# Properties of special parallelograms

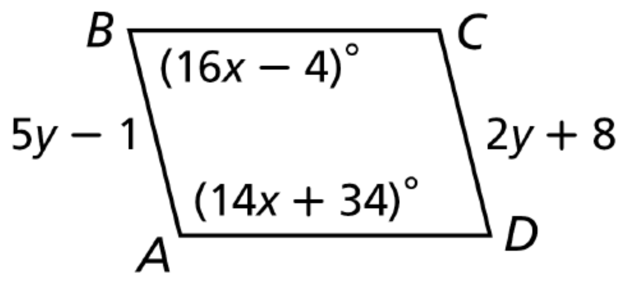
## Warm Up

**Solve for *x*.**

**1.** $16x - 3 = 12x + 13$ 4

**2.** $2x - 4 = 90$ 47

***ABCD* is a parallelogram. Find each measure.**




**3.** *CD* 14

**4.** m∠*C* 104°

# Objectives

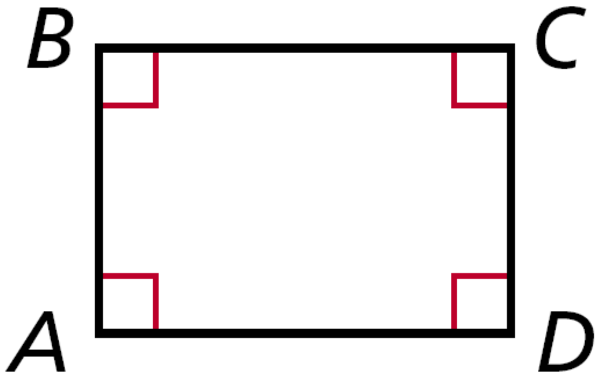
Prove and apply properties of rectangles, rhombuses, and squares.

Use properties of rectangles, rhombuses, and squares to solve problems.

# *Vocabulary*

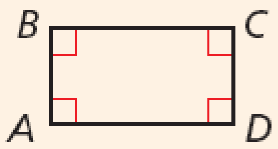
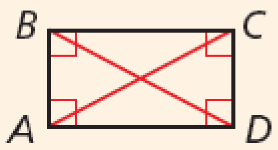
rectangle
rhombus
square

A second type of special quadrilateral is a *rectangle*.
A **rectangle** is a quadrilateral with four right angles.




Rectangle *ABCD*

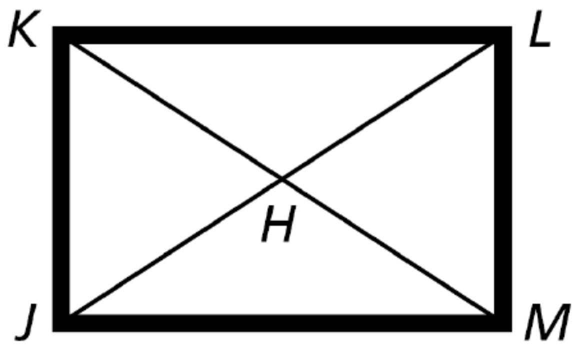
## Theorems: Properties of Rectangles

| THEOREM | | HYPOTHESIS | CONCLUSION |
|---|---|---|---|
| **6-4-1** | If a quadrilateral is a rectangle, then it is a parallelogram. (rect. → ▱) |   | $ABCD$ is a parallelogram. |
| **6-4-2** | If a parallelogram is a rectangle, then its diagonals are congruent. (rect. → diags. ≅) |   | $\overline{AC} \cong \overline{BD}$ |

Since a rectangle is a parallelogram by Theorem 6-4-1, a rectangle "inherits" all the properties of parallelograms that you learned in Lesson 6-2.

## Example 1: Craft Application

**A woodworker constructs a rectangular picture frame so that *JK* = 50 cm and *JL* = 86 cm. Find *HM*.**




$\overline{KM} \cong \overline{JL}$ — *Rect. → diags. ≅*

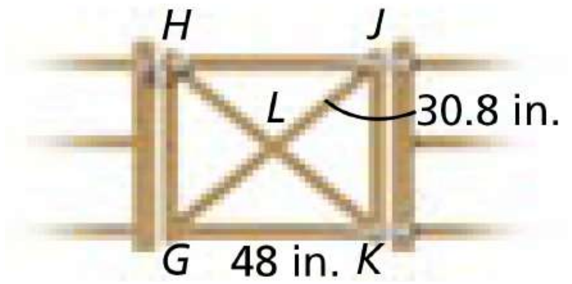
$KM = JL = 86$ — *Def. of ≅ segs.*

$HM = \frac{1}{2}KM$ — *▱ → diags. bisect each other*

$HM = \frac{1}{2}(86) = 43 \text{ cm}$ — *Substitute and simplify.*

## Check It Out! Example 1a

**Carpentry The rectangular gate has diagonal braces.**
**Find *HJ*.**

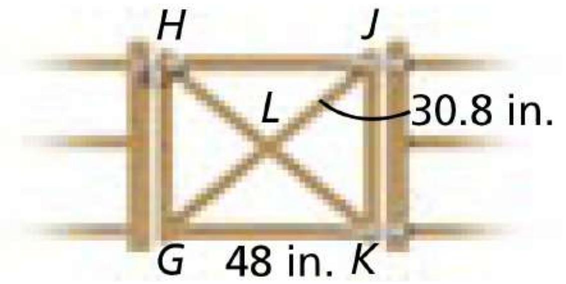



$\overline{HJ} \cong \overline{GK}$ *Rect. → diags. ≅*

$HJ = GK = 48$ *Def. of ≅ segs.*

## Check It Out! Example 1b

**Carpentry** **The rectangular gate has diagonal braces. Find *HK*.**



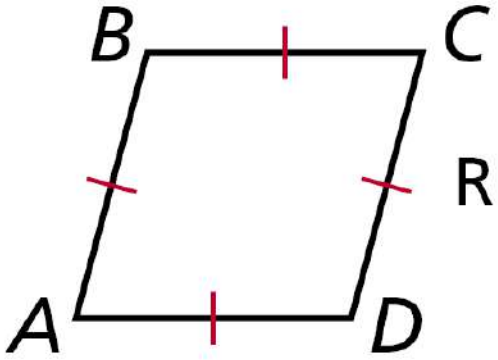

$\overline{HK} \cong \overline{JG}$ — *Rect. → diags. ≅*

$\overline{JL} \cong \overline{LG}$ — *Rect. → diagonals bisect each other*

$JL = LG$ — *Def. of ≅ segs.*

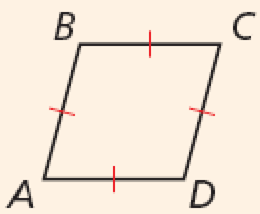
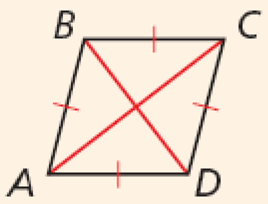
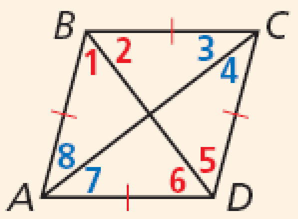
$JG = 2JL = 2(30.8) = 61.6$ — *Substitute and simplify.*

A *rhombus* is another special quadrilateral. A **rhombus** is a quadrilateral with four congruent sides.




Rhombus *ABCD*

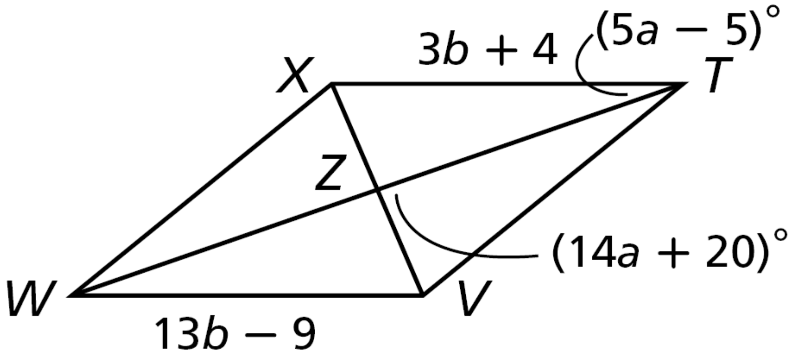
**Theorems** Properties of Rhombuses

| THEOREM | | HYPOTHESIS | CONCLUSION |
|---|---|---|---|
| **6-4-3** | If a quadrilateral is a rhombus, then it is a parallelogram. (rhombus → ▱) |  | *ABCD* is a parallelogram. |
| **6-4-4** | If a parallelogram is a rhombus, then its diagonals are perpendicular. (rhombus → diags. ⊥) |  | $\overline{AC} \perp \overline{BD}$ |
| **6-4-5** | If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. (rhombus → each diag. bisects opp. ∠s) |  | $\angle 1 \cong \angle 2$<br>$\angle 3 \cong \angle 4$<br>$\angle 5 \cong \angle 6$<br>$\angle 7 \cong \angle 8$ |

Like a rectangle, a rhombus is a parallelogram. So you can apply the properties of parallelograms to rhombuses.

# Example 2A: Using Properties of Rhombuses to Find Measures

***TVWX*** **is a rhombus.**
**Find *TV*.**



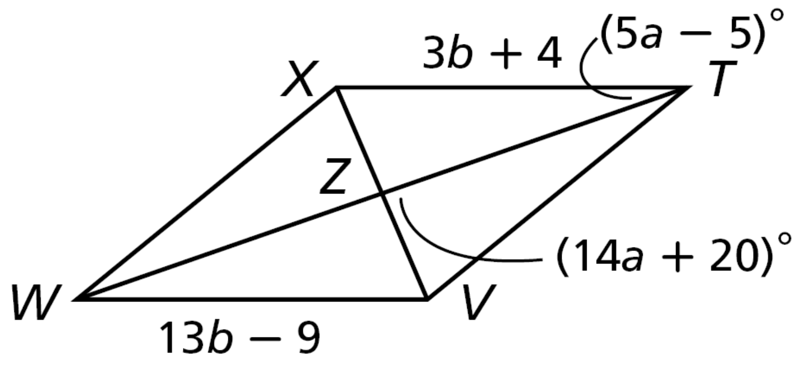

| | |
|---|---|
| $WV = XT$ | *Def. of rhombus* |
| $13b - 9 = 3b + 4$ | *Substitute given values.* |
| $10b = 13$ | *Subtract 3b from both sides and add 9 to both sides.* |
| $b = 1.3$ | *Divide both sides by 10.* |

# Example 2A Continued

| | |
|---|---|
| $TV = XT$ | *Def. of rhombus* |
| $TV = 3b + 4$ | *Substitute 3b + 4 for XT.* |
| $TV = 3(1.3) + 4 = 7.9$ | *Substitute 1.3 for b and simplify.* |

# Example 2B: Using Properties of Rhombuses to Find Measures

***TVWX*** **is a rhombus.**
**Find m∠*VTZ*.**




m∠*VZT* = 90° — *Rhombus → diag. ⊥*

14*a* + 20 = 90° — *Substitute 14a + 20 for m∠VTZ.*

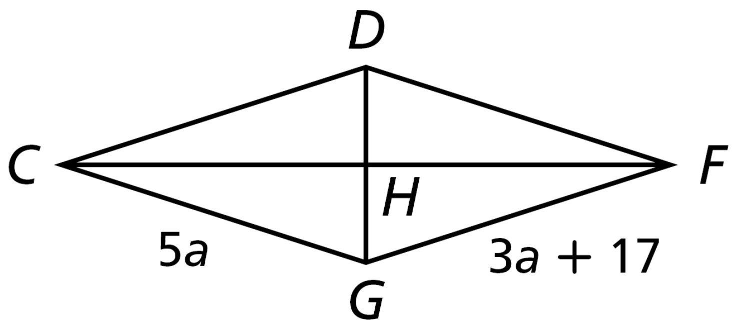
*a* = 5 — *Subtract 20 from both sides and divide both sides by 14.*

## Example 2B Continued

| | |
|---|---|
| $m\angle VTZ = m\angle ZTX$ | *Rhombus → each diag. bisects opp. ∠s* |
| $m\angle VTZ = (5a - 5)°$ | *Substitute 5a – 5 for m∠VTZ.* |
| $m\angle VTZ = [5(5) - 5)]° = 20°$ | *Substitute 5 for a and simplify.* |

## Check It Out! Example 2a

***CDFG* is a rhombus.**
**Find *CD*.**

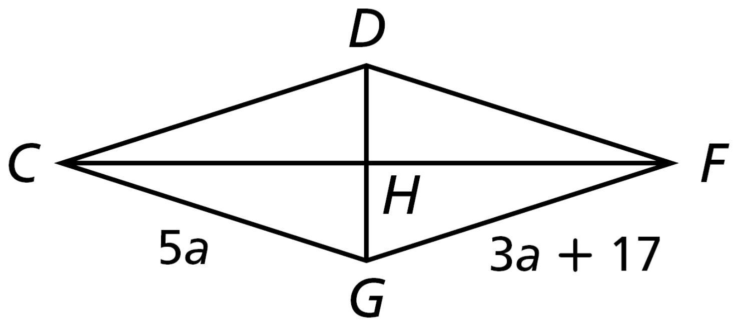



| | |
|---|---|
| $CG = GF$ | *Def. of rhombus* |
| $5a = 3a + 17$ | *Substitute* |
| $a = 8.5$ | *Simplify* |
| $GF = 3a + 17 = 42.5$ | *Substitute* |
| $CD = GF$ | *Def. of rhombus* |
| $CD = 42.5$ | *Substitute* |

# Check It Out! Example 2b

***CDFG* is a rhombus.**
**Find the measure.**

**m$\angle$*GCH* if m$\angle$*GCD* = (*b* + 3)°**
**and m$\angle$*CDF* = (6*b* – 40)°**




$m\angle GCD + m\angle CDF = 180°$ *Def. of rhombus*

$b + 3 + 6b - 40 = 180°$ *Substitute.*

$7b = 217°$ *Simplify.*

$b = 31°$ *Divide both sides by 7.*

# Check It Out! Example 2b Continued

$m\angle GCH + m\angle HCD = m\angle GCD$

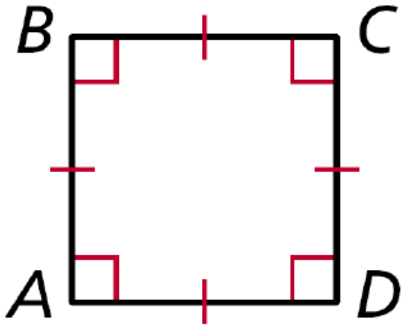
$2m\angle GCH = m\angle GCD$ — *Rhombus → each diag. bisects opp. ∠s*

$2m\angle GCH = (b + 3)$ — *Substitute.*

$2m\angle GCH = (31 + 3)$ — *Substitute.*

$m\angle GCH = 17°$ — *Simplify and divide both sides by 2.*

A **square** is a quadrilateral with four right angles and four congruent sides. In the exercises, you will show that a square is a parallelogram, a rectangle, and a rhombus. So a square has the properties of all three.



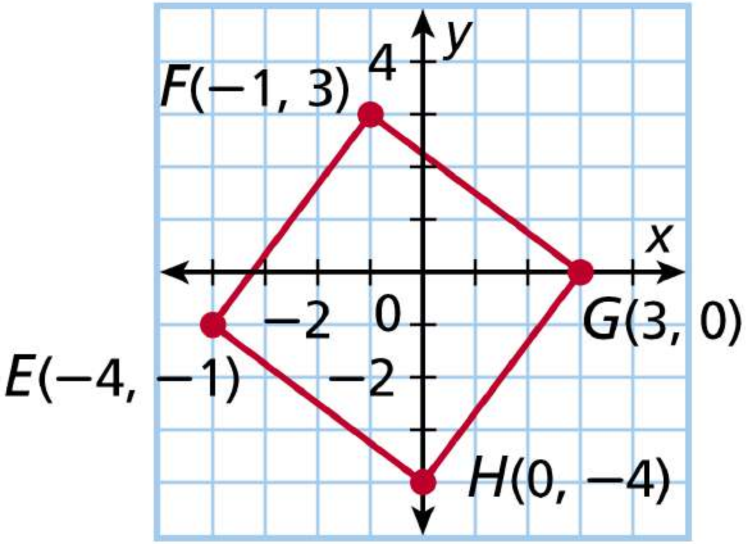
Square *ABCD*

**Helpful Hint**

Rectangles, rhombuses, and squares are sometimes referred to as *special parallelograms*.

# Example 3: Verifying Properties of Squares

**Show that the diagonals of square *EFGH* are congruent perpendicular bisectors of each other.**



## Example 3 Continued

**Step 1** Show that $\overline{EG}$ and $\overline{FH}$ are congruent.

$$EG = \sqrt{[3-(-4)]^2 + [0-(-1)]^2} = \sqrt{50}$$

$$FH = \sqrt{[0-(-1)]^2 + (-4-3)^2} = \sqrt{50}$$

Since $EG = FH$, $\overline{EG} \cong \overline{FH}$.

## Example 3 Continued

**Step 2** Show that $\overline{EG}$ and $\overline{FH}$ are perpendicular.

$$\text{slope of } \overline{EG} = \frac{0-(-1)}{3-(-4)} = \frac{1}{7}$$

$$\text{slope of } \overline{FH} = \frac{-4-3}{0-(-1)} = -7$$

Since $\left(\frac{1}{7}\right)(-7) = -1$, $\overline{EG} \perp \overline{FH}$.

## Example 3 Continued

**Step 3** Show that $\overline{EG}$ and $\overline{FH}$ are bisect each other.

$$\text{mdpt. of } \overline{EG}: \left(\frac{-4+3}{2}, \frac{-1+0}{2}\right) = \left(-\frac{1}{2}, -\frac{1}{2}\right)$$

$$\text{mdpt. of } \overline{FH}: \left(\frac{-1+0}{2}, \frac{3+(-4)}{2}\right) = \left(-\frac{1}{2}, -\frac{1}{2}\right)$$

Since $\overline{EG}$ and $\overline{FH}$ have the same midpoint, they bisect each other.

The diagonals are congruent perpendicular bisectors of each other.

## Check It Out! Example 3

**The vertices of square *STVW* are *S*(–5, –4), *T*(0, 2), *V*(6, –3) , and *W*(1, –9) . Show that the diagonals of square *STVW* are congruent perpendicular bisectors of each other.**

$SV = TW = \sqrt{122}$ so, $\overline{SV} \cong \overline{TW}$.

$$\text{slope of } \overline{SV} = \frac{1}{11}$$

$$\text{slope of } \overline{TW} = -11$$

$$\overline{SV} \perp \overline{TW}$$

## Check It Out! Example 3 Continued

**Step 1** Show that $\overline{SV}$ and $\overline{TW}$ are congruent.

$$SV = \sqrt{[6-(-5)]^2 + (-3-(-4))^2} = \sqrt{122}$$

$$TW = \sqrt{[1-0]^2 + (-9-2)^2} = \sqrt{122}$$

Since $SV = TW$, $\overline{SV} \cong \overline{TW}$.

## Check It Out! Example 3 Continued

**Step 2** Show that $\overline{SV}$ and $\overline{TW}$ are perpendicular.

$$\text{slope of } \overline{SV} = \frac{-3-(-4)}{6-(-5)} = \frac{1}{11}$$

$$\text{slope of } \overline{TW} = \frac{-9-2}{1-0} = -11$$

$$\text{Since } \left(\frac{1}{11}\right)(-11) = -1,\ \overline{SV} \perp \overline{TW}.$$

## Check It Out! Example 3 Continued

**Step 3** Show that $\overline{SV}$ and $\overline{TW}$ bisect each other.

$$\text{mdpt. of } \overline{SV}: \left(\frac{-5+6}{2}, \frac{-4+(-3)}{2}\right) = \left(\frac{1}{2}, -\frac{7}{2}\right)$$

$$\text{mdpt. of } \overline{TW}: \left(\frac{0+1}{2}, \frac{2+(-9)}{2}\right) = \left(\frac{1}{2}, -\frac{7}{2}\right)$$

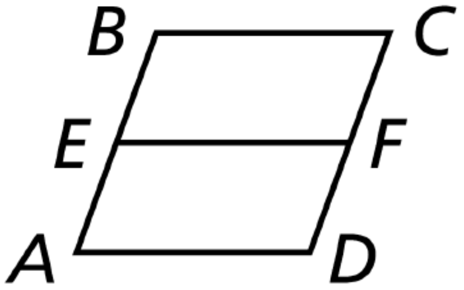
Since $\overline{SV}$ and $\overline{TW}$ have the same midpoint, they bisect each other.

The diagonals are congruent perpendicular bisectors of each other.

## Example 4: Using Properties of Special Parallelograms in Proofs

**Given:** ***ABCD*** **is a rhombus.** ***E*** **is the midpoint of** $\overline{AB}$**, and** ***F*** **is the midpoint of** $\overline{CD}$.

**Prove:** ***AEFD*** **is a parallelogram.**



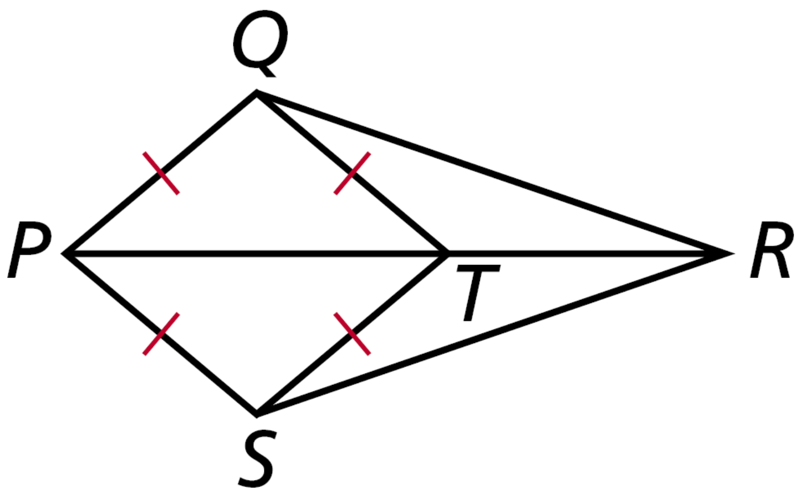
# Example 4 Continued

| Statements | Reasons |
|---|---|
| 1. *ABCD* is a rhombus. *E* is the midpoint of $\overline{AB}$ . *F* is the midpoint of $\overline{CD}$ | 1. Given. |
| 2. *ABCD* is a parallelogram. | 2. Rhombus → ▱ |
| 3. $\overline{AE} \parallel \overline{FD}$ | 3. Def. of ▱ |
| 4. $\overline{AB} \cong \overline{CD}$ | 4. ▱ → opp. sides ≅ |
| 5. $AB = CD$ | 5. Def. of ≅ |
| 6. $AE = \frac{1}{2}AB, FD = \frac{1}{2}CD$ | 6. Def. of mdpt. |
| 7. $FD = \frac{1}{2}AB$ | 7. Subst. |
| 8. $AE = FD$ | 8. Trans. Prop. of = |
| 9. $\overline{AE} \cong \overline{FD}$ | 9. Def. of ≅ |
| 10. *AEFD* is a parallelogram. | 10. Quad. with 1 pair of opp. sides ≅ and \|\| → ▱ |

## Check It Out! Example 4

**Given:** *PQTS* is a rhombus with diagonal $\overline{PR}$.

**Prove:** $\overline{RQ} \cong \overline{RS}$

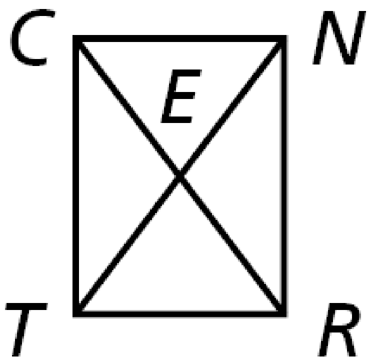


## Check It Out! Example 4 Continued

| Statements | Reasons |
| --- | --- |
| **1.** *PQTS* is a rhombus. | **1.** Given. |
| **2.** $\overline{PT}$ bisects $\angle QPS$ | **2.** Rhombus → each diag. bisects opp. ∠s |
| **3.** $\angle QPR \cong \angle SPR$ | **3.** Def. of ∠ bisector. |
| **4.** $\overline{PQ} \cong \overline{PS}$ | **4.** Def. of rhombus. |
| **5.** $\overline{PR} \cong \overline{PR}$ | **5.** Reflex. Prop. of ≅ |
| **6.** $\triangle QPR \cong \triangle SPR$ | **6.** SAS |
| **7.** $\overline{RQ} \cong \overline{RS}$ | **7.** CPCTC |

# Lesson Quiz: Part I

**A slab of concrete is poured with diagonal spacers. In rectangle *CNRT*, *CN* = 35 ft, and *NT* = 58 ft. Find each length.**

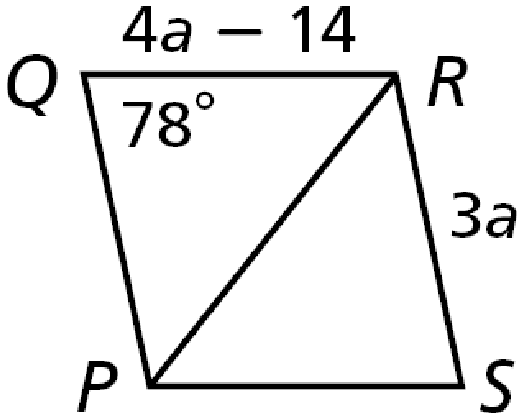



**1.** *TR* 35 ft

**2.** *CE* 29 ft

## Lesson Quiz: Part II

***PQRS* is a rhombus. Find each measure.**




**3.** *QP*

42

**4.** m∠*QRP*

51°

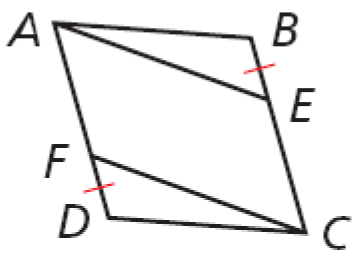
## Lesson Quiz: Part III

**5.** The vertices of square *ABCD* are *A*(1, 3), *B*(3, 2), *C*(4, 4), and *D*(2, 5). Show that its diagonals are congruent perpendicular bisectors of each other.

$AC = BD = \sqrt{10}$, so $\overline{AC} \cong \overline{BD}$. Slope of $\overline{AC} = \frac{1}{3}$, and slope of $\overline{BD} = -3$, so $\overline{AC} \perp \overline{BD}$. (2.5, 3.5) is the mdpt. of $\overline{AC}$ and $\overline{BD}$, so $\overline{AC}$ and $\overline{BD}$ bisect each other.

# Lesson Quiz: Part IV

**6. Given:** *ABCD* is a rhombus. $\overline{DF} \cong \overline{BE}$
**Prove:** $\triangle ABE \cong \triangle CDF$




1. *ABCD* is a rhombus. (Given)
2. $\overline{AB} \cong \overline{CD}$ (Def. of rhombus)
3. *ABCD* is a ▱ (Rhombus → ▱)
4. $\angle B \cong \angle D$ (▱ → opp. ∠s ≅)
5. $\overline{DF} \cong \overline{BE}$ (Given)
6. $\triangle ABE \cong \triangle CDF$ (SAS)