Putting Vectors to Use

Many different problems can be solved using vectors. Any situation that involves quantities with both magnitude and direction can be represented using vectors. For each problem below, draw and label a diagram, then use what you know about vectors to answer the question.
Show all work on a separate piece of paper.

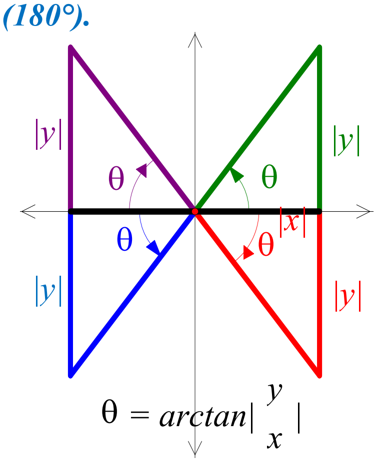
Comment:

When finding the direction of a vector (or, later, the argument of a complex number), we must make a decision about the possible answers. Some will allow these angles to be in [0°, 360°), whereas others use [-180°, 360°). To avoid some confusion with negative angles, throughout this unit we will use the interval [0°, 360°).

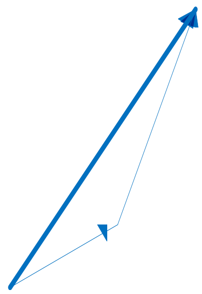
Regardless of the interval used, conversions must be made in certain situations. There are several ways to do this, some of which retain the sign of the components (e.g., using arctan(5/-4) for the vector $\langle 4,-5 \rangle$). Here, signs are ignored and only the quadrant of the vector's terminal point (when in standard position) is needed to tell us how to convert the angle to get it to be in the interval [0°, 360°):

Quadrant I:	$\arctan(\lvert y/x \rvert)$
Quadrant II:	$180° - \arctan(\lvert y/x \rvert)$
Quadrant III:	$180° + \arctan(\lvert y/x \rvert)$
Quadrant IV:	$360° - \arctan(\lvert y/x \rvert)$

The reasoning behind this can be shown easily with the following diagram, which does not require any negative angles. arctan(|y/x|) is used in each triangle, and students must simply decide whether they are moving "forward" (counterclockwise) or "backward" (clockwise) from the positive-x axis (0° and 360°) or the negative x-axis (180°).



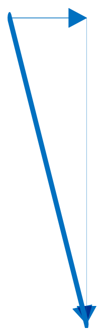
1. A ship leaves port and travels 49 miles at a standard position angle of 30°. The ship then travels for 89 miles in a standard position angle of 70°. At that point, the ship drops anchor. A helicopter, beginning from the same port, needs to join the ship as quickly as possible. Tell the helicopter's pilot how to get to the ship.



	horizontal	*vertical*
first portion	*49cos30° ≈ 42.44 mi*	*49sin30° ≈ 24.50 mi*
second portion	*89cos70° ≈ 30.44 mi*	*89sin70° ≈ 83.63 mi*
total	*+72.88 mi (East)*	*+108.13 mi (North)*

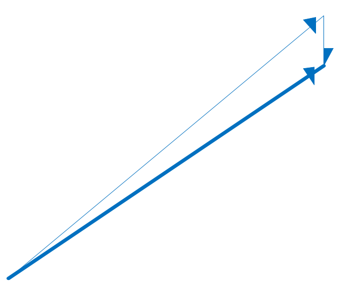
magnitude: $\sqrt{(72.88^2 + 108.13^2)} = 130.40$ ***mi***
direction: arctan(108.13/72.88) = 56.02°

2. You jump into a river intending to swim straight across to the other side. But when you start swimming, you realize the current is traveling 4 miles per hour due south. You are trying to swim due East at 1 mile per hour, but the current is pulling on you. If you don't make any adjustment for the current, how far from your starting point will you be in 15 minutes?



resultant velocity = swimming + current
magnitude: $\sqrt{(1^2 + 4^2)} = \sqrt{17}$ ***mi / hr*** $= \sqrt{17}/60$ ***miles / minute***
after 15 minutes: $(\sqrt{17}/60)(15) = \sqrt{17}/4 \approx 1.03$ ***miles***
Alternative solution method:
In 15 minutes, swimming without a current would get me (15/60)(1) = 0.25 miles East of my starting point.
In 15 minutes, the current pulls me (15/60)(4) = 1 mi S of my starting point.
The total distance from my starting point is given by the Pyth. Thm.:
$\sqrt{(0.25^2 + 1^2)} \approx 1.03$ ***miles***

3. A plane is traveling at 400 mph along a path 40° North of East. A strong wind begins to blow at 50 mph from North to South. If no adjustment is made for the wind, what are the resulting bearing and groundspeed of the plane?



	horizontal	*vertical*
original velocity	*400cos40°= 306.42 mph*	*400sin40°= 257.12 mph*
wind	*0 mph*	*-50 mph*
resulting velocity	*306.42 mph*	*207.12 mph*

bearing: arctan(207.12/306.42) = 34.06°
groundspeed: $\sqrt{(306.42^2 + 207.12^2)} = 369.85$ ***mph***

4. A motorboat traveling from one shore to the other at a rate of 5 m/s east encounters a current flowing at a rate of 3.5 m/s north.

a. What is the resultant velocity?

$\sqrt{(5^2 + 3.5^2)} = 6.10$ ***m/s***

b. If the width of the river is 60 meters wide, then how much time does it take the boat to travel to the opposite shore?

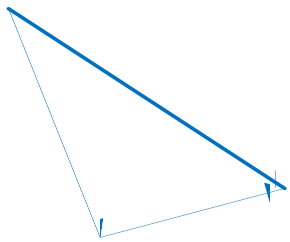
(60 m) / (5 m/s) = 12 s

c. What distance downstream does the boat reach the opposite shore?

(3.5 m/s) (12 s) = 42 m

5. A ship sails 12 hours at a speed of 8 knots (nautical miles per hour) at a heading of 68° <u>south</u> of east. It then turns to a heading of 15° <u>north</u> of east and travels for 5 hours at 15 knots.

a. Find the resultant displacement vector. Give your answer in component form.



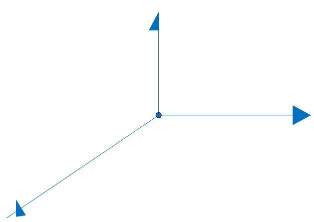
	horizontal	***vertical***
first portion: ***96 miles, 68° S of E***	***96cos(-68°)*** ***35.96 mi***	***96sin(-68°)*** ***-89.01 mi***
second portion: ***75 miles, 15° N of E***	***75cos15°*** ***72.44 mi***	***75sin15°*** ***19.41 mi***
total	***+108.40 mi*** ***(East)***	***-69.60 mi*** ***(South)***

109.40 miles East, 69.60 miles South or <108.4, 69.6>

b. Convert your answer to magnitude-direction form.

magnitude: $\sqrt{(108.4^2 + 69.6^2)} = 128.8$ ***miles***
direction: $360° + \arctan(-69.6/108.4) = 327.3°$

6. In three-person tug-of-war, three ropes are tied at a point. Adam is pulling due East with a force of 600 Newtons, Barry is pulling due North with a force of 400 Newtons, and Cal is pulling the third rope. The knot in the middle is not moving. What are the direction and magnitude of Cal's effort?



	horizontal	***vertical***
Adam	***600 N***	***0 N***
Barry	***0 N***	***400 N***
Cal ***0 – (Adam + Barry)***	***0 – 600*** ***-600 N***	***0 – 400*** ***-400 N***
total = 0 ***(knot doesn't move)***	***0 N***	***0 N***

direction: $180° + \arctan(-400/-600) = 213.7°$
magnitude: $\sqrt{(600^2 + 400^2)} = 200\sqrt{13} \approx 721$ ***Newtons***