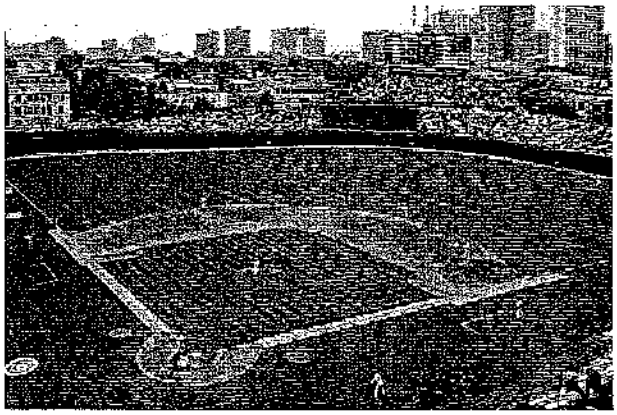
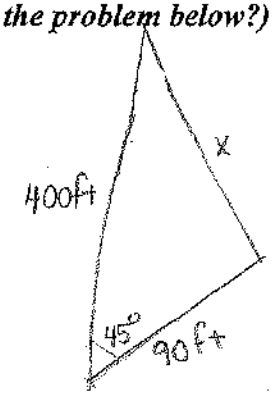
Name: Key **Date:** _____

PROVING THE LAW OF COSINES

- During a baseball game an outfielder caught a ball hit to dead center field, 400 feet from home plate. If the distance from home plate to first base is 90 feet, how far does the outfielder have to throw the ball to get it to first base?



1. Model the problem with a picture in the space below. Be sure to label information that you know. ***(Hint: The bases form a diamond/square – how can you use this information to help model the problem below?)***



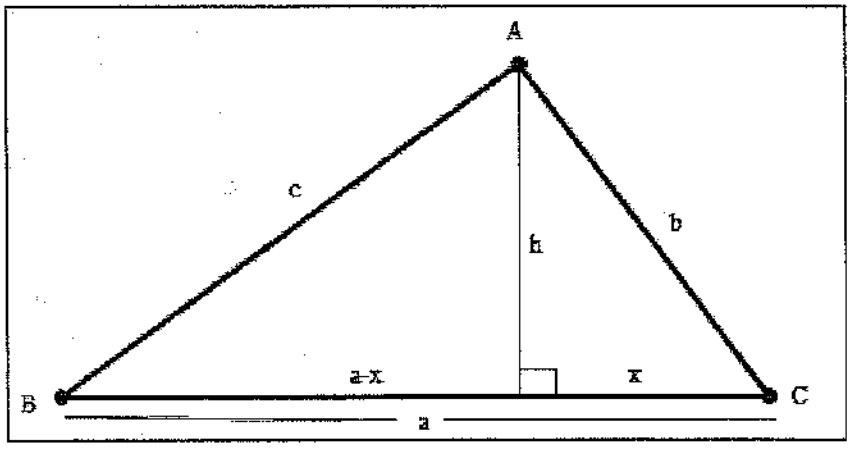
2. Do you have enough information to solve the problem? no

- If not, what is missing? you need more information

- Typically, you have solved triangles that are right triangles. This is a case where we ***do not*** have a ***right triangle*** to solve. We know ***two sides*** and ***one included angle.***

- In this task, you will develop a method for solving triangles like this using trigonometry. We will come back to the baseball example later. For now, consider the triangle below.

- Assume we ***ONLY KNOW*** measurements for segments *a*, *b*, and angle *C*.



Step 1: What does segment *h* represent? height of ΔABC

What are its properties in this model? 1) it is an altitude

2) It creates two triangles that have a rt. $\angle$.

What does it do to the large triangle? It creates two triangles.

Step 2: Write an equation that represents c^2. **(using the left side of the triangle only)**

$h^2 + (a-x)^2 = c^2$

Explain the method you used. The Pythagorean Theorem

Step 3: Now write an equation that represents h^2 in terms of b and x. (***in other words, b and x should be in your equation about h²***) $h^2 = b^2 - x^2$

Substitute this expression into the expression you wrote in step 2. $h^2 - x^2 + (a-x)^2 = c^2$

$b^2 - x^2 + a^2 - 2ax + x^2 = c^2$

Now expand and simplify in the space below:

$c^2 = b^2 + a^2 - 2ax$

Step 4: Now write an expression that represents x in terms of the angle C. (***in other words, write an equation that contains angle C when representing x. Use the right side of the triangle only)***

$\cos C = x/b$

$x = b\cos C$

Substitute this expression into the equation you wrote in step 3. $c^2 = b^2 + a^2 - 2a(b\cos C)$

Simplify completely in the space below:

*Your final answer is one of three formulas that make up the **Law of Cosines**. Each of the formulas can be derived in the same way you derived this one by working with each vertex and the other heights of the triangle.*

Law of Cosines
Let a, b, and c be the lengths of the legs of a triangle opposite angles A, B, and C. Then,

$$a^2 = b^2 + c^2 - 2bc(cosA)$$
$$b^2 = a^2 + c^2 - 2ac(cosB)$$
$$c^2 = a^2 + b^2 - 2ab(cosC)$$

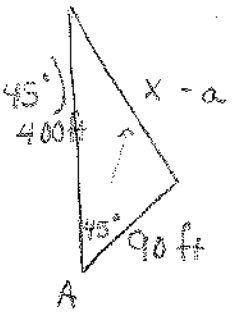
These formulas can be used to solve for unknown lengths and angles in a triangle.

3. Now solve the baseball problem at the beginning of this task using the Law of Cosines in the space below:

$a^2 = (400)^2 + (90)^2 - 2(400)(90)(\cos 45°)$

$\sqrt{a^2} = \sqrt{107555.4581}$

$a = 342.3$ ft



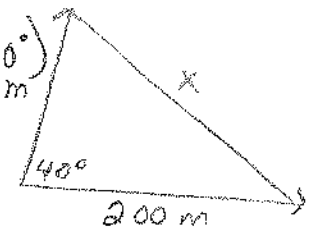
Here are a few problems to help you apply the Law of Cosines.

4. Two airplanes leave an airport, and the angle between their flight paths is 40°. An hour later, one plane has traveled 300 miles while the other has traveled 200 miles. How far apart are the planes at this time?

$a^2 = (300)^2 + (200)^2 - 2(300)(200)(\cos 40°)$

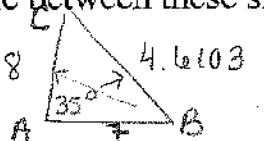
$\sqrt{a^2} = \sqrt{38074.66083}$

$a = 195.127$ miles



5. A triangle has sides of 8 and 7 and the angle between these sides is 35°. Solve the triangle. (Find all missing angles and sides.)

$a^2 = 8^2 + 7^2 - 2(8)(7)(\cos 35°)$ $\sqrt{a^2} = \sqrt{21.25497}$ $a = 4.6103$



$8^2 = 7^2 + 21.25497 - 2(7)(4.6103)\cos$

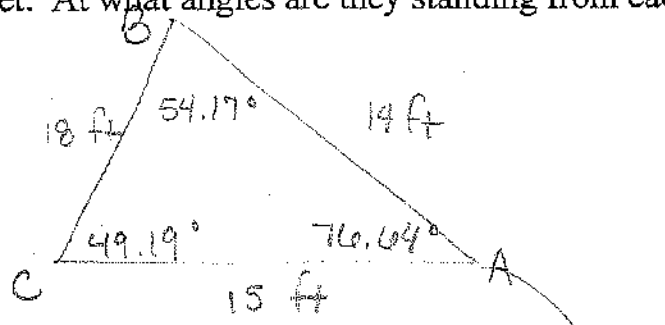
$\angle B = 84.438°$

$7^2 = 8^2 + 21.25497 - 2(8)(4.6103)(\cos$

$-36.25497 = -73.76648(\cos C)$ $\quad \cos C = .49149$ $\quad \cos^{-1}(.49149) = \angle C$

$\angle C = 60.56°$

6. Three soccer players are practicing on a field. The triangle they create has side lengths of 18, 14, and 15 feet. At what angles are they standing from each other?



7. Is it possible to know two sides of a triangle and the included angle and not be able to solve for the third side? No

$18^2 = 15^2 + 14^2 - 2(15)(14)(\cos A)$

$\angle A = 76.64°$

$15^2 = 18^2 + 14^2 - 2(18)(14)(\cos B)$

$\angle B = 54.17°$

$180 - 54.17 - 76.64 = \angle C$

$\angle C = 49.19°$