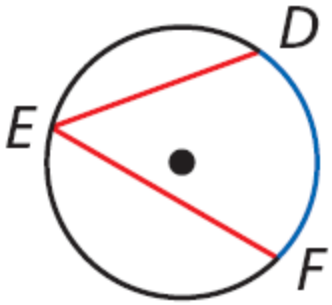
Inscribed Angles

Vocabulary

inscribed angle
intercepted arc
subtend

Inscribed Angles



$\angle DEF$ is an **inscribed angle**.

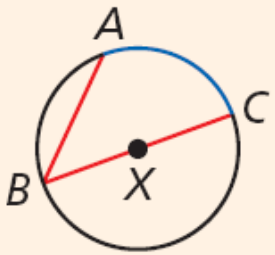
$\overset{\frown}{DF}$ is the **intercepted arc**.

$\overset{\frown}{DF}$ subtends $\angle DEF$.

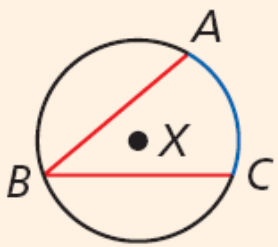
Theorem 11-4-1 **Inscribed Angle Theorem**

The measure of an inscribed angle is half the measure of its intercepted arc.

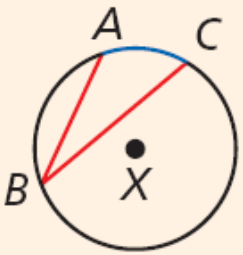
$$m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$$



Case 1



Case 2



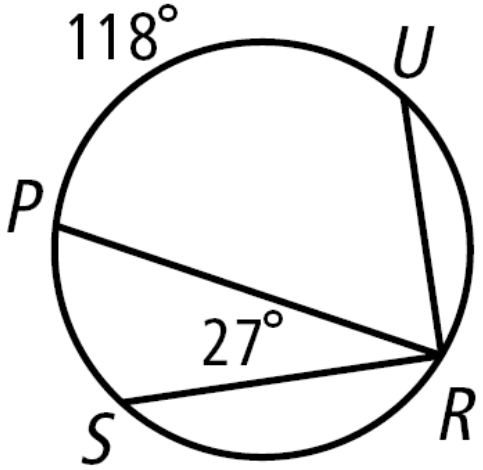
Case 3

Inscribed Angles

Example 1A: Finding Measures of Arcs and Inscribed Angles

Find each measure.

m∠*PRU*

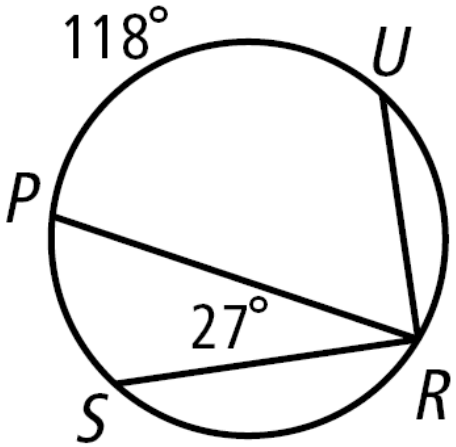


$$m\angle PRU = \frac{1}{2}m\overset{\frown}{PU}$$ *Inscribed ∠ Thm.*

$$= \frac{1}{2}(118°) = 59°$$ *Substitute 118 for $m\overset{\frown}{PU}$.*

Example 1B: Finding Measures of Arcs and Inscribed Angles

Find each measure.



$\overset{\frown}{mSP}$

$m\angle SRP = \frac{1}{2}\overset{\frown}{mSP}$ *Inscribed ∠ Thm.*

$27° = \frac{1}{2}\overset{\frown}{mSP}$ *Substitute 27 for m∠ SRP.*

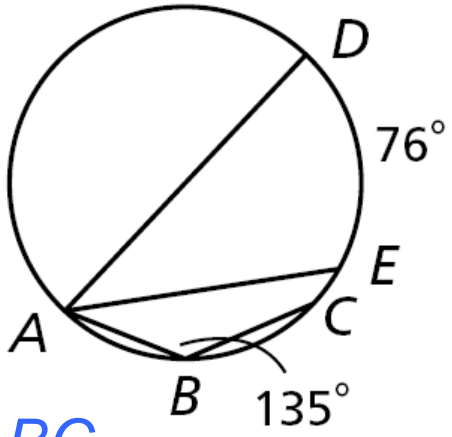
$\overset{\frown}{mSP} = 54°$ *Multiply both sides by 2.*

Inscribed Angles

Check It Out! Example 1a

Find each measure.

$\overset{\frown}{mADC}$



$m\angle ABC = \frac{1}{2}\overset{\frown}{mADC}$ *Inscribed ∠ Thm.*

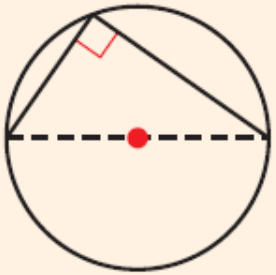
$135° = \frac{1}{2}\overset{\frown}{mADC}$ *Substitute 135 for m∠ ABC.*

$270° = \overset{\frown}{mADC}$ *Multiply both sides by 2.*

Inscribed Angles

Theorem 11-4-3

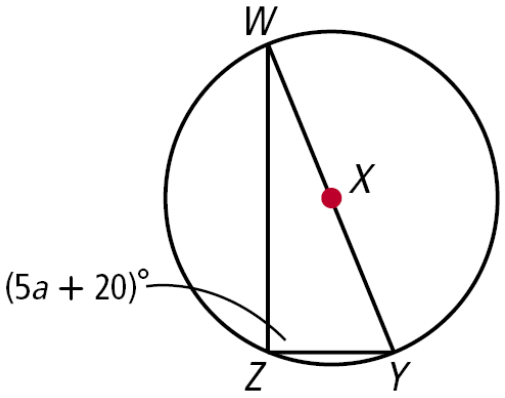
An inscribed angle subtends a semicircle if and only if the angle is a right angle.



Inscribed Angles

Example 3A: Finding Angle Measures in Inscribed Triangles

Find *a*.



$\angle WZY$ is a right angle — *$\angle WZY$ is inscribed in a semicircle.*

$m\angle WZY = 90°$ — *Def of rt. $\angle$*

$5a + 20 = 90$ — *Substitute $5a + 20$ for $m\angle WZY$.*

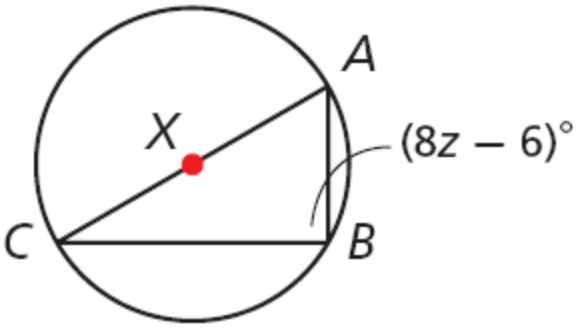
$5a = 70$ — *Subtract 20 from both sides.*

$a = 14$ — *Divide both sides by 5.*

Inscribed Angles

Check It Out! Example 3a

Find *z*.



$\angle ABC$ is a right angle — *$\angle ABC$ is inscribed in a semicircle.*

$m\angle ABC = 90°$ — *Def of rt. $\angle$*

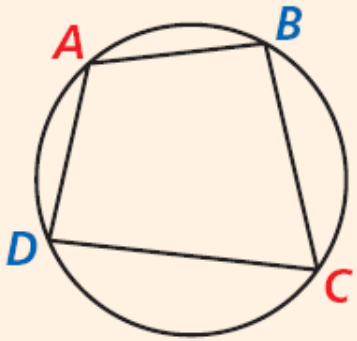
$8z - 6 = 90$ *Substitute.*

$8z = 96$ *Add 6 to both sides.*

$z = 12$ *Divide both sides by 8.*

Inscribed Angles

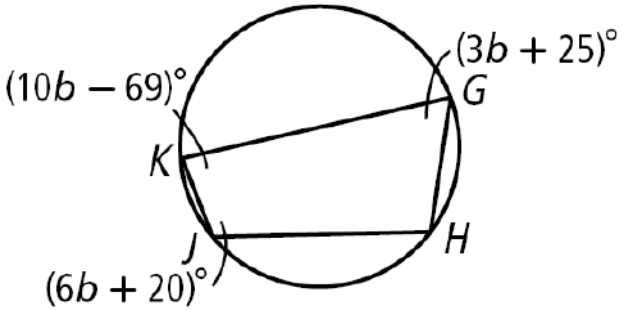
Theorem 11-4-4

THEOREM	HYPOTHESIS	CONCLUSION
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.	 *ABCD* is inscribed in ⊙*E*.	∠*A* and ∠*C* are supplementary. ∠*B* and ∠*D* are supplementary.

Inscribed Angles

Example 4: Finding Angle Measures in Inscribed Quadrilaterals

Find the angle measures of *GHJK*.



Step 1 Find the value of *b*.

$m\angle G + m\angle J = 180°$ *GHJK is inscribed in a ⊙.*

$3b + 25 + 6b + 20 = 180$ *Substitute the given values.*

$9b + 45 = 180$ *Simplify.*

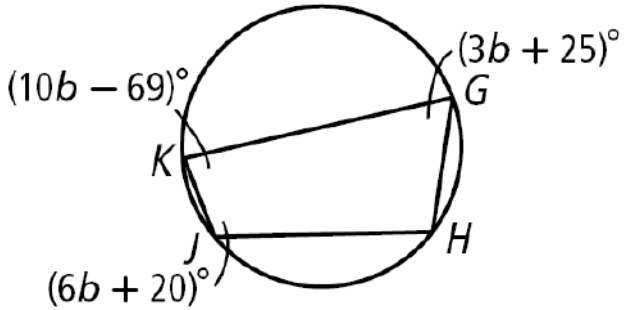
$9b = 135$ *Subtract 45 from both sides.*

$b = 15$ *Divide both sides by 9.*

Inscribed Angles

Example 4 Continued

Step 2 Find the measure of each angle.



$m\angle G = 3(15) + 25 = 70°$ *Substitute 15 for b*

$m\angle J = 6(15) + 20 = 110°$ *in each expression.*

$m\angle K = 10(15) - 69 = 81°$

$m\angle H + m\angle K = 180°$ *∠H and ∠K are supp.*

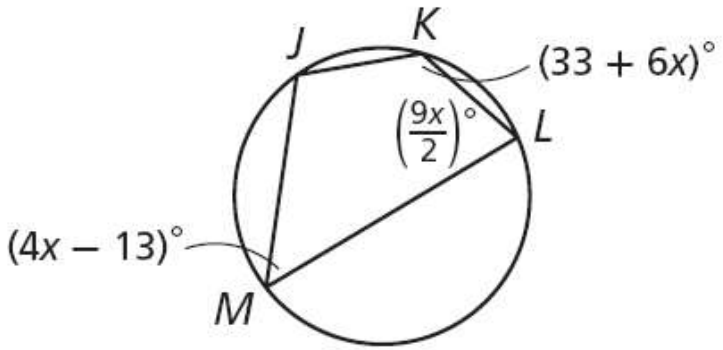
$m\angle H + 81° = 180°$ *Substitute 81 for m∠K.*

$m\angle H = 99°$ *Subtract 81 from both sides*

Inscribed Angles

Check It Out! Example 4

Find the angle measures of *JKLM*.



Step 1 Find the value of *b*.

$m\angle M + m\angle K = 180°$ *JKLM is inscribed in a ⊙.*

$4x - 13 + 33 + 6x = 180$ *Substitute the given values.*

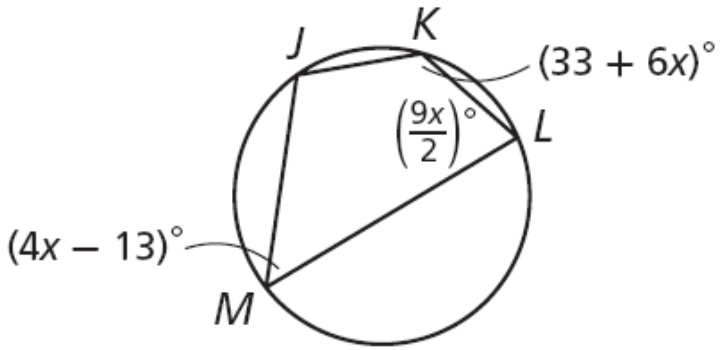
$10x + 20 = 180$ *Simplify.*

$10x = 160$ *Subtract 20 from both sides.*

$x = 16$ *Divide both sides by 10.*

Check It Out! Example 4 Continued

Find the angle measures of *JKLM*.



Step 2 Find the measure of each angle.

$m\angle M = 4(16) - 13 = 51°$

$m\angle K = 33 + 6(16) = 129°$

$$m\angle L = \frac{9(16)}{2} = 72°$$

$m\angle J = 360° - 252° = 108°$

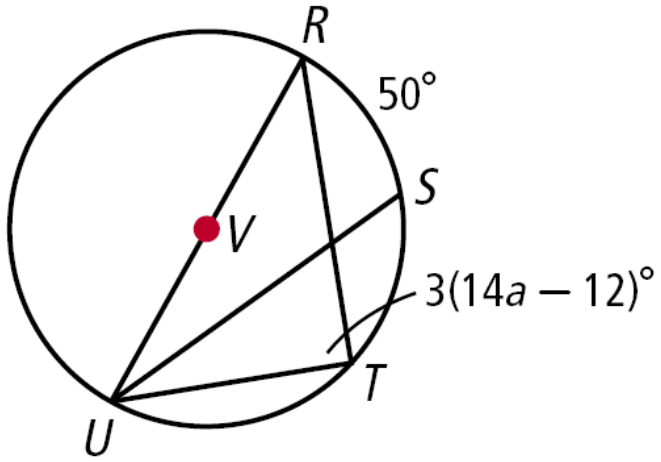
Inscribed Angles

Lesson Quiz: Part I

Find each measure.

1. $\angle RUS$ 25°

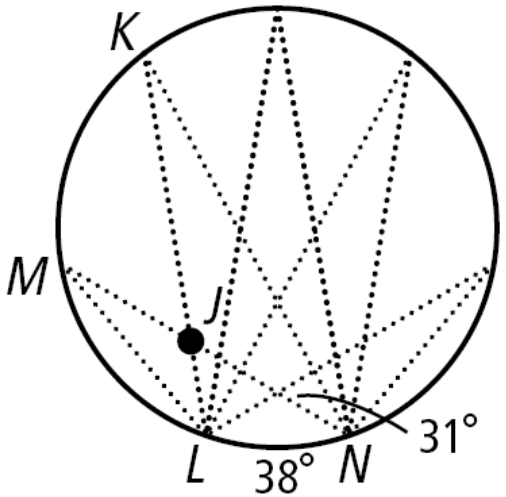
2. a 3



Inscribed Angles

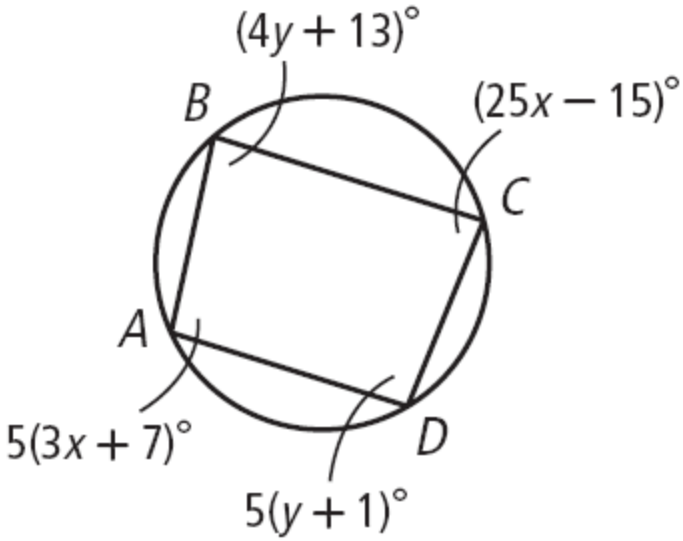
Lesson Quiz: Part II

3. A manufacturer designs a circular ornament with lines of glitter as shown. Find m∠*KJN.*



130°

4. Find the angle measures of *ABCD*.



m∠ *A* = 95°

m∠ *B* = 85°

m∠ *C* = 85°

m∠ *D* = 95°