Quadratic models

Warm Up

Solve each system of equations.

1. $\begin{cases} 3a + b = -5 \\ 2a - 6b = 30 \end{cases}$ $a = 0, b = -5$

2. $\begin{cases} 9a + 3b = 24 \\ a + b = 6 \end{cases}$ $a = 1, b = 5$

3. $\begin{cases} 4a - 2b = 8 \\ 2a - 5b = 16 \end{cases}$ $a = \frac{1}{2}, b = -3$

Objectives

Use quadratic functions to model data.

Use quadratic models to analyze and predict.

Vocabulary

quadratic model

quadratic regression

Recall that you can use differences to analyze patterns in data. For a set of ordered parts with equally spaced *x*-values, a quadratic function has constant nonzero second differences, as shown below.

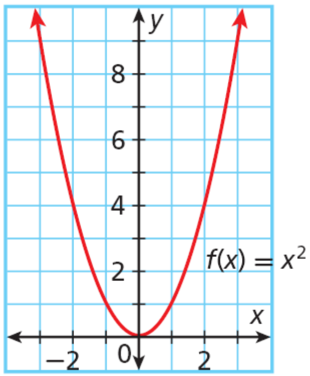
Equally spaced *x*-values

x	−3	−2	−1	0	1	2	3
$f(x) = x^2$	9	4	1	0	1	4	9

1st differences: −5, −3, −1, 1, 3, 5

2nd differences: 2, 2, 2, 2, 2

Constant 2nd differences



Example 1A: Identifying Quadratic Data

Determine whether the data set could represent a quadratic function. Explain.

x	1	3	5	7	9
y	–1	1	7	17	31

Find the first and second differences.

Equally spaced x-values

x	1	3	5	7	9
y	–1	1	7	17	31

1st 2 6 10 14

2nd 4 4 4

Quadratic function: second differences are constant for equally spaced x-values

Example 1B: Identifying Quadratic Data

Determine whether the data set could represent a quadratic function. Explain.

x	3	4	5	6	7
y	1	3	9	27	81

Find the first and second differences.

Equally spaced *x*-values

x	3	4	5	6	7
y	1	3	9	27	81

1st 2 6 18 54

2nd 4 12 36

Not a Quadratic function: second differences are not constant for equally spaced x-values

Check It Out! Example 1a

Determine whether the data set could represent a quadratic function. Explain.

x	3	4	5	6	7
y	11	21	35	53	75

Find the first and second differences.

Equally spaced *x*-values

x	3	4	5	6	7
y	11	21	35	53	75

1st 10 14 18 22

2nd 4 4 4

Quadratic function: second differences are constant for equally spaced *x*-values

Check It Out! Example 1b

Determine whether the data set could represent a quadratic function. Explain.

x	10	9	8	7	6
y	6	8	10	12	14

Find the first and second differences.

Equally spaced *x*-values

x	10	9	8	7	6
y	6	8	10	12	14

1st	2	2	2	2
2nd	0	0	0	

Not a quadratic function: first differences are constant so the function is linear.

Just as two points define a linear function, three noncollinear points define a quadratic function. You can find three coefficients *a*, *b*, and *c*, of $f(x) = ax^2 + bx + c$ by using a system of three equations, one for each point. The points do not need to have equally spaced *x*-values.

Reading Math

Collinear points lie on the same line. *Noncollinear* points *do not* all lie on the same line.

Example 2: Writing a Quadratic Function from Data

Write a quadratic function that fits the points (1, –5), (3, 5) and (4, 16).

Use each point to write a system of equations to find a, b, and c in $f(x) = ax^2 + bx + c$.

(x, y)	$f(x) = ax^2 + bx + c$	System in a, b, c
(1, –5)	$-5 = a(1)^2 + b(1) + c$	$a + b + c = -5$ ❶
(3, 5)	$5 = a(3)^2 + b(3) + c$	$9a + 3b + c = 5$ ❷
(4, 16)	$16 = a(4)^2 + b(4) + c$	$16a + 4b + c = 16$ ❸

Example 2 Continued

Subtract equation ❷ by equation ❶ to get ❹.

❷ $9a + 3b + c = 5$

❶ $a + b + c = -5$

❹ $8a + 2b + 0c = 10$

Subtract equation ❸ by equation ❶ to get ❺.

❸ $16a + 4b + c = 16$

❶ $a + b + c = -5$

❺ $15a + 3b + 0c = 21$

Example 2 Continued

Solve equation ❹ and equation ❺ for *a* and *b* using elimination.

❺ $2(15a + 3b = 21)$	→	$30a + 6b = 42$	*Multiply by 2.*
❹ $-3(8a + 2b = 10)$	→	$-24a - 6b = -30$	*Multiply by –3.*
		$6a + 0b = 12$	*Subtract.*
		$a = 2$	*Solve for a.*

Example 2 Continued

Substitute 2 for a into equation ❹ or equation ❺ to get b.

$$8(2) + 2b = 10$$
$$2b = -6$$
$$b = -3$$

$$15(2) + 3b = 21$$
$$3b = -9$$
$$b = -3$$

Example 2 Continued

Substitute $a = 2$ and $b = -3$ into equation ❶ to solve for c.

$$(2) + (-3) + c = -5$$

$$-1 + c = -5$$

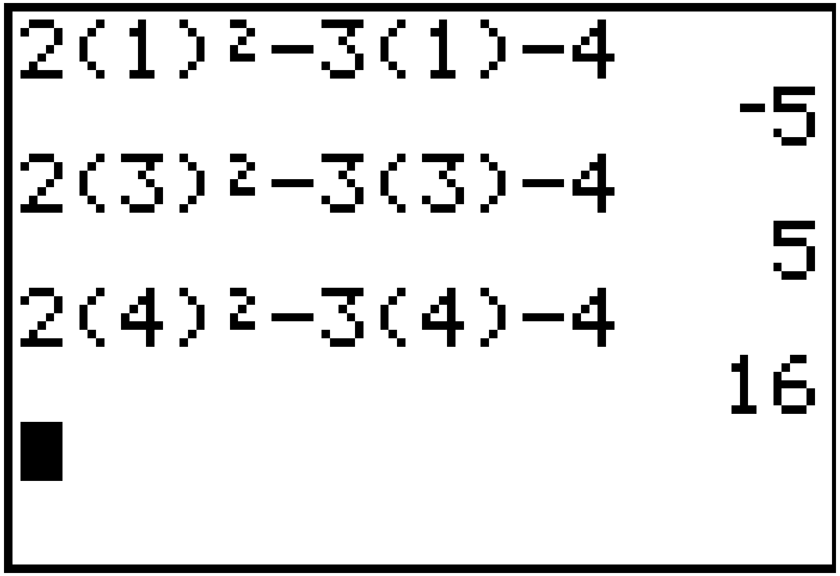
$$c = -4$$

Write the function using $a = 2$, $b = -3$ and $c = -4$.

$$f(x) = ax^2 + bx + c \longrightarrow f(x) = 2x^2 - 3x - 4$$

Example 2 Continued

Check Substitute or create a table to verify that (1, –5), (3, 5), and (4, 16) satisfy the function rule.



Check It Out! Example 2

Write a quadratic function that fits the points (0, –3), (1, 0) and (2, 1).

Use each point to write a system of equations to find a, b, and c in $f(x) = ax^2 + bx + c$.

(x,y)	$f(x) = ax^2 + bx + c$	System in a, b, c	
(0, –3)	$-3 = a(0)^2 + b(0) + c$	$c = -3$	❶
(1, 0)	$0 = a(1)^2 + b(1) + c$	$a + b + c = 0$	❷
(2, 1)	$1 = a(2)^2 + b(2) + c$	$4a + 2b + c = 1$	❸

Check It Out! Example 2 Continued

Substitute $c = -3$ from equation ❶ into both equation ❷ and equation ❸.

❷ $a + b + c = 0$

$a + b - 3 = 0$

$a + b = 3$ ❹

❸ $4a + 2b + c = 1$

$4a + 2b - 3 = 1$

$4a + 2b = 4$ ❺

Check It Out! Example 2 Continued

Solve equation ❹ and equation ❺ for b using elimination.

❹ $4(a + b) = 4(3)$ → $4a + 4b = 12$ *Multiply by 4.*

❺ $4a + 2b = 4$ → $-(4a + 2b = 4)$ *Subtract.*

$0a + 2b = 8$ *Solve for b.*

$b = 4$

Check It Out! Example 2 Continued

Substitute 4 for *b* into equation ❹ or equation ❺ to find *a.*

❹ $a + b = 3$

$a + 4 = 3$

$a = -1$

❺ $4a + 2b = 4$

$4a + 2(4) = 4$

$4a = -4$

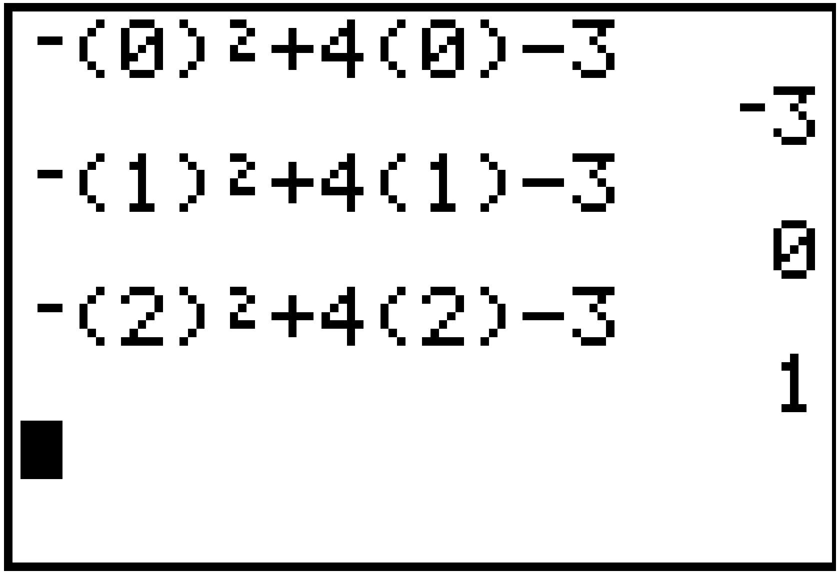
$a = -1$

Write the function using $a = -1$, $b = 4$, and $c = -3$.

$$f(x) = ax^2 + bx + c \longrightarrow f(x) = -x^2 + 4x - 3$$

Check It Out! Example 2 Continued

Check Substitute or create a table to verify that (0, –3), (1, 0), and (2, 1) satisfy the function rule.



You may use any method that you studied in Chapters 3 or 4 to solve the system of three equations in three variables. For example, you can use a matrix equation as shown.

$$\begin{cases} c = 5 \\ 4a + 2b + c = 1 \\ 9a + 3b + c = 2 \end{cases} \rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 5 \end{bmatrix}$$

A **quadratic model** is a quadratic function that represents a real data set. Models are useful for making estimates.

In Chapter 2, you used a graphing calculator to perform a *linear regression* and make predictions. You can apply a similar statistical method to make a quadratic model for a given data set using **quadratic regression**.

Helpful Hint

The coefficient of determination R^2 shows how well a quadratic function model fits the data. The closer R^2 is to 1, the better the fit. In a model with $R^2 \approx 0.996$, which is very close to 1, the quadratic model is a good fit.

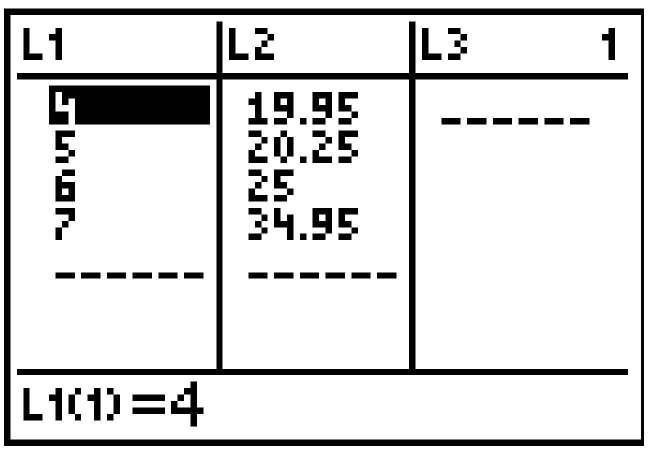
Example 3: Consumer Application

The table shows the cost of circular plastic wading pools based on the pool's diameter. Find a quadratic model for the cost of the pool, given its diameter. Use the model to estimate the cost of the pool with a diameter of 8 ft.

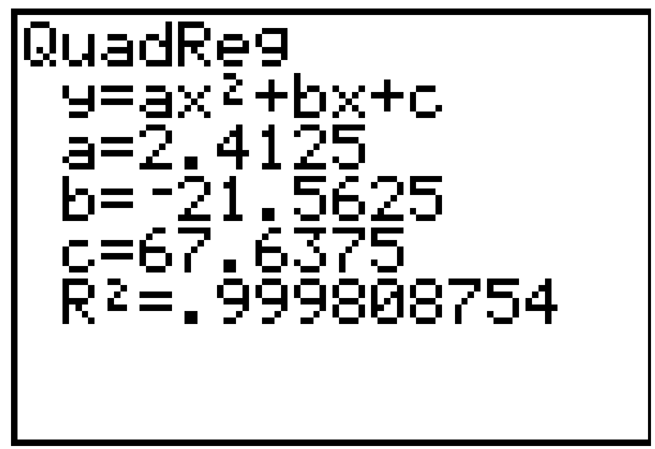
Diameter (ft)	4	5	6	7
Cost	$19.95	$20.25	$25.00	$34.95

Example 3 Continued

Step 1 Enter the data into two lists in a graphing calculator.

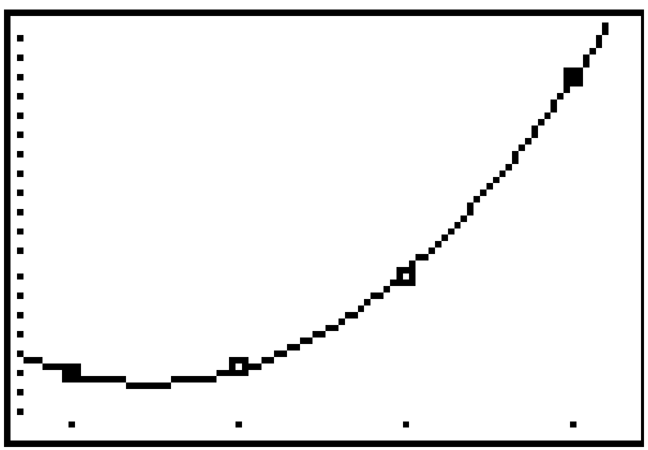


Step 2 Use the quadratic regression feature.

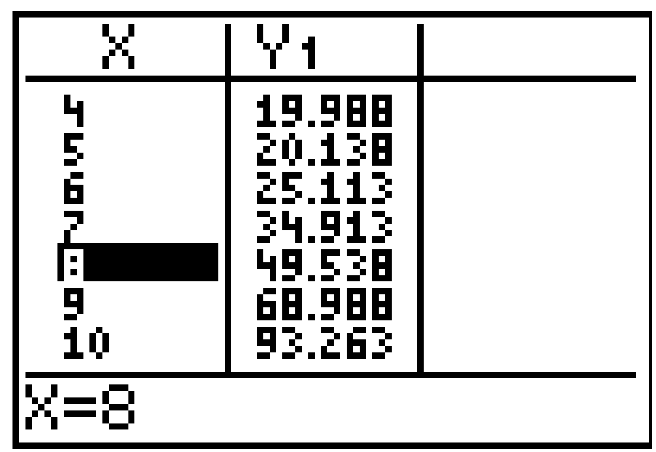


Example 3 Continued

Step 3 Graph the data and function model to verify that the model fits the data.



Step 4 Use the table feature to find the function value x = 8.



Example 3 Continued

A quadratic model is $f(x) \approx 2.4x^2 - 21.6x + 67.6$, where x is the diameter in feet and $f(x)$ is the cost in dollars. For a diameter of 8 ft, the model estimates a cost of about $49.54.

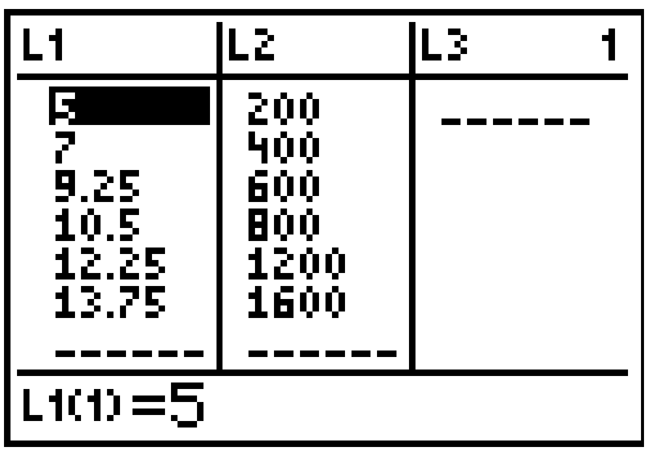
Check It Out! Example 3

The tables shows approximate run times for 16 mm films, given the diameter of the film on the reel. Find a quadratic model for the reel length given the diameter of the film. Use the model to estimate the reel length for an 8-inch-diameter film.

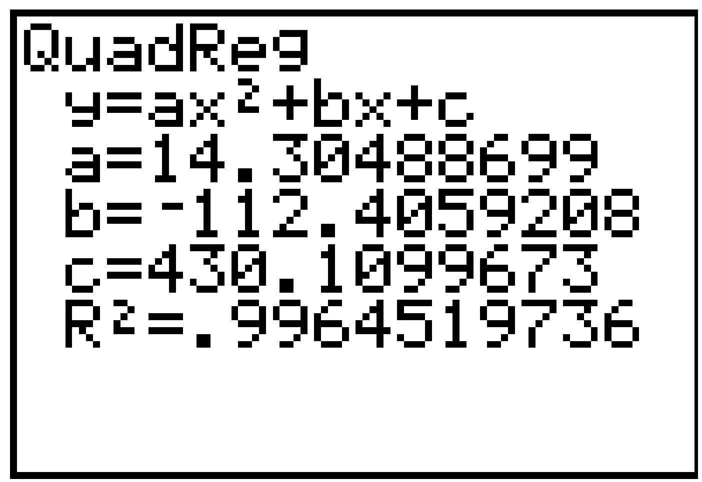
Film Run Times (16 mm)		
Diameter (in)	Reel Length (ft)	Run Time (min)
5	200	5.55
7	400	11.12
9.25	600	16.67
10.5	800	22.22
12.25	1200	33.33
13.75	1600	44.25

Check It Out! Example 4 Continued

Step 1 Enter the data into two lists in a graphing calculator.

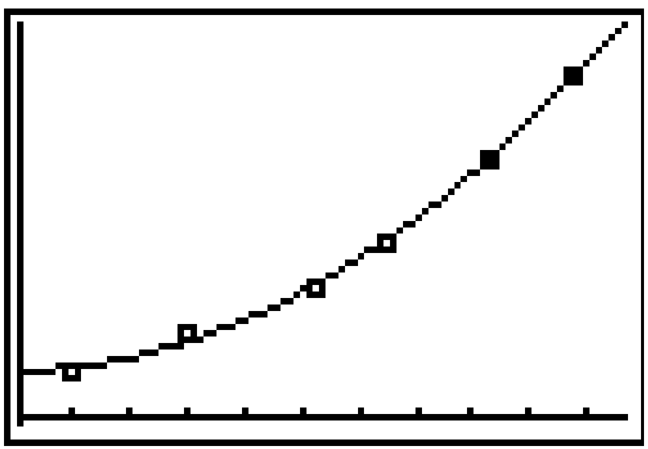


Step 2 Use the quadratic regression feature.



Check It Out! Example 4 Continued

Step 3 Graph the data and function model to verify that the model fits the data.



Step 4 Use the table feature to find the function value *x* = 8.

X	Y_1
4	209.36
5	225.7
6	270.65
7	344.21
8	446.38
9	577.15
10	736.54

X=8

Check It Out! Example 4 Continued

A quadratic model is $L(d) \approx 14.3d^2 - 112.4d + 430.1$, where d is the diameter in inches and $L(d)$ is the reel length. For a diameter of 8 in., the model estimates the reel length to be about 446 ft.

Lesson Quiz: Part I

Determine whether each data set could represent a quadratic function.

1.

x	5	6	7	8	9
y	5	8	13	21	34

not quadratic

2.

x	2	3	4	5	6
y	1	11	25	43	65

quadratic

3. Write a quadratic function that fits the points (2, 0), (3, –2), and (5, –12).

$f(x) = -x^2 + 3x - 2$

Lesson Quiz: Part II

4. The table shows the prices of an ice cream cake, depending on its side. Find a quadratic model for the cost of an ice cream cake, given the diameter. Then use the model to predict the cost of an ice cream cake with a diameter of 18 in.

Diameter (in.)	**Cost**
6	\$7.50
10	\$12.50
15	\$18.50

$f(x) \approx -0.011x^2 + 1.43x - 0.67;$

$\approx \$21.51$