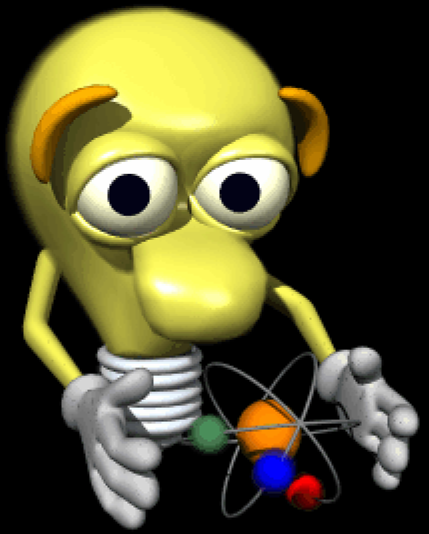
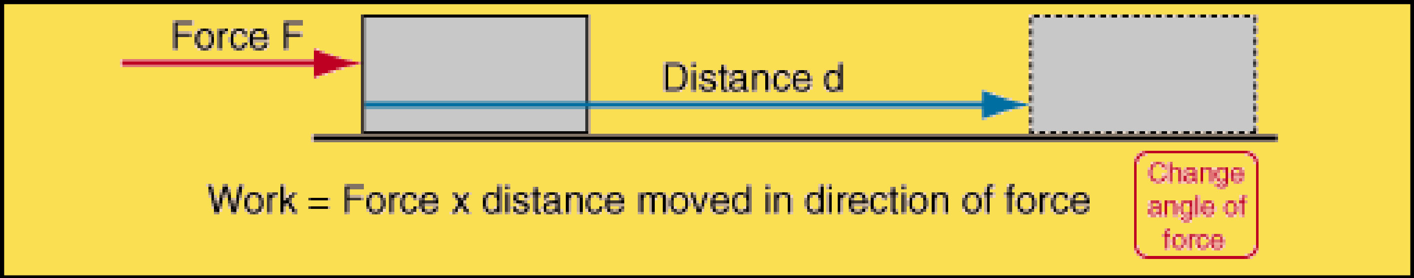
Work & Energy



4.1 Work Done by a Constant Force

When force and displacement are in the same direction (parallel)



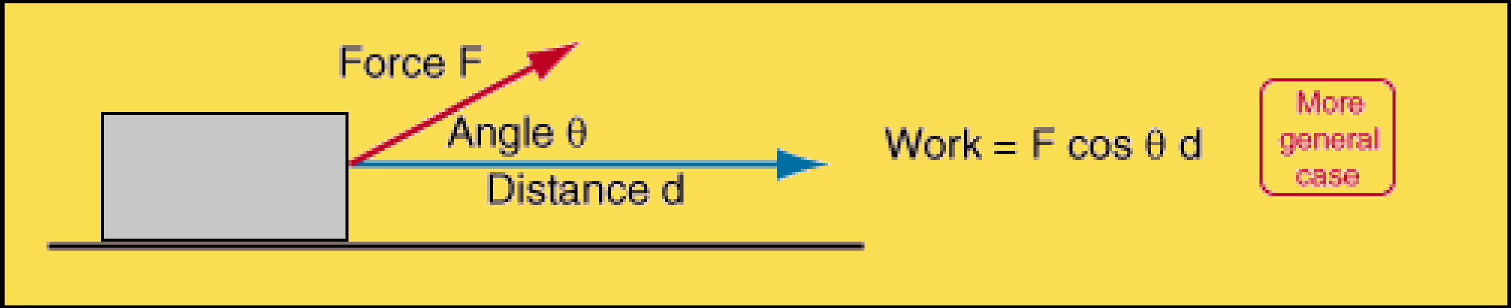
The equation is

$$\boxed{W = Fx}$$

Work Applet

4.1Work Done by a Constant Force

If Force is at an angle



The more general Equation is

$$W = F\cos\theta x$$

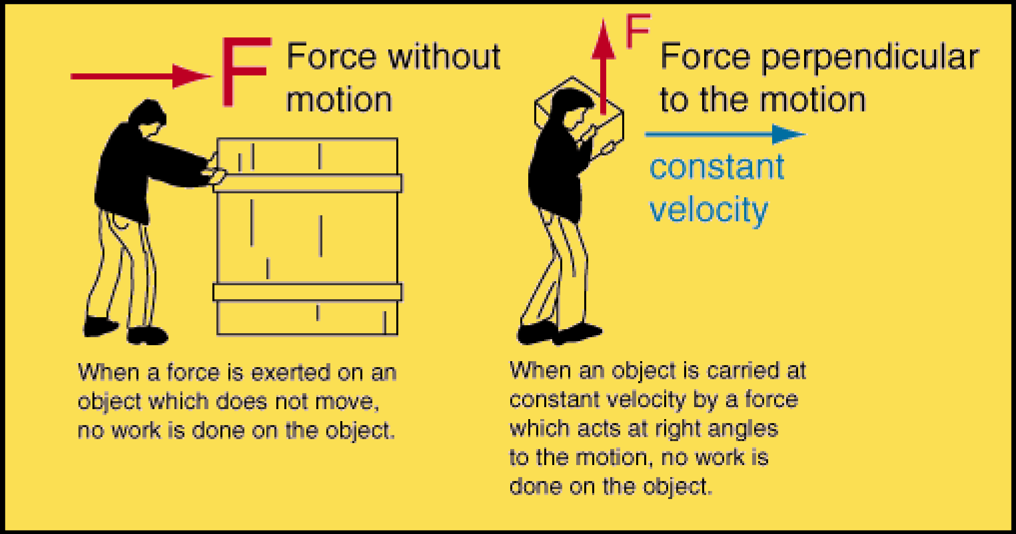
More commonly written

$$W = Fx\cos\theta$$

4.1 Work Done by a Constant Force

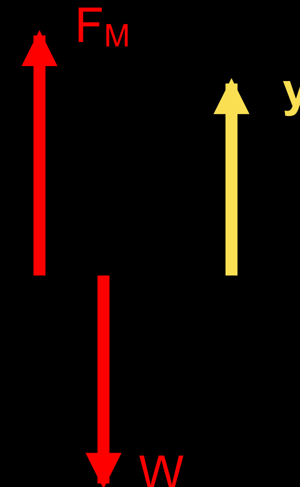
Work is done only if displacement is in the direction of force

Two situation where no work is done



4.1 Work Done by a Constant Force

Negative work is done, if the force is opposite to the direction of the displacement



Force of man – positive work

Force of gravity – negative work

4.1 Work Done by a Constant Force

A car of mass m=2000 kg coasts down a hill inclined at an angle of 30^o below the horizontal. The car is acted on by three forces; (i) the normal force; (ii) a force due to air resistance (F=1000 N); (iii) the force of weight. Find work done by each force, and the total work done on the car as it travels a distance of 25 m down the slope.

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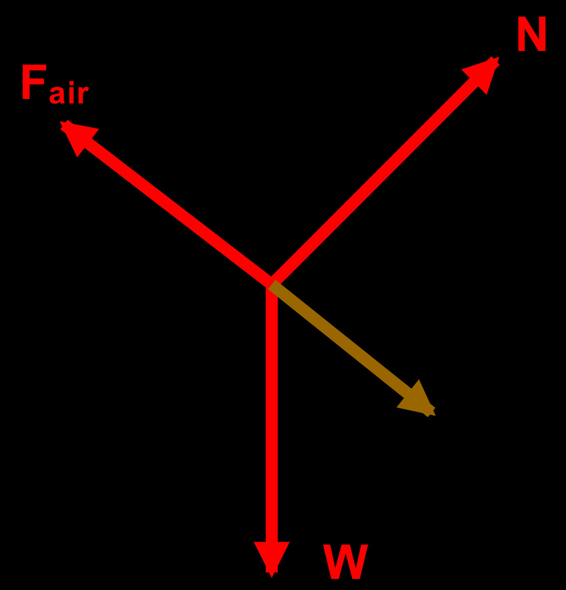
Work done by the Normal

$$W_N = Fx\cos\theta$$

$$W_N = Nx\cos 90$$

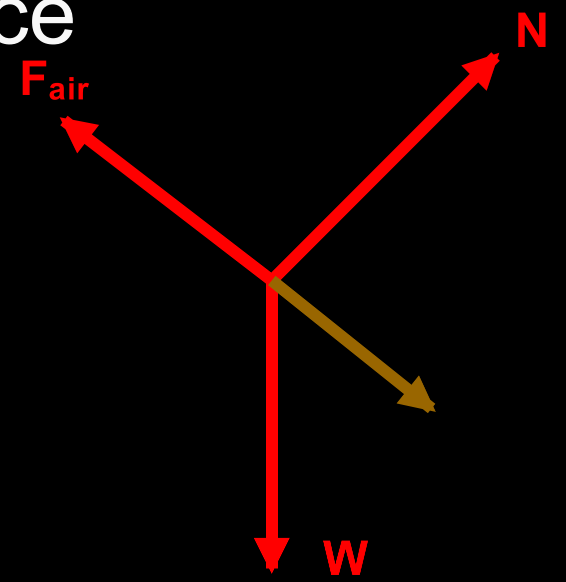
$$W_N = 0$$



4.1 Work Done by a Constant Force

A car of mass m=2000 kg coasts down a hill inclined at an angle of 30° below the horizontal. The car is acted on by three forces; (i) the normal force; (ii) a force due to air resistance (F=1000 N); (iii) the force of weight. Find work done by each force, and the total work done on the car as it travels a distance of 25 m down the slope.

Work done by the Air Resistance



$$W_F = Fx\cos\theta$$

$$W_F = 1000(25)\cos 180$$

$$W_F = -25000J$$

$$W_N = 0$$

4.1 Work Done by a Constant Force

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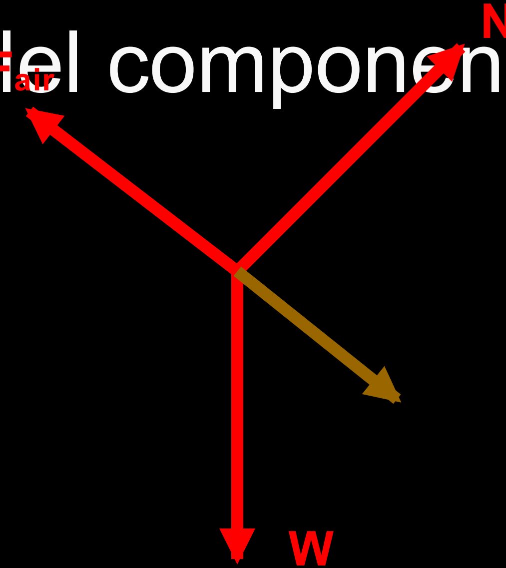
Work done by the Weight (parallel component)

$$W_W = mg\sin\theta x$$

$$W_W = (2000)(9.8)\cos(60)(25)$$

$$W_F = 245000J$$

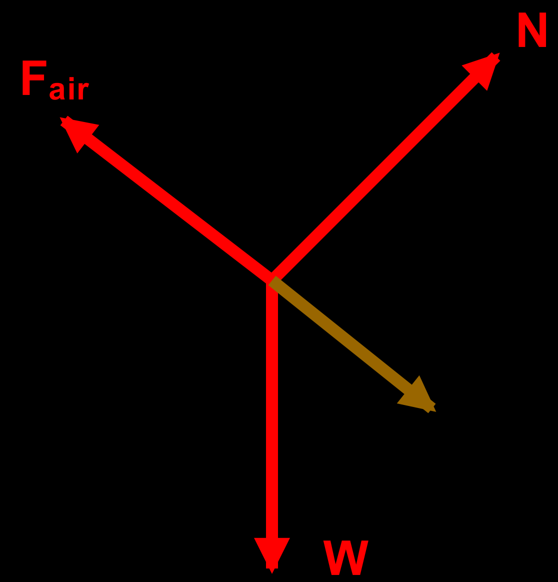


$$W_N + W_F = 0 - 25000 = -25000J$$

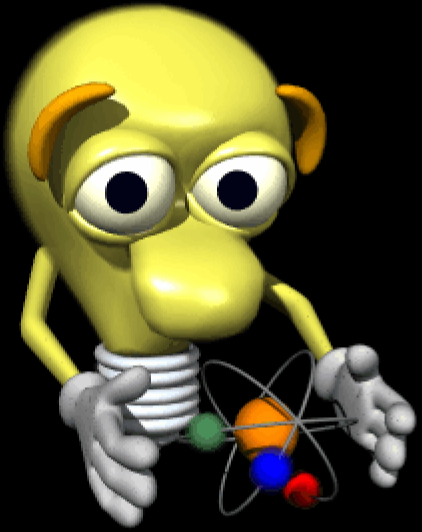
4.1 Work Done by a Constant Force

A car of mass m=2000 kg coasts down a hill inclined at an angle of 30^o below the horizontal. The car is acted on by three forces; (i) the normal force; (ii) a force due to air resistance (F=1000 N); (iii) the force of weight. Find work done by each force, and the total work done on the car as it travels a distance of 25 m down the slope.

Total Work



$$W_N + W_F + W_W = 0 - 25000 + 245000 = 220000 J$$



4.2 Work Kinetic Energy Theorem

Suppose an object falls from the side of building.

The acceleration is calculated as

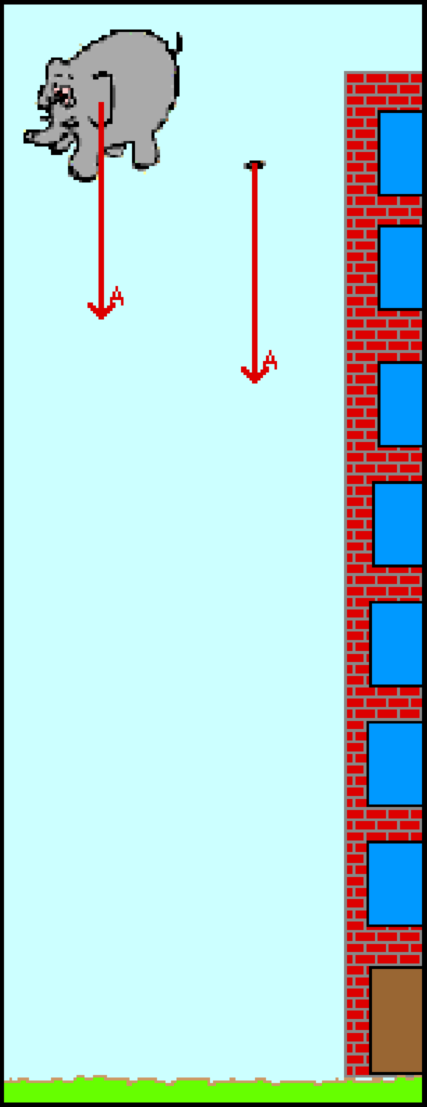
$$F = ma$$

$$a = \frac{F}{m}$$

We can also solve acceleration as related to velocity

$$v^2 = v_0^2 + 2ax$$

$$a = \frac{v^2 - v_0^2}{2x}$$



4.2 Work Kinetic Energy Theorem

Combining the two equations

$$F = ma$$

$$a = \frac{F}{m}$$

$$v^2 = v_0^2 + 2ax$$

$$a = \frac{v^2 - v_0^2}{2x}$$

$$\frac{F}{m} = \frac{v^2 - v_0^2}{2x}$$

With a little manipulation

$$Fx = \tfrac{1}{2} m (v^2 - v_0^2)$$

$$Fx = \tfrac{1}{2} m v^2 - \tfrac{1}{2} m v_0^2$$

4.2 Work Kinetic Energy Theorem

Combine with our definition of work

$$W_{tot} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

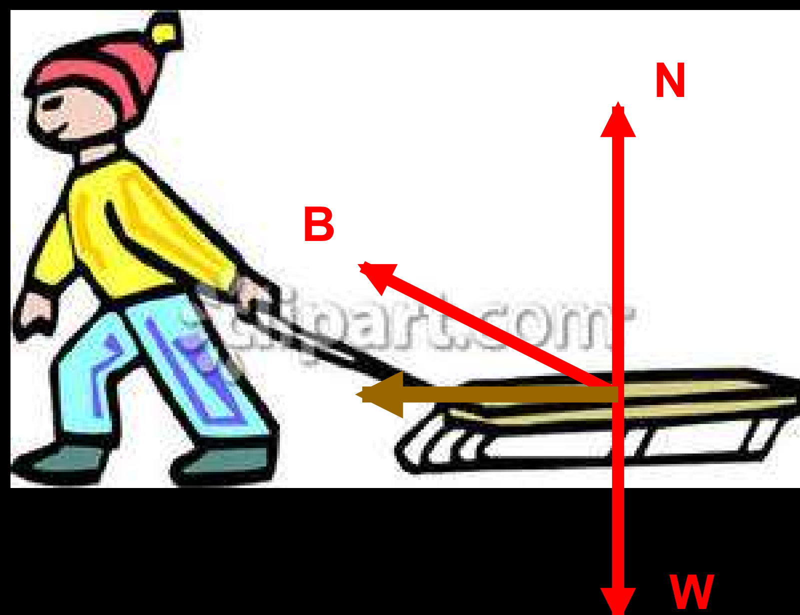
This is the work kinetic energy theorem

The quantity $\frac{1}{2}mv^2$ is called kinetic energy

$$K = \frac{1}{2}mv^2$$

4.2 Work Kinetic Energy Theorem

A child is pulling a sled with a force of 11N at 29º above the horizontal. The sled has a mass of 6.4 kg. Find the work done by the boy and the final speed of the sled after it moves 2 m. The sled starts out moving at 0.5 m/s.



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Work done by boy

$$W = Fx\cos\theta$$

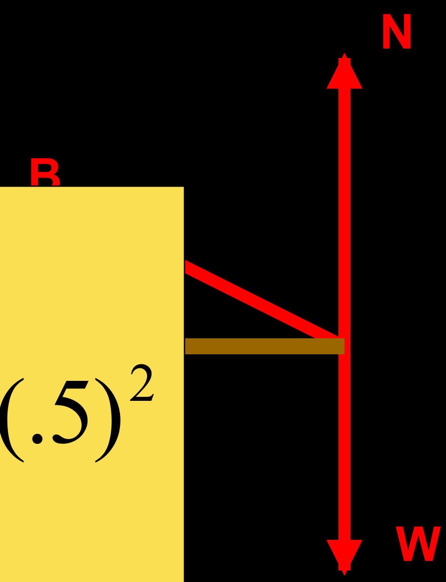
$$W = (11)(2)\cos(29) = 19.2J$$

Velocity

$$W = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2$$

$$19.2 = \tfrac{1}{2}(6.4)v^2 - \tfrac{1}{2}(6.4)(.5)^2$$

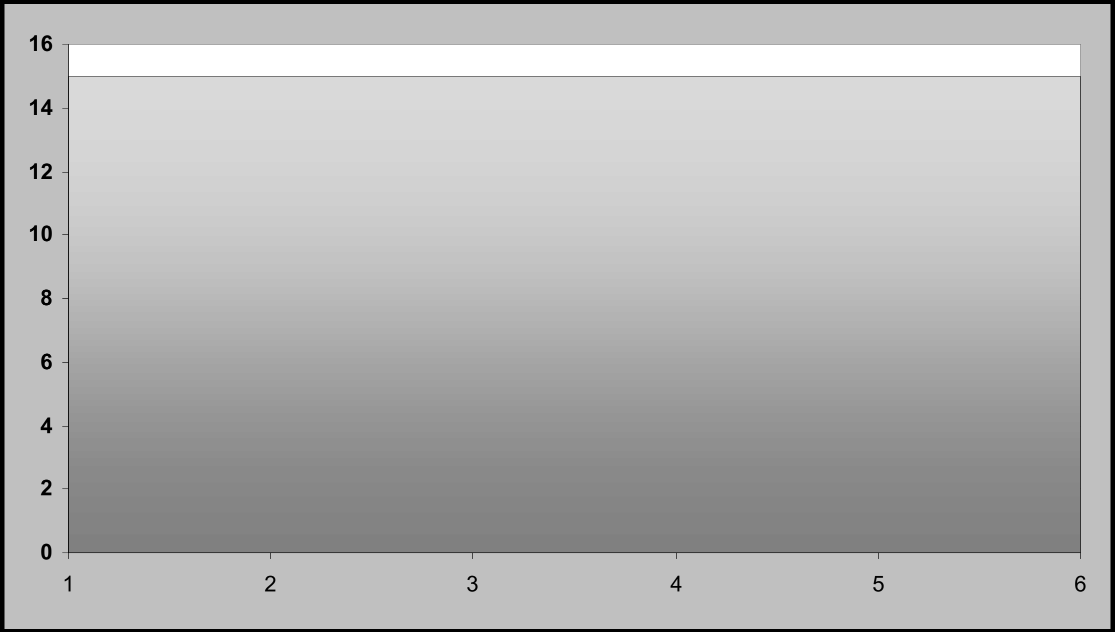
$$v = 2.5\tfrac{m}{s}$$



4.2 Work Kinetic Energy Theorem

4.3 Work Done by a Variable Force

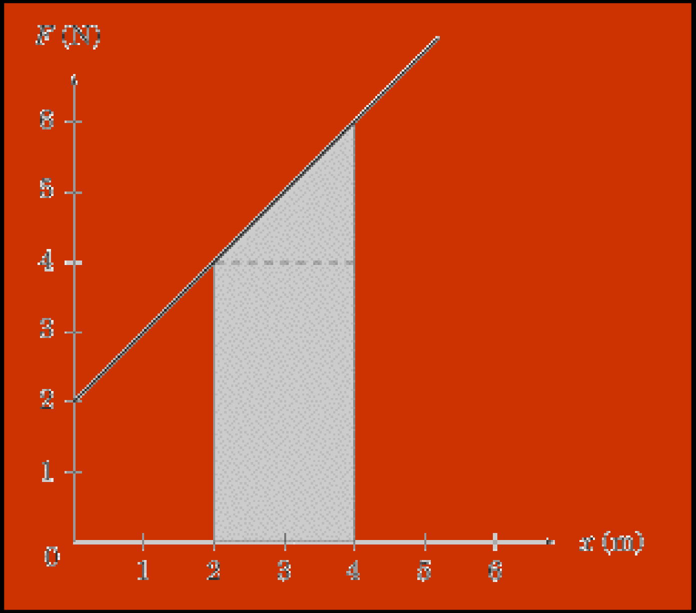
If we make a graph of constant force and displacement



The area under the line is work

4.3 Work Done by a Variable Force

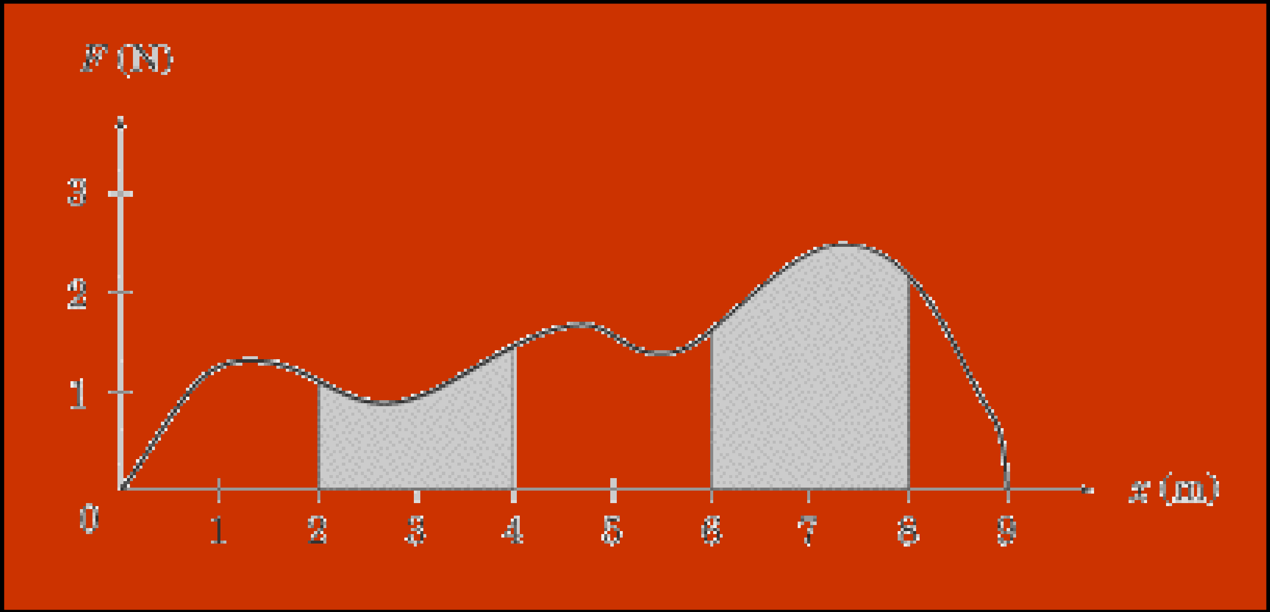
If the force varies at a constant rate



The area is still work

4.3 Work Done by a Variable Force

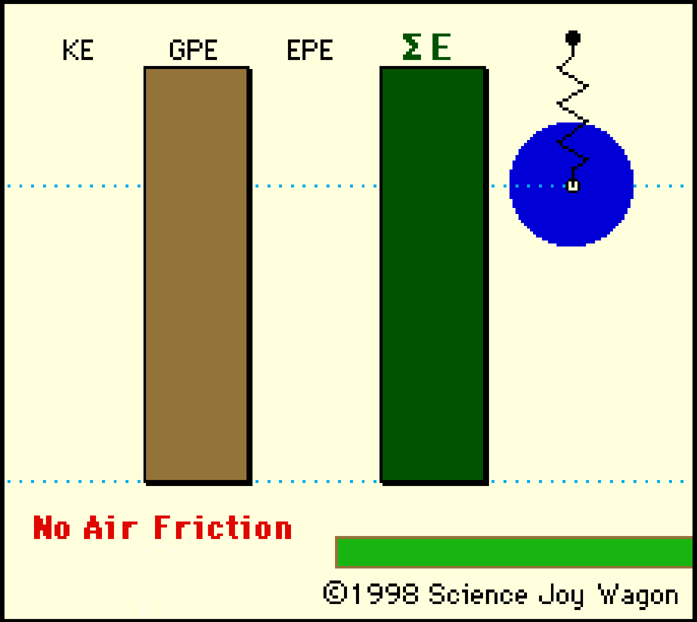
For a more complex variation



The area is still work. Calculated using calculus.

4.3 Work Done by a Variable Force

Springs are important cases of variable forces



The force is $F = kx$

4.3 Work Done by a Variable Force

If we graph this

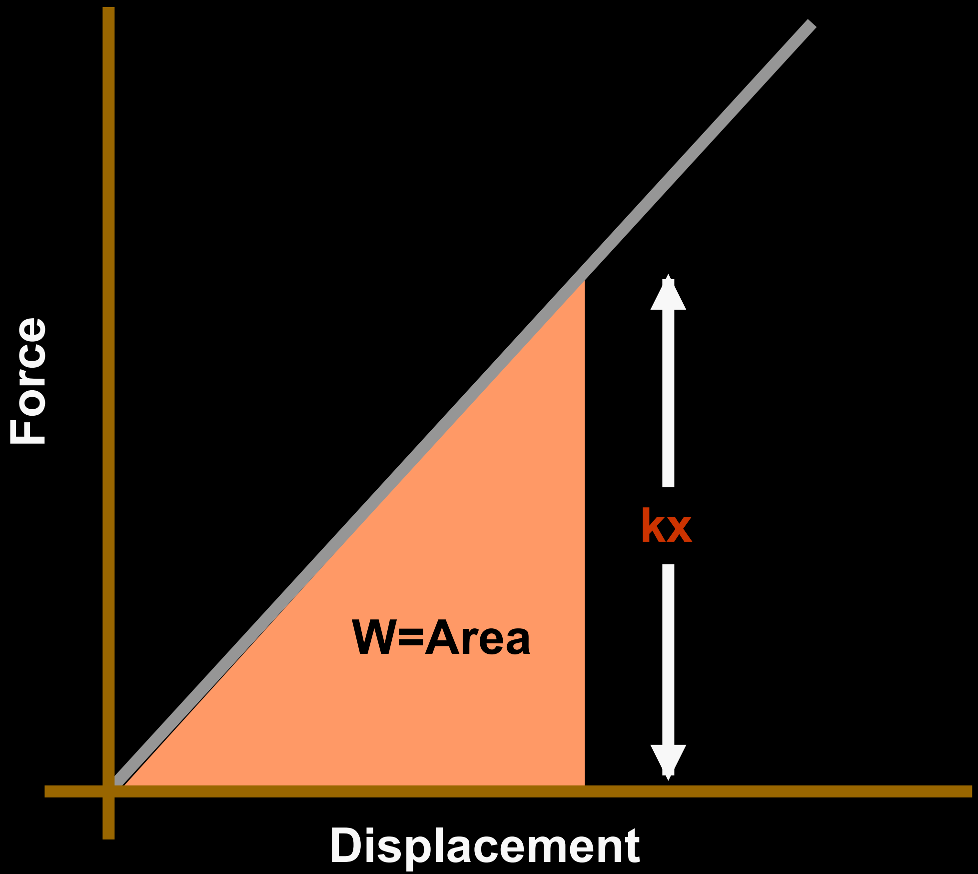
The equation for W is

$$W = \tfrac{1}{2}Fx$$

$$W = \tfrac{1}{2}(kx)x$$

$$W = \tfrac{1}{2}kx^2$$

$$U_s = \tfrac{1}{2}kx^2$$



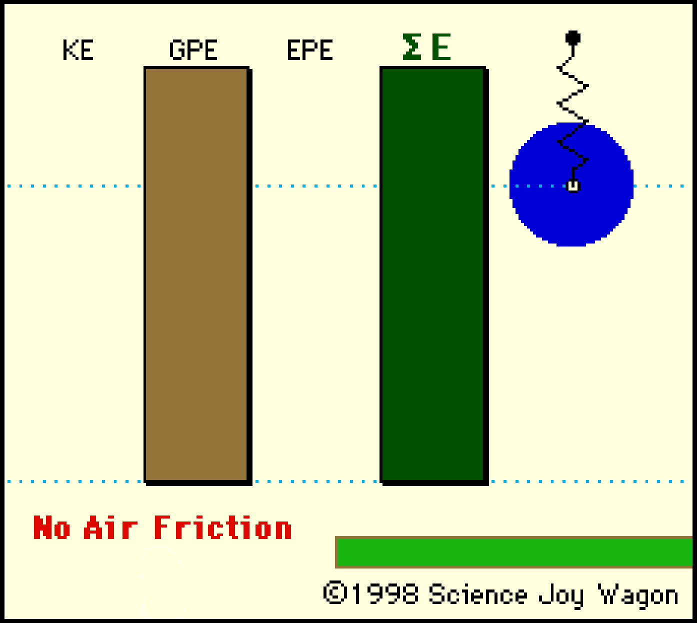
4.3 Work Done by a Variable Force

So for a spring

Spring potential is greatest at maximum displacement

Force is maximum at maximum displacement

Velocity is greatest at equilibrium (all energy is now kinetic)

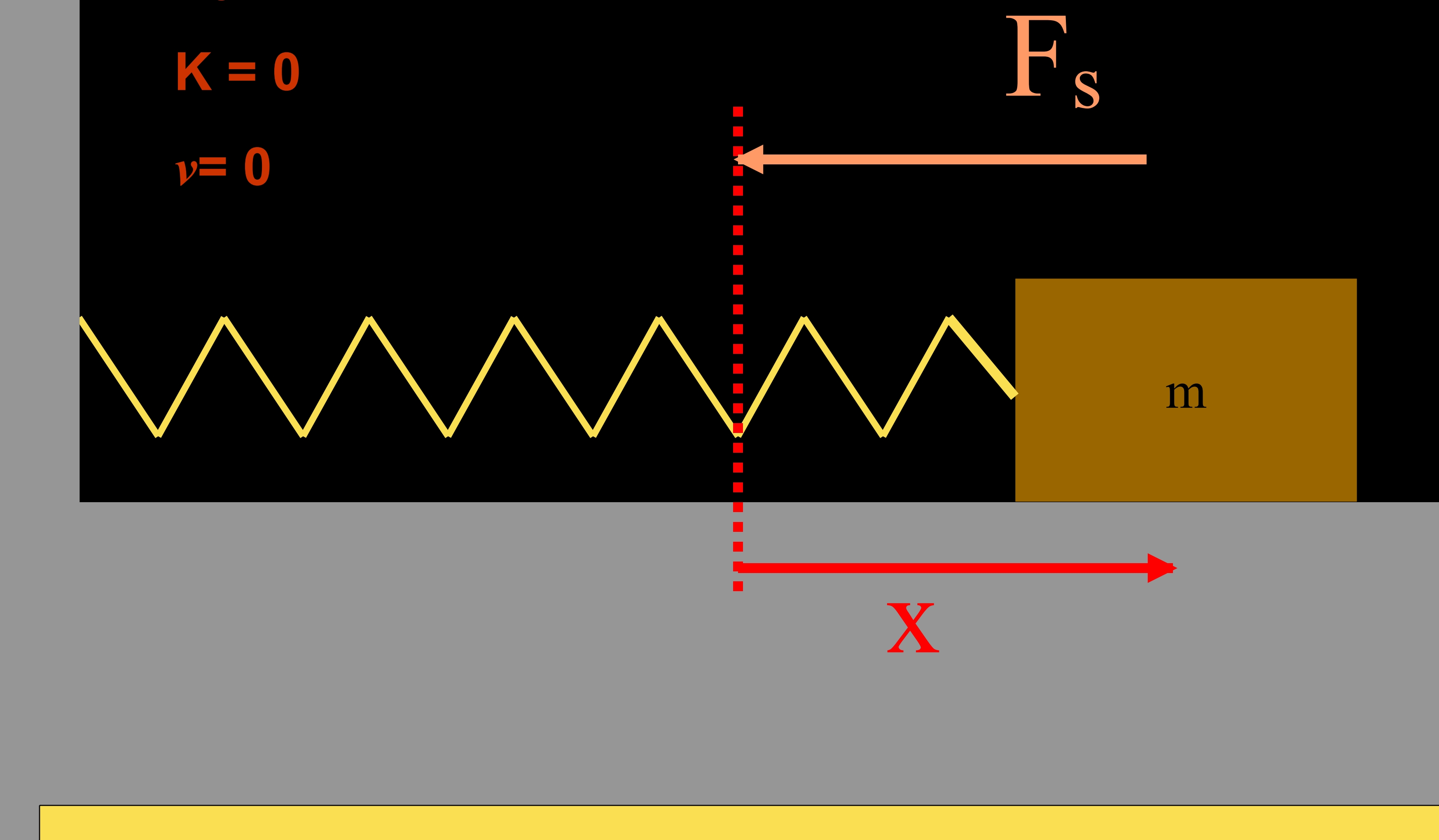


$F = \max$

$U_s = \max$

$K = 0$

$v = 0$



4.3 Work Done by a Variable Force

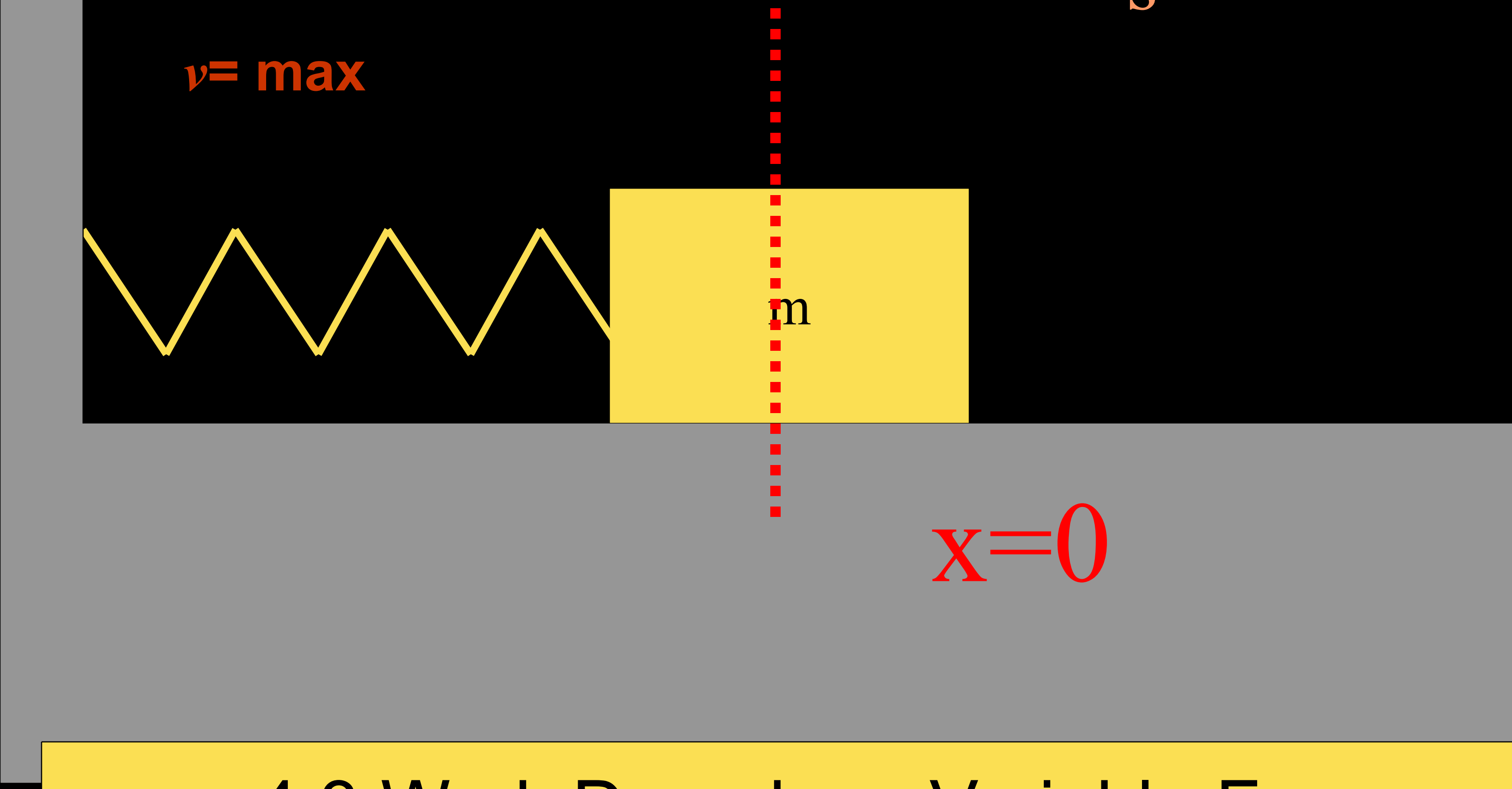
$F = 0$

$U_s = 0$

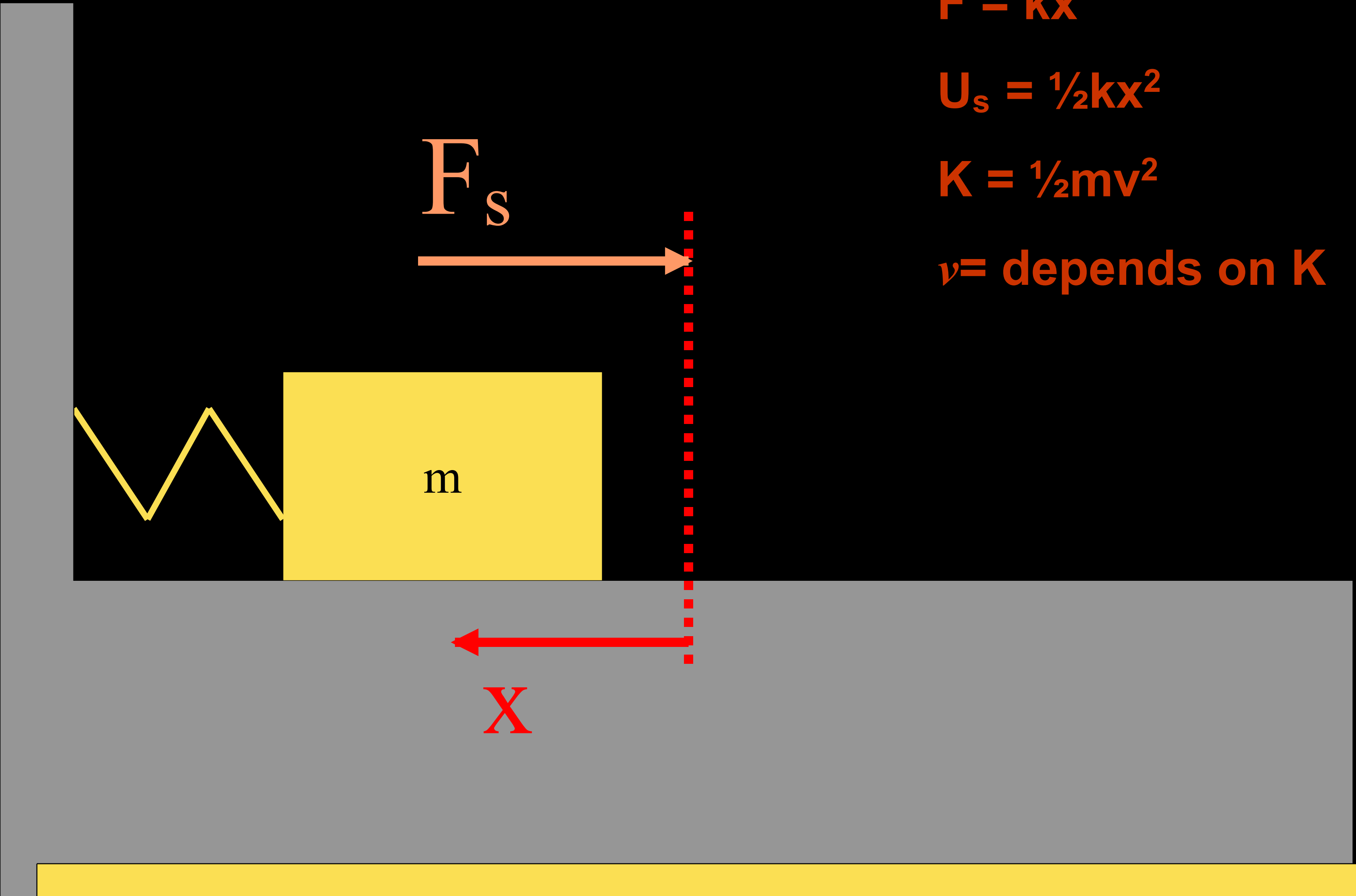
$K = max$

$v = max$

$F_s = 0$



4.3 Work Done by a Variable Force



$F = kx$

$U_s = \frac{1}{2}kx^2$

$K = \frac{1}{2}mv^2$

v= depends on K

4.3 Work Done by a Variable Force

4.4 Power

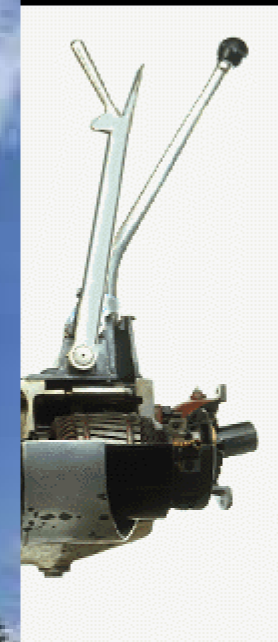
Power

How fast work is done

$$P = \frac{W}{t} = \frac{Fx}{t} = Fv$$

4.4 Power

Measured in watts (j/s)



4.4 Power

You are out driving in your Porsche and you want to pass a slow moving truck. The car has a mass of 1300 kg and nees to accelerate from 13.4 m/s to 17.9 m/s in 3 s. What is the minimum power needed?

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$$P = \frac{W}{t}$$

$$W = \tfrac{1}{2} m v^2 - \tfrac{1}{2} m v_0^2$$

$$W = \tfrac{1}{2}(1300)(17.9)^2 - \tfrac{1}{2}(1300)(13.4)^2$$

$$W = 91550 J$$

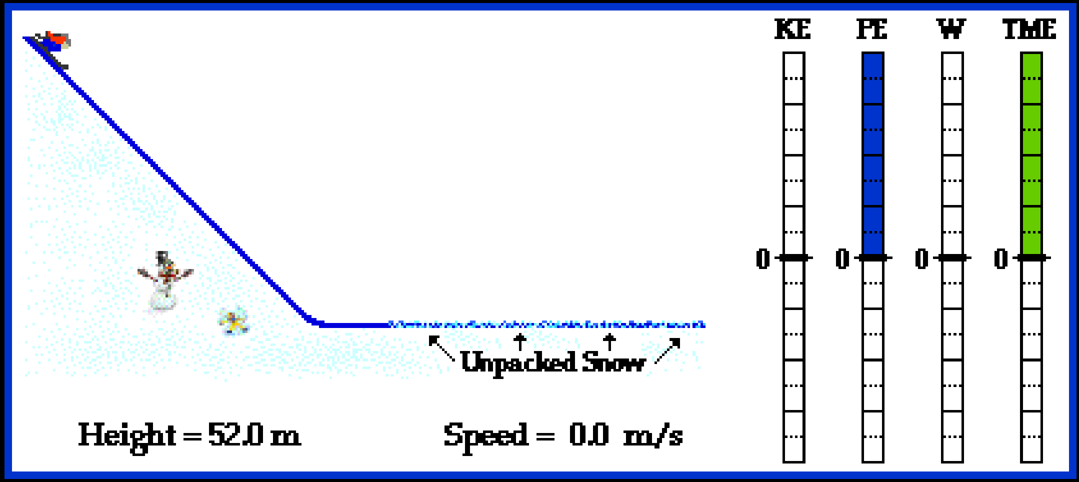
$$P = \frac{91550}{3} = 30500 W$$

$$P = 41 hp$$

4.4 Power

4.5 Conservative and Nonconservative Forces

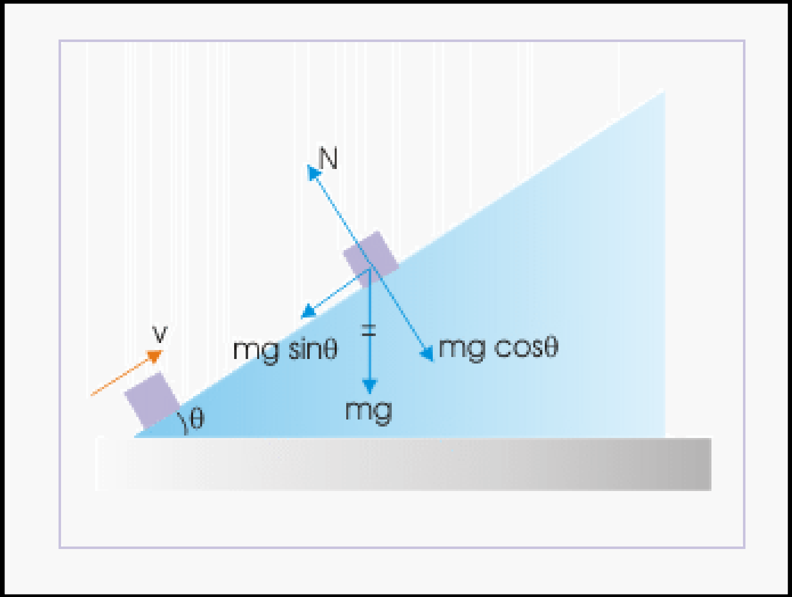
Conservative force – energy is stored and can be released



Nonconservative – energy can not be recovered as kinetic energy (loss due to friction)

4.5 Conservative and Nonconservative Forces

If a slope has no friction

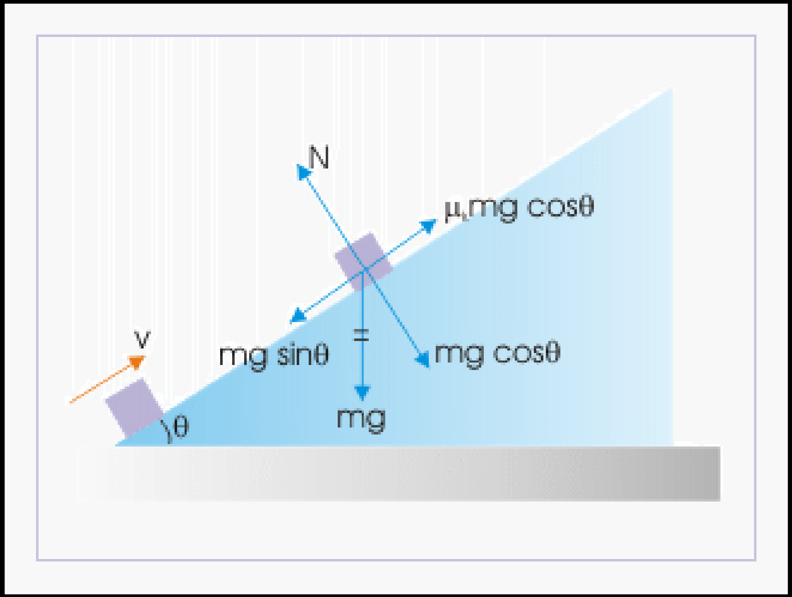


All the energy is stored

This situation is conservative

4.5 Conservative and Nonconservative Forces

If a slope has friction

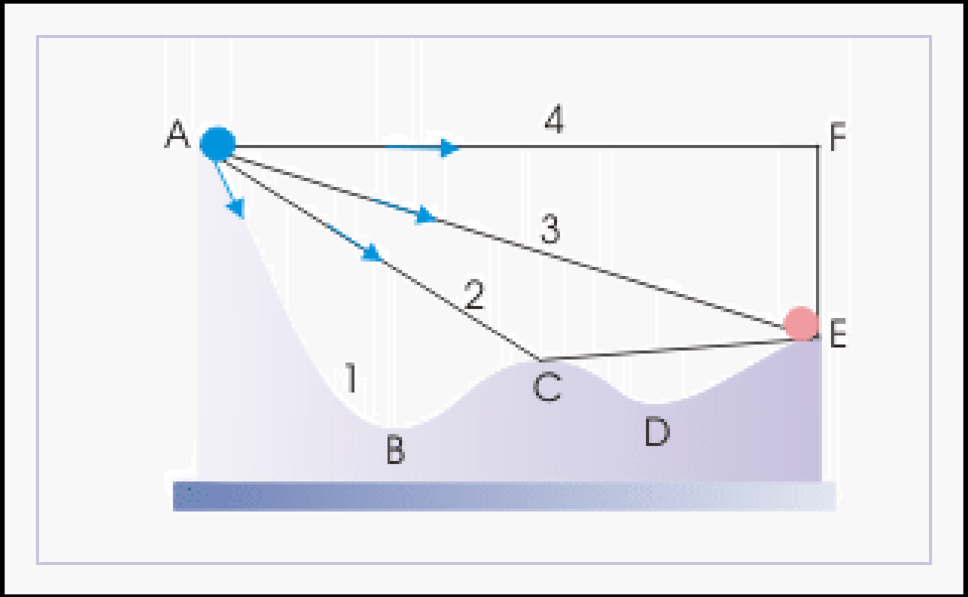


Some energy is lost as heat

It is nonconservative

4.5 Conservative and Nonconservative Forces

Conservative forces are independent of the pathway



Friction (nonconservative) more work as the distance increases

4.5 Conservative and Nonconservative Forces

4.6 Potential Energy

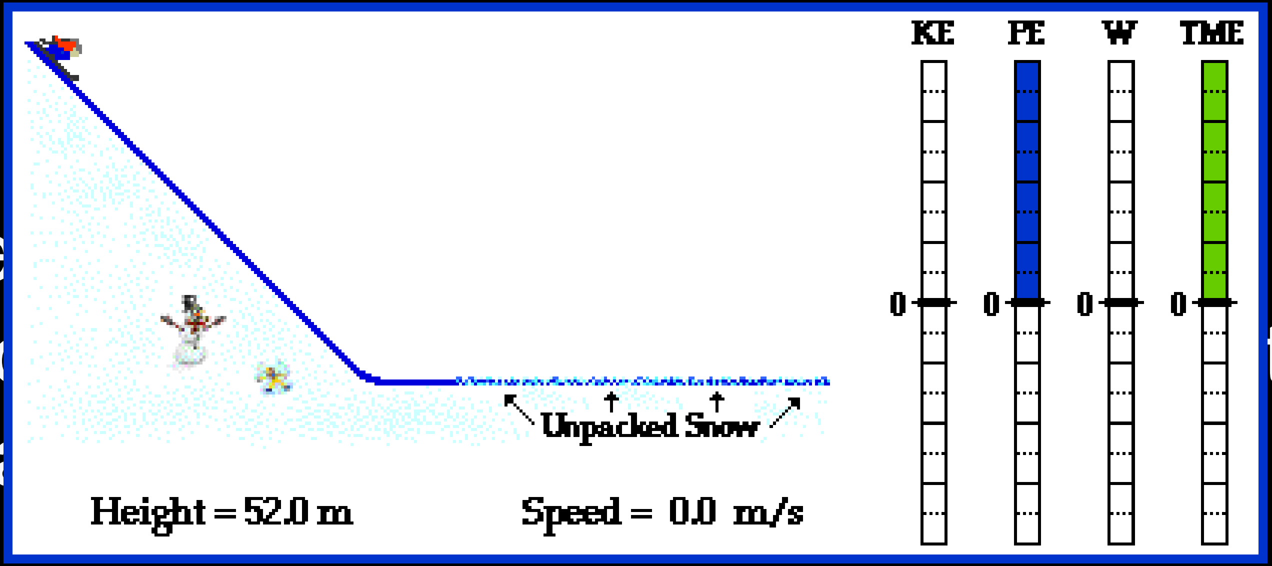
Work is transfer of energy

If an object is lifted, the work done is

This e energy

if the to its

origina



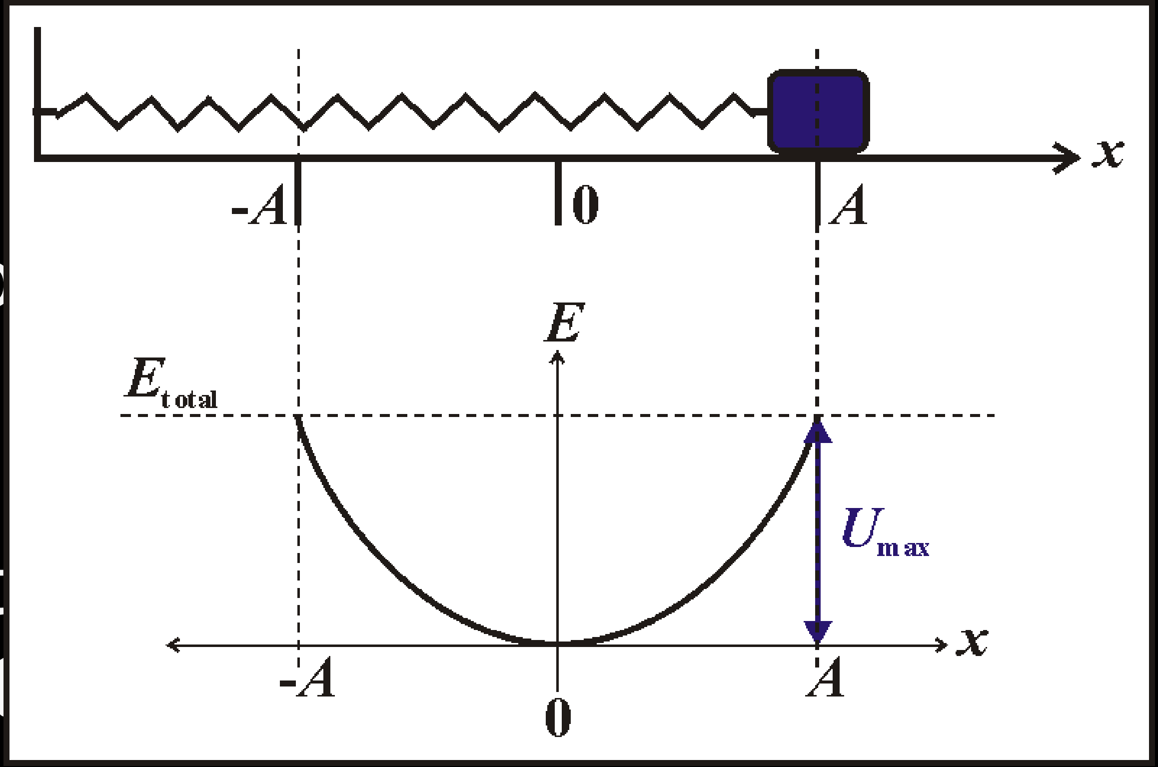
Stored Energy is called Potential Energy so

$$U_g = mgy$$

4.6 Potential Energy

Using t

The wo



When t is

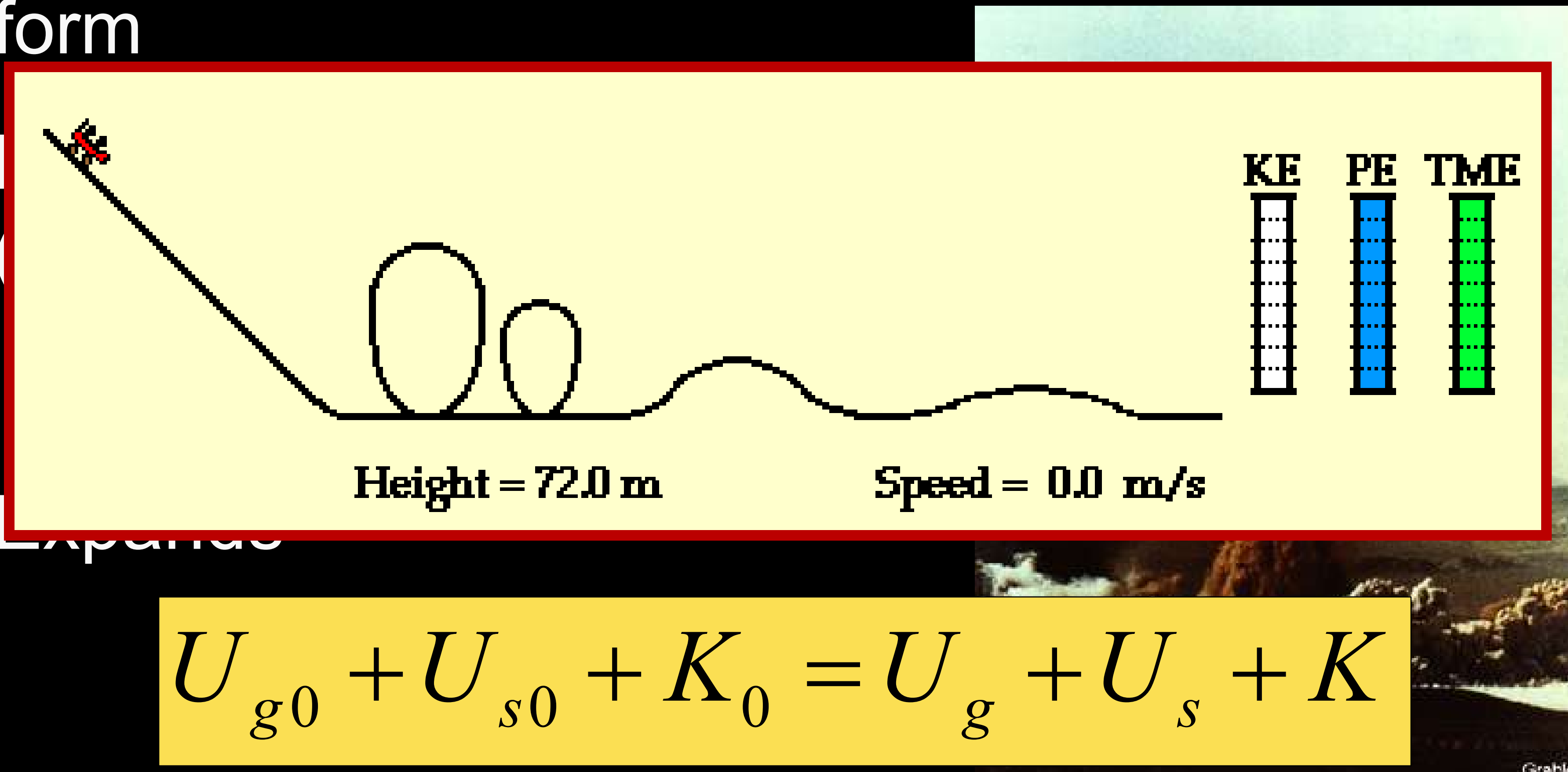
convert

$$U_s = \frac{1}{2}kx^2$$

4.6 Potential Energy

4.7 Conservation of Mechanical Energy

Law of conservation of energy – energy can not be created or destroyed, it only changes form

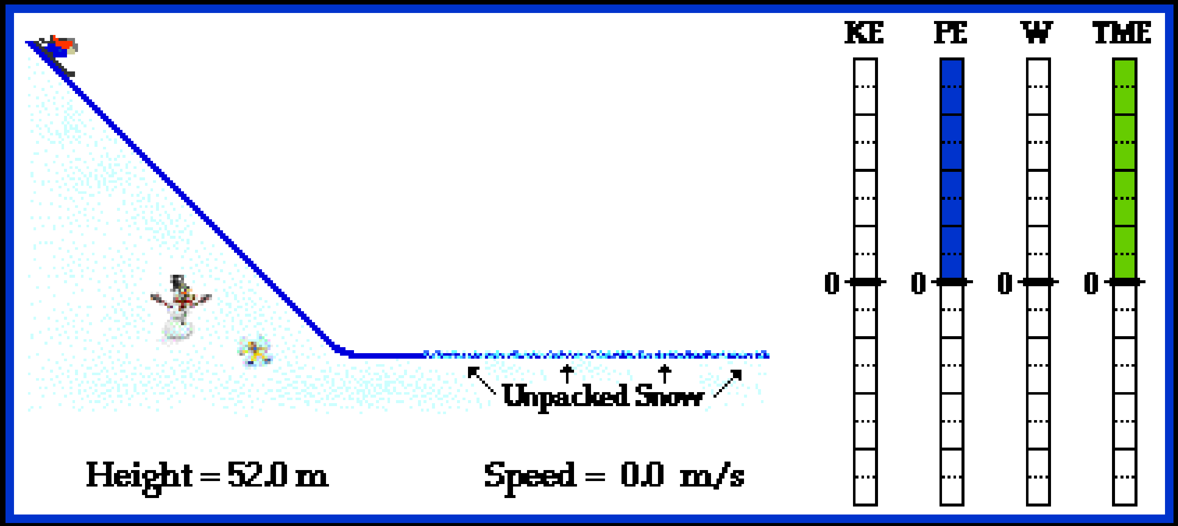


$$U_{g0} + U_{s0} + K_0 = U_g + U_s + K$$

4.6 Potential Energy

4.8 Work Done by Nonconservative Forces

Nonconservative forces take energy away from the total mechanical energy



At the end, total mechanical energy decreases because of the work done by friction

We can think of this as energy that is used up (although it just goes into a nonmechanical form)

$$E_0 = E + W_f$$

Expand this and our working equation becomes

$$U_{g0} + U_{s0} + K_0 = U_g + U_s + K + W_f$$

4.7 Work Done by Nonconservative Forces

Remember that the work due to friction is still defined as

$$W = Fx$$

$$W = fx$$

$$W = \mu N x$$

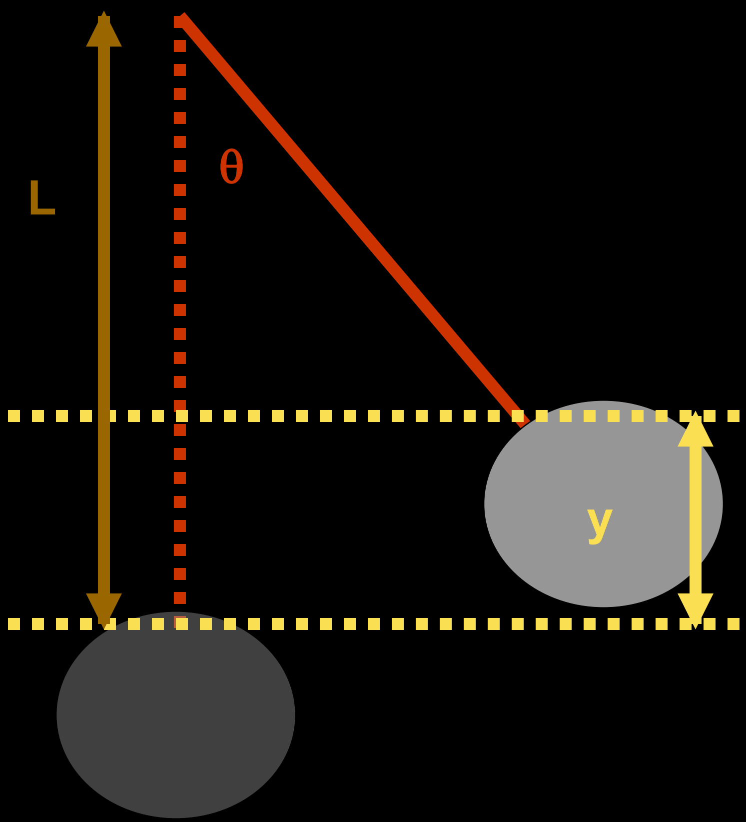
$$W = \mu m g x$$

One other helpful concept for the energy of a pendulum

How do you calculate y for potential energy?

If an angle is given and the string is defined a L

The height of the triangle is



$L\cos\theta$

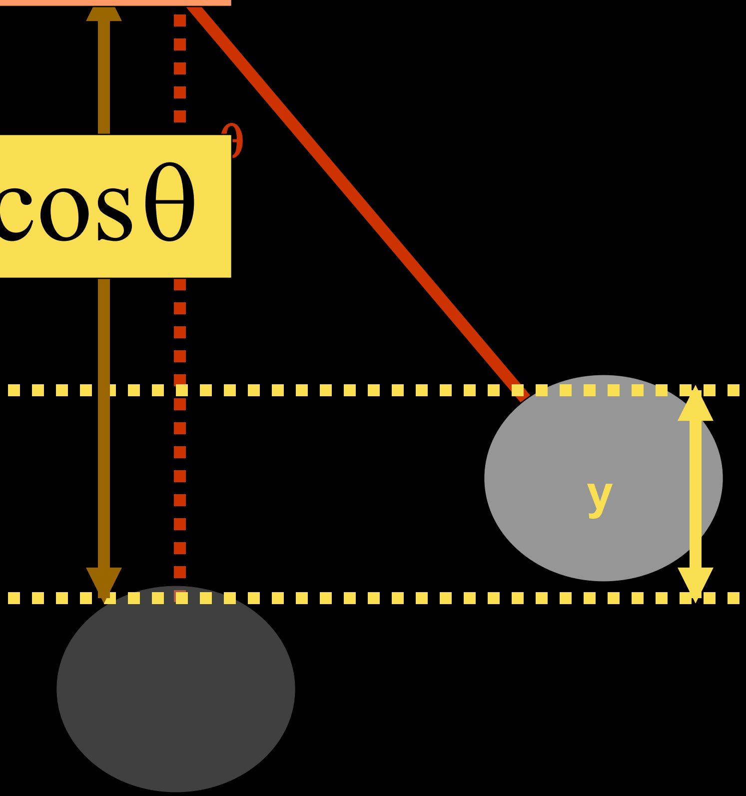
4.7 Work Done by Nonconservative Forces

Now the change in height is

$$L = y + L\cos\theta$$

Which we can rewrite as

$$y = L - L\cos\theta$$



$$L\cos\theta$$

4.7 Work Done by Nonconservative Forces