Pythagorean Theorem

Warm Up

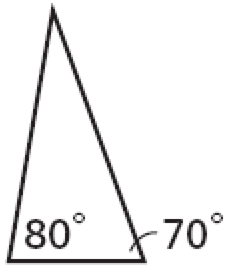
Lesson Presentation

Lesson Quiz

Warm Up

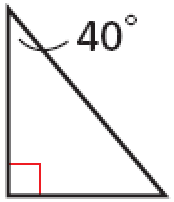
Classify each triangle by its angle measures.

1.



acute

2.



right

3. Simplify $\left(2\sqrt{3}\right)^2$. 12

4. If a = 6, b = 7, and c = 12, find $a^2 + b^2$ and find c^2. Which value is greater?

85; 144; c^2

Objectives

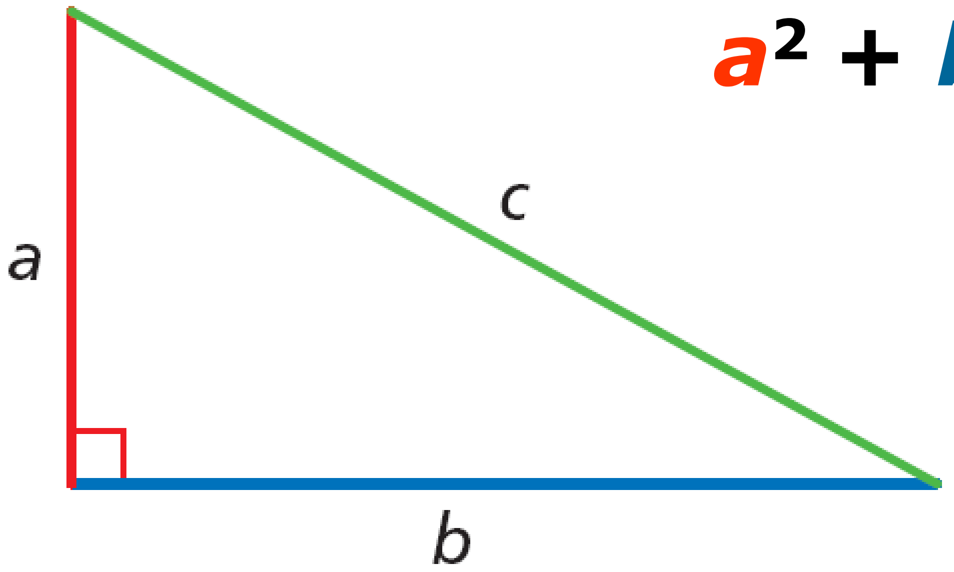
Use the Pythagorean Theorem and its converse to solve problems.

Use Pythagorean inequalities to classify triangles.

Vocabulary

Pythagorean triple

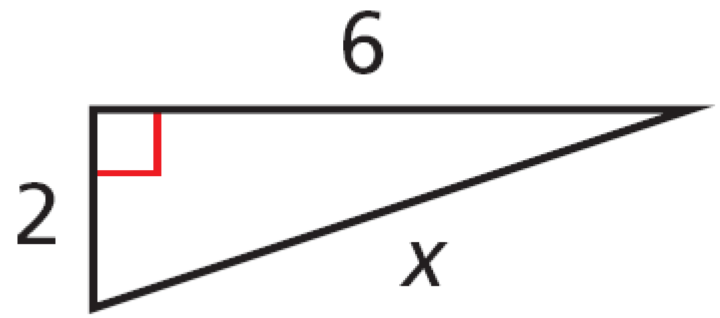
The Pythagorean Theorem is probably the most famous mathematical relationship. As you learned in Lesson 1-6, it states that in a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse.



$$a^2 + b^2 = c^2$$

Example 1A: Using the Pythagorean Theorem

Find the value of *x*. Give your answer in simplest radical form.



$a^2 + b^2 = c^2$ *Pythagorean Theorem*

$2^2 + 6^2 = x^2$ *Substitute 2 for a, 6 for b, and x for c.*

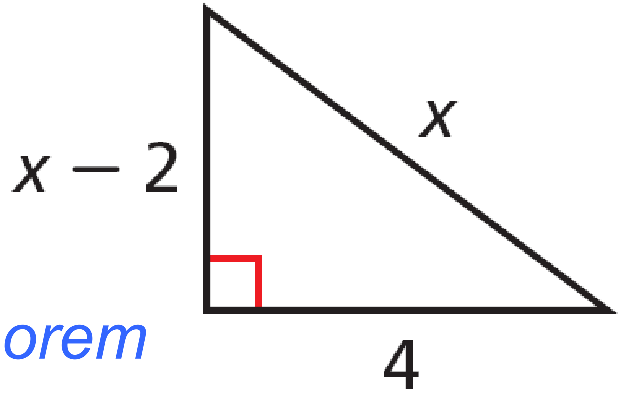
$40 = x^2$ *Simplify.*

$\sqrt{40} = x$ *Find the positive square root.*

$x = \sqrt{(4)(10)} = 2\sqrt{10}$ *Simplify the radical.*

Example 1B: Using the Pythagorean Theorem

Find the value of *x*. Give your answer in simplest radical form.



$a^2 + b^2 = c^2$ *Pythagorean Theorem*

$(x - 2)^2 + 4^2 = x^2$ *Substitute x – 2 for a, 4 for b, and x for c.*

$x^2 - 4x + 4 + 16 = x^2$ *Multiply.*

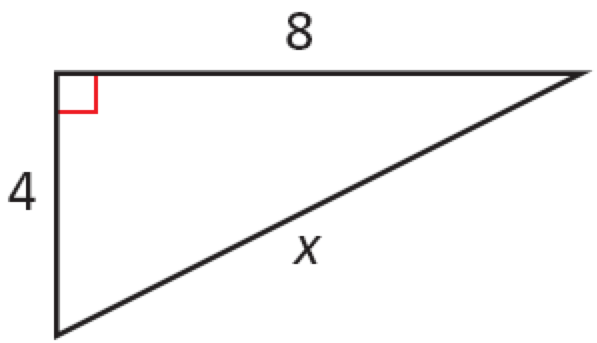
$-4x + 20 = 0$ *Combine like terms.*

$20 = 4x$ *Add 4x to both sides.*

$5 = x$ *Divide both sides by 4.*

Check It Out! Example 1a

Find the value of *x*. Give your answer in simplest radical form.



$a^2 + b^2 = c^2$ *Pythagorean Theorem*

$4^2 + 8^2 = x^2$ *Substitute 4 for a, 8 for b, and x for c.*

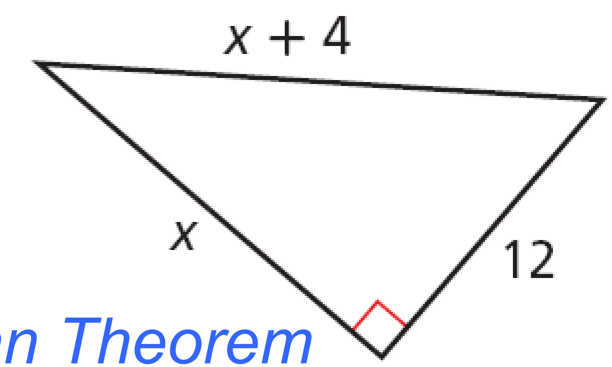
$80 = x^2$ *Simplify.*

$\sqrt{80} = x$ *Find the positive square root.*

$x = \sqrt{(16)(5)} = 4\sqrt{5}$ *Simplify the radical.*

Check It Out! Example 1b

Find the value of *x*. Give your answer in simplest radical form.



$a^2 + b^2 = c^2$ — *Pythagorean Theorem*

$x^2 + 12^2 = (x + 4)^2$ — *Substitute x for a, 12 for b, and x + 4 for c.*

$x^2 + 144 = x^2 + 8x + 16$ — *Multiply.*

$128 = 8x$ — *Combine like terms.*

$16 = x$ — *Divide both sides by 8.*

Example 2: Crafts Application

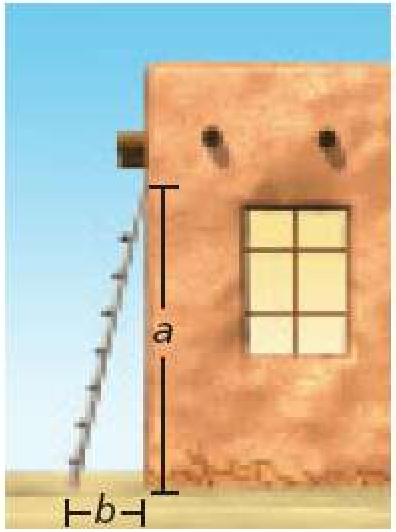
Randy is building a rectangular picture frame. He wants the ratio of the length to the width to be 3:1 and the diagonal to be 12 centimeters. How wide should the frame be? Round to the nearest tenth of a centimeter.

Let ℓ and w be the length and width in centimeters of the picture. Then $\ell:w = 3:1$, so $\ell = 3w$.

Example 2 Continued

$a^2 + b^2 = c^2$	*Pythagorean Theorem*
$(3w)^2 + w^2 = 12^2$	*Substitute 3w for a, w for b, and 12 for c.*
$10w^2 = 144$	*Multiply and combine like terms.*
$w^2 = \frac{144}{10}$	*Divide both sides by 10.*
$w = \sqrt{\frac{144}{10}} \approx 3.8 \text{ cm}$	*Find the positive square root and round.*

Check It Out! Example 2



What if...? According to the recommended safety ratio of 4:1, how high will a 30-foot ladder reach when placed against a wall? Round to the nearest inch.

Let x be the distance in feet from the foot of the ladder to the base of the wall. Then $4x$ is the distance in feet from the top of the ladder to the base of the wall.

Check It Out! Example 2 Continued

$$a^2 + b^2 = c^2$$

Pythagorean Theorem

$$(4x)^2 + x^2 = 30^2$$

Substitute 4x for a, x for b, and 30 for c.

$$17x^2 = 900$$

Multiply and combine like terms.

$$x^2 = \frac{900}{17}$$

$$x = \sqrt{\frac{900}{17}} \approx 7.28$$

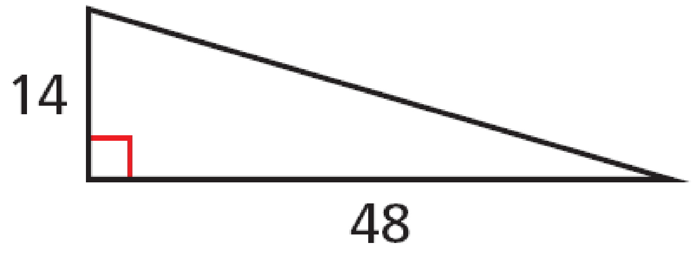
Since $4x$ is the distance in feet from the top of the ladder to the base of the wall, $4(7.28) \approx 29$ ft 1 in.

A set of three nonzero whole numbers a, b, and c such that $a^2 + b^2 = c^2$ is called a **Pythagorean triple**.

Common Pythagorean Triples			
3, 4, 5	5, 12, 13,	8, 15, 17	7, 24, 25

Example 3A: Identifying Pythagorean Triples

Find the missing side length. Tell if the side lengths form a Pythagorean triple. Explain.

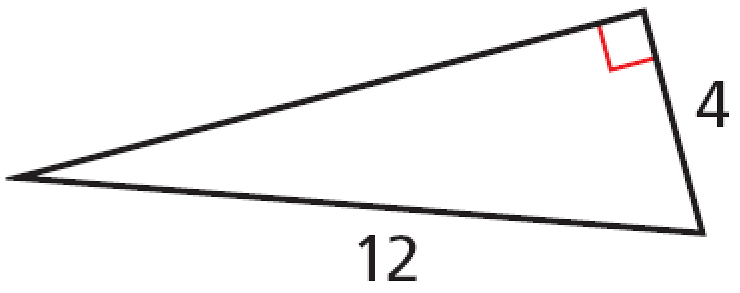


$a^2 + b^2 = c^2$	*Pythagorean Theorem*
$14^2 + 48^2 = c^2$	*Substitute 14 for a and 48 for b.*
$2500 = c^2$	*Multiply and add.*
$50 = c$	*Find the positive square root.*

The side lengths are nonzero whole numbers that satisfy the equation $a^2 + b^2 = c^2$, so they form a Pythagorean triple.

Example 3B: Identifying Pythagorean Triples

Find the missing side length. Tell if the side lengths form a Pythagorean triple. Explain.



$a^2 + b^2 = c^2$ *Pythagorean Theorem*

$4^2 + b^2 = 12^2$ *Substitute 4 for a and 12 for c.*

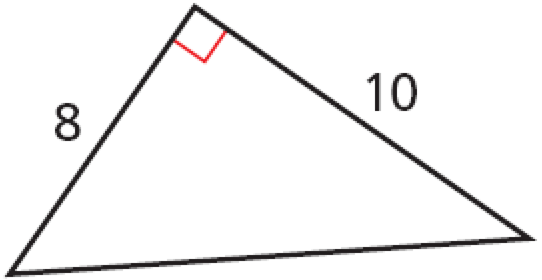
$b^2 = 128$ *Multiply and subtract 16 from both sides.*

$b = \sqrt{128} = 8\sqrt{2}$ *Find the positive square root.*

The side lengths do not form a Pythagorean triple because $8\sqrt{2}$ is not a whole number.

Check It Out! Example 3a

Find the missing side length. Tell if the side lengths form a Pythagorean triple. Explain.



$a^2 + b^2 = c^2$ *Pythagorean Theorem*

$8^2 + 10^2 = c^2$ *Substitute 8 for a and 10 for b.*

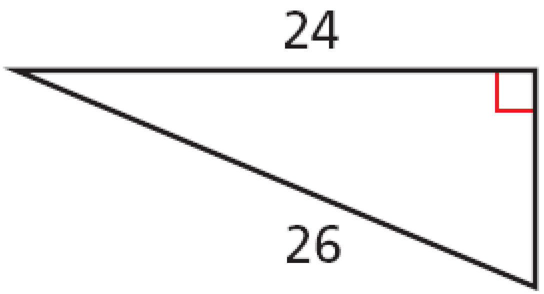
$164 = c^2$ *Multiply and add.*

$c = 2\sqrt{41}$ *Find the positive square root.*

The side lengths do not form a Pythagorean triple because $2\sqrt{41}$ is not a whole number.

Check It Out! Example 3b

Find the missing side length. Tell if the side lengths form a Pythagorean triple. Explain.



$a^2 + b^2 = c^2$ *Pythagorean Theorem*

$24^2 + b^2 = 26^2$ *Substitute 24 for a and 26 for c.*

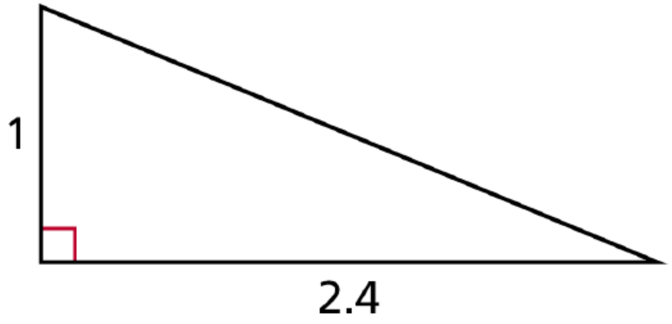
$b^2 = 100$ *Multiply and subtract.*

$b = 10$ *Find the positive square root.*

The side lengths are nonzero whole numbers that satisfy the equation $a^2 + b^2 = c^2$, so they form a Pythagorean triple.

Check It Out! Example 3c

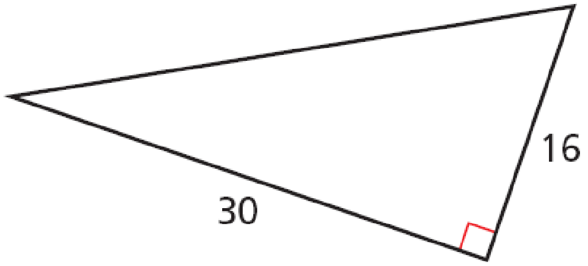
Find the missing side length. Tell if the side lengths form a Pythagorean triple. Explain.



No. The side length 2.4 is not a whole number.

Check It Out! Example 3d

Find the missing side length. Tell if the side lengths form a Pythagorean triple. Explain.



$a^2 + b^2 = c^2$ *Pythagorean Theorem*

$30^2 + 16^2 = c^2$ *Substitute 30 for a and 16 for b.*

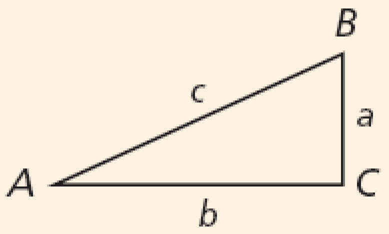
$c^2 = 1156$ *Multiply.*

$c = 34$ *Find the positive square root.*

Yes. The three side lengths are nonzero whole numbers that satisfy Pythagorean's Theorem.

The converse of the Pythagorean Theorem gives you a way to tell if a triangle is a right triangle when you know the side lengths.

Theorems 5-7-1 **Converse of the Pythagorean Theorem**

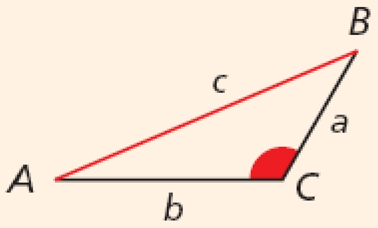
THEOREM	HYPOTHESIS	CONCLUSION
If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.	 $a^2 + b^2 = c^2$	$\triangle ABC$ is a right triangle.

You can also use side lengths to classify a triangle as acute or obtuse.

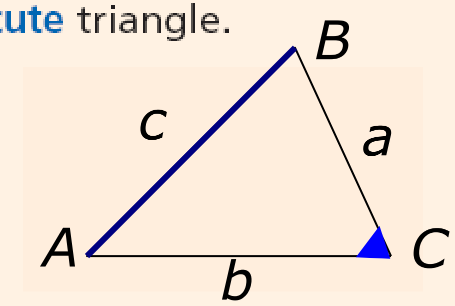
Theorems 5-7-2 **Pythagorean Inequalities Theorem**

In $\triangle ABC$, c is the length of the longest side.

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is an **obtuse** triangle.

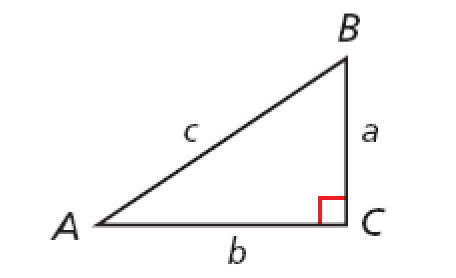


If $c^2 < a^2 + b^2$, then $\triangle ABC$ is an **acute** triangle.

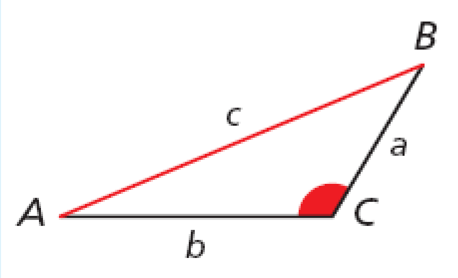


To understand why the Pythagorean inequalities are true, consider ΔABC.

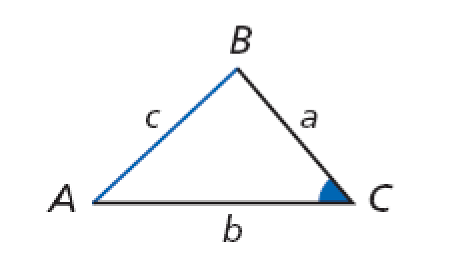
If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle by the Converse of the Pythagorean Theorem. So $m\angle C = 90°$.



If $c^2 > a^2 + b^2$, then c has increased. By the Converse of the Hinge Theorem, $m\angle C$ has also increased. So $m\angle C > 90°$.



If $c^2 < a^2 + b^2$, then c has decreased. By the Converse of the Hinge Theorem, $m\angle C$ has also decreased. So $m\angle C < 90°$.



Remember!

By the Triangle Inequality Theorem, the sum of any two side lengths of a triangle is greater than the third side length.

Example 4A: Classifying Triangles

Tell if the measures can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.

5, 7, 10

Step 1 Determine if the measures form a triangle.

By the Triangle Inequality Theorem, 5, 7, and 10 can be the side lengths of a triangle.

Example 4A Continued

Step 2 Classify the triangle.

$c^2 \stackrel{?}{=} a^2 + b^2$	*Compare c^2 to $a^2 + b^2$.*
$10^2 \stackrel{?}{=} 5^2 + 7^2$	*Substitute the longest side for c.*
$100 \stackrel{?}{=} 25 + 49$	*Multiply.*
$100 > 74$	*Add and compare.*

Since $c^2 > a^2 + b^2$, the triangle is obtuse.

Example 4B: Classifying Triangles

Tell if the measures can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.

5, 8, 17

Step 1 Determine if the measures form a triangle.

Since $5 + 8 = 13$ and $13 \not> 17$, these cannot be the side lengths of a triangle.

Check It Out! Example 4a

Tell if the measures can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.

7, 12, 16

Step 1 Determine if the measures form a triangle.

By the Triangle Inequality Theorem, 7, 12, and 16 can be the side lengths of a triangle.

Check It Out! Example 4a Continued

Step 2 Classify the triangle.

$c^2 \stackrel{?}{=} a^2 + b^2$	*Compare c^2 to $a^2 + b^2$.*
$16^2 \stackrel{?}{=} 12^2 + 7^2$	*Substitute the longest side for c.*
$256 \stackrel{?}{=} 144 + 49$	*Multiply.*
$256 > 193$	*Add and compare.*

Since $c^2 > a^2 + b^2$, the triangle is obtuse.

Check It Out! Example 4b

Tell if the measures can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.

11, 18, 34

Step 1 Determine if the measures form a triangle.

Since $11 + 18 = 29$ and $29 \not> 34$, these cannot be the sides of a triangle.

Check It Out! Example 4c

Tell if the measures can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.

3.8, 4.1, 5.2

Step 1 Determine if the measures form a triangle.

By the Triangle Inequality Theorem, 3.8, 4.1, and 5.2 can be the side lengths of a triangle.

Check It Out! Example 4c Continued

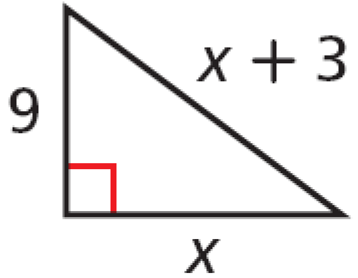
Step 2 Classify the triangle.

$c^2 \stackrel{?}{=} a^2 + b^2$	*Compare c^2 to $a^2 + b^2$.*
$5.2^2 \stackrel{?}{=} 3.8^2 + 4.1^2$	*Substitute the longest side for c.*
$27.04 \stackrel{?}{=} 14.44 + 16.81$	*Multiply.*
$27.04 < 31.25$	*Add and compare.*

Since $c^2 < a^2 + b^2$, the triangle is acute.

Lesson Quiz: Part I

1. Find the value of *x*.



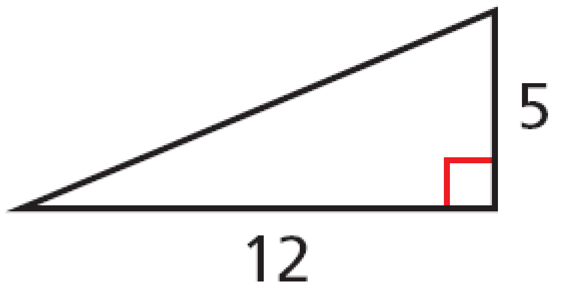
12

2. An entertainment center is 52 in. wide and 40 in. high. Will a TV with a 60 in. diagonal fit in it? Explain.

yes; $\sqrt{40^2 + 52^2} \approx 65.6 > 60$

Lesson Quiz: Part II

3. Find the missing side length. Tell if the side lengths form a Pythagorean triple. Explain.



13; yes; the side lengths are nonzero whole numbers that satisfy Pythagorean's Theorem.

4. Tell if the measures 7, 11, and 15 can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.

yes; obtuse