Applying Properties of Similar Triangles

Warm Up

Lesson Presentation

Lesson Quiz

Warm Up :

Solve each proportion.

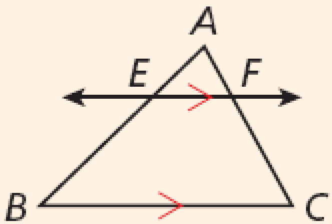
1. $\frac{12}{15} = \frac{AB}{20}$ $AB = 16$

$\frac{9.5}{QR} = \frac{3.8}{4.2}$ $QR = 10.5$

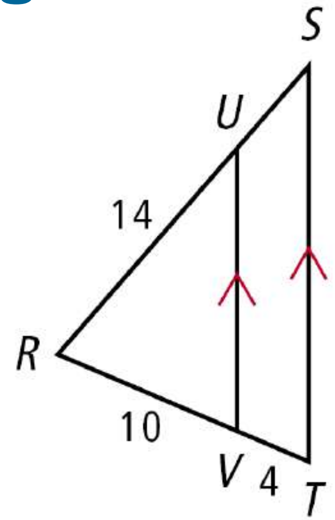
3. $\frac{x-5}{20} = \frac{x+3}{30}$ $x = 21$

$\frac{y+7}{2y-4} = \frac{3.5}{2.8}$ $y = 8$

Theorem 7-4-1 Triangle Proportionality Theorem

THEOREM	HYPOTHESIS	CONCLUSION
If a line parallel to a side of a triangle intersects the other two sides, then it divides those sides proportionally.	 $\overline{EF} \parallel \overline{BC}$	$\frac{AE}{EB} = \frac{AF}{FC}$

Example 1: Finding the Length of a Segment



Find *US*.

It is given that $\overline{ST} = \overline{UV}$, so $\frac{US}{RU} = \frac{VT}{RV}$ by the Triangle Proportionality Theorem.

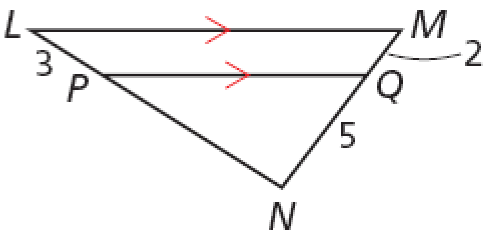
$$\frac{US}{14} = \frac{4}{10}$$ *Substitute 14 for RU, 4 for VT, and 10 for RV.*

$$US(10) = 56$$ *Cross Products Prop.*

$$US = \frac{56}{10}, \text{ or } 5\frac{3}{5}$$ *Divide both sides by 10.*

Check It Out! Example 1

Find *PN*.



Use the Triangle Proportionality Theorem.

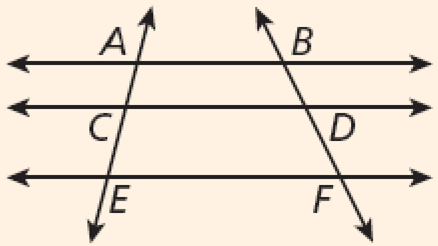
$$\frac{LP}{PN} = \frac{MQ}{QN}$$

$$\frac{3}{PN} = \frac{2}{5}$$ *Substitute in the given values.*

$2PN = 15$ *Cross Products Prop.*

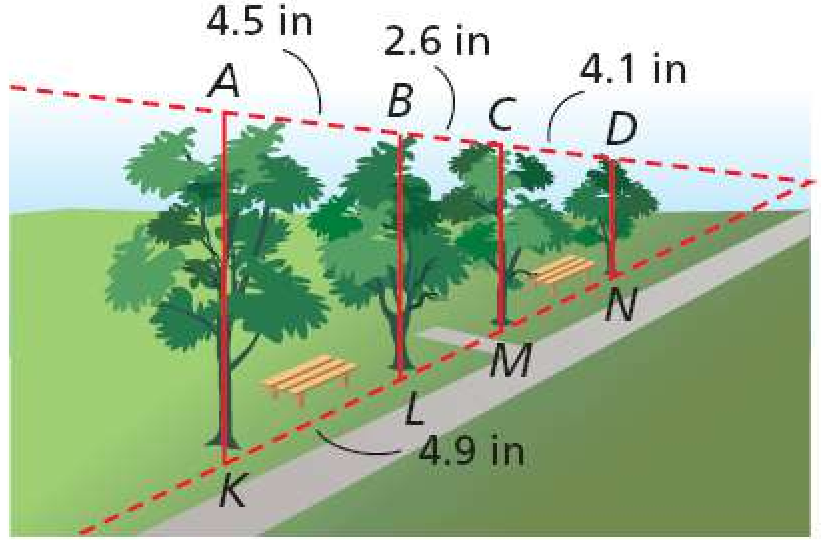
$PN = 7.5$ *Divide both sides by 2.*

Corollary 7-4-3 Two-Transversal Proportionality

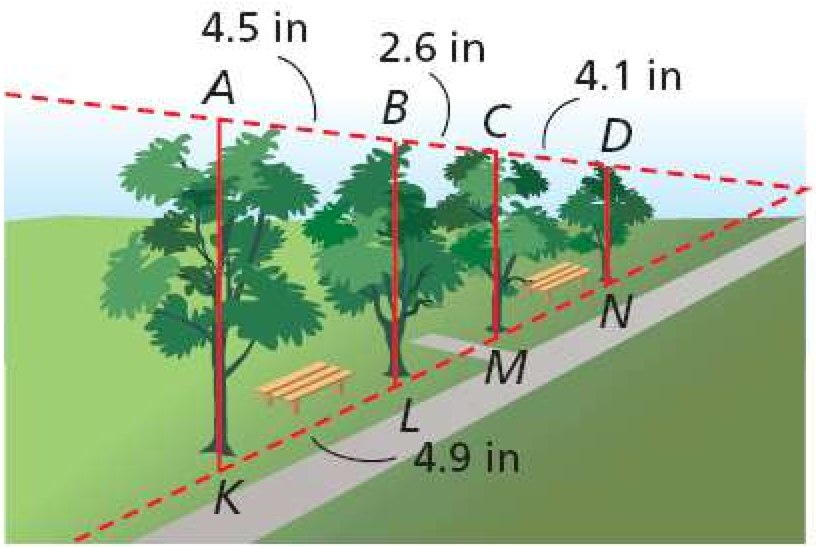
THEOREM	HYPOTHESIS	CONCLUSION
If three or more parallel lines intersect two transversals, then they divide the transversals proportionally.		$\frac{AC}{CE} = \frac{BD}{DF}$

Example 3: Art Application

Suppose that an artist decided to make a larger sketch of the trees. In the figure, if *AB* = 4.5 in., *BC* = 2.6 in., *CD* = 4.1 in., and *KL* = 4.9 in., find *LM* and *MN* to the nearest tenth of an inch.



Example 3 Continued



$\overline{AK} \parallel \overline{BL} \parallel \overline{CM} \parallel \overline{DN}$ *Given*

$\frac{KL}{LM} = \frac{AB}{BC}$ *2-Trans. Proportionality Corollary*

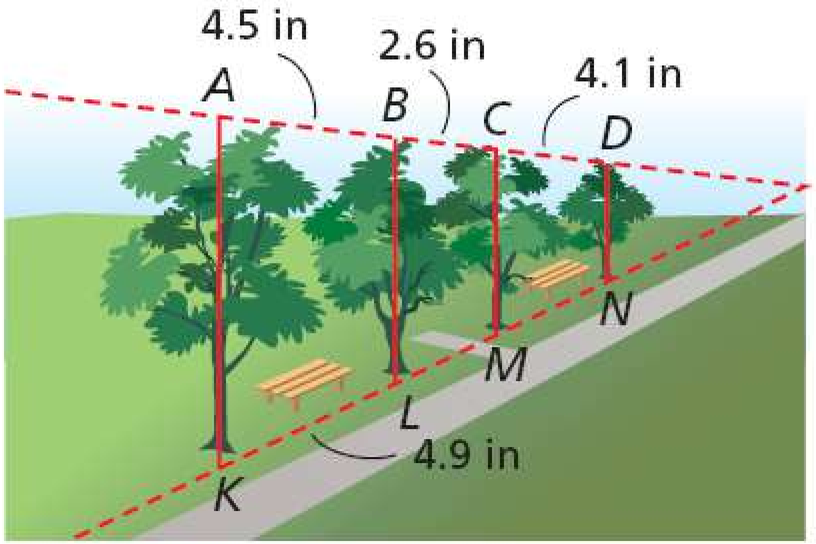
$\frac{4.9}{LM} = \frac{4.5}{2.6}$ *Substitute 4.9 for KL, 4.5 for AB, and 2.6 for BC.*

$4.5(LM) = 4.9(2.6)$ *Cross Products Prop.*

$LM \approx 2.8$ in. *Divide both sides by 4.5.*

Example 3 Continued

$$\frac{KL}{MN} = \frac{AB}{CD}$$ *2-Trans. Proportionality Corollary*



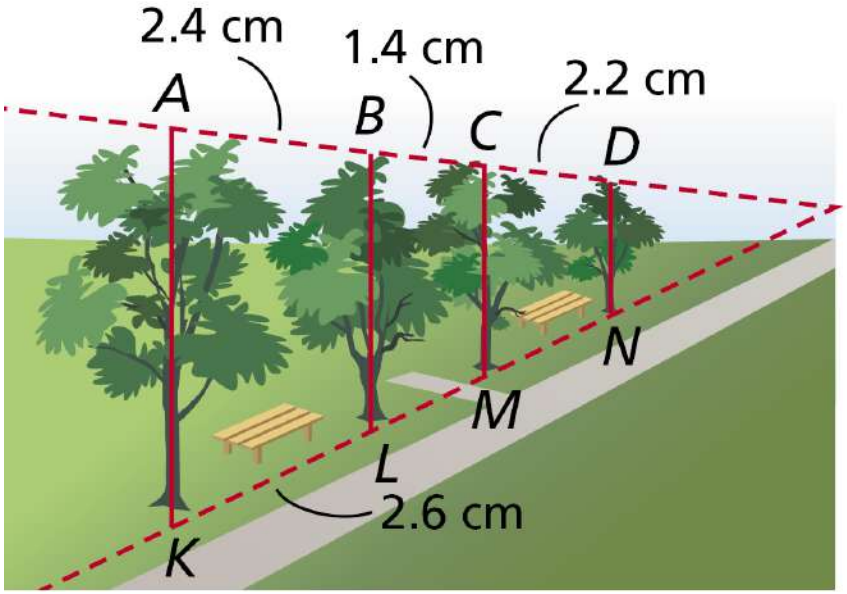
$$\frac{4.9}{MN} = \frac{4.5}{4.1}$$ *Substitute 4.9 for KL, 4.5 for AB, and 4.1 for CD.*

$4.5(MN) = 4.9(4.1)$ *Cross Products Prop.*

$MN \approx 4.5$ in. *Divide both sides by 4.5.*

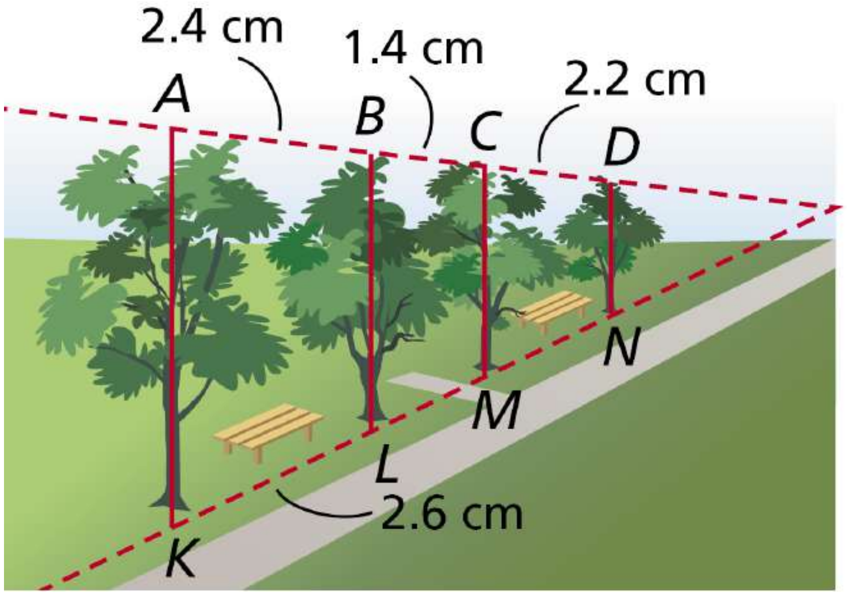
Check It Out! Example 3

Use the diagram to find *LM* and *MN* to the nearest tenth.



Check It Out! Example 3 Continued

$\overline{AK} \parallel \overline{BL} \parallel \overline{CM} \parallel \overline{DN}$ *Given*



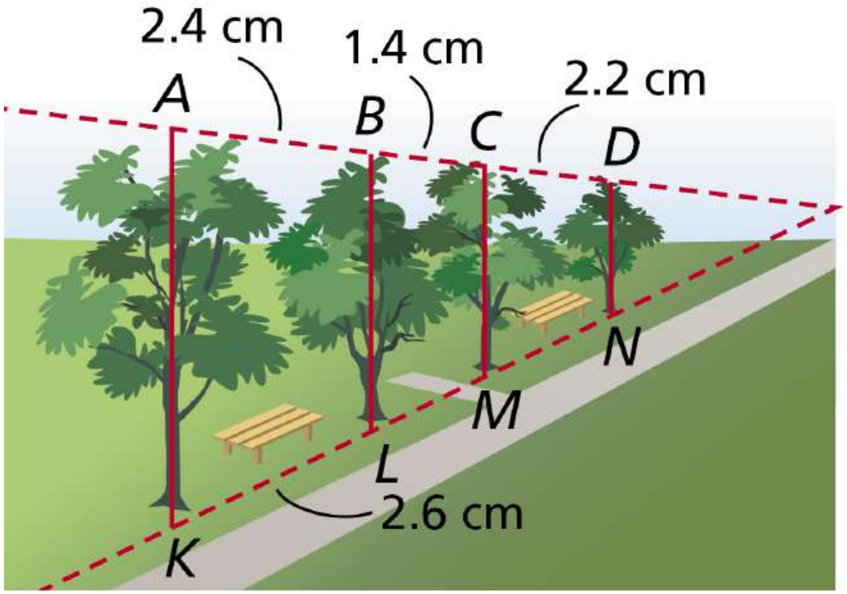
$\frac{KL}{LM} = \frac{AB}{BC}$ *2-Trans. Proportionality Corollary*

$\frac{2.6}{LM} = \frac{2.4}{1.4}$ *Substitute 2.6 for KL, 2.4 for AB, and 1.4 for BC.*

$2.4(LM) = 1.4(2.6)$ *Cross Products Prop.*

$LM \approx 1.5$ cm *Divide both sides by 2.4.*

Check It Out! Example 3 Continued



$$\frac{KL}{MN} = \frac{AB}{CD}$$ *2-Trans. Proportionality Corollary*

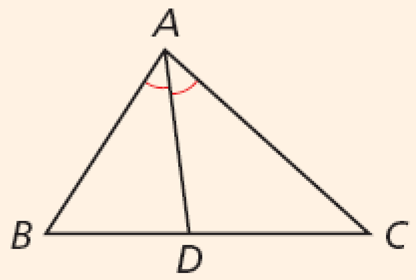
$$\frac{2.6}{MN} = \frac{2.4}{2.2}$$ *Substitute 2.6 for KL, 2.4 for AB, and 2.2 for CD.*

$2.4(MN) = 2.2(2.6)$ *Cross Products Prop.*

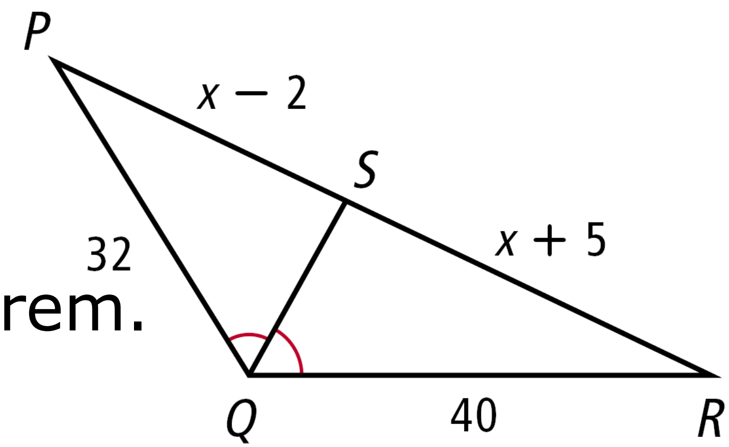
$MN \approx 2.4$ cm *Divide both sides by 2.4.*

The previous theorems and corollary lead to the following conclusion.

Theorem 7-4-4 **Triangle Angle Bisector Theorem**

THEOREM	HYPOTHESIS	CONCLUSION
An angle bisector of a triangle divides the opposite side into two segments whose lengths are proportional to the lengths of the other two sides. (△ ∠ Bisector Thm.)		$\frac{BD}{DC} = \frac{AB}{AC}$

Example 4: Using the Triangle Angle Bisector Theorem



Find *PS* and *SR*.

$\frac{PS}{SR} = \frac{QP}{QR}$ by the Δ ∠ Bisector Theorem.

$\frac{x-2}{x+5} = \frac{32}{40}$ *Substitute the given values.*

$40(x - 2) = 32(x + 5)$ *Cross Products Property*

$40x - 80 = 32x + 160$ *Distributive Property*

Example 4 Continued

$40x - 80 = 32x + 160$

$8x = 240$ *Simplify.*

$x = 30$ *Divide both sides by 8.*

Substitute 30 for x.

$PS = x - 2$

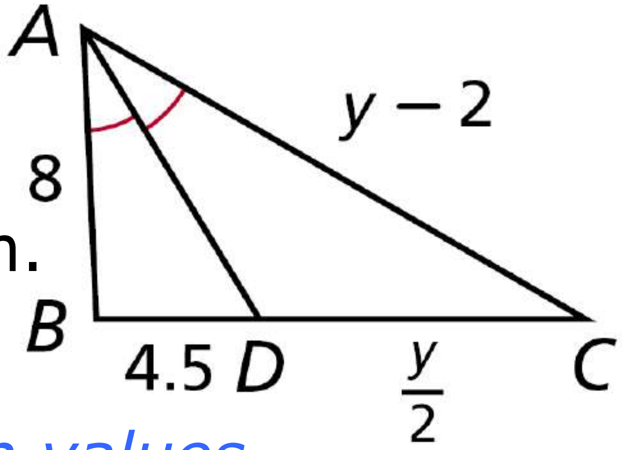
$= 30 - 2 = 28$

$SR = x + 5$

$= 30 + 5 = 35$

Check It Out! Example 4

Find *AC* and *DC*.



$\frac{BD}{DC} = \frac{AB}{AC}$ by the Δ ∠ Bisector Theorem.

$\frac{4.5}{\frac{y}{2}} = \frac{8}{y-2}$ *Substitute in given values.*

$4y = 4.5y - 9$ *Cross Products Theorem*

$-0.5y = -9$ *Simplify.*

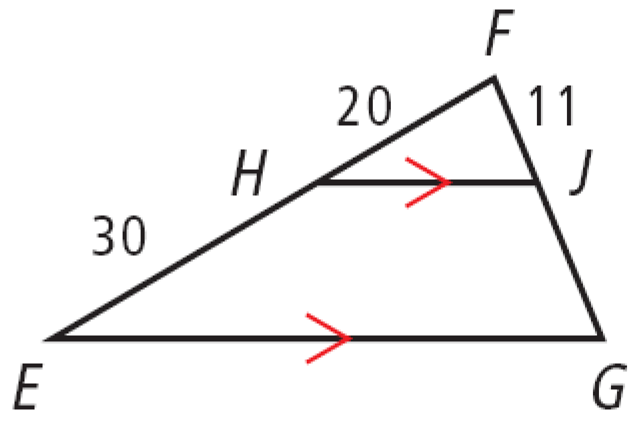
$y = 18$ *Divide both sides by –0.5.*

So $DC = 9$ and $AC = 16$.

Lesson Quiz: Part I

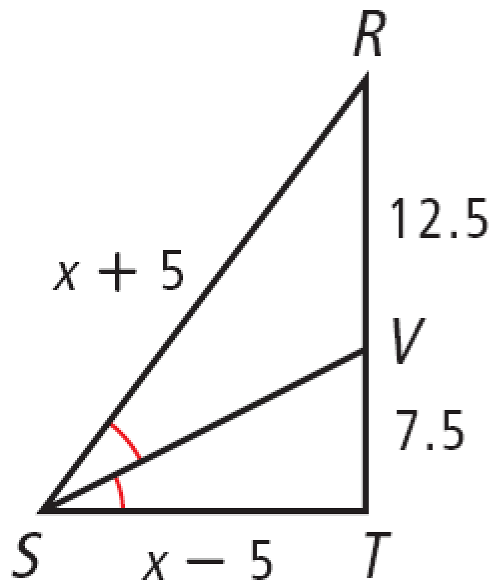
Find the length of each segment.

1. $\overline{JG}$



$\frac{33}{2}$ or $16\frac{1}{2}$

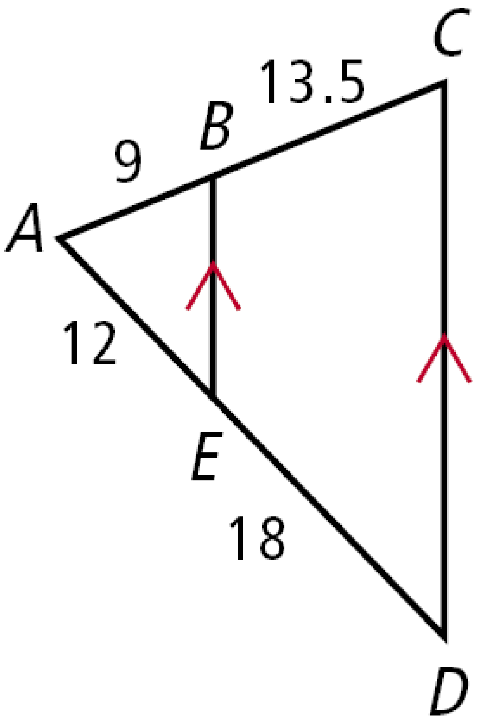
$\overline{SR}$ and $\overline{ST}$



$SR = 25, ST = 15$

Lesson Quiz: Part II

3. Verify that $\overline{BE}$ and $\overline{CD}$ are parallel.



$\frac{BC}{AB} = \frac{13.5}{9} = 1.5$ $\frac{ED}{AE} = \frac{18}{12} = 1.5$

Since $\frac{BC}{AB} = \frac{ED}{AE}$, $\overline{BE} \parallel \overline{CD}$ by the Converse of the Δ Proportionality Thm.