Name: ……………………………… Period: ……Date: ……………Seat#: …… Mr.Bala Thevar/Mr.Andrews

Unit 2 – Graphs of Polynomial Functions - Study Guide

<u>KEY STANDARDS ADDRESSED:</u>

MM3A1. Students will analyze graphs of polynomial functions of higher degree.

a. Graph simple polynomial functions as translations of the function $f(x) = ax^n$.
b. Understand the effects of the following on the graph of a polynomial function: degree, lead coefficient, and multiplicity of real zeros.
c. Determine whether a polynomial function has symmetry and whether it is even, odd, or neither.
d. Investigate and explain characteristics of polynomial functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior.

<u>Vocabulary and Definitions</u>

<u>Polynomial function</u>
A ***polynomial function*** is defined as a function,
$f(x)= a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1x + a_n$, where the coefficients an, a_{n-1}, a_{n-2}, ….a_1, a_0 are real numbers, and the exponents n, n-1, n-2, …, 2, 1, 0 are non-negative integers.

<u>Degree</u> The ***degree of a polynomial*** (n) is equal to the greatest exponent of its variable.

<u>End Behavior</u> The value of $f(x)$ as x approaches negative infinity ($-\infty$) and infinity ($+\infty$) describes the ***end behavior*** of a polynomial function.

<u>Zero</u> If f(x) is a polynomial function, then the values of x for which f(x) = 0 are called the ***zeros*** of the function. Graphically these are the x intercepts. Numbers that are zeros of a polynomial function are also solutions to polynomial equations.

<u>Root</u> Solutions to polynomial equations are called ***roots***.

<u>Multiplicity</u> The ***multiplicity of a root*** refers to the number of times a root occurs at a given point of a polynomial equation.

<u>Relative minimum(Local minimum)</u> A ***relative minimum*** is a point on the graph where the function is increasing as you move away from the point in the positive and negative direction along the horizontal axis.
[The y-coordinate of the turning point]

<u>Relative maximum(Local Maximum)</u> A ***relative maximum*** is a point on the graph where the function is decreasing as you move away from the point in the positive and negative direction along the horizontal axis.
[The y-coordinate of the turning point]

<u>Relative Extrema</u> ***Relative extrema*** refers to relative minimum and relative maximum points.

Note: **Absolute Extrema**
Can ever have an **absolute maximum** and an **absolute minimum** in the same function? If so sketch the graph with both. If not, why not?

*For **odd degree polynomial functions**, absolute maximum or absolute minimum values do not exist. Because the end behaviors are opposite, one end approaches to positive infinity (+ ∞) and the other end approaches to negative infinity (- ∞). So the highest and lowest points are not defined but rather reach to infinity.*

*For **even degree polynomial** functions the end behavior is the same, both approaching to positive infinity (+ ∞) or negative infinity (- ∞). So an even degree polynomial will have an absolute maximum or an absolute minimum, but not both.*

MM3A1.a

1.

Sketch the graph of the basic parabola $f(x) = x^2$.

Then sketch the following graphs using transformation of the above basic graph.

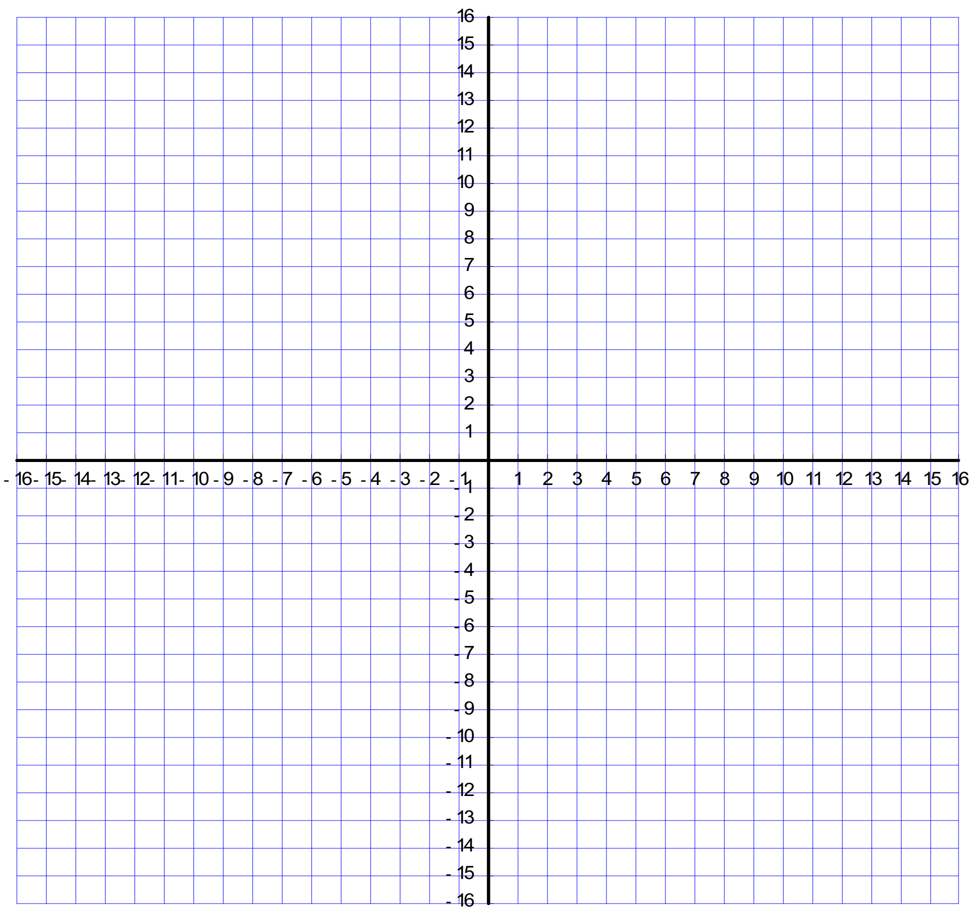
(a). $g(x) = (x - 5)^2$ shift ___units ______

(b). $h(x) = x^2 + 3$ shift ___units ______

(c). $k(x) = (x + 6)^2 + 4$ shift ____units _____ & _____units ______

(d). $j(x) = (x - 3)^2 - 7$ shift ____units ______ & ____units _______

$y = x^2$	
x	y
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	



MM3A1.a

2. Write the following polynomial functions in Standard Form. Identify the degree and leading coefficient of each polynomial.

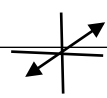
	Polynomial	Standard Form (Highest to lowest degree)	Degree	Leading Coefficient
1.	$f(x) = x^3 + 3x^2 - 5x + 2$			
2.	$g(x) = 5x - 7x^2 + 1$			
3.	$h(x) = x^3 - 3x^2 - x^5 - 4$			
4.	$k(x) = 3 + 2x$			

MM3A1. c

3. Is the **degree** even or odd?

Is the leading coefficient positive or negative?

Does the graph rise or fall on the left? On the right? Sketch the graph.

Polynomial function	Degree	Lead Coefficient	End Behavior	
$f(x) = ax^n$ Graph	Even/Odd	Positive/Negative	Left End As x → -∞	Right End As x → ∞
1. $y = x$ 	1 odd	1 positive	y → -∞ Falls	y → ∞ Rises
2. $y = x^2$				
5. $y = x^3$				
7. $y = -x^2$				
9. $y = -x^3$				

MM3A1. c

4. Using the table below and your handout of the following eight polynomial functions, classify the functions by their **symmetry**.

Function	Symmetry about the y axis?	Symmetry about the origin?	Even, Odd, or Neither?
$f(x) = x^2 + 2x^1$	No	No	Neither
$g(x) = -2x^2 + x$			
$h(x) = x^3 - x^1$	No	Yes	Odd
$j(x) = -x^3 + 2x^2 + 3x$			
$k(x) = x^4 - 5x^2 + 4$	yes	no	Even
$l(x) = -(x^4 - 5x^2 + 4)$			
$m(x) = \frac{1}{2}(x^5 + 4x^4 - 7x^3 - 22x^2 + 24x)$			

n(x) = $-\frac{1}{2}(x^5 + 4x^4 - 7x^3 - 22x^2 + 24x)$			

MM3A1. d

5. Without graphing, find the **x-intercepts** and **y-intercepts** for the graph of each equation. Check your answer by graphing.

   a. y = - 0.25 (x + 1.5) (x + 6)

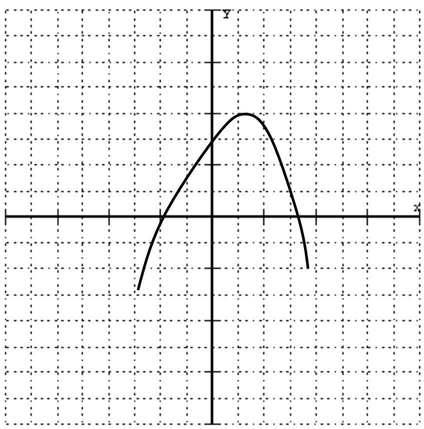
   b. y = 3 (x - 4) (x - 4)

   c. y = - 2 (x - 3) (x + 2) (x + 5)

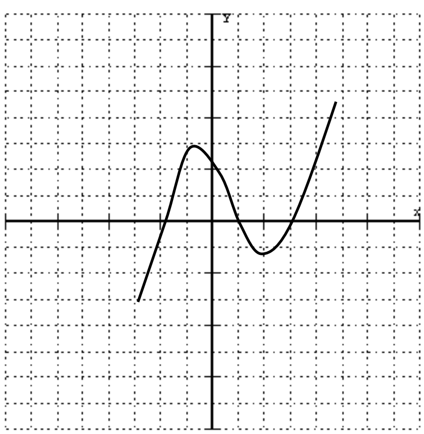
MM3A1.d

6. Write the **factored form of the polynomial function** for each graph. Don't forget the vertical scale factor (Lead Coefficient).

   a.



   b.



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7. Calculate the finite differences for each table and find the degree and the leading coefficient of the polynomial function that models the following data.

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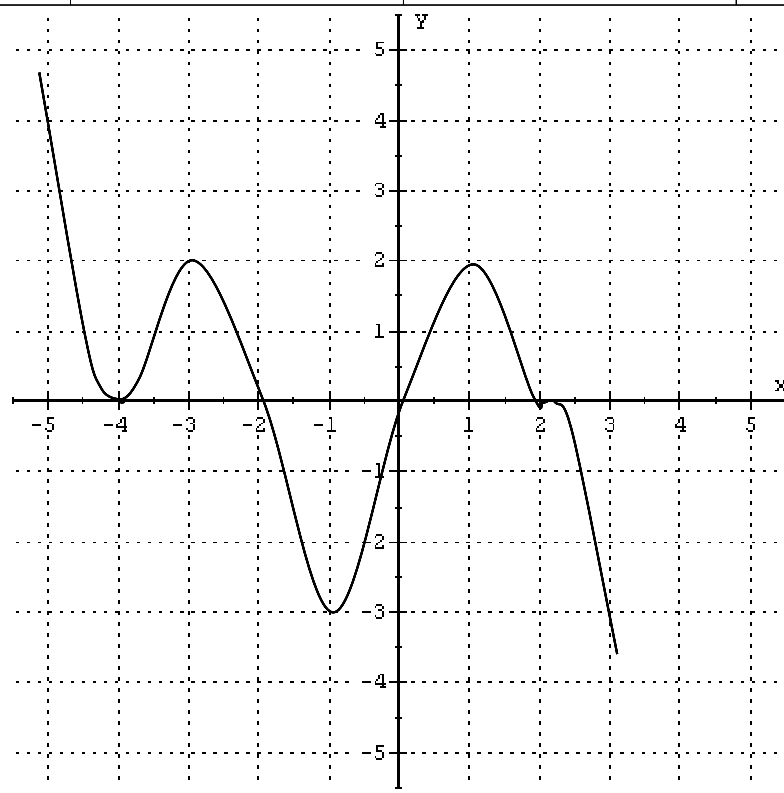
x	Y	D_1	D_2	D_3
0	12			
2	-4			
4	- 164			
6	-612			
8	-1492			
10	-2948			
12	-5124			

M3 A1. d



The above graph is a complete graph of a polynomial function. Answer the following questions.

a. Is the degree of the polynomial function even or odd? Why?

b. Is the leading coefficient of the polynomial function positive or negative? Why?

c. Name the zeros of the polynomial function.

d. Write a polynomial function with a suitable leading coefficient a = 1 or a = -1.

e. Find the degree of the polynomial function.

f. Write the domain and range of the polynomial function.

g. Write the end behavior of the polynomial function.

MM3A1.d

Each of these is the graph of polynomial function with leading coefficient a = 1 or a = -1.

9.
a. Write the polynomial function in factored form.

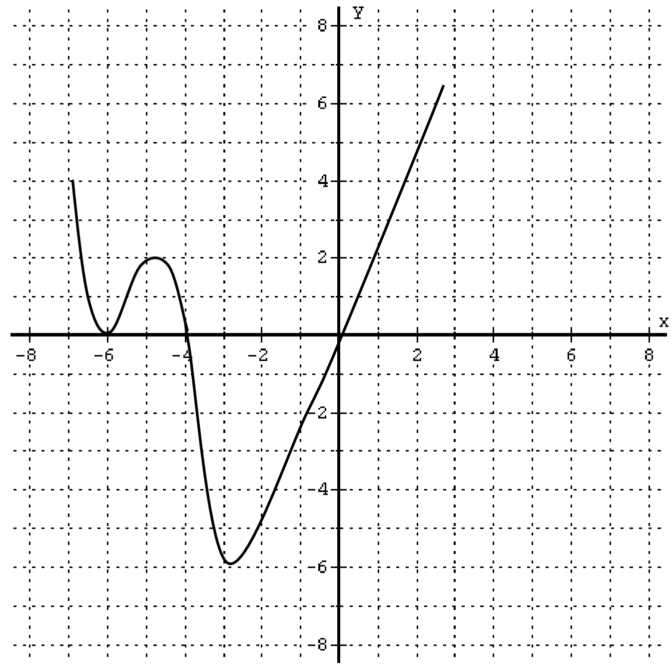
b. Name the zeros of the polynomial function (multiple zeros write that many times).

c. What is the degree of the polynomial function?

d. How many extreme (relative maximum or minimum) values does the graph have?

e. What is the relation between the degree of the polynomial function and the number of extreme values?

f. Does the graph have any absolute maximum or minimum value? If yes, what is that value?



MM3A1.d
10.
a. Write the polynomial function in factored form.

b. Name the zeros of the polynomial function (multiple zeros write that many times).

c. What is the degree of the polynomial function?

d. How many extreme (relative maximum or minimum) values does the graph have?

e. What is the relation between the degree of the polynomial function and the number of extreme values?

f. Does the graph have any absolute maximum or minimum value? If yes, what is that value?