Properties of arcs and chords

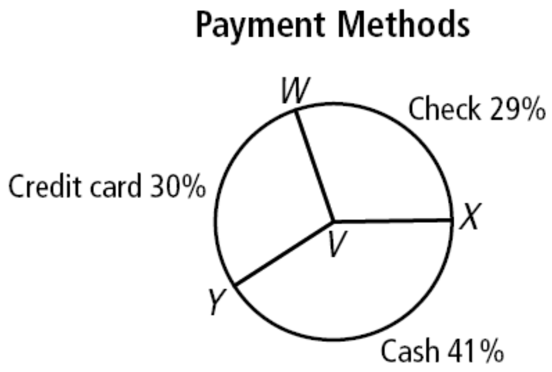
Warm Up

1. What percent of 60 is 18? 30

2. What number is 44% of 6? 2.64

3. Find m$\angle WVX$.

104.4°



Objectives

Apply properties of arcs.

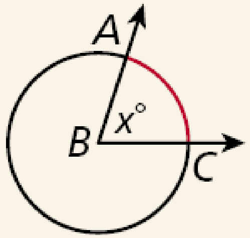
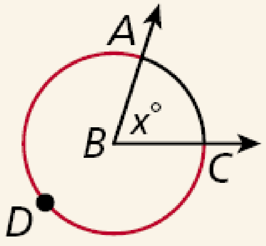
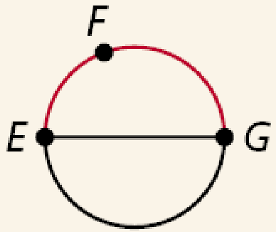
Apply properties of chords.

Vocabulary

central angle	semicircle
arc	adjacent arcs
minor arc	congruent arcs
major arc	

A **central angle** is an angle whose vertex is the center of a circle. An **arc** is an unbroken part of a circle consisting of two points called the endpoints and all the points on the circle between them.

Arcs and Their Measure

ARC	MEASURE	DIAGRAM
A **minor arc** is an arc whose points are on or in the interior of a central angle.	The measure of a minor arc is equal to the measure of its central angle. $m\overset{\frown}{AC} = m\angle ABC = x°$	
A **major arc** is an arc whose points are on or in the exterior of a central angle.	The measure of a major arc is equal to 360° minus the measure of its central angle. $m\overset{\frown}{ADC} = 360° - m\angle ABC$ $= 360° - x°$	
If the endpoints of an arc lie on a diameter, the arc is a **semicircle**.	The measure of a semicircle is equal to 180°. $m\overset{\frown}{EFG} = 180°$	

Writing Math

Minor arcs may be named by two points. Major arcs and semicircles must be named by three points.

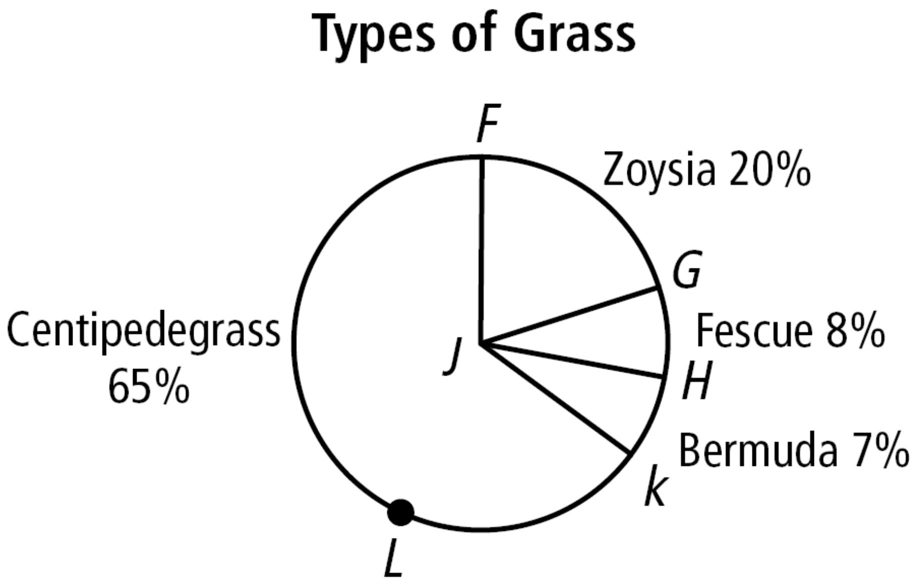
Example 1: Data Application

The circle graph shows the types of grass planted in the yards of one neighborhood. Find $m\widehat{KLF}$.

$$m\widehat{KLF} = 360° - m\angle KJF$$

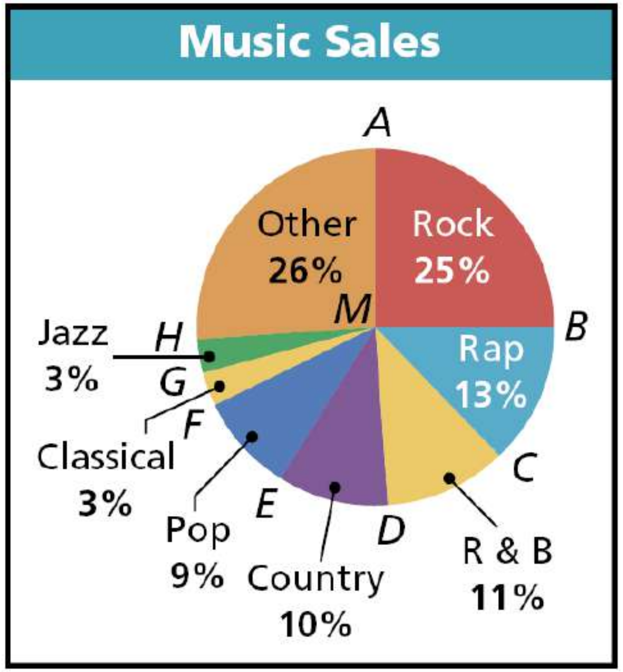
$$m\angle KJF = 0.35(360°)$$
$$= 126°$$

$$m\widehat{KLF} = 360° - 126°$$
$$= 234°$$



Check It Out! Example 1

Use the graph to find each of the following.



a. m∠FMC

m∠FMC = 0.30(360°)

= 108°

Central ∠ is 30% of the ⊙.

b. m$\widehat{AHB}$ = 360° – m∠AMB

m∠AHB = 360° – 0.25(360°)

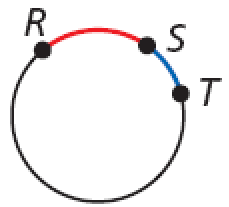
= 270°

c. m∠EMD = 0.10(360°)

= 36°

Central ∠ is 10% of the ⊙.

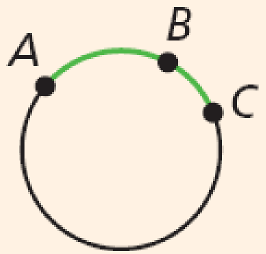
Adjacent arcs are arcs of the same circle that intersect at exactly one point. $\overset{\frown}{RS}$ and $\overset{\frown}{ST}$ are adjacent arcs.



Postulate 11-2-1 **Arc Addition Postulate**

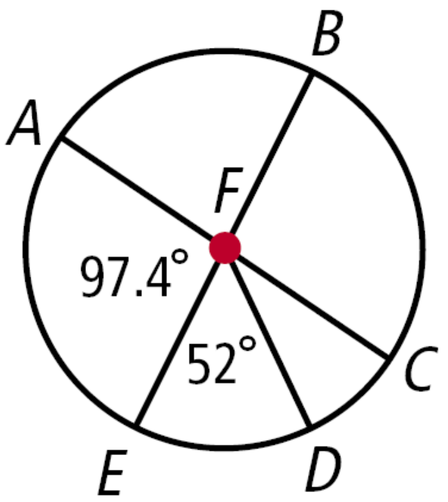
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$$



Example 2: Using the Arc Addition Postulate

Find $m\overset{\frown}{BD}$.



$m\overset{\frown}{BC} = 97.4°$ *Vert. ∠s Thm.*

$m\angle CFD = 180° - (97.4° + 52°)$

$= 30.6°$ *Δ Sum Thm.*

$m\overset{\frown}{CD} = 30.6°$ *$m\angle CFD = 30.6°$*

$m\overset{\frown}{BD} = m\overset{\frown}{BC} + m\overset{\frown}{CD}$ *Arc Add. Post.*

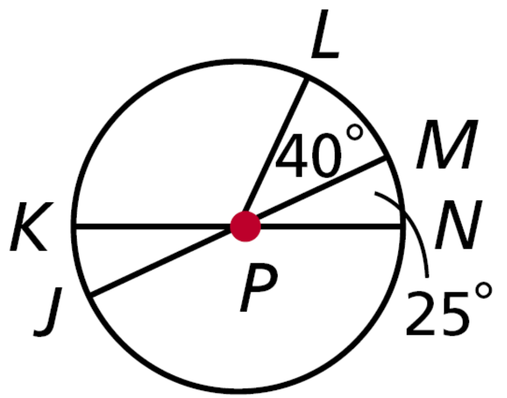
$= 97.4° + 30.6°$ *Substitute.*

$= 128°$ *Simplify.*

Check It Out! Example 2a

Find each measure.

$m\overset{\frown}{JKL}$



$m\angle KPL = 180° - (40 + 25)°$

$m\overset{\frown}{KL} = 115°$

$m\overset{\frown}{JKL} = m\overset{\frown}{JK} + m\overset{\frown}{KL}$ *Arc Add. Post.*

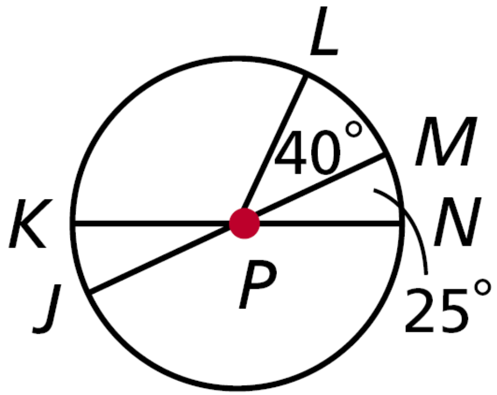
$= 25° + 115°$ *Substitute.*

$= 140°$ *Simplify.*

Check It Out! Example 2b

Find each measure.

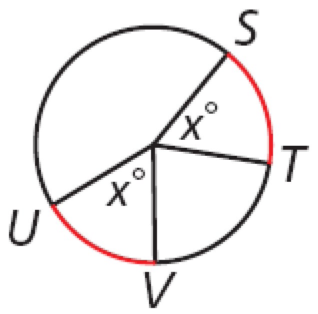
$\mathbf{m\widehat{LJN}}$



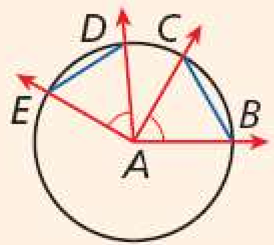
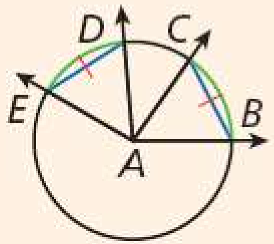
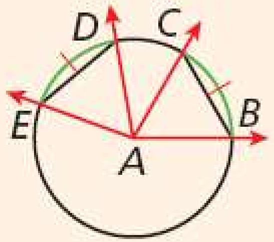
$$m\widehat{LJN} = 360° - (40 + 25)°$$

$$= 295°$$

Within a circle or congruent circles, **congruent arcs** are two arcs that have the same measure. In the figure $\overset{\frown}{ST} \cong \overset{\frown}{UV}$.

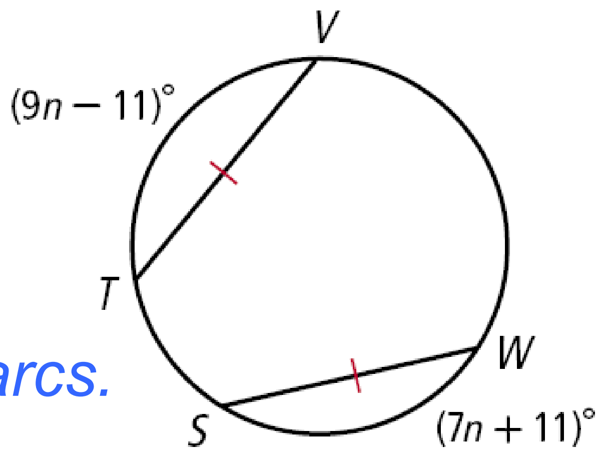


Theorem 11-2-2

THEOREM	HYPOTHESIS		CONCLUSION
In a circle or congruent circles:			
(1) Congruent central angles have congruent chords.		$\angle EAD \cong \angle BAC$	$\overline{DE} \cong \overline{BC}$
(2) Congruent chords have congruent arcs.		$\overline{ED} \cong \overline{BC}$	$\overset{\frown}{DE} \cong \overset{\frown}{BC}$
(3) Congruent arcs have congruent central angles.		$\overset{\frown}{ED} \cong \overset{\frown}{BC}$	$\angle DAE \cong \angle BAC$

Example 3A: Applying Congruent Angles, Arcs, and Chords

$\overline{TV} \cong \overline{WS}$. Find $m\widehat{WS}$.

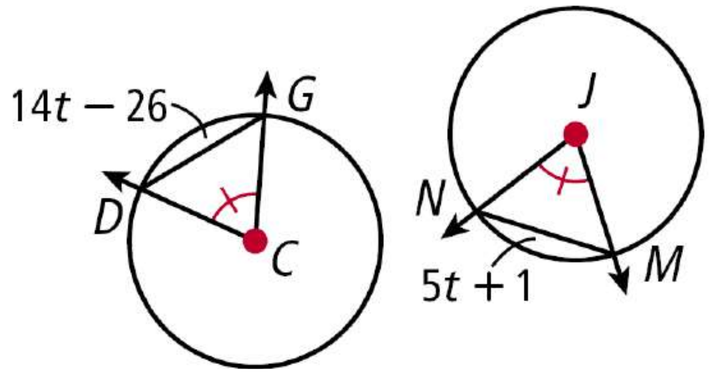


$\widehat{TV} \cong \widehat{WS}$	*≅ chords have ≅ arcs.*
$m\widehat{TV} = m\widehat{WS}$	*Def. of ≅ arcs*
$9n - 11 = 7n + 11$	*Substitute the given measures.*
$2n = 22$	*Subtract 7n and add 11 to both sides.*
$n = 11$	*Divide both sides by 2.*
$m\widehat{WS} = 7(11) + 11$	*Substitute 11 for n.*
$= 88°$	*Simplify.*

Example 3B: Applying Congruent Angles, Arcs, and Chords

$\odot C \cong \odot J$, and $m\angle GCD \cong m\angle NJM$. Find *NM*.

$\overset{\frown}{GD} \cong \overset{\frown}{NM}$	$\angle GCD \cong \angle NJM$
$\overline{GD} \cong \overline{NM}$	$\cong$ arcs have $\cong$ chords.
$GD = NM$	Def. of $\cong$ chords



Example 3B Continued

⊙C ≅ ⊙J, and m∠GCD ≅ m∠NJM. Find NM.

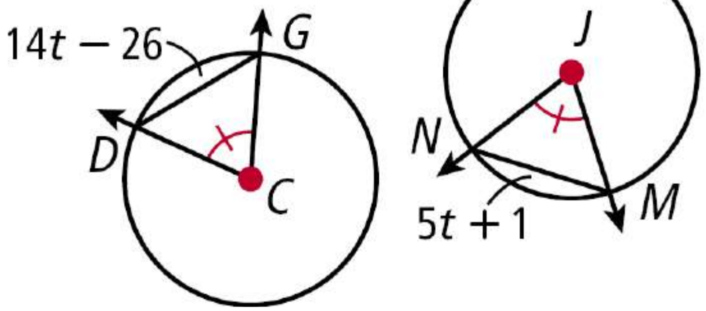
$14t - 26 = 5t + 1$ *Substitute the given measures.*

$9t = 27$ *Subtract 5t and add 26 to both sides.*

$t = 3$ *Divide both sides by 9.*

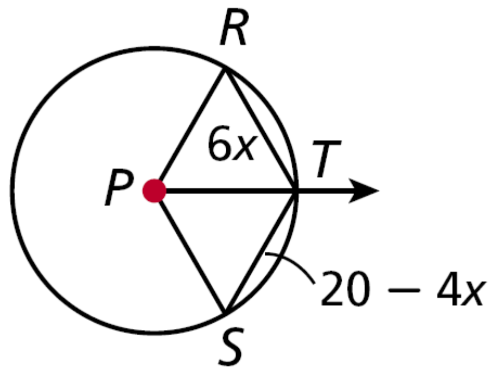
$NM = 5(3) + 1$ *Substitute 3 for t.*

$= 16$ *Simplify.*



Check It Out! Example 3a

$\overrightarrow{PT}$ **bisects** $\angle RPS$**. Find** RT**.**



$\angle RPT \cong \angle SPT$

$m\overset{\frown}{RT} \cong m\overset{\frown}{TS}$

$RT = TS$

$6x = 20 - 4x$

$10x = 20$ *Add 4x to both sides.*

$x = 2$ *Divide both sides by 10.*

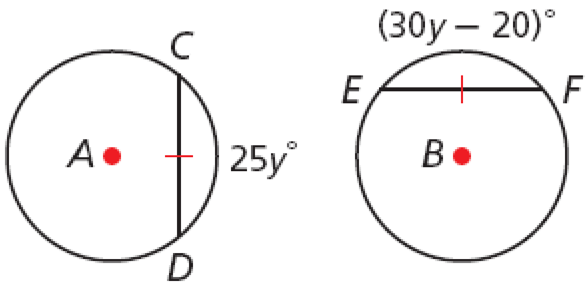
$RT = 6(2)$ *Substitute 2 for x.*

$RT = 12$ *Simplify.*

Check It Out! Example 3b

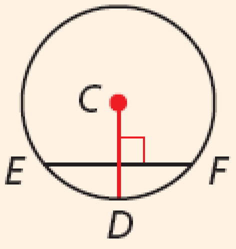
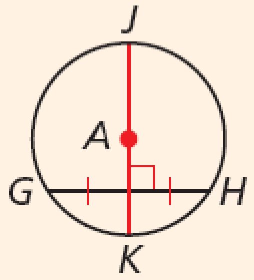
Find each measure.

$\odot A \cong \odot B$, and $\overline{CD} \cong \overline{EF}$. Find $m\overset{\frown}{CD}$.



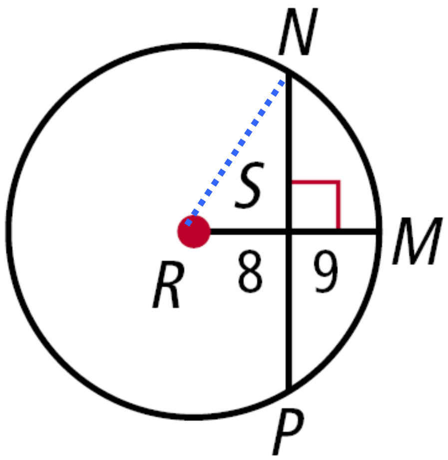
$m\overset{\frown}{CD} = m\overset{\frown}{EF}$	*≅ chords have ≅ arcs.*
$25y° = (30y - 20)°$	*Substitute.*
$20 = 5y$	*Subtract 25y from both sides. Add 20 to both sides.*
$4 = y$	*Divide both sides by 5.*
$CD = 25(4)$	*Substitute 4 for y.*
$m\overset{\frown}{CD} = 100°$	*Simplify.*

Theorems

THEOREM	HYPOTHESIS	CONCLUSION
11-2-3 In a circle, if a radius (or diameter) is perpendicular to a chord, then it bisects the chord and its arc.	 $\overline{CD} \perp \overline{EF}$	$\overline{CD}$ bisects $\overline{EF}$ and $\overset{\frown}{EF}$.
11-2-4 In a circle, the perpendicular bisector of a chord is a radius (or diameter).	 $\overline{JK}$ is $\perp$ bisector of $\overline{GH}$.	$\overline{JK}$ is a diameter of $\odot A$.

Example 4: Using Radii and Chords

Find *NP*.



Step 1 Draw radius $\overline{RN}$.

$RN = 17$ *Radii of a ⊙ are ≅.*

Step 2 Use the Pythagorean Theorem.

$SN^2 + RS^2 = RN^2$

$SN^2 + 8^2 = 17^2$ *Substitute 8 for RS and 17 for RN.*

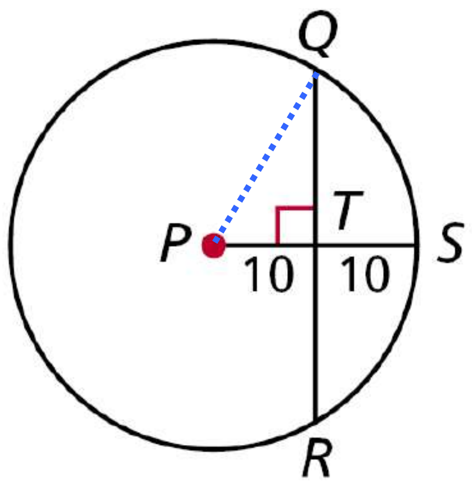
$SN^2 = 225$ *Subtract 8^2 from both sides.*

$SN = 15$ *Take the square root of both sides.*

Step 3 Find *NP*.

$NP = 2(15) = 30$ *$\overline{RM} \perp \overline{NP}$, so $\overline{RM}$ bisects $\overline{NP}$.*

Check It Out! Example 4



Find QR to the nearest tenth.

Step 1 Draw radius $\overline{PQ}$.

$PQ = 20$ *Radii of a ⊙ are ≅.*

Step 2 Use the Pythagorean Theorem.

$TQ^2 + PT^2 = PQ^2$

$TQ^2 + 10^2 = 20^2$ *Substitute 10 for PT and 20 for PQ.*

$TQ^2 = 300$ *Subtract 10^2 from both sides.*

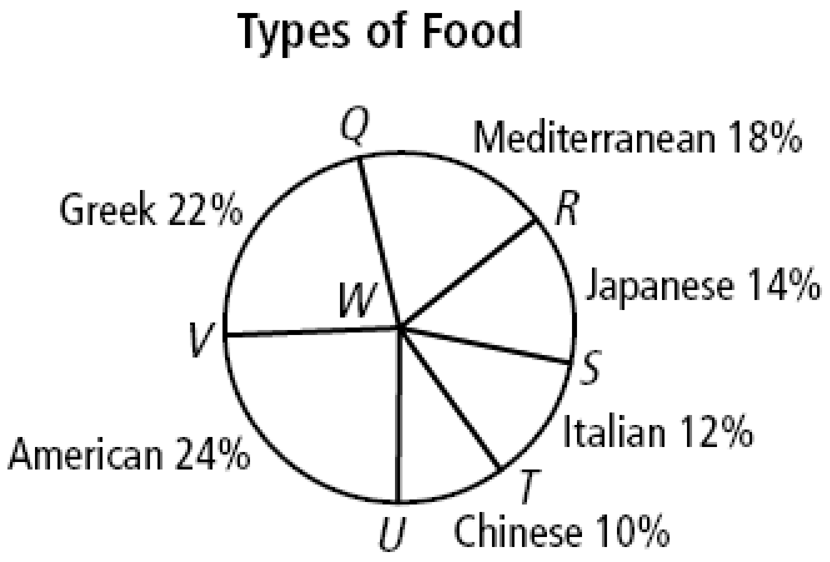
$TQ \approx 17.3$ *Take the square root of both sides.*

Step 3 Find QR.

$QR = 2(17.3) = 34.6$ *$\overline{PS} \perp \overline{QR}$, so $\overline{PS}$ bisects $\overline{QR}$.*

Lesson Quiz: Part I

1. The circle graph shows the types of cuisine available in a city. Find $m\overset{\frown}{TRQ}$.



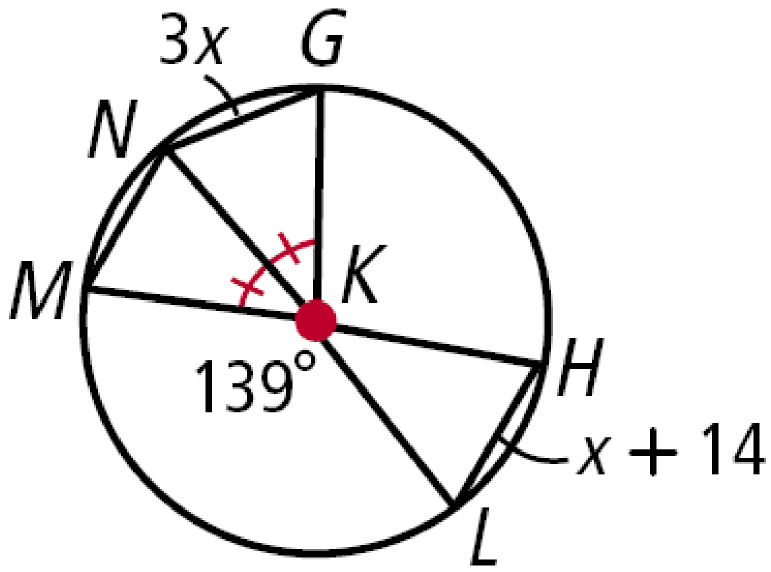
158.4°

Lesson Quiz: Part II

Find each measure.

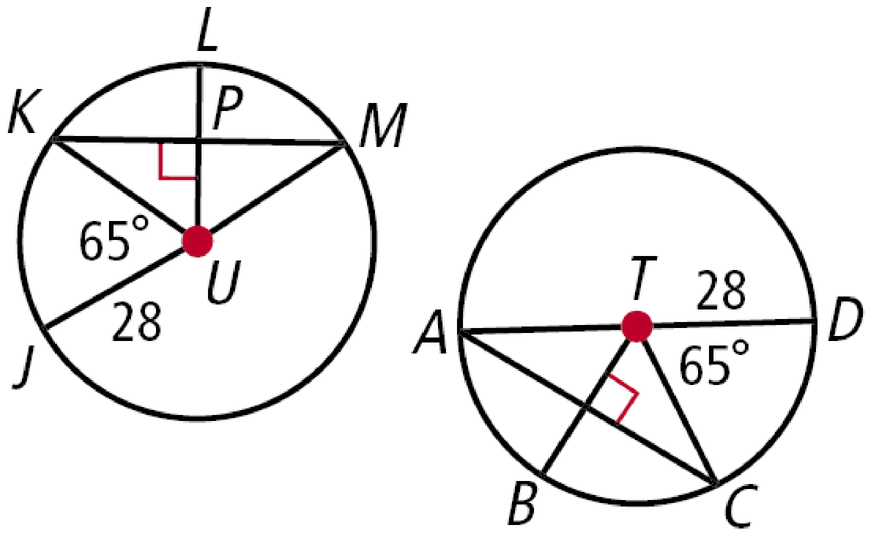
2. $\overset{\frown}{NGH}$ 139°

3. HL 21



Lesson Quiz: Part III

4. $\odot T \cong \odot U$, and $AC = 47.2$. Find PL to the nearest tenth.



≈ 12.9