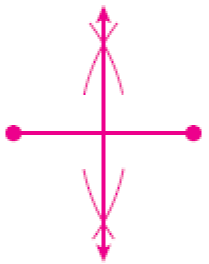
Perpendicular bisector and angle bisector

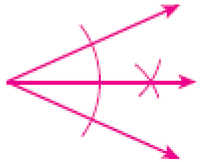
Warm Up

Construct each of the following.

1. A perpendicular bisector.



2. An angle bisector.



3. Find the midpoint and slope of the segment (2, 8) and (–4, 6).

$(-1, 7), \frac{1}{3}$

Objectives

Prove and apply theorems about perpendicular bisectors.

Prove and apply theorems about angle bisectors.

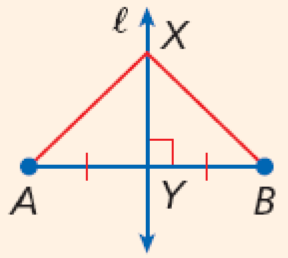
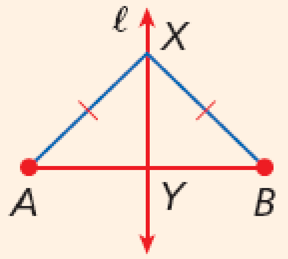
Vocabulary

equidistant

locus

When a point is the same distance from two or more objects, the point is said to be **equidistant** from the objects. Triangle congruence theorems can be used to prove theorems about equidistant points.

Theorems — Distance and Perpendicular Bisectors

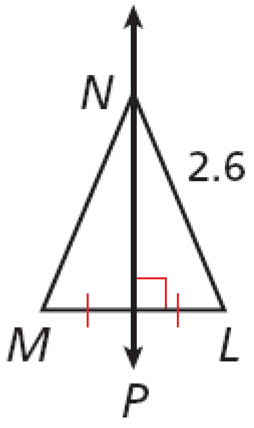
THEOREM	HYPOTHESIS	CONCLUSION
5-1-1 **Perpendicular Bisector Theorem** If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.	 $\overline{XY} \perp \overline{AB}$ $\overline{YA} \cong \overline{YB}$	$XA = XB$
5-1-2 **Converse of the Perpendicular Bisector Theorem** If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.	 $XA = XB$	$\overline{XY} \perp \overline{AB}$ $\overline{YA} \cong \overline{YB}$

A **locus** is a set of points that satisfies a given condition. The perpendicular bisector of a segment can be defined as the locus of points in a plane that are equidistant from the endpoints of the segment.

Example 1A: Applying the Perpendicular Bisector Theorem and Its Converse

Find each measure.

MN



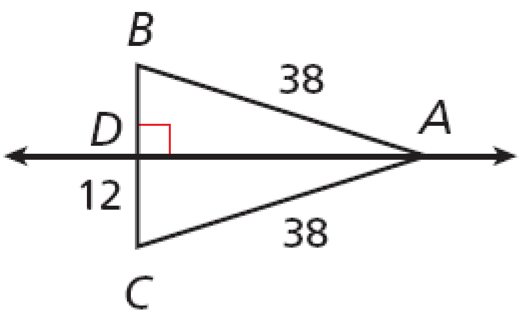
$MN = LN$ *$\perp$ Bisector Thm.*

$MN = 2.6$ *Substitute 2.6 for LN.*

Example 1B: Applying the Perpendicular Bisector Theorem and Its Converse

Find each measure.

BC



Since $AB = AC$ and $\ell \perp \overline{BC}$, is the perpendicular bisector of $\overline{BC}$ by the Converse of the Perpendicular Bisector Theorem.

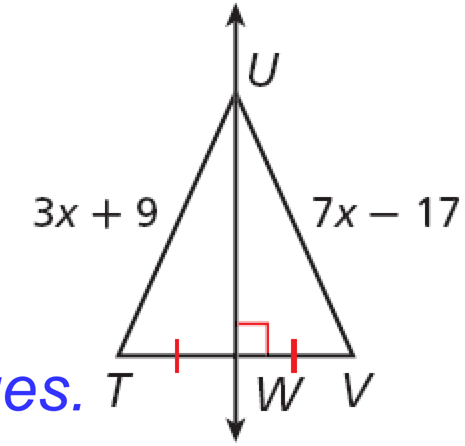
$BC = 2CD$ *Def. of seg. bisector.*

$BC = 2(12) = 24$ *Substitute 12 for CD.*

Example 1C: Applying the Perpendicular Bisector Theorem and Its Converse

Find each measure.

TU



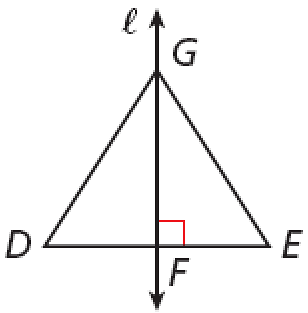
$TU = UV$	*⊥ Bisector Thm.*
$3x + 9 = 7x - 17$	*Substitute the given values.*
$9 = 4x - 17$	*Subtract 3x from both sides.*
$26 = 4x$	*Add 17 to both sides.*
$6.5 = x$	*Divide both sides by 4.*

So $TU = 3(6.5) + 9 = 28.5$.

Check It Out! Example 1a

Find the measure.

Given that line ℓ is the perpendicular bisector of $\overline{DE}$ and *EG* = 14.6, find *DG*.



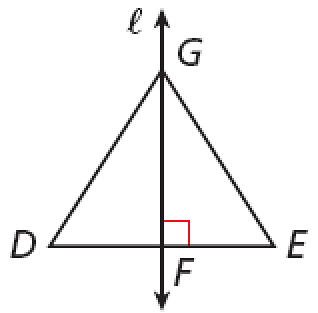
$DG = EG$ — *⊥ Bisector Thm.*

$DG = 14.6$ — *Substitute 14.6 for EG.*

Check It Out! Example 1b

Find the measure.

Given that *DE* = 20.8, *DG* = 36.4, and *EG* =36.4, find *EF*.



Since $DG = EG$ and $\ell \perp \overline{DE}$, is the perpendicular bisector of $\overline{DE}$ by the Converse of the Perpendicular Bisector Theorem.

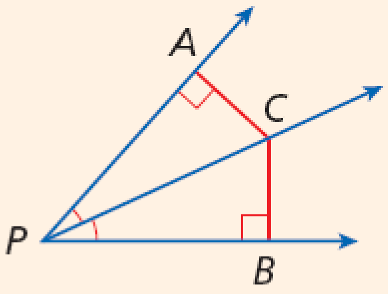
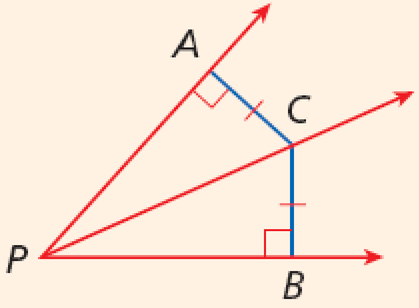
$DE = 2EF$ *Def. of seg. bisector.*

$20.8 = 2EF$ *Substitute 20.8 for DE.*

$10.4 = EF$ *Divide both sides by 2.*

Remember that the distance between a point and a line is the length of the perpendicular segment from the point to the line.

Theorems — Distance and Angle Bisectors

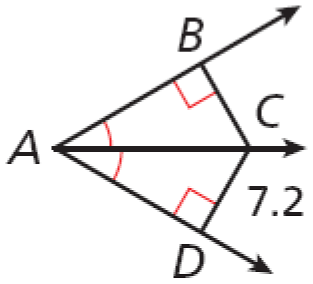
THEOREM	HYPOTHESIS	CONCLUSION
5-1-3 Angle Bisector Theorem If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.	 $\angle APC \cong \angle BPC$	$AC = BC$
5-1-4 Converse of the Angle Bisector Theorem If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.	 $AC = BC$	$\angle APC \cong \angle BPC$

Based on these theorems, an angle bisector can be defined as the locus of all points in the interior of the angle that are equidistant from the sides of the angle.

Example 2A: Applying the Angle Bisector Theorem

Find the measure.

BC



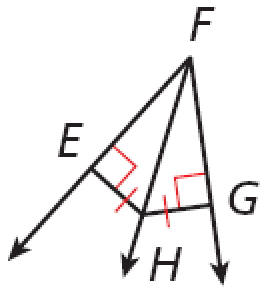
BC = *DC*   *∠ Bisector Thm.*

BC = 7.2   *Substitute 7.2 for DC.*

Example 2B: Applying the Angle Bisector Theorem

Find the measure.

m∠*EFH*, given that m∠*EFG* = 50°.



Since $EH = GH$, $\overline{EH} \perp \overline{FE}$ and $\overline{GH} \perp \overline{FG}$, $\overrightarrow{FH}$ bisects $\angle EFG$ by the Converse of the Angle Bisector Theorem.

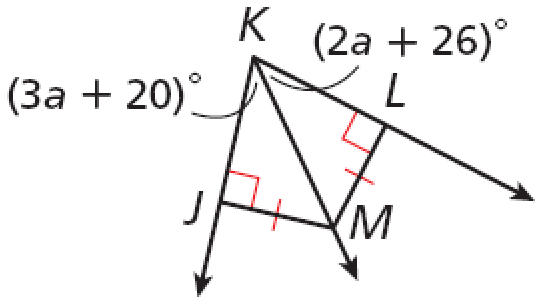
$$m\angle EFH = \frac{1}{2} m\angle EFG$$ *Def. of ∠ bisector*

$$m\angle EFH = \frac{1}{2}(50°) = 25°$$ *Substitute 50° for m∠EFG.*

Example 2C: Applying the Angle Bisector Theorem

Find m∠*MKL*.

Since, $JM = LM$, $\overline{KJ} \perp \overline{JM}$ and $\overline{KL} \perp \overline{LM}$, $\overrightarrow{KM}$ bisects $\angle JKL$ by the Converse of the Angle Bisector Theorem.

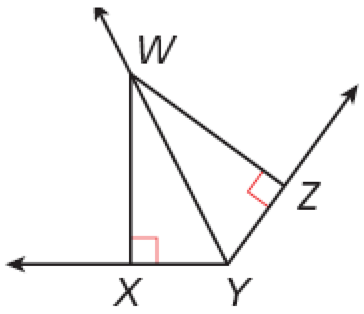


$m\angle MKL = m\angle JKM$	*Def. of ∠ bisector*
$3a + 20 = 2a + 26$	*Substitute the given values.*
$a + 20 = 26$	*Subtract 2a from both sides.*
$a = 6$	*Subtract 20 from both sides.*

So $m\angle MKL = [2(6) + 26]° = 38°$

Check It Out! Example 2a

Given that $\overrightarrow{YW}$ bisects $\angle XYZ$ and WZ = 3.05, find WX.



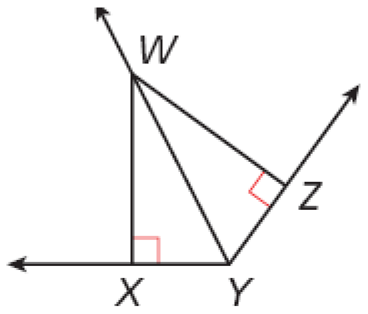
$WX = WZ$ — $\angle$ *Bisector Thm.*

$WX = 3.05$ — *Substitute 3.05 for WZ.*

So $WX = 3.05$

Check It Out! Example 2b

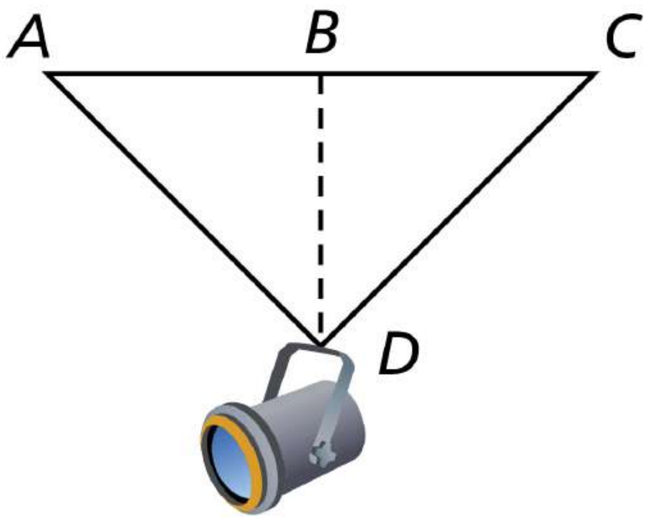
Given that $m\angle WYZ = 63°$, $XW = 5.7$, and $ZW = 5.7$, find $m\angle XYZ$.



$m\angle WYZ + m\angle WYX = m\angle XYZ$	
$m\angle WYZ = m\angle WYX$	$\angle$ Bisector Thm.
$m\angle WYZ + m\angle WYZ = m\angle XYZ$	Substitute $m\angle WYZ$ for $m\angle WYX$.
$2m\angle WYZ = m\angle XYZ$	Simplify.
$2(63°) = m\angle XYZ$	Substitute 63° for $m\angle WYZ$.
$126° = m\angle XYZ$	Simplfiy .

Example 3: Application

John wants to hang a spotlight along the back of a display case. Wires $\overline{AD}$ and $\overline{CD}$ are the same length, and *A* and *C* are equidistant from *B*. How do the wires keep the spotlight centered?

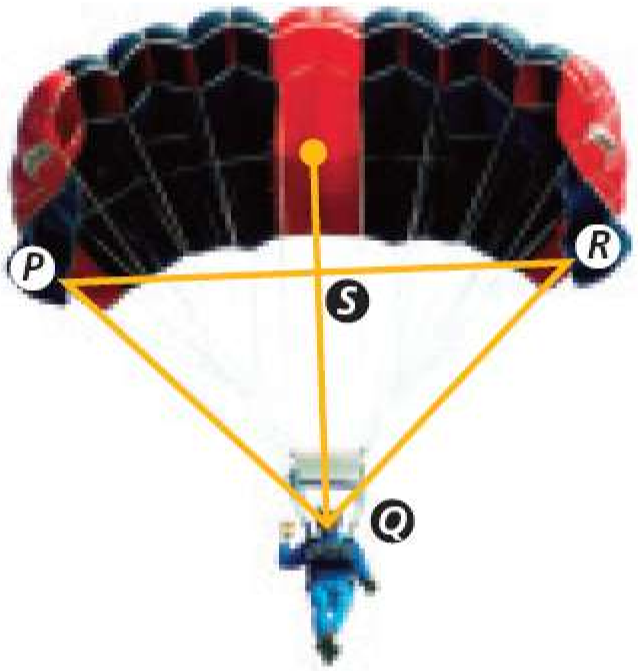


It is given that $\overline{AD} \cong \overline{CD}$. So *D* is on the perpendicular bisector of $\overline{AC}$ by the Converse of the Angle Bisector Theorem. Since *B* is the midpoint of $\overline{AC}$, $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$. Therefore the spotlight remains centered under the mounting.

Check It Out! Example 3

S is equidistant from each pair of suspension lines. What can you conclude about $\overrightarrow{QS}$?

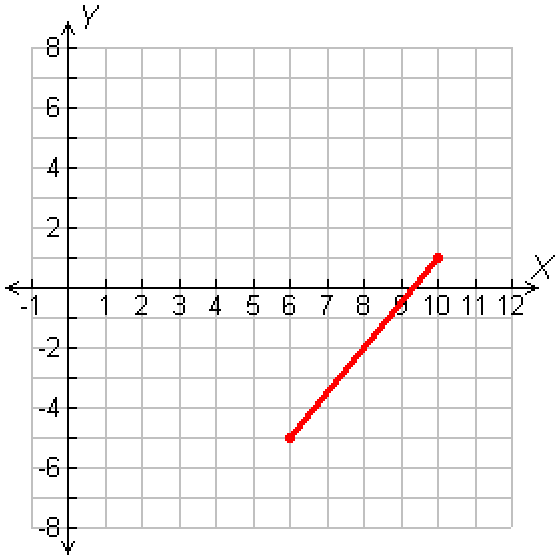
$\overrightarrow{QS}$ bisects $\angle PQR$.



Example 4: Writing Equations of Bisectors in the Coordinate Plane

Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints *C*(6, –5) and *D*(10, 1).

Step 1 Graph $\overline{CD}$.



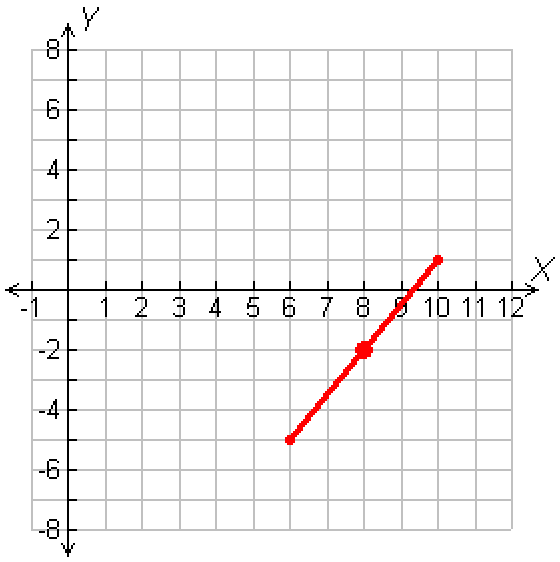
The perpendicular bisector of $\overline{CD}$ is perpendicular to $\overline{CD}$ at its midpoint.

Example 4 Continued

Step 2 Find the midpoint of $\overline{CD}$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$ *Midpoint formula.*

$$\text{mdpt. of } \overline{CD} = \left(\frac{6+10}{2}, \frac{-5+1}{2}\right) = (8, -2)$$



Example 4 Continued

Step 3 Find the slope of the perpendicular bisector.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$ *Slope formula.*

$$\text{slope of } \overline{CD} = \frac{1-(-5)}{10-6} = \frac{6}{4} = \frac{3}{2}$$

Since the slopes of perpendicular lines are opposite reciprocals, the slope of the perpendicular bisector is $-\frac{2}{3}$.

Example 4 Continued

Step 4 Use point-slope form to write an equation.

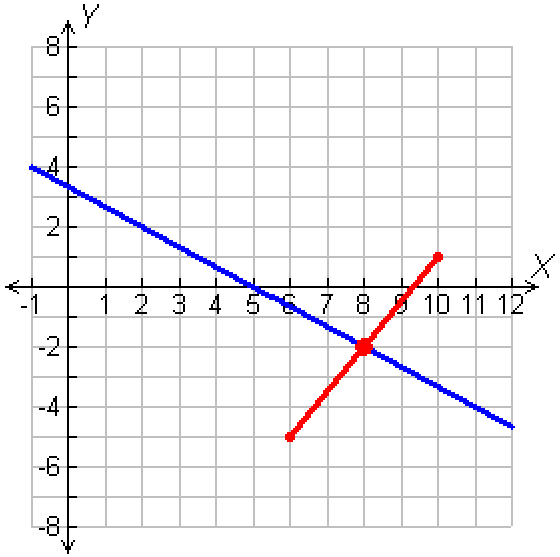
The perpendicular bisector of $\overline{CD}$ has slope $-\frac{2}{3}$ and passes through (8, –2).

$y - y_1 = m(x - x_1)$ — *Point-slope form*

$$y + 2 = -\frac{2}{3}(x - 8)$$

Substitute –2 for y_1, $-\frac{2}{3}$ for m, and 8 for $\wedge_1$.

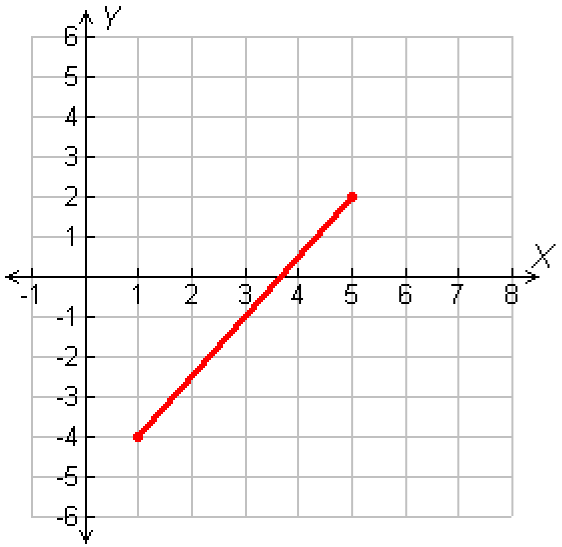
Example 4 Continued



Check It Out! Example 4

Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints *P*(5, 2) and *Q*(1, –4).

Step 1 Graph $\overline{PQ}$.



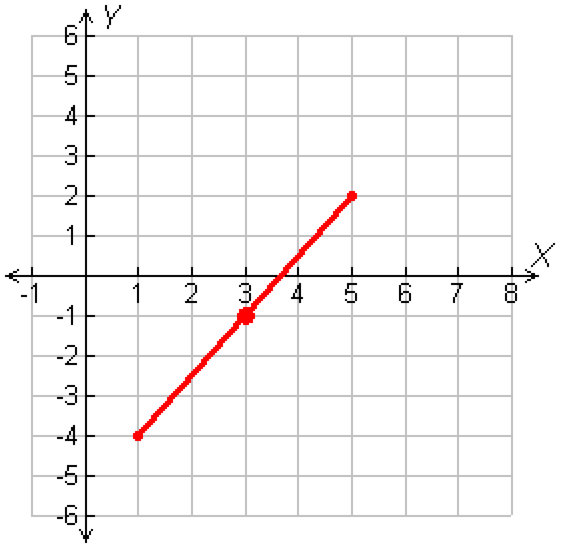
The perpendicular bisector of $\overline{PQ}$ is perpendicular to $\overline{PQ}$ at its midpoint.

Check It Out! Example 4 Continued

Step 2 Find the midpoint of $\overline{PQ}$.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$ *Midpoint formula.*

$$\overline{PQ} = \left(\frac{5+1}{2}, \frac{2+(-4)}{2}\right) = (3, -1)$$



Check It Out! Example 4 Continued

Step 3 Find the slope of the perpendicular bisector.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$ *Slope formula.*

$$\text{slope of } \overline{PQ} = \frac{-4-2}{1-5} = \frac{-6}{-4} = \frac{3}{2}$$

Since the slopes of perpendicular lines are opposite reciprocals, the slope of the perpendicular bisector is $-\frac{2}{3}$.

Check It Out! Example 4 Continued

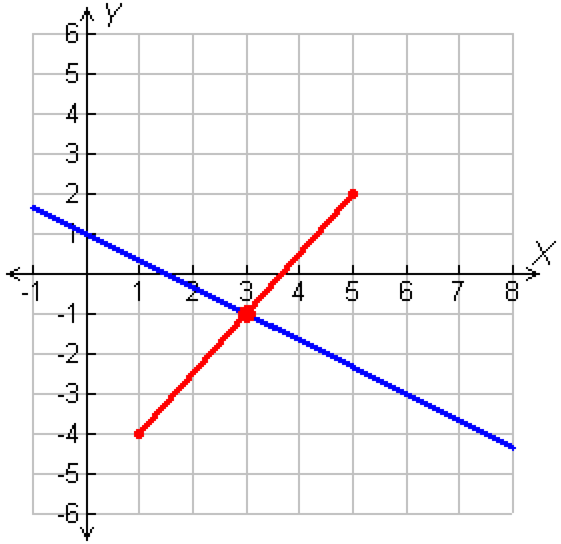
Step 4 Use point-slope form to write an equation.

The perpendicular bisector of *PQ* has slope $-\frac{2}{3}$ and passes through (3, –1).

$y - y_1 = m(x - x_1)$ *Point-slope form*

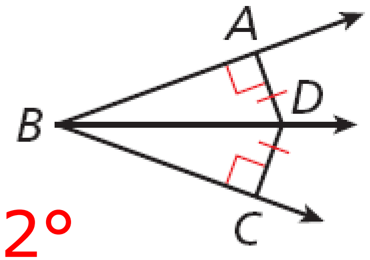
$y - (-1) = -\frac{2}{3}(x - 3)$ *Substitute*

$y + 1 = -\frac{2}{3}(x - 3)$



Lesson Quiz: Part I

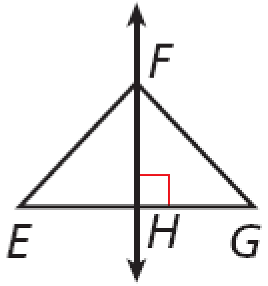
Use the diagram for Items 1–2.



1. Given that m$\angle ABD$ = 16°, find m$\angle ABC$. 32°

2. Given that m$\angle ABD$ = $(2x + 12)°$ and m$\angle CBD$ = $(6x - 18)°$, find m$\angle ABC$. 54°

Use the diagram for Items 3–4.



3. Given that $\overline{FH}$ is the perpendicular bisector of $\overline{EG}$, $EF = 4y - 3$, and $FG = 6y - 37$, find FG. 65

4. Given that $EF = 10.6$, $EH = 4.3$, and $FG = 10.6$, find EG. 8.6

Lesson Quiz: Part II

5. Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints $X(7, 9)$ and $Y(-3, 5)$.

$$y - 7 = -\frac{5}{2}(x - 2)$$