Name: ____________________________ **Date: __________________**

Properties of Matrices: Walk Like a Mathematician Learning Task.1

Matrices allow us to perform many useful mathematical tasks which ordinarily require large number of computations. Some types of problems which can be done efficiently with matrices include solving systems of equations, finding the area of triangles given the coordinates of the vertices, finding equations for graphs given sets of ordered pairs, and determining information contained in vertex edge graphs. In order to address these types of problems, it is necessary to understand more about matrix operations and properties; and, to use technology to perform some of the computations.

Matrix operations have many of the same properties as real numbers. There are more restrictions on matrices than on real numbers, however, because of the rules governing matrix addition, subtraction, and multiplication. Some of the real number properties which are more useful when considering matrix properties are listed below.

Let a, b, and c be real numbers

	ADDITION PROPERTIES	MULTIPLICATION PROPERTIES
COMMUTATIVE	a + b = b + a	ab = ba
ASSOCIATIVE	(a + b) + c = a + (b + c)	(ab)c = a(bc)
IDENTITY	There exists a unique real number zero, 0, such that a + 0 = 0 + a = a	There exists a unique real number one, 1, such that a * 1 = 1 * a = a
INVERSE	For each real number a, there is a unique real number - a such that a + (-a) = (-a) + a = 0	For each nonzero real number a, there is a unique real number $\frac{1}{a}$ such that $a(\frac{1}{a}) = (\frac{1}{a})a = 1$

State the Property of Real Numbers

*3+4= 4+3*________________________________

*(18+24) +5= 18+(24+5)*__________________________

*8+0=8*________________________________

(2+4)+3=2+(4+3) ________________________________

*5+(-5)=0*________________________________

*3x-3x=0*________________________________

*3*4=4*3*________________________________

*(18*24)*5= 18*(24*5)*__________________________

*8*1=8*________________________________

*(2*4)*3=2*(4*3)* ________________________________

8(⅛)=1*________________________________

The following is a set of matrices without row and column labels. Use these matrices to complete the problems.

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 2 & 4 & 1 \\ 0 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 5 \\ -2 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 0 & -1 \\ 1/3 & 2/3 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} 3 & 4 & -1 \\ 0 & -2 & 3 \\ 5 & 1 & 2 \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} 5 & -2 \\ -1 & 0 \end{bmatrix} \quad \mathbf{J} = \begin{bmatrix} 3 & 0 & 2 \\ -4 & 1 & -1 \\ 6 & 4 & 3 \end{bmatrix}$$

I. Commutative of Addition Investigation

a). Find A+F= and F+A=

b.) Find B+G and G+B=

c.) Is Matrix Addition Commutative? Explain__________________________________

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II. Associative of Addition Investigation

a.) Find (A+F) + H = and A+ (F+H) =

b.) Find (B+G) +J = and B+ (G+J) =

III. Commutative of Multiplication Investigation

a.) Find AF= and FA=

b.) Find BG= and GB=

c.) Is Matrix Multiplication Commutative?

Explain____

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IV. Associative of Multiplication Investigation

a.) Find (AF) H = and A (FH) =

b.) Find (BG) J = and B (GJ) =

c.) Is Matrix Multiplication Commutative?
Explain____

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V. Zero and Identity Matrices

a.) Do matrices have an additive identity (zero) or **zero matrix**?

If so, what does a **zero matrix** look like? Give three examples of zero matrices (different dimensions).

Is a **zero matrix** unique and does it satisfy the property $a + 0 = 0 + a = a$?

b.) Do matrices have a **one** or an **identity matrix (*I*),** for multiplication?

If so what does an **identity matrix** look like? Give three examples of identity matrices (different dimensions).

Is it unique; and, does it satisfy the property $a * 1 = 1 * a = a$?

VI. Inverse Matrices

a.) Consider matrices A and D from the original set of matrices. A and D are **inverse matrices**.

> In order for a matrix to have an inverse, it must satisfy two conditions.
>
> 1. The matrix must be a square matrix.
>
> 2. No row of the matrix can be a multiple of any other row.
>
> Both D and G are 2x2 matrices; and, the rows in D are not multiples of each other. The same is true of G.
>
> The notation normally used for a matrix and its inverse is D and D^{-1} or G and G^{-1}.

The product of two inverse matrices should be the identity matrix, **I**.

Find A*D and D*A.

b.) The following formula can be used to find the inverse of a 2x2 matrix. Given matrix A where the rows of A are not multiples of each other:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{then} \quad A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

For higher order matrices, we will use technology to find inverses.

Find the inverse of matrix J from the matrices listed above. Verify that J and J^{-1} are inverses.

VII. Determinants of Matrices

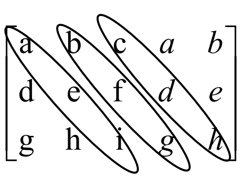
a.) Another type of matrix operation is finding the determinant. Only square matrices have determinants. To find determinants of 2x2 matrices by hand use the following procedure.

$$\text{determinant}(A) = \det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

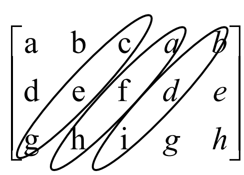
To find determinants of 3x3 matrices use the following procedure: given matrix $B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$,

rewrite the matrix and repeat columns 1 and 2 to get $\begin{bmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{bmatrix}$. Now multiply and

combine products according to the following patterns.



and



to give $\det(B) = aei + bfg + cdh - ceg - afh - bdi$.

We will also use expansion by minors and technology to find determinants of matrices.

b.) The determinant of a matrix can be used to **find the area of a triangle**. If (x_1, y_1), (x_2, y_2), and (x_3, y_3) are vertices of a triangle, the area of the triangle is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

Problem 1

Given a triangle with vertices (-1, 0), (1, 3), and (5, 0), find the area using the determinant formula. Verify that the area you found is correct using geometric formulas.

Problem 2

Given a triangle with vertices (-3, 4), (2, 3), and (1, -4), find the area using the determinant formula. Verify that the area you found is correct using geometric formulas.

Problem 3

Suppose you are finding the area of a triangle with vertices (-1, -1), (4, 7), and (9, -6). You find the area of the triangle to be -52.5 and your partner works the same problem and gets +52.5. After checking both solutions, you each have done your work correctly. How can you explain this discrepancy?

Problem 4

Suppose another triangle with vertices (1, 1), (4, 2), and (7, 3) gives an area of 0. What do you know about the triangle and the points?

Problem 5

A gardener is trying to find a triangular area behind his house that encloses 1750 square feet. He has placed the first two fence posts at (0, 50) and at (40, 0). The final fence post is on the property line at y = 100. Find the point where the gardener can place the final fence post.