Rational and Irrational Numbers

Rational Numbers

A rational number is any number that can be expressed as the ratio of two integers.

$$\frac{a}{b}$$

All terminating and repeating decimals can be expressed in this way so they are irrational numbers.

Examples

$$\frac{4}{5} \qquad 2\frac{2}{3} = \frac{8}{3} \qquad 6 = \frac{6}{1} \qquad -3 = -\frac{3}{1} \qquad 2.7 = \frac{27}{10}$$

$$0.7 = \frac{7}{10} \qquad 0.625 = \frac{5}{8} \qquad 34.56 = \frac{3456}{100}$$

$$0.\dot{3} = \frac{1}{3} \qquad 0.\dot{2}\dot{7} = \frac{3}{11} \qquad 0.\dot{1}4285\dot{7} = \frac{1}{7}$$

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Show that the terminating decimals below are rational.

$0.6 \rightarrow \frac{6}{10}$

$3.8 \rightarrow \frac{38}{10}$

$56.1 \rightarrow \frac{561}{10}$

$3.45 \rightarrow \frac{345}{100}$

$2.157 \rightarrow \frac{2157}{1000}$

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By looking at the previous examples can you spot a quick method of determining the rational number for any given repeating decimal.

$$0.\dot{3} \qquad 0.\dot{4}\dot{5} \qquad 0.\dot{2}7\dot{3} \qquad 0.\dot{1}23\dot{4}$$

$$\frac{3}{9} \qquad \frac{45}{99} \qquad \frac{273}{999} \qquad \frac{1234}{9999}$$

Rational and Irrational Numbers

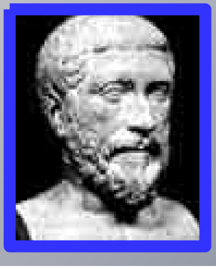
Irrational Numbers

An irrational number is any number that cannot be expressed as the ratio of two integers.

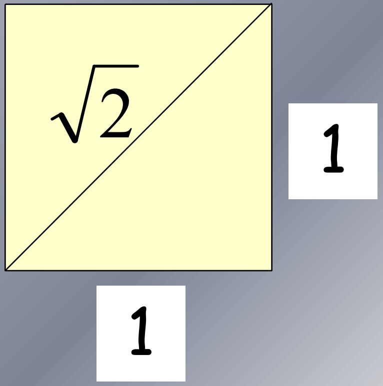
$$\cancel{\frac{a}{b}}$$

The history of irrational numbers begins with a discovery by the Pythagorean School in ancient Greece. A member of the school discovered that the diagonal of a unit square could not be expressed as the ratio of any two whole numbers. The motto of the school was "All is Number" (by which they meant whole numbers). Pythagoras believed in the absoluteness of whole numbers and could not accept the discovery. The member of the group that made it was Hippasus and he was sentenced to death by drowning.

(See slide 19/20 for more history)



Pythagoras



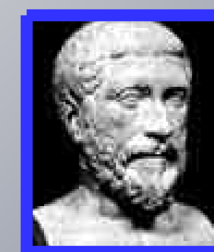
Rational and Irrational Numbers

Irrational Numbers

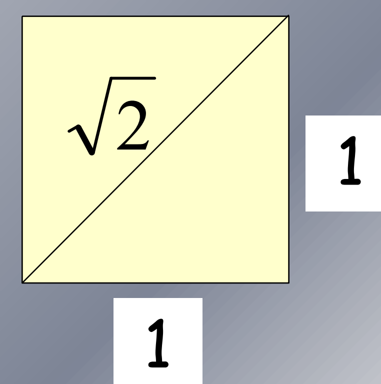
An irrational number is any number that cannot be expressed as the ratio of two integers.

$$\cancel{\frac{a}{b}}$$

Intuition alone may convince you that all points on the "Real Number" line can be constructed from just the infinite set of rational numbers, after all between **any** two rational numbers we can always find another. It took mathematicians hundreds of years to show that the majority of Real Numbers are in fact irrational. The **rationals** and **irrationals** are needed together in order to complete the continuum that is the set of "Real Numbers".



Pythagoras



Rational and Irrational Numbers

Multiplication and division of surds.

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

For example: $\sqrt{36} = \sqrt{4 \times 9} = \sqrt{4} \times \sqrt{9} = 2 \times 3 = 6$

and $\sqrt{50} = \sqrt{5 \times 10} = \sqrt{5} \times \sqrt{10}$

also $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ for example $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$

and $\sqrt{\frac{6}{7}} = \frac{\sqrt{6}}{\sqrt{7}}$

Rational and Irrational Numbers

Example questions

a Show that $\sqrt{3} \times \sqrt{12}$ is rational

$$\sqrt{3} \times \sqrt{12} = \sqrt{3 \times 12} = \sqrt{36} = 6 \qquad \text{rational}$$

b Show that $\dfrac{\sqrt{45}}{\sqrt{5}}$ is rational

$$\frac{\sqrt{45}}{\sqrt{5}} = \sqrt{\frac{45}{5}} = \sqrt{9} = 3 \qquad \text{rational}$$

Rational and Irrational Numbers

Questions

State whether each of the following are rational or irrational.

a $\sqrt{6} \times \sqrt{7}$

irrational

b $\sqrt{20} \times \sqrt{5}$

rational

c $\sqrt{27} \times \sqrt{3}$

rational

d $\sqrt{4} \times \sqrt{3}$

irrational

e $\dfrac{\sqrt{32}}{\sqrt{8}}$

rational

f $\dfrac{\sqrt{44}}{\sqrt{11}}$

rational

g $\dfrac{\sqrt{18}}{\sqrt{2}}$

rational

h $\dfrac{\sqrt{25}}{\sqrt{5}}$

irrational

Rational and Irrational Numbers

Combining Rationals and Irrationals

Determine whether the following are rational or irrational.

(a) 0.73 — rational

(b) $\sqrt{2}$ — irrational

(c) 0.666.... — rational

(d) 3.142 — rational

(e) $\sqrt{12.25}$ — irrational

(f) $\sqrt{7}$ — irrational

(g) $4+\sqrt{5}$ — irrational

(h) $(\sqrt[3]{2})^3+1$ — rational

(i) $16^{\frac{1}{2}}$ — rational

(j) $(\sqrt[3]{2})^2$ — irrational

(j) $(\sqrt{3}+1)(\sqrt{3+1})$ — irrational

(k) $(\sqrt{6}+1)(\sqrt{6}-1)$ — rational

(l) $(1+\sqrt{2})(1-\sqrt{2})$ — rational