warmest and coldest temperature for the month.
3. Find the differences between the record high and record low temperatures for each day of the month
4. Clearly explain procedures used for adding and subtracting integers. Give examples.

Rubric for Performance Task

	4	3	2	1
Calculations	All calculations are correct	Most calculations are correct	Some calculations are correct	Few calculations are correct
Explanation	Clearly, coherently explains procedures for adding and subtracting integers using mathematical language	Explains procedures for adding and subtracting integers but does not use mathematical language	Attempts to explain procedures for adding and subtracting integers but explanation is incomplete	The student's explanation contains conceptual misunderstandings

Other Evidence

Unit Test (attached)

Informal Classroom Observation

Individual/Group Whiteboards

Classroom Discussion

Think, Pair, Share

"Quickwrite"

Daily Exit Ticket

Weekly Quiz

Student Self-Assessment and Reflection

Students will have the opportunity to reflect on their learning at the end of the unit before the unit test. As part of the reflection, the students will answer the essential questions, will define vocabulary words, will reflect on concepts which they feel strongest and weakest on, and will relate concepts in the unit to the real world. The self-reflection will help conclude the unit, and will also help the teacher to plan appropriate review activities before the test.

Stage 3 – Learning Plan
(Design learning activities to align with Stage 1 and Stage 2 expectations)

_____

Key learning events needed to achieve unit goals

Lesson 1

Objective: I can identify and describe relationships between subsets of rational numbers

Resource: Spring Board Middle School Mathematics 2, Unit 1, Activity 1.1

Vocabulary: Counting Numbers, Whole Numbers, Integers & Rational Numbers

1. Hook/Inquiry Activity - Teacher places numbers in a Venn diagram on the board as students try to determine in pairs how the numbers are grouped. Teacher leads class discussion about how the numbers are grouped.
2. Direct Instruction- Teacher leads direct instruction on different "groups" of numbers including counting numbers, whole numbers, integers and rational numbers as students take notes. Students will add this vocabulary to the interactive word wall in the classroom as well as a personal word wall in their folder.
3. Guided Practice- Students practice grouping and comparing different numbers through the use of Venn diagrams, "quickwrites" describing the relationship between numbers, and compare/contrast with their partner. Teacher walks around to observe and leads classroom in a debrief discussion.
4. Exit Slip- Students are able to identify an example from each subset of numbers discussed and are able to compare and contrast the similarities and differences between subsets of numbers

Lesson 2

Objective: I can compare and order integers on the number line

I can connect integers to real world situations

Resource: Spring Board Middle School Mathematics 2, Unit 1, Activity 1.2

1. Warm-Up- Students will place numbers into different subsets of rational numbers
2. Hook- Pictures & Observations of Death Valley. How are the peaks/canyons of Death Valley similar to positive and negative numbers?
3. Activity – Students will plot elevations on a number line in order to compare positive and negative elevations. Students will use the number line to make generalizations about the order of integers on the number line. Teacher will lead classroom discussion to confirm that all students were able to draw generalizations. (For example, numbers increase as you go right on the number line, and -100 is actually less than -50)
4. Guided Practice – Students will compare and order sets of real world integers on individual white boards
5. Exit Slip- Students will compare and order integers

Lesson 3

Objective: I can find opposite of an integer and explain what opposite means

I can understand why the sum of opposites is 0

Resource: Spring Board Middle School Mathematics 2, Unit 1, Activity 1.2

Vocabulary: Opposites

1. Warm-Up- Students will compare and order integers
2. Guided Inquiry- Students will plot opposite elevations on a number line. Students will describe the relationship between the two numbers.
3. Direct Instruction- Teacher will explain what opposite numbers are as students take notes. Teacher will use a real world scenario with opposites and explain why the sum is zero. Students will add the word "opposite" to the interactive word wall in the classroom and in their folder.

4. Guided Inquiry- Students will be given integer chips and a number line in addition to a guided inquiry activity. Students will prove using integer chips and a number line that all opposite numbers' sum is zero. Teacher will lead classroom discussion at the end of the activity

5. Exit Slip- Student will think of a real-world pair of opposite numbers and explain why the sum of the numbers is zero.

Lesson 4

Objective: I can find absolute value of a number and explain what it means
Vocabulary: Absolute Value

1. Warm-Up- Give an example of opposites in the real world. What is the sum of opposites? How do you know?

2. Direct Instruction- Teacher will explain what absolute value is as students take notes. Teacher will check for understanding after notes. Students will add the word "absolute value" to the interactive word wall in the classroom and in their folder.

3. Activity – Students will be given several different number lines with numbers plotted. They will have to determine which number on the number line has the greater absolute value. The number lines will progress from basic integers to more challenging number lines including fractions, decimals and variables. Teacher will lead classroom discussion on each number line in which students will need to defend their answers about which number has the greatest absolute value and why.

4. Exit Slip- Students will find absolute value of a number, and will also determine the greater absolute value when given two numbers on a number line

Lesson 5

Objective: I can add integers using a number line
Resource: Resource: Spring Board Middle School Mathematics 2, Unit 1, Activity 1.2

1. Warm- Up- Find opposite and absolute value of an integer. Explain why your answer is correct.

2. Activity- Students will think of adding integers in terms of hiking. They will follow rules for which direction to hike on the number line depending on the sign in front of the number. Students will roll number cubes to create a problem, and solve the problem using the number line. Students will then look for patterns to begin understanding rules for adding integers.

3. Classroom Debrief- Teacher will debrief class on patterns that they noticed and clarify students' understanding of the rules for adding integers

4. Partner Practice- Students will apply understanding of adding integers to real-world problems requiring them to add integers

5. Independent Practice- Students will independently apply understanding of adding integers to real-world problems

6. Exit Slip- Students will solve an adding integers problem using a number line, and will solve a real-world problem involving integer addition by using the "rules" for adding integers

Lesson 6

Objective: I can subtract integers using a number line
Resource: Resource: Spring Board Middle School Mathematics 2, Unit 1, Activity 1.2

1. Warm- Up- Adding integers
2. Activity- Students will use the number line to find the answer to a problem requiring them to subtract integers. Students will then figure out how they could represent each subtraction problem as an addition problem. Students will use patterns to come to the conclusion that subtracting an integer is the same as "adding the opposite."
3. Classroom Debrief- Teacher will debrief class on patterns that they noticed and clarify students' understanding of rules for subtracting integers
4. Partner Practice- Students will apply understanding of subtracting integers to real-world problems
5. Independent Practice- Students will independently apply understanding of subtracting integers to real-world problems
6. Exit Slip- Students will solve a subtracting integers problem using a number line, and will solve a real-world problem involving integers using any method of their choosing

Lesson 7

Objective: I can solve multi-step, real-world problems involving both addition and subtraction of integers

Resource: Resource: Spring Board Middle School Mathematics 2, Unit 1, Activity 1.2

1. Warm-Up- Students will review adding/subtracting integers
2. Activity- In pairs, students will complete multi-step real-world problems involving both addition and subtraction of integers. Students will draw out their solution to the problem on a large group white board. Students will also be given a numerical expression and will have to think of a real-world scenario to represent that situation. Students will share their answers with the class through classroom discussion
3. Independent Practice- Students will work on adding/subtracting multi-step integer problems independently
4. Exit Slip- Students will work on a few problems similar in nature to prove final mastery

Resources

- Spring Board Middle School Mathematics 2 Text book
- Teacher created activities, quizzes, and worksheets, as necessary
- Integer Chips & Number lines
- Individual/ Large Group White Boards
- Number Cubes

Teaching Tips

- Students may get confused about which direction they should be moving on the number line when adding and subtracting integers. It is important that the teacher is clear about number line directions before students begin the activity with adding integers. The teacher may want to model a few examples before students begin. Similarly, when students begin subtracting integers, teacher may want to model again how to use the number line to find a difference.
- Students may also begin to confuse adding and subtracting integers if they have not fully mastered the rules for adding integers before they begin subtracting integers. The teacher should ensure full mastery of lesson 6 before moving on to lesson 7.

Differentiation

- During the classroom activities, students will be given optional extension questions if they finish before the rest of the class
- Struggling students will be given a chart of rules for adding integers to keep on their desk if needed
- Students will work with partners and groups to provide additional support while learning
- Teacher will spend time strategically with groups that require additional modeling and support
- Activities appeal to kinesthetic learners by including hands-on activities with number cubes and white boards
- Classroom discussion will benefit auditory learners
- Visual learners will benefit from the reading and writing in the activities

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.

This Unit represents integers in a number of different ways to aide students' understanding. Students learn how to add and subtract integers using integer chips, number lines, rules that they generate, and also through real-world representations of integers. The Unit supports different types of learners as described in the differentiation section above.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- **8th Grade Technology Literacy** - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century(SETDA, 2003).

Students will interact with technology by using the Smart Board primarily during the Warm-Up. Students will be called up individually to interact with the Smart Board to show their answer to a Warm-Up problem or a problem during the classroom activity. Students will also use technology during the Performance Task. They will need to access a reliable website to obtain weather information in order to complete the necessary calculations for the task.

Content Connections
Content Standards integrated within instructional strategies

Use of vocabulary, reading and writing is embedded within all of the activities.

The Spring Board Activity on integers is linked to geography since it discusses Death Valley and other places in the world with high and low elevations. During this activity I show students pictures of many of the places discussed in the activity to give them a better understanding of the places mentioned in the activity.

Name:
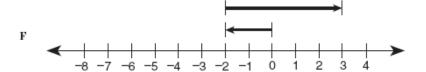
Date:
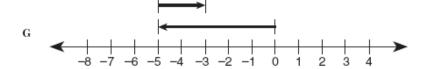
Unit 1 Test: Introduction to Rational Numbers
Adding/Subtracting Integers
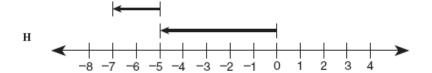
Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **CC.7.NS.1**
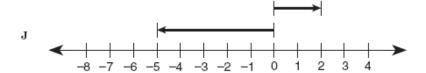
1.

Which model represents $-5 + 2$?
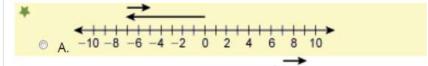
2. Which number line represents the expression $(-5)-(-3)$?

A.

B.

C.

D.

3.

☐ **MSM I, Unit 1**

Which number line represents the expression $(-7)-(-2)$?

▼ **(Hide Anwers)**

○ A.

○ B.

○ C.

○ D.

Describe situations in which opposite quantities combine to make 0.. **CC.7.NS.1a**

4. Describe a situation in the real world where two opposite numbers combine to make 0.

_____
_____
_____
_____
_____
_____
_____

Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real world contexts.
CC.7.NS.1b
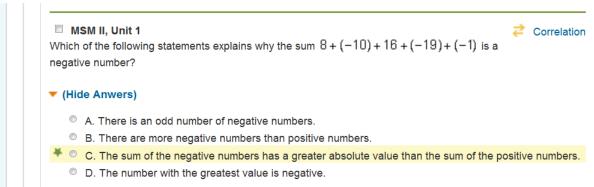
5. Shelby made the following deposits and withdrawals in one month. What was the overall change in her bank account balance?

-40　　+120　-60　　+110
+130　-80　　+108　-60
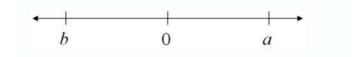
A. -$20
B. $0
C. +$228
D. -$228

6. Which problem could be solved by computing $-5 + (-11)$?

A. The temperature started at $-5° F$, then it dropped by $11°$. What is the new temperature?

B. The temperature started out at $-11° F$, then it went up by $5°$. What is the new temperature?

C. The temperature started out at $-5° F$. Two hours later, it was $11° F$. How much did the temperature increase?

D. The temperature started at $-5° F$. Two hours later it was $-11° F$. How much did the temperature change?

7.

☐ **MSM II, Unit 1**　　　　　　　　　　　　　　　　　　⇄ Correlation

Which of the following statements explains why the sum $8 + (-10) + 16 + (-19) + (-1)$ is a negative number?

▼ **(Hide Anwers)**

   ○ A. There is an odd number of negative numbers.

   ○ B. There are more negative numbers than positive numbers.

✱ ○ C. The sum of the negative numbers has a greater absolute value than the sum of the positive numbers.

   ○ D. The number with the greatest value is negative.

8. On the number line above, the numbers a and b are the same distance from 0. What is $a+b$? Explain how you know.

Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **CC.7.NS.1c**

9. Which of the following has the greatest absolute value?

A. -9

B. 7

C. -2

D. 6

10. Re-write the following subtraction problem as addition, then find the solution.

-15 – 25= _____ + _____= _____

11. Ocean water freezes at about $-2^\circ c$. Fresh water freezes at $0^\circ c$. Antifreeze, a liquid used to cool most car engines, freezes at $-64^\circ c.$

Imagine that the temperature is exactly at the freezing point for ocean water. How many degrees must the temperature drop for the antifreeze to turn to ice?

12.

The low temperature on Sunday was −9°F. The high temperature on Sunday was 14 degrees warmer than the low temperature.

What was the high temperature on Sunday?

Answer _____ °F

The low temperature on Monday was 6 degrees warmer than Sunday's low of −9°F. The low temperature on Tuesday was 3 degrees warmer than Monday's low. What was the low temperature on Tuesday?

Show your work.

Answer _____ °F

13. The record of expenses and income for this year's jamboree for the Organized Otters is shown below. The group was trying to raise $150. How much more money will they need to raise in order to reach their goal? Be sure to show your work.

Expenses	Income
-20	25
-15	40
-45	50
-10	5
-8	25

Curriculum Framework for Mathematics

School: Reach Academy for Girls   Curricular Tool: SpringBoard   Grade: 8   Teacher:

Standards Alignment	Unit Concept Big Ideas	Esssntial Questions Student Learning Targets	Assessments
Unit One: Numerical Relationships **Timeline : 6 weeks**			
Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. **CC.8.NS.1** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.* **CC.8.NS.2** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.* **CC.8.EE.1** Use square root and cube root symbols to represent solutions to equations of the form $x2 = p$ and $x3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **CC.8.EE.2** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate	Numbers that are not rational can be given an approximate value. The properties of number systems and their relationships remain consistent when applied to integer exponents	**Essential Questions:** Why is it important to understand the procedures for working with different kinds of numbers? Why is scientific notation necessary and helpful for expressing large and small numbers? **Learning Targets:** I can know and apply properties of exponents I can evaluate numbers in exponential form with exponents that are zero, one or negative numbers I can understand and apply square roots and cube roots to solve algebraic equations I can understand the difference between rational and irrational numbers I can approximate the value of an irrational square root I can write small and large	**Formative Assessments:** Informal Classroom Observation Individual Whiteboards Classroom Discussion Think, Pair, Share "Quickwrite" Daily Exit Ticket Weekly Quiz Embedded Assessment **Summative Assessments:** Unit Test Performance Task Rubrics

E-151

Standards Alignment	Unit Concept / Big Ideas	Essential Questions / Student Learning Targets	Assessments
very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.* **CC.8.EE.3** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. **CC.8.EE.4**		**Student Learning Targets** numbers in scientific notation I can choose units of appropriate size for measurements of very large or very small quantities I can operate with numbers in scientific notation	
Unit Two: Functions & Linear Equations **Timeline: 8 weeks**			
Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1] **CC.8.F.1** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* **CC.8.F.2** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line;	Define, evaluate and compare linear and nonlinear relationships Interpret linear equations in tables, equations and graphs in order to understand real-world scenarios	**Essential Questions:** What are functions and what is the difference between a linear and nonlinear function? How can graphs of linear relationships be used to interpret solutions of real world problems? **Learning Targets:** I can understand the difference between linear and nonlinear patterns in tables, graphs and equations I can define and identify	**Formative Assessments:** Informal Classroom Observation Individual Whiteboards Classroom Discussion Think, Pair, Share "Quickwrite" Daily Exit Ticket Weekly Quiz Embedded Assessment **Summative Assessments:** Unit Test Performance Task Rubrics

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[1] Function notation is not required in Grade 8.

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
give examples of functions that are not linear. *For example, the function A = s² giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* **CC.8.F.3**		relationships that are functions I can find rate of change or slope of a linear function and interpret the slope in the context of the problem	
Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **CC.8.F.4**		I can find the initial value of a linear function and interpret its meaning in the context of the problem I can write linear equations in the form y=mx+b	
Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **CC.8.F.5**		I can solve linear equations with one solution, no solutions or infinitely many solutions I can solve linear equations that involve distributive property or combining like terms	
Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **CC.8.EE.5**			
Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y = mx* for a line through the origin and the equation *y = mx + b* for a line intercepting the vertical axis at *b*. **CC.8.EE.6**			
Give examples of linear equations in one variable			

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). **CC.8.EE.7a** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **CC.8.EE.7b**			

Unit Three: Applications of Linear Equations- Bivariate Data & Systems of Equations
Timeline: 8 weeks

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **CC.8.SP.1** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. **CC.8.SP.2** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* **CC.8.SP.3** Understand that patterns of association can also	Linear equations, systems of equations, linear functions, and their understanding of slope of a line can be used to analyze situations and solve problems Apply knowledge of linear equations to help classify, describe and analyze trends in bivariate data	**Essential Questions:** How can a trend line help us to interpret a bivariate data set? How does solving systems of equations help us to solve real-world problems? **Learning Targets:** I can collect and interpret bivariate data using a scatter plot I can draw a trend line and use it to better explain the bivariate data I can solve systems of linear equations by graphing I can solve systems of linear equations algebraically	**Formative Assessments:** Informal Classroom Observation Individual Whiteboards Classroom Discussion Think, Pair, Share "Quickwrite" Daily Exit Ticket Weekly Quiz Embedded Assessment **Summative Assessments:** Unit Test Performance Task Rubrics

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?* **CC.8.SP.4**		I can solve real world problems using systems of equations	
Analyze and solve pairs of simultaneous linear equations. **CC.8.EE.8**			
Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. **CC.8.EE.8a**			
Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* **CC.8.EE.8b**			
Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* **CC.8.EE.8c**			

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
Unit Four: Angle Relationships & Transformations **Timeline: 7 weeks**			
Verify experimentally the properties of rotations, reflections, and translations: **CC.8.G.1** Lines are taken to lines, and line segments to line segments of the same length. **CC.8.G.1a** Angles are taken to angles of the same measure. **CC.8.G.1b** Parallel lines are taken to parallel lines. **CC.8.G.1c** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. **CC.8.G.2** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. **CC.8.G.3** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. **CC.8.G.4** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange*	Use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations to describe and analyze two-dimensional figures and solve real-world problems Understand congruence and similarity using physical models, transparencies, or geometry software	**Essential Questions:** What are transformations and how are they useful in solving real-world problems? How can we prove angle relationships that exist in triangles and in parallel lines? **Learning Targets:** I can identify angle relationships and find missing angles in parallel lines cut by a transversal I can find the sum of interior and exterior angles of a triangle I can rotate, reflect, translate and dilate figures on the coordinate grid I can describe the effects of transforming a figure I can identify and explain why figures are similar I can find missing sides and angles in similar figures	**Formative Assessment:** Informal Classroom Observation Individual Whiteboards Classroom Discussion Think, Pair, Share "Quickwrite" Daily Exit Ticket Weekly Quiz Embedded Assessment **Summative Assessments:** Unit Test Performance Task

Standards Alignment	Unit Concept Big Ideas	Essential Questions Student Learning Targets	Assessments
three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. **CC.8.G.5**			
Unit Five: Pythagorean Theorem & 3-Dimensional Geometry **Timeline: 6 weeks**			
Explain a proof of the Pythagorean Theorem and its converse. **CC.8.G.6** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **CC.8.G.7** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **CC.8.G.8** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. **CC.8.G.9**	The Pythagorean Theorem, the special relationship between side lengths of a right triangle, can be used to find and describe unknown length Surface area and volume of three-dimensional figures is based on other measurements of the figure such as height, base, and parallel bases	**Essential Questions:** How can the Pythagorean Theorem help find unknown lengths in two and three dimensional figures? How do changes in dimensions of a geometric figure affect area, surface area, and volume? **Learning Targets:** I can understand and apply Pythagorean Theorem to find missing side length in a right triangle I can apply Pythagorean Theorem to find distance between two points on the coordinate plane I can apply Pythagorean Theorem to find missing lengths in a 3-D figure I can prove Pythagorean Theorem I can find surface area and volume of cones, cylinders and spheres I can explain how changes in dimensions affect surface area and volume	**Formative Assessments:** Informal Classroom Observation Individual Whiteboards Classroom Discussion Think, Pair, Share "Quickwrite" Daily Exit Ticket Weekly Quiz Embedded Assessment **Summative Assessments:** Unit Test Performance Task

Unit Title:	**Numerical Relationships**
Designed by:	**Caroline Strunk**
District:	**Thomas Edison Charter School, used with permission by the Reach Academy for Girls**
Content Area:	**Mathematics**
Grade Level(s):	**8th Grade**

_____

Summary of Unit

This Unit allows students to explore, understand, and compute with different types of numbers. Specifically, students will work with irrational and rational numbers, numbers in exponential form, and numbers in scientific notation. Although students may have had exposure to some of these number forms before, concepts such as irrational numbers will be completely new. As students work through the unit they will gain an appreciation for where these numbers appear in the real world as well as why it is helpful to express numbers in various forms.

<div align="center">

Stage 1 – Desired Results
What students will know, do, and understand

</div>

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Delaware Content Standards

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. **CC.8.NS.1**

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). _For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations._ **CC.8.NS.2**

Know and apply the properties of integer exponents to generate equivalent numerical expressions. _For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$._ **CC.8.EE.1**

Use square root and cube root symbols to represent solutions to equations of the form $x_2 = p$ and $x_3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **CC.8.EE.2**

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. _For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger._ **CC.8.EE.3**

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. **CC.8.EE.4**

Big Idea(s)

Numbers that are not rational can be given an approximate value.

The properties of number systems and their relationships remain consistent when applied to integer exponents.

Unit Enduring Understanding(s)

At the end of this unit, students will be able to understand and operate with rational/irrational numbers, numbers in exponential form, and numbers in scientific notation. Furthermore, students will be able to explain where numbers in these forms are often found in the real world, and why it is helpful to write numbers in various forms.

Unit Essential Questions(s)

Why is it important to understand the procedures for working with different kinds of numbers? Why is scientific notation necessary and helpful for expressing large and small numbers?

Knowledge and Skills

Students will know…
The following vocabulary:
- Exponent
- Base
- Rational Numbers
- Irrational Numbers
- Scientific Notation

Students will be able to…
- Know and apply properties of exponents
- Evaluate numbers in exponential form with exponents that are zero, one, or negative
- Understand and apply square roots and cube roots to solve algebraic equations
- Articulate the difference between rational and irrational numbers
- Approximate the value of an irrational square root
- Write small and large numbers in scientific notation
- Choose units of appropriate size for measurements of very large or very small quantities
- Operate with numbers in scientific notation

Stage 2 – Assessment Evidence
Evidence that will be collected to determine whether or not Desired Results are achieved

_____

Suggested Performance/Transfer Task(s)

On May 15, 2005, a group of volunteers met in Las Vegas to create what would become the world's largest cake. In order for the cake to qualify for the *Guinness Book of World Records*, it had to meet the following qualifications:
- Contain traditional ingredients in the correct proportions
- Be prepared in the same manner as a normal-sized cake

- Be prepared according to appropriate hygiene standards and
- Be totally edible and safe to eat

To create the world's largest cake, the organizers had to begin with a recipe for one cake and scale the ingredients accordingly. A sample recipe for one cake is shown below.

2 cups flour
1 1/3 tablespoon baking powder
¼ teaspoon salt
2/3 cup sugar
2 eggs
¾ cup milk
½ teaspoon vanilla

1. If a volunteer prepared for 15 cakes, determine the amount of sugar and milk needed.

2. Are the amounts of the ingredients used rational or irrational numbers? Explain your reasoning.

3. The finished cake weighed in at 130,000 pounds, breaking the world record of 128,000 pounds.
   a. Express each of these values in scientific notation

   b. Explain the process you would use to find the product of the numbers in scientific notation.

4. If cake A weighs 3×10^6 ounces and cake B weighs 7×10^7 ounces, how many times as heavy as cake A is cake B?

Rubric for Performance Task

	4	3	2	1
Calculations	All calculations are correct	Most calculations are correct with one exception	Attempts calculations but makes more than one mistake	Few calculations are correct
Explanation	Explanations are clear, concise and use mathematical language	Explanations are clear and concise	Attempts to explain but explanations are incomplete	The student's explanations contain conceptual misunderstandings

Other Evidence

- Informal Classroom Observation
- Individual/Group Whiteboards
- Classroom Discussion
- Think, Pair, Share
- "Quickwrite"
- Daily Exit Ticket
- Weekly Quiz
- End of Unit Test (attached)

Student Self-Assessment and Reflection

Students will have the opportunity to reflect on their learning at the end of the unit before the unit test. As part of the reflection, the students will answer the essential questions, will define vocabulary words, will reflect on concepts which they feel strongest and weakest on, and will relate concepts in the unit to the real world. The self-reflection will conclude the unit, and will also help the teacher to plan appropriate review activities before the test.

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Key learning events needed to achieve unit goals

Lesson 1
Objective: I can apply the properties of exponents
Resource: SpringBoard Middle School Math 3
Vocabulary: Exponent, Base

1. Hook: Students will read the riddle "As I was going to St. Ives" and start discussing possible answers to the riddle.
2. Activity: The riddle leads into a discussion of exponents. The teacher will review the meaning of exponent and base and students will post the words on the interactive word wall. Next, students will review writing exponents in expanded form, exponential form and standard form. Following this review, students work on a guided inquiry activity leading them to understand the properties of multiplying two numbers in exponential form, dividing two numbers in exponential form, and multiplying a power by another power.
3. Classroom Debrief: Teacher will lead classroom discussion to ensure that all students understand the properties of exponents.
4. Independent Practice: Students will apply rules independently
5. Exit Slip: Fink check to assess mastery

Lesson 2

Objective: I can simplify numbers in exponential form with exponents including zero and negative numbers.

1. Warm-Up: Students will apply properties of exponents
2. Activity: Students will work in groups of three on a guided inquiry activity in which they will discover why a number raised to the power of 0 is one, and will learn that a number raised to a negative exponent is a fraction. The teacher will lead a classroom debrief discussion to ensure that all students understand the concept.
3. Guided Practice: Students will work on individual white boards to complete problems involving exponents with zero and one while also applying properties of exponents learned in Lesson 1.
4. Exit Slip: Students will complete a few problems to assess final mastery

Lesson 3

Objective: I can explain square roots and cubed roots as the inverse of squaring and cubing
   I can solve algebraic equations involving squares and cubes

1. Warm-Up: Apply properties of exponents, review how inverse operations are used to solve equations
2. Direct Instruction: Teacher will talk about inverse operations used in algebra to undo multiplication or addition, and will explain that the inverse operation of squaring a number

is taking a square root. Similarly, the inverse operation of cubing a number is taking the cube root. Teacher will model several problems on the board showing how to solve algebraic equations by using square roots and cube roots.

3. Guided Practice: Students will work on white boards to find the solution to an equation such as $x^2 = 25$. Teacher will include some rational and irrational answers, and ask students to make observations about the answers as a precursor to the next lesson on rational/irrational numbers.

4. Independent Practice/Exit Slip: Students will practice independently and complete a few problems to assess final mastery.

Lesson 4

Objective: I can articulate the difference between rational and irrational numbers
Resource: SpringBoard Middle School Math 3
Vocabulary: Rational and Irrational Numbers

1. Warm-Up: Solve algebraic equations using square roots and cube roots
2. Hook: A piece of paper can theoretically be folded an infinite number of times. This will lead to a discussion about irrational numbers.
3. Direct Instruction: Teacher will discuss the difference between rational and irrational numbers. Students will add rational and irrational numbers to the interactive word wall.
4. Guided Practice: Students will take turns coming to the interactive SmartBoard to classify numbers as "rational" or "irrational." Students will need to explain their thinking to the class as they make their decision.
5. Independent Practice: Students will classify numbers as rational or irrational and explain their thinking.
6. Exit Slip: Students will complete a few problems to assess final mastery.

Lesson 5

Objective: I can approximate the value of an irrational square root
Resource: SpringBoard Middle School Math 3

1. Warm-Up: Students will articulate the difference between rational and irrational numbers and give an example of each
2. Guided Inquiry Activity: Students will work through a guided inquiry activity in which they will learn how to estimate the value of an irrational square root by using perfect squares. Teacher will monitor the groups and assist as needed. After approximating the value of the square root, students will plot the values on a number line.
3. Guided Practice: Students will work on white boards to practice approximating the value of a square root and representing the number on the number line.
4. Independent Practice/Exit Slip: Final practice and final check for mastery

Lesson 6

Objective: I can write small and large numbers in scientific notation
Resource: SpringBoard Middle School Math 3
Vocabulary: Scientific Notation

1. Warm-Up: Approximating values of irrational square roots
2. Direct Instruction: Teacher will introduce scientific notation as a way to simplify numbers that are very large or very small. Students will add the word scientific notation to the interactive word wall. Teacher will model that scientific notation involves multiplying a number by factors of 10 and model an example.
3. Guided Practice: Students will convert numbers form standard from into scientific notation
4. Guided Inquiry: Students will discover how scientific notation is represented differently for large numbers and small numbers by comparing and contrasting numbers in scientific notation in a table. Students will draw conclusions from the activity to learn how to write small numbers in scientific notation. Teacher will lead a quick classroom discussion to ensure that all students have reached this generalization.
5. Independent Practice: Students will practice converting large and small numbers to scientific notation
6. Exit Slip: Final check for mastery

Lesson 7

Objective: I can choose units of appropriate size for measurements of very large or very small quantities

1. Warm-Up: Converting numbers in scientific notation
2. Activity: Students will explore the scale of the Universe Applet online (http://www.newgrounds.com/portal/view/525347) and will complete corresponding activity. Students will investigate and record the sizes of various objects and planets in the Universe, taking special note of the units used to measure those objects. Based on the investigation, students will need to predict the sizes of other objects and defend why their prediction is accurate. Teacher will lead a classroom discussion to debrief predictions and make sure that student thinking is reasonable.
3. Independent Practice/Exit Slip: Students will predict an appropriate size and measurement for objects and defend their reasoning.

Lesson 8

Objective: I can multiply and divide numbers in scientific notation
Resource: SpringBoard Middle School Math 3

1. Warm-Up: Choosing a reasonable unit measurement for real-world objects, review of commutative property & properties of exponents
2. Activity: Students will work on an inquiry-based activity that guides students to understand how to multiply numbers in scientific notation based on the commutative property and properties of exponents. Students will also build on knowledge of properties of exponents to learn how to divide numbers in scientific notation. Teacher will lead a debrief discussion to ensure that all students understand how to multiply and divide numbers in scientific notation.
3. Guided Practice: Students will work on white boards to practice multiplying and dividing numbers in scientific notation
4. Independent Practice/Exit Slip: Individual practice and final check for mastery

Resources and Teaching Tips

Resources

- Spring Board Middle School Mathematics 3 Text book
- Teacher created activities, quizzes, and worksheets, as necessary
- Scale of the Universe Applet
- SMART Board
- Individual/Group White Boards

Teaching Tips

- It is important when teaching scientific notation that students understand that scientific notation is another way to write a number multiplied by factors of 10 repeatedly. This is important so that students do not get caught up in "tricks' of converting scientific notation while lacking a conceptual understanding.
- Some students may have knowledge gaps in their understanding of measurement. Therefore, it may be necessary to review basic units of measurement before beginning Lesson 7.

Differentiation

- During the classroom activities, students will be given optional extension questions if they finish before the rest of the class
- Students will work with partners and groups to provide additional support while learning
- Teacher will spend time strategically with groups that require additional modeling and support during guided inquiry activities
- Classroom discussion will benefit auditory learners
- Visual learners will benefit from the reading and writing during the activities
- Kinesthetic learners will benefit form the paper-folding irrational numbers activity, and the online investigative activity

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

21st Century Learning

This unit focuses on teaching students primarily through guided inquiry lessons. The lessons require students to construct their own understanding by beginning with prior knowledge and completing scaffolded questions to come to a new understanding. Students then apply what they learn to new questions and situations in the independent practice. A strong emphasis on guided inquiry requires students to be active learners throughout class in addition to working cooperatively with classmates.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

Technology is integrated throughout the unit in different ways. First, students will interact with the SMART Board during the warm up each day as we review the warm up problems. Students will also have the opportunity to interact with technology during the guided practice in Lesson 4 as students learn the difference between rational and irrational numbers. Finally, in order to learn about appropriate units of measurement, students will explore an online applet about the relative size of objects in the universe.

Content Connections

Content Standards integrated within instructional strategies

Use of vocabulary, reading, and writing is embedded within all of the activities.

Students will learn about science as we discuss scientific notation. For example, in order for students to conceptualize small numbers in scientific notation we will talk about cells, and we will talk about the size of planets in the universe in order for them to conceptualize large numbers in scientific notation.

Name:

Date:

Numerical Relationships- Unit 1 Test

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number. **CC.8.NS.1**

1.

Which of the following is *not* a rational number?

F -0.75

G 0

H $\sqrt{4}$

J $\sqrt{15}$

2. Decide whether each of the following numbers is rational or irrational. If it is rational, explain how you know.

a. 0.333..

c. π=3.141592…

b. √2=1.414213…

d. $\dfrac{1}{7}$ = 0.142857

Use rational approximations of irrational numbers to compare the size of irrational numbers locate them approximately on a number line diagram and estimate the value of expressions (e.g. π^2). *For example by truncating the decimal expansion of $\sqrt{2}$ show that $\sqrt{2}$ is between 1 and 2 then between 1.4 and 1.5 and explain how to continue on to get better approximations.*
CC.8.NS.2

3.

Which of the following lists is in order
from least to greatest?

A. $\sqrt{3}$, π, 3
B. $\sqrt{3}$, 3, π
C. 3, π, $\sqrt{3}$
D. 3, $\sqrt{3}$, π

4.

Between which two whole numbers is $\sqrt{33}$?
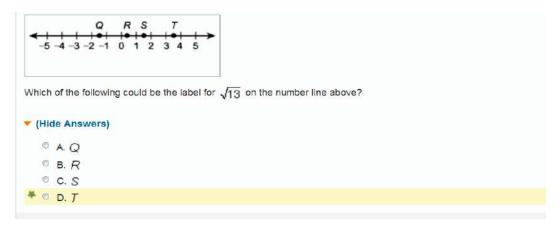
F   32 and 34
G   16 and 17
H   6 and 7
J   5 and 6

5.

Which of the following inequalities
is true?

A. $\sqrt{3} > \pi$
B. $\sqrt{9} > \pi$
C. $\sqrt{5} > 2$
D. $\sqrt{6} > 3$

6.

Which of the following could be the label for $\sqrt{13}$ on the number line above?

▼ (Hide Answers)

   ○ A. Q
   ○ B. R
   ○ C. S
✸ ○ D. T

Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^5 = 3^3 = 1/3^3 = 1/27$.* **CC.8.EE.1**

7. Which of the following is equal to $\dfrac{5^6}{5^2}$?

A. 5^3

B. 5^4

C. 5^8

D. 5^{12}

8. Which of the following gives the general rule for simplifying the product of two powers with the same base?

A. Multiply the bases, and multiply the exponents
B. Multiply the bases, and add the exponents.
C. Keep the base the same, and multiply the exponents
D. Keep the base the same, and add the exponents.

9. Which of the following is equal to $4^3 \cdot 4^5$?

A. 4^3 B. 4^5

C. 4^2 D. 4^8

10.

Evaluate 11^{-2}.

A. $-\dfrac{1}{121}$ B. 121

C. -121 D. $\dfrac{1}{121}$

Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$ where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **CC.8.EE.2**

11. Without a calculator, find the value of x in each equation below.

a) $x^2 = 25$ b) $x^2 = 100$

c) $x^3 = 27$ d) $x^3 = 125$

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. *For example estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 and determine that the world population is more than 20 times larger.* **CC.8.EE.3**

12. Which of the following represents the value of the number pointed to by the arrow?

450,384,000

A. 8×10^4

B. 8

C. 84

D. 8×100

13. Which of the following is a reasonable prediction for the population of the United States?

    A. 1×10^2

    B. 3×10^{-8}

    C. 3×10^8

    D. 8×10^3

14. Which of the following is 20 times larger than 3×10^3?

A. 60×10

B. 6×10^3

C. 60×10^2

D. 6×10^4

Perform operations with numbers expressed in scientific notation including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or v very small quantities (e.g. use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. **CC.8.EE.4**

15. **Which of the following is a correct restatement of the product**
$(3.1 \times 10^4) \cdot (7.58 \times 10^2)$?

A. $(3.1 \times 7.58) \cdot (10^4 \times 10^2)$

B. $(3.1 + 7.58) \cdot (10^4 \times 10^2)$

C. $(3.1 \times 7.58) \cdot (10^4 + 10^2)$

D. $(3.1 + 7.58) \cdot (10^4 + 10^2)$

16. Simplify $\dfrac{5.0 \times 10^8}{2.5 \times 10^3}$

A. 2.0×10^{11}

B. 2.0×10^5

C. 12.5×10^5

17.

A new computer can perform 6.0×10^8 computations in 1 second. How many computations can the computer perform in 60 seconds?

A. 1.0×10^7

B. 1.0×10^9

C. 3.6×10^9

D. 3.6×10^{10}

18. Which of the following is the same as 3.5 x 10^{-3}?

A) 3,500
B) .0035
C) .00045
D. 35,000