Pre-Calculus ELT Overview

Essential Learning Target	*Standards*	*Prerequisite Skills*
1. I can use right triangle trigonometry to solve real life problems involving right triangles.	**MGSE9-12.G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	***MGSE8.G.7*** *I can apply the Pythagorean Theorem to determine unknown side lengths in right triangles.*
2. I can use special right triangles to find exact Sine and Cosine values on the 16 point Unit circle and apply them to solve problems involving all 6 trigonometric functions.	**Extend the domain of trigonometric functions using the unit circle** **MGSE9-12.F.TF.3** Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π - x, π + x, and 2π - x in terms of their values for x, where x is any real number. **MGSE9-12.F.TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **MGSE9-12.F.TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	***MGSE9-12.G.SRT.6*** *I can use angle relationships in special right triangles to find missing side lengths.*
3. I can create and use graphs of transformations of sine, cosine, and tangent functions to solve problems.	**Analyze functions using different representations** **MGSE9-12.F.IF.7e** Graph ~~exponential and logarithmic functions, showing intercepts and end behavior, and~~ trigonometric functions, showing period, midline, and amplitude. **MGSE9-12.F.IF.7** Graph functions expressed algebraically and show key features of the graph both by hand and by using technology. **Interpret functions that arise in applications in terms of the context** **MGSE9-12.F.IF.4** Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts, interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	***MGSE9-12.F.BF.3*** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. I can determine the trigonometric function that best models a situation based on period, amplitude, frequency, and midline.	**Model periodic phenomena with trigonometric functions** **MGSE9-12.F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **Extend the domain of trigonometric functions using the unit circle** **MGSE9-12.F.TF.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions	***MGSE9-12.F.IF.7e*** *Graph ~~exponential and logarithmic functions, showing intercepts and end behavior, and~~ trigonometric functions, showing period, midline, and amplitude.*
5. I can describe how to restrict the domain of a trigonometric function so that its inverse is a function.	**Model periodic phenomena with trigonometric functions** **MGSE9-12.F.TF.6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	***MGSE9-12.F.IF.1*** *Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns*

		to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).
6. I can use inverse trigonometric functions to solve trigonometric equations.	**<u>Model periodic phenomena with trigonometric functions</u>** **MGSE9-12.F.TF.7** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	***MGSE9-12.F.BF.4c*** *Read values of an inverse function from a graph or a table, given that the function has an inverse.*
7. I can use the *SAS formula for the area of a triangle* to find the area of any triangle given only two sides and an included angle.	**<u>Apply trigonometry to general triangles</u>** **MGSE9-12.G.SRT.9** Derive the formula A = (1/2)ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	***MGSE9-12.G.SRT.8*** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.* ***MGSE9-12.A.CED.4*** *Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. Examples: Rearrange Ohm's law V = IR to highlight resistance R; Rearrange area of a circle formula A = π r 2 to highlight the radius r.*
8. I can use the *Law of Sines* and/or *Law of Cosines* to solve problems in real life contexts.	**<u>Apply trigonometry to general triangles</u>** **MGSE9-12.G.SRT.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces) **MGSE9-12.G.SRT.10** Prove the Laws of Sines and Cosines and use them to solve problems.	***MGSE9-12.A.CED.4*** *Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. Examples: Rearrange Ohm's law V = IR to highlight resistance R; Rearrange area of a circle formula A = π r 2 to highlight the radius r.*
9. I can use the Pythagorean identity to determine the exact values of trigonometric functions when given one	**<u>Prove and apply trigonometric identities</u>** **MGSE9-12.F.TF.8** Prove the Pythagorean identity $(\sin A)^2 + (\cos A)^2 = 1$ and use it to find sin A, cos A, or tan A, given sin A, cos A, or tan A, and the quadrant of the angle.	***MGSE8.G.7*** *Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and*

trigonometric function and the quadrant of the angle.		*mathematical problems in two and three dimensions.*
10. I can use trigonometric identities to find exact Sine and Cosine values NOT on the 16 point Unit circle and apply them to solve problems involving all 6 trigonometric functions.	**MGSE9-12.F.TF.9** Prove addition, subtraction, double and half-angle formulas for sine, cosine, and tangent and use them to solve problems.	***MGSE9-12.G.SRT.8*** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*
11. I can perform the following operations on matrices of appropriate dimensions: addition, subtraction, scalar multiplication, and matrix multiplication.	**Perform operations on matrices and use matrices in applications** **MGSE9-12.N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions. **MGSE9-12.N.VM.6** Use matrices to represent and manipulate data, e.g., transformations of vectors. **MGSE9-12.N.VM.7** Multiply matrices by scalars to produce new matrices. **MGSE9-12.N.VM.9** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **MGSE9-12.N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **MGSE9-12.N.VM.12** Work with 2 X 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.	***MGSE9-12.G.CO.2*** *Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).*
12. I can use the inverse of matrices to solve real life problems involving systems of equations.	**Solve systems of equations** **MGSE9-12.A.REI.9** Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). **MGSE9-12.A.REI.8** Represent a system of linear equations as a single matrix equation in a vector variable	***MGSE9-12.A.CED.3*** *Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.*
13. I can convert complex numbers in rectangular form to polar form and vice-versa.	**Use properties of rational and irrational numbers.** **Represent complex numbers and their operations on the complex plane** **MGSE9-12.N.CN.4** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **MGSE9-12.N.CN.3** Find the conjugate of a complex number; use the conjugate to find the absolute value (modulus) and quotient of complex numbers. **Represent complex numbers and their operations on the complex plane**	***MGSE8.G.8*** *Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.*

	MGSE9-12.N.CN.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}i)^3 = 8$ *because* $(-1 + \sqrt{3}i)$ *has modulus 2 and argument 120°.* **MGSE9-12.N.CN.6** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	
14. I can perform operations on vectors. (N.VM.4-5)	**<u>Perform operations on vectors</u>** **MGSE9-12.N.VM.4** Add and subtract vectors. **MGSE9-12.N.VM.5** Multiply a vector by a scalar. **<u>Represent and model with vector quantities</u>** **MGSE9-12.N.VM.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*). **MGSE9-12.N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **<u>Perform operations on vectors</u>** **MGSE9-12.N.VM4a** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **MGSE9-12.N.VM4b** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **MGSE9-12.N.VM4c** Understand vector subtraction ***v*** – ***w*** as ***v*** + (–***w***), where (–***w***) is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. **MGSE9-12.N.VM.5a** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. **MGSE9-12.N.VM.5b** Compute the magnitude of a scalar multiple c***v*** using ||c***v***|| = |c|***v.*** Compute the direction of c***v*** knowing that when |c|***v*** = 0, the direction of c***v*** is either along ***v*** (for c > 0) or against ***v*** (for c < 0). **MGSE9-12.N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	***MGSE9-12.G.GPE.7*** *I can find the distance between coordinates using the distance formula*
15. I can use vectors to solve problems involving velocity and direction.	**<u>Represent and model with vector quantities</u>** **MGSE9-12.N.VM.3** Solve problems involving velocity and other quantities that can be represented by vectors.	***MGSE7.RP.2d*** *I can explain what a point (x, y) on the graph of a line means in terms of the situation.*
16. I can use permutations and combinations to compute probabilities of compound events and solve problems.	**<u>Use the rules of probability to compute probabilities of compound events in a uniform probability model</u>** **MGSE9-12.S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems. **MGSE9-12.S.CP.8** Apply the general Multiplication Rule in a uniform probability model, P(A and B) = [P(A)]x[P(B|A)] =[P(B)]x[P(A|B)], and interpret the answer in terms of the model.	***MGSE9-12.S.CP.2*** *Understand that if two events A and B are independent, the probability of A and B occurring together is the product of their probabilities, and that if the probability of two events A and B occurring together is the product of*

		their probabilities, the two events are independent.
17. I can calculate expected values and use them to solve probability problems.	**Use probability to evaluate outcomes of decisions** **MGSE9-12.S.MD.5** Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. **MGSE9-12.S.MD.5a** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* **MGSE9-12.S.MD.5b** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* **MGSE9-12.S.MD.6** Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **MGSE9-12.S.MD.7** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). **Calculate expected values and use them to solve problems** **MGSE9-12.S.MD.1** Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. **MGSE9-12.S.MD.2** Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. **MGSE9-12.S.MD.3** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* **MGSE9-12.S.MD.4** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*	***MGSE7.SP.5*** *Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.*
18. I can translate between the algebraic and graphical representations of a parabola.	**Translate between the geometric description and the equation for a conic section** **MGSE9-12.G.GPE.2** Derive the equation of a parabola given a focus and directrix.	***MGSE9-12.A.REI.4a*** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from ax2 + bx + c = 0.*
19. I can translate between the algebraic and graphical representations of an ellipse and hyperbola.	**Translate between the geometric description and the equation for a conic section** **MGSE9-12.G.GPE.3** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **Solve systems of equations**	***MGSE9-12.G.GPE.1*** *Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius*

	MGSE9-12.A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.	*of a circle given by an equation.*