FACTORS, ROOTS, AND ZEROES

Solving a Polynomial Equation

Rearrange the terms to have zero on one side:

$$x^2 + 2x = 15 \quad \Rightarrow \quad x^2 + 2x - 15 = 0$$

Factor:

Set each factor equal to zero and solve:

The only way that $x^2 + 2x - 15$ can $= 0$ is if $x = -5$ or $x = 3$

Zeros of a Polynomial Function

A Polynomial Function is usually written in function notation or in terms of x and y.

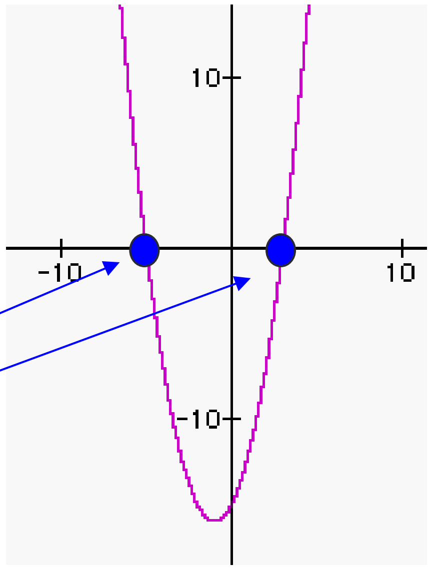
$$f(x) = x^2 + 2x - 15 \quad \text{or} \quad y = x^2 + 2x - 15$$

The Zeros of a *Polynomial Function* are the *solutions* to the equation you get when you set the polynomial equal to zero.

Graph of a Polynomial Function

Here is the graph of our polynomial function:

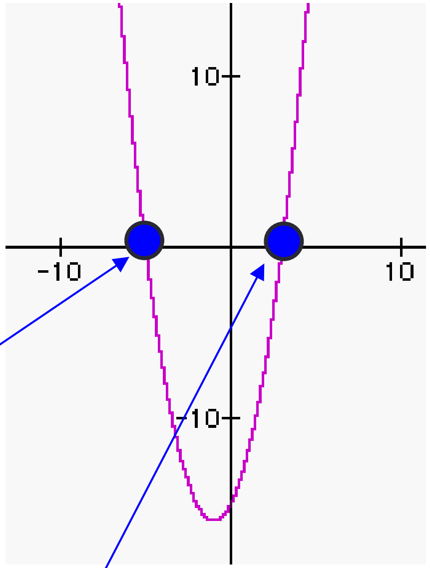
$$y = x^2 + 2x - 15$$



The *Zeros* of the Polynomial are the values of x when the polynomial equals zero. In other words, the *Zeros* are the x-values where *y equals zero*.

x-Intercepts of a Polynomial

The points where y = 0 are called the *x*-intercepts of the graph.



The *x*-intercepts for our graph are the points...

(-5, 0) and (3, 0)

Factors, Roots, Zeros

For our *Polynomial Function*:

$$y = x^2 + 2x - 15$$

The ***Factors*** are: $(x + 5)$ & $(x - 3)$

The ***Roots/Solutions*** are: $x = -5$ and 3

The ***Zeros*** are at: $(-5, 0)$ and $(3, 0)$

Factor the following. . . .(Hint: use factoring by grouping.)

$x^3 – 12x^2 – 4x + 24 = 0$

This equation factors to:

$(x - 2)(x + 2)(x - 12)= 0$

The roots therefore are: -2, 2, 12

Take a closer look at the original equation and our roots:

$x^3 - 5x^2 - 2x + \underline{24} = 0$

The roots therefore are: $\underline{-2, 2, 12}$

What do you notice?

-2, 2, and 12 all go into the last term, 24!

This leads us to the Rational Root Theorem

$$Y = a_0 x^n + a_1 x^{n-1} + \ldots + a_{n-1} x + a_n$$

- For a polynomial,
- If p/q is a root of the polynomial,
- then p is a factor of an
- and q is a factor of a_0

Example (RRT)

1. For polynomial $x^3 + x^2 - 3x - 3 = 0$ Here p = -3 and q = 1

Possible roots are $\dfrac{\text{Factors of -3}}{\text{Factors of 1}} \rightarrow \dfrac{\pm 3, \pm 1}{\pm 1}$ Or 3,-3, 1, -1

2. For polynomial $3x^3 + 9x^2 + 4x + 12 = 0$ Here p = 12 and q = 3

Possible roots are $\dfrac{\text{Factors of 12}}{\text{Factors of 3}} \rightarrow \dfrac{\pm 12, \pm 6, \pm 3, \pm 2, \pm 1 \pm 4}{\pm 1, \pm 3}$

Or $\underbrace{\pm 12, \pm 4, \pm 6, \pm 2, \pm 3, \pm 1, \pm 2/3, \pm 1/3, \pm 4/3}$

Wait a second . . . Where did all of these come from???

Let's look at our solutions

<u>±12, ±6 , ±3 , ± 2 , ±1, ±4</u>
±1 , ±3

Note that $\pm$ 2 is listed twice; we only consider it as one answer

Note that $\pm$ 1 is listed twice; we only consider it as one answer

Note that $\pm$ 4 is listed twice; we only consider it as one answer

$$\frac{\pm 12}{1} = \pm 12$$
$$\frac{\pm 6}{1} = \pm 6$$
$$\frac{\pm 3}{1} = \pm 3$$
$$\frac{\pm 2}{1} = \pm 2$$
$$\frac{\pm 1}{1} = \pm 1$$
$$\frac{\pm 4}{1} = \pm 4$$

$$\frac{\pm 12}{3} = \pm 4$$
$$\frac{\pm 6}{3} = \pm 2$$
$$\frac{\pm 3}{3} = \pm 1$$
$$\frac{\pm 2}{3} = \pm \frac{2}{3}$$
$$\frac{\pm 1}{3} = \pm \frac{1}{3}$$
$$\frac{\pm 4}{3} = \pm \frac{4}{3}$$

That is where our 9 possible answers come from!

Let's Try One

Find the POSSIBLE roots of $x^3 - 5x^2 - 4x + 20=0$

Identify p = $\underline{+}\,20,\ \underline{+}\,10,\ \underline{+}\,5,\ \underline{+}\,4,\ \underline{+}\,2,\ \underline{+}\,1$

Identify q = $\underline{+}\,1$

Identify all possible combinations of p/q = $\underline{+}\,20,\ \underline{+}\,10,\ \underline{+}\,5,\ \underline{+}\,4,\ \underline{+}\,2,\ \underline{+}\,1$

That's a lot of answers!

- Obviously $x^3 - 5x^2 - 4x + 20=0$ does not have all of those roots as answers.

- Remember: these are only **POSSIBLE** roots. We take these roots and figure out what answers actually WORK.

Steps for solving polynomial equations.

1. Develop your possible roots using p/q
2. Use synthetic division with your possible roots to find an actual root. If you started with a 4^{th} degree, that makes the dividend a cubic polynomial.
3. Continue the synthetic division trial process with the resulting cubic. Don't forget that roots can be used more than once.
4. Once you get to a quadratic, use factoring techniques or the quadratic formula to get to the other two roots.

Let's Try One

Find the roots of $2x^3 - x^2 + 2x - 1$

Take this in parts. First find the possible roots. Then determine which root actually works.

Using the Polynomial Theorems

FACTOR and SOLVE $x^3 - 5x^2 + 8x - 6 = 0$

Find p and q

- p = -6
- q = 1

By RRT, the only rational root is of the form…

- $\dfrac{\text{Factors of } p}{\text{Factors of } q}$

Using the Polynomial Theorems

FACTOR and SOLVE $x^3 - 5x^2 + 8x - 6 = 0$

Factors

- Factors of -6 = ±1, ±2, ±3, ±6
  Factors of 1 = ±1

Possible roots

- -6, 6, -3, 3, -2, 2, 1, and -1

Using the Polynomial Theorems

FACTOR and SOLVE $x^3 - 5x^2 + 8x - 6 = 0$

Test each root

X	$x^3 - 5x^2 + 8x - 6$
-6	-450
6	78
3	0 THIS IS YOUR ROOT
-3	-102
2	-2
-2	-50
1	-2
-1	-20

Synthetic division

$$\begin{array}{c|ccc|c} \boxed{3} & 1 & -5 & 8 & -6 \\ & \downarrow & 3 & -6 & 6 \\ \hline & 1 & -2 & 2 & 0 \\ & \downarrow & \downarrow & \downarrow & \end{array}$$

$1x^2 + -2x + 2$

Using the Polynomial Theorems

FACTOR and SOLVE $x^3 - 5x^2 + 8x - 6 = 0$

Rewrite

- $x^3 - 5x^2 + 8x - 6$

$= (x - 3)(x^2 - 2x + 2)$

Factor more and solve

- $(x - 3)(x^2 - 2x + 2)$

$\downarrow$ $\downarrow$

X= 3 Quadratic Formula

$$x = 1 \pm i$$

- Roots are $3,\ 1 \pm i$

Find each of the roots, classify them and show the factors.

Example: $x^3 - 5x^2 - 4x + 20$

p/q = $\underline{+}20, \underline{+}10, \underline{+}5, \underline{+}4, \underline{+}2, \underline{+}1$

$$\begin{array}{r|rrrr} 2 & 1 & -5 & -4 & 20 \\ & & 2 & -6 & -20 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

$$x^2 - 3x - 10$$
$$(x-5)(x+3)$$
$$(x-2)(x-5)(x+3)$$

Roots are x = 2 (Rational), x=5 (Rational), x= -3 (Rational)