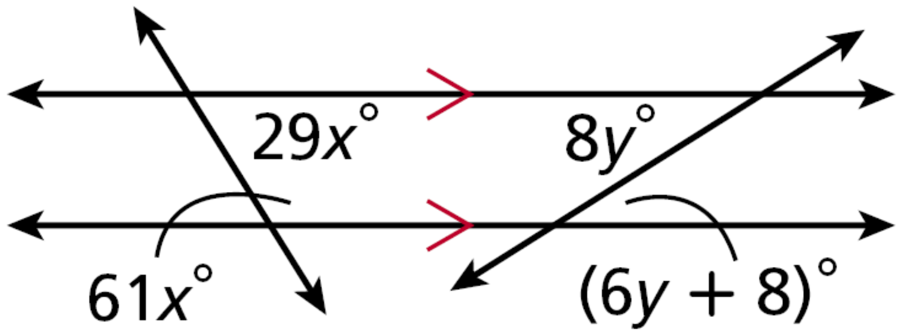
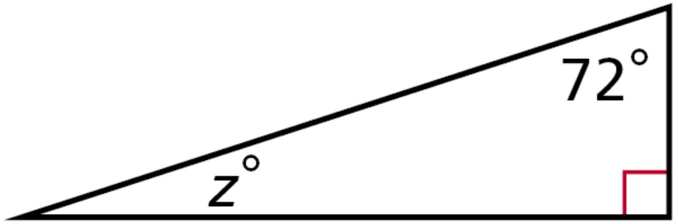
Properties of parallelograms

Warm Up

Find the value of each variable.





1. x 2

2. y 4

3. z 18

Objectives

Prove and apply properties of parallelograms.

Use properties of parallelograms to solve problems.

Vocabulary

parallelogram

Any polygon with four sides is a quadrilateral. However, some quadrilaterals have special properties. These *special quadrilaterals* are given their own names.

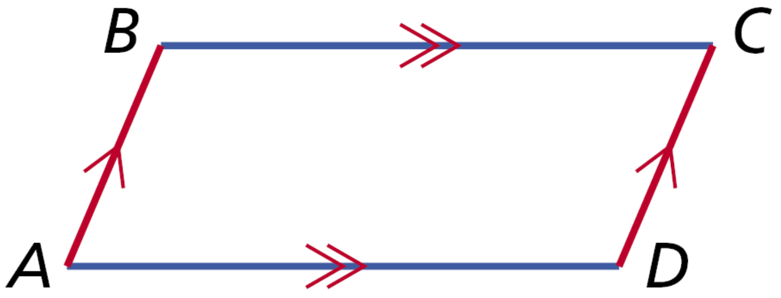
Helpful Hint

Opposite sides of a quadrilateral do not share a vertex. Opposite angles do not share a side.

A quadrilateral with two pairs of parallel sides is a **parallelogram**. To write the name of a parallelogram, you use the symbol ▱.

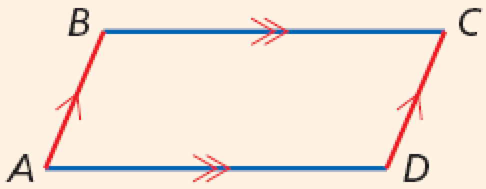
Parallelogram $ABCD$

▱$ABCD$

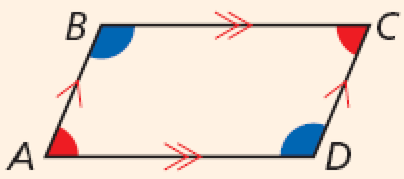
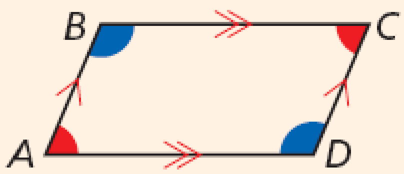
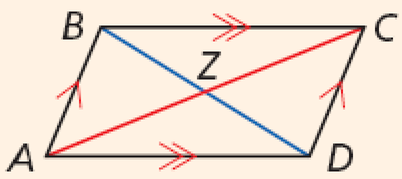


$\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DA}$

Theorem 6-2-1 Properties of Parallelograms

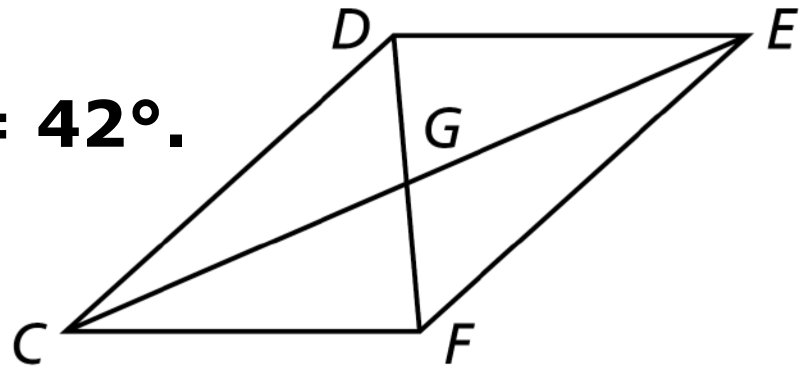
THEOREM	HYPOTHESIS	CONCLUSION
If a quadrilateral is a parallelogram, then its opposite sides are congruent. ($\square \rightarrow$ opp. sides $\cong$)		$\overline{AB} \cong \overline{CD}$ $\overline{BC} \cong \overline{DA}$

Theorems — Properties of Parallelograms

THEOREM	HYPOTHESIS	CONCLUSION
6-2-2 If a quadrilateral is a parallelogram, then its opposite angles are congruent. ($\square \rightarrow$ opp. $\angle$s $\cong$)		$\angle A \cong \angle C$ $\angle B \cong \angle D$
6-2-3 If a quadrilateral is a parallelogram, then its consecutive angles are supplementary. ($\square \rightarrow$ cons. $\angle$s supp.)		$m\angle A + m\angle B = 180°$ $m\angle B + m\angle C = 180°$ $m\angle C + m\angle D = 180°$ $m\angle D + m\angle A = 180°$
6-2-4 If a quadrilateral is a parallelogram, then its diagonals bisect each other. ($\square \rightarrow$ diags. bisect each other)		$\overline{AZ} \cong \overline{CZ}$ $\overline{BZ} \cong \overline{DZ}$

Example 1A: Properties of Parallelograms

In ▱*CDEF*, *DE* = 74 mm,
***DG* = 31 mm, and m∠*FCD* = 42°.**
Find *CF*.



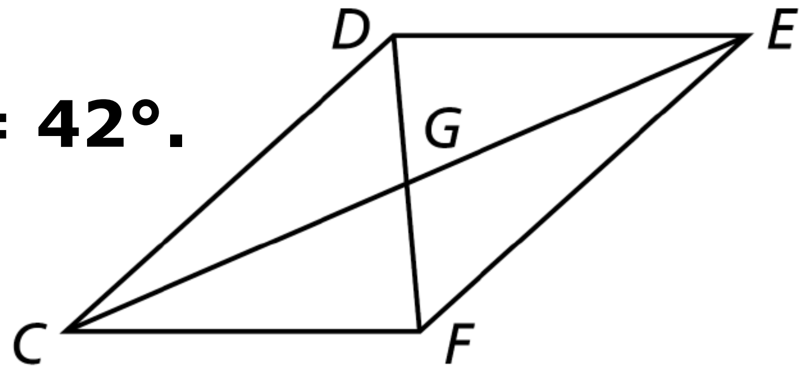
$\overline{CF} \cong \overline{DE}$ ▱ → *opp. sides* ≅

CF = *DE* *Def. of ≅ segs.*

CF = 74 mm *Substitute 74 for DE.*

Example 1B: Properties of Parallelograms

In ▱*CDEF*, *DE* = 74 mm, *DG* = 31 mm, and m∠*FCD* = 42°. Find m∠*EFC*.



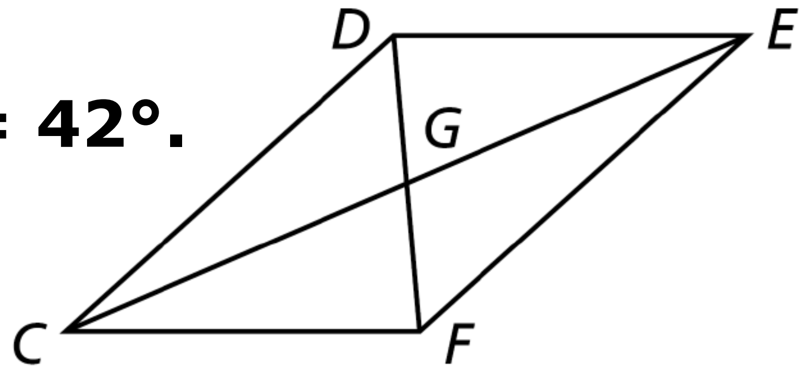
$m\angle EFC + m\angle FCD = 180°$ ▱ → *cons. ∠s supp.*

$m\angle EFC + 42 = 180$ *Substitute 42 for m∠FCD.*

$m\angle EFC = 138°$ *Subtract 42 from both sides.*

Example 1C: Properties of Parallelograms

In ▱*CDEF*, *DE* = 74 mm, *DG* = 31 mm, and m∠*FCD* = 42°. Find *DF*.



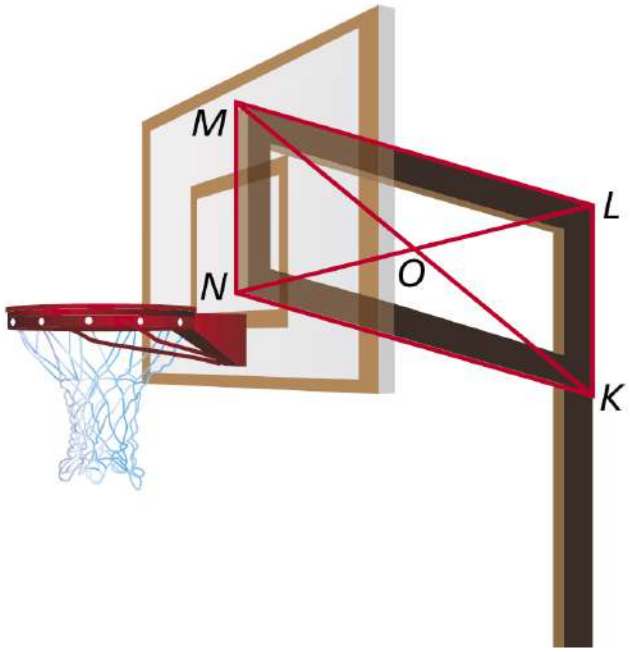
$DF = 2DG$ ▱ → *diags. bisect each other.*

$DF = 2(31)$ *Substitute 31 for DG.*

$DF = 62$ *Simplify.*

Check It Out! Example 1a

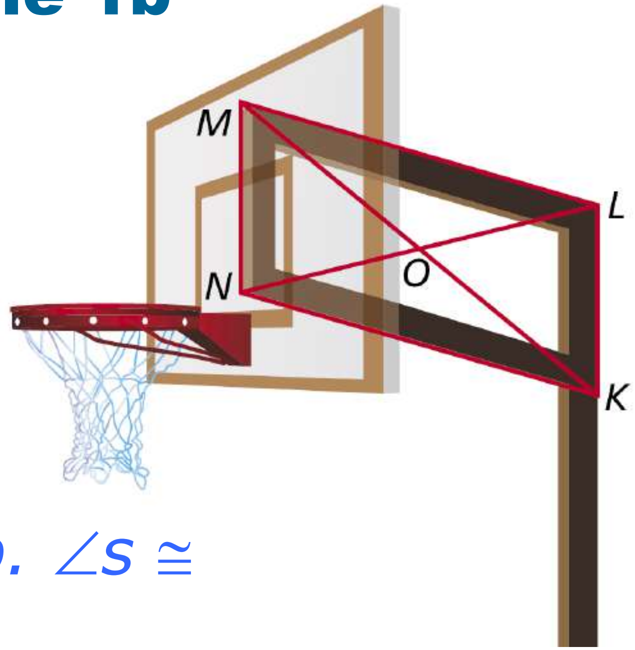
In ▱*KLMN*, *LM* = 28 in., *LN* = 26 in., and m∠*LKN* = 74°. Find *KN*.



$\overline{LM} \cong \overline{KN}$	▱ → opp. sides ≅
$LM = KN$	Def. of ≅ segs.
$LM = 28$ in.	Substitute 28 for DE.

Check It Out! Example 1b

In ▱*KLMN*, *LM* = 28 in., *LN* = 26 in., and m∠*LKN* = 74°. Find m∠*NML*.



$\angle NML \cong \angle LKN$ — ▱ → *opp.* ∠*s* ≅

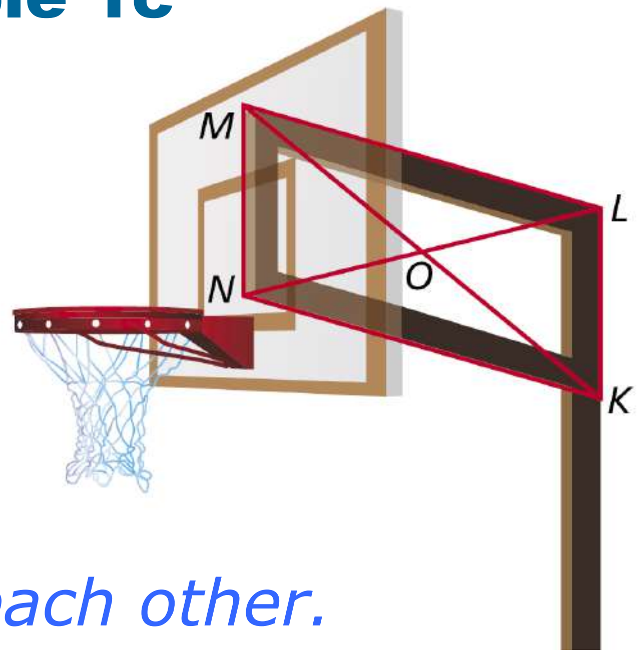
$m\angle NML = m\angle LKN$ — *Def. of* ≅ ∠*s.*

$m\angle NML = 74°$ — *Substitute 74° for m∠LKN.*

Def. of angles.

Check It Out! Example 1c

In ▱*KLMN*, *LM* = 28 in., *LN* = 26 in., and m∠*LKN* = 74°. Find *LO*.



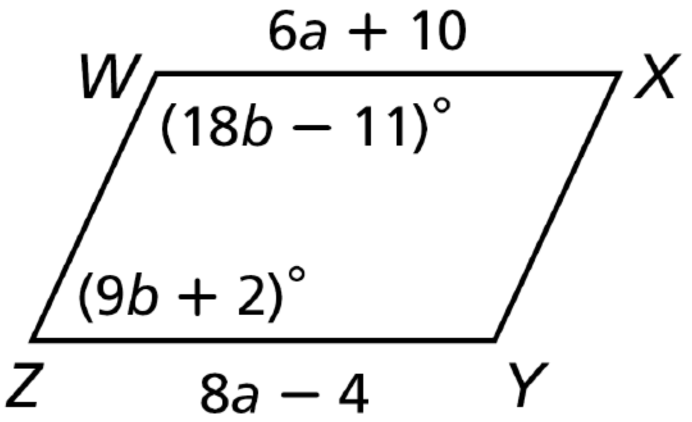
LN = 2*LO* ▱ → *diags. bisect each other.*

26 = 2*LO* *Substitute 26 for LN.*

LO = 13 in. *Simplify.*

Example 2A: Using Properties of Parallelograms to Find Measures

WXYZ **is a parallelogram.**
Find ***YZ*****.**



$\overline{YZ} \cong \overline{XW}$ ▱ → opp. ∠s ≅

$YZ = XW$ Def. of ≅ segs.

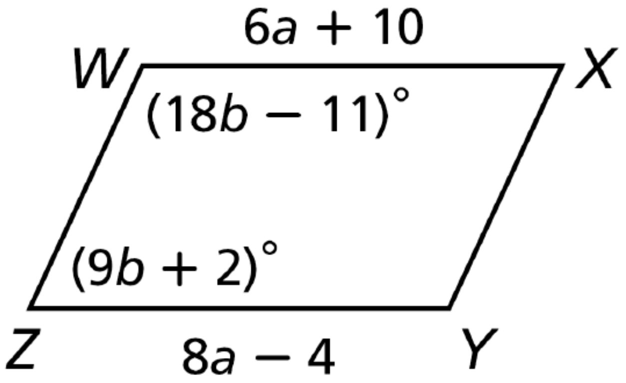
$8a - 4 = 6a + 10$ Substitute the given values.

$2a = 14$ Subtract 6a from both sides and add 4 to both sides.

$a = 7$ Divide both sides by 2.

$YZ = 8a - 4 = 8(7) - 4 = 52$

Example 2B: Using Properties of Parallelograms to Find Measures



WXYZ **is a parallelogram.**
Find m∠*Z* .

$m\angle Z + m\angle W = 180°$ ▱ → *cons. ∠s supp.*

$(9b + 2) + (18b - 11) = 180$ *Substitute the given values.*

$27b - 9 = 180$ *Combine like terms.*

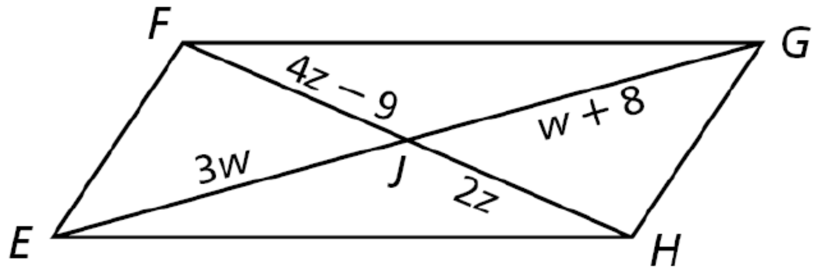
$27b = 189$ *Add 9 to both sides.*

$b = 7$ *Divide by 27.*

$m\angle Z = (9b + 2)° = [9(7) + 2]° = 65°$

Check It Out! Example 2a

EFGH **is a parallelogram.**
Find *JG*.



$\overline{EJ} \cong \overline{JG}$ ▱ → *diags. bisect each other.*

$EJ = JG$ *Def. of ≅ segs.*

$3w = w + 8$ *Substitute.*

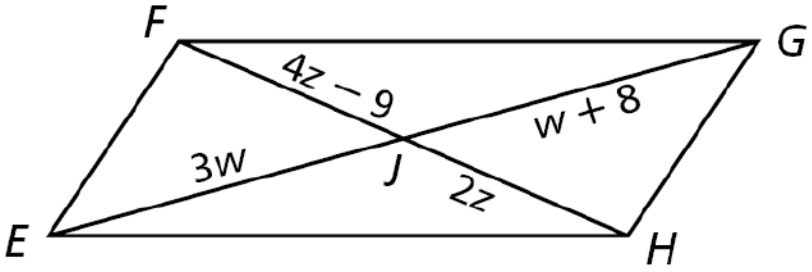
$2w = 8$ *Simplify.*

$w = 4$ *Divide both sides by 2.*

$JG = w + 8 = 4 + 8 = 12$

Check It Out! Example 2b

EFGH **is a parallelogram.**
Find *FH*.



$\overline{FJ} \cong \overline{JH}$ ▱ → *diags. bisect each other.*

$FJ = JH$ *Def. of ≅ segs.*

$4z - 9 = 2z$ *Substitute.*

$2z = 9$ *Simplify.*

$z = 4.5$ *Divide both sides by 2.*

$FH = (4z - 9) + (2z) = 4(4.5) - 9 + 2(4.5) = 18$

Remember!

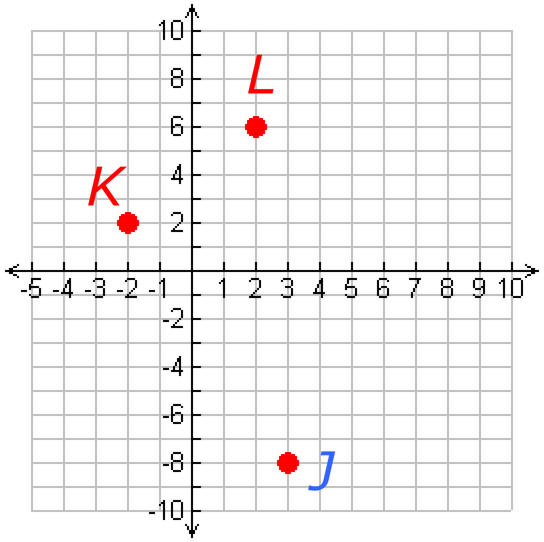
When you are drawing a figure in the coordinate plane, the name *ABCD* gives the order of the vertices.

Example 3: Parallelograms in the Coordinate Plane

Three vertices of ▱*JKLM* are *J*(3, –8), *K*(–2, 2), and *L*(2, 6). Find the coordinates of vertex *M*.

Since *JKLM* is a parallelogram, both pairs of opposite sides must be parallel.

Step 1 Graph the given points.

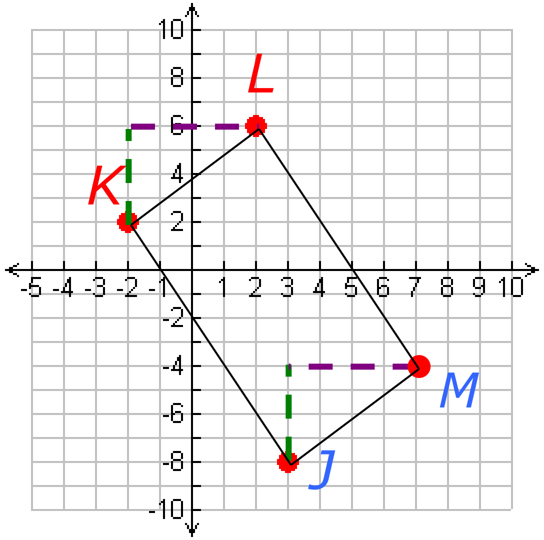


Example 3 Continued

Step 2 Find the slope of $\overline{KL}$ by counting the units from *K* to *L*.

The rise from 2 to 6 is 4.

The run of –2 to 2 is 4.



Step 3 Start at *J* and count the same number of units.

A rise of 4 from –8 is –4.

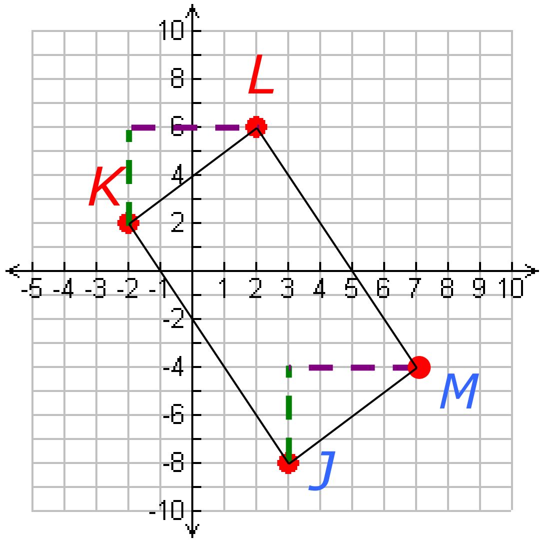
A run of 4 from 3 is 7. Label (7, –4) as vertex *M*.

Example 3 Continued

Step 4 Use the slope formula to verify that $\overline{LM} \parallel \overline{KJ}$.

$$\text{slope of } \overline{LM} = \frac{-4-6}{7-2} = \frac{-10}{5} = -2$$

$$\text{slope of } \overline{KJ} = \frac{-8-2}{3-(-2)} = \frac{-10}{5} = -2$$



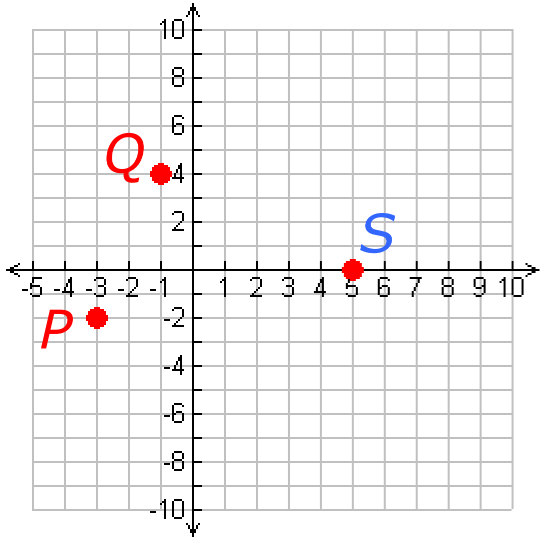
The coordinates of vertex *M* are (7, –4).

Check It Out! Example 3

Three vertices of ▱*PQRS* are *P*(–3, –2), *Q*(–1, 4), and *S*(5, 0). Find the coordinates of vertex *R*.

Since *PQRS* is a parallelogram, both pairs of opposite sides must be parallel.

Step 1 Graph the given points.

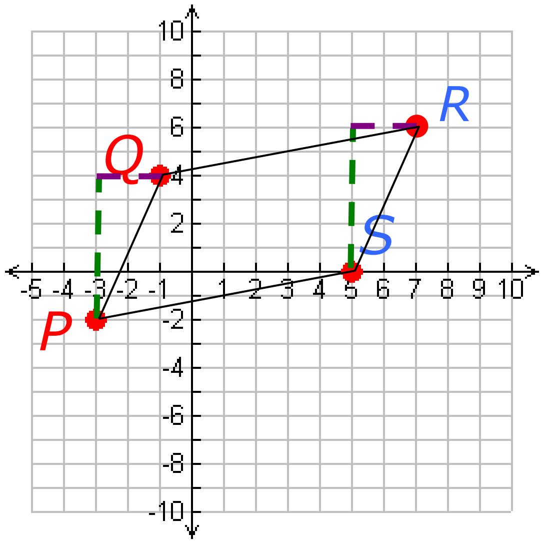


Check It Out! Example 3 Continued

Step 2 Find the slope of $\overline{PQ}$ by counting the units from *P* to *Q*.

The rise from –2 to 4 is 6.

The run of –3 to –1 is 2.



Step 3 Start at *S* and count the same number of units.

A rise of 6 from 0 is 6.

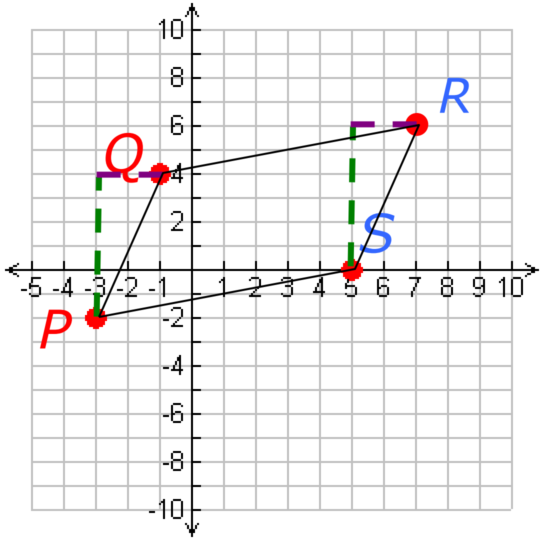
A run of 2 from 5 is 7. Label (7, 6) as vertex *R*.

Check It Out! Example 3 Continued

Step 4 Use the slope formula to verify that $\overline{PQ} \parallel \overline{SR}$.

$$\text{slope of } \overline{PQ} = \frac{-2-4}{-3-(-1)} = \frac{-6}{-2} = 3$$

$$\text{slope of } \overline{SR} = \frac{0-6}{5-7} = \frac{-6}{-2} = 3$$



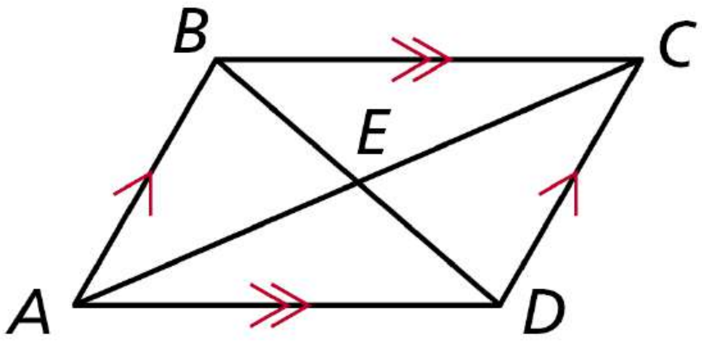
The coordinates of vertex *R* are (7, 6).

Example 4A: Using Properties of Parallelograms in a Proof

Write a two-column proof.

Given: ABCD is a parallelogram.

Prove: $\Delta AEB \cong \Delta CED$



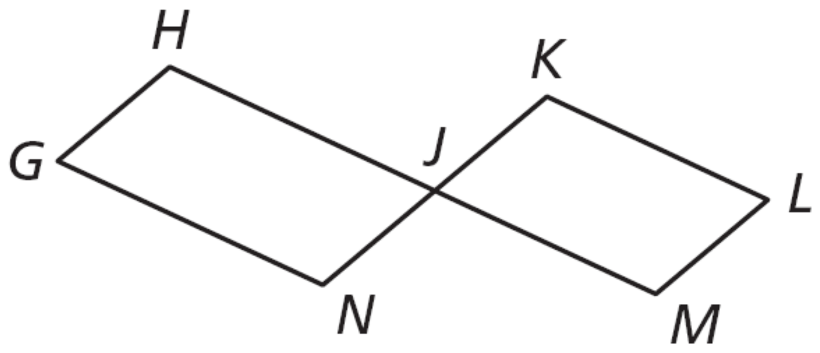
Example 4A Continued

Proof:

Statements	Reasons
1. *ABCD* is a parallelogram	**1.** Given
2. $\overline{AB} \cong \overline{CD}$	**2.** ▱ → opp. sides ≅
3. $\overline{AE} \cong \overline{CE}, \overline{BE} \cong \overline{DE}$	**3.** ▱ → diags. bisect each other
4. $\triangle AEB \cong \triangle CED$	**4.** SSS *Steps 2, 3*

Example 4B: Using Properties of Parallelograms in a Proof

Write a two-column proof.



Given: *GHJN* and *JKLM* are parallelograms. *H* and *M* are collinear. *N* and *K* are collinear.

Prove: $\angle H \cong \angle M$

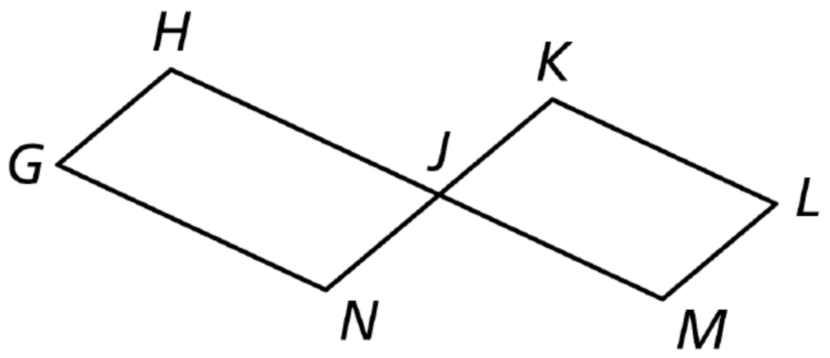
Example 4B Continued

Proof:

Statements	Reasons
1. *GHJN* and *JKLM* are parallelograms.	**1.** Given
2. $\angle H$ and $\angle HJN$ are supp. $\angle M$ and $\angle MJK$ are supp.	**2.** ▱ $\rightarrow$ cons. $\angle$s supp.
3. $\angle HJN \cong \angle MJK$	**3.** Vert. $\angle$s Thm.
4. $\angle H \cong \angle M$	**4.** $\cong$ Supps. Thm.

Check It Out! Example 4

Write a two-column proof.



Given: *GHJN* and *JKLM* are parallelograms.
H and *M* are collinear. *N* and *K* are collinear.

Prove: $\angle N \cong \angle K$

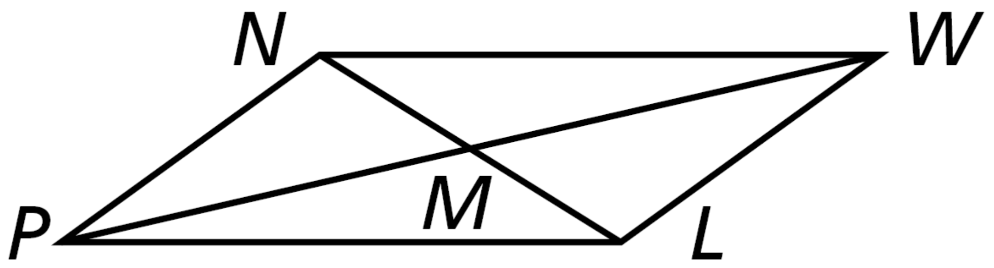
Check It Out! Example 4 Continued

Proof:

Statements	Reasons
1. *GHJN* and *JKLM* are parallelograms.	**1.** Given
2. $\angle N$ and $\angle HJN$ are supp. $\angle K$ and $\angle MJK$ are supp.	**2.** ▱ → cons. $\angle$s supp.
3. $\angle HJN \cong \angle MJK$	**3.** Vert. $\angle$s Thm.
4. $\angle N \cong \angle K$	**4.** $\cong$ Supps. Thm.

Lesson Quiz: Part I

In ▱*PNWL*, *NW* = 12, *PM* = 9, and m∠*WLP* = 144°. Find each measure.



1. *PW*

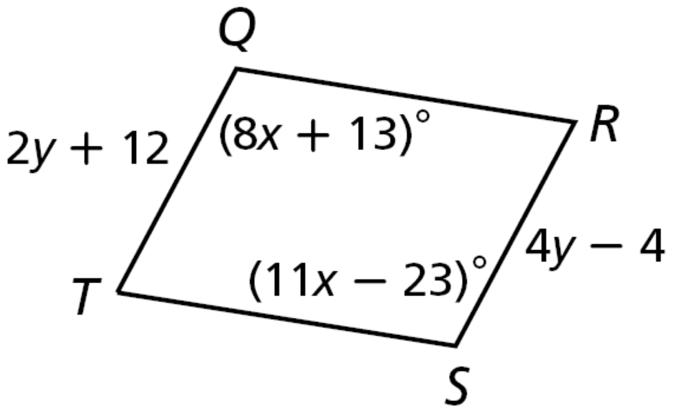
18

2. m∠*PNW*

144°

Lesson Quiz: Part II

***QRST* is a parallelogram. Find each measure.**



2. TQ

28

3. $m\angle T$

71°

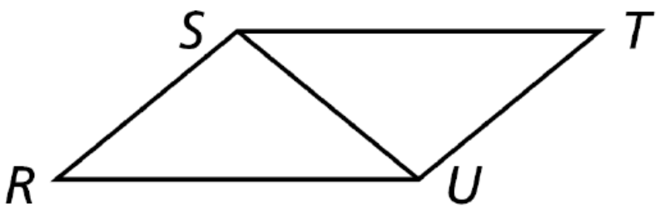
Lesson Quiz: Part III

5. Three vertices of ▱*ABCD* are *A* (2, –6), *B* (–1, 2), and *C*(5, 3). Find the coordinates of vertex *D*.

(8, –5)

Lesson Quiz: Part IV

6. Write a two-column proof.



Given: *RSTU* is a parallelogram.
Prove: $\Delta RSU \cong \Delta TUS$

Statements	Reasons
1. *RSTU* is a parallelogram.	**1.** Given
2. $\overline{RU} \cong \overline{TS}$; $\overline{RS} \cong \overline{UT}$	**2.** ▱ → cons. $\angle$s $\cong$
3. $\angle R \cong \angle T$	**3.** ▱ → opp. $\angle$s $\cong$
4. $\Delta RSU \cong \Delta TUS$	**4.** SAS