

Permutations and Combinations Learning Task

Name _____

Date _____

Common Core State Standard

MCC9-12.S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Model with mathematics
4. Attend to precision

Part 1: Fundamental Counting Principle

a) A deli has a lunch special which consists of a sandwich, soup, and a dessert for \$4.99. They offer the following choices:

Sandwich – chicken salad, turkey, ham, or roast beef

Soup – tomato, chicken noodle, or broccoli cheddar

Dessert – cookie or pie

Use a diagram to determine the number of different lunch combinations. Then, use the Fundamental Counting Principle to determine the number of different lunch combinations.

Diagram:

Fundamental Counting Principle:

b) Karl has 5 shirts, 3 pairs of pants, and 2 sweaters in his closet. How many different outfits that consist of a shirt, pair of pants, and sweater can he make?

c) If you roll a dice, then toss a coin, how many different outcomes could you get?

d) A license plate in Canada consists of:

LETTER, LETTER, LETTER, NUMBER, NUMBER, NUMBER

How many different license plates can be created?

e) A padlock has a 4-digit combination using the digits 0 – 9. How many different padlock combinations are there if repetition of the numbers is allowed?

Part 2: Permutations and Combinations

Permutations

Definition: A **permutation** is an ordered arrangement of n objects (people, numbers, letters, etc.) The **order** of the objects matters – **a different order creates a different outcome**.

a) There are 8 people running a race. How many different outcomes for the race are there?

The example above requires you to multiply a series of descending natural numbers: $8*7*6*5*4*3*2*1$. This can be written as $8!$ and read as “8 factorial”.

$8!$ means $8*7*6*5*4*3*2*1$.

$5! = 5*4*3*2*1 = 120$

$9! = 9*8*7*6*5*4*3*2*1 = 362,880$

It is generally accepted that $0! = 1$.

What is $6!$?

Now, what if you had to calculate $20!$? Do you want to enter all of those numbers into your calculator? The factorial key on your calculator can be found by:

OK, now that we know what factorial means, let's revisit the race problem from above and change it a little bit.

b) There are 8 people competing in a race. In how many different ways can first, second, and third place medals be awarded?

If we want to use the factorial notation described above, we would start with $8!$ or $8*7*6*5*4*3*2*1$. However, we know that we want to stop multiplying after 6 so we divide by $5!$ or $5*4*3*2*1$.

$$\frac{8 * 7 * 6 * 5 * 4 * 3 * 2 * 1}{5 * 4 * 3 * 2 * 1}$$

Let's look at the formula:

Permutation Formula (no repetition allowed)

${}_n P_r = P(n, r) = \frac{n!}{(n-r)!}$ where n is the number of things you choose from and r is the actual number of things you choose.

In our race example, there are 8 people to choose from which would represent n and we are choosing 3 of them to win first, second, and third place which would represent r :

$${}_8 P_3 = P(8, 3) = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8 * 7 * 6 * 5 * 4 * 3 * 2 * 1}{5 * 4 * 3 * 2 * 1} = 336$$

c) Twelve skiers are competing in the final round of the Olympic freestyle skiing aerial competition. In how many ways can 3 of the skiers finish first, second, and third to win the gold, silver, and bronze medals?

Using your calculator: To compute a permutation using your calculator, do the following:

d) A relay race team has 4 runners who run different parts of the race. There are 16 students on your track team. How many different ways can your coach select students to compete in the race?

e) The school yearbook has an editor-in-chief and an assistant editor-in-chief. The staff of the yearbook has 15 students. In how many different ways can students be chosen for these 2 positions?

It is important to note that when you use the formula, repetition is not allowed. In other words, you can't have the same person win first and second place.

Another Case to Consider

f) How many different ways can the letters HTAM be arranged to create four-letter “words”?

But, what if some of the letters repeated?

g) In how many ways can the letters in CLASSES be rearranged to create 7 letter “words”? Since the letter S repeats 3 times, some of the permutations will be the same so we will have to eliminate them.

h) How many ways can the letters in MISSISSIPPI be arranged to create 11-letter “words”?

Combinations

Definition: A **combination** is an arrangement of objects in which **order does NOT matter**.

Let's consider the following. You have three people – 1, 2, and 3. Here are the possibilities:

Order Does Matter (Permutation)	Order Does Not Matter (Combination)
1 2 3	1 2 3
1 3 2	
2 1 3	
2 3 1	
3 1 2	
3 2 1	

The permutations have 6 times as many possibilities as the combinations.

Let's look at the formula we just learned and use it to calculate the number of permutations of the numbers 1, 2, and 3:

$${}_3P_3 = P(3,3) = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3*2*1}{1} = 6 \text{ (as shown in the chart)}$$

So, to get the number of combinations we have to divide the number of permutations by 6 or 3!. Basically all we are doing is taking the permutation formula and reducing it by $r!$ to eliminate the duplicates. If you were to start with 4 numbers, there would be 24 or 4! times more permutations than combinations so you would start with the permutation formula and then divide by 4!.

This leads us to the combination formula:

Combination Formula

${}_nC_r = C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$ where n is the number of things you choose from and r is the actual number of things you choose. (no repetition allowed)

In our example above:

$${}_3C_3 = C(3,3) = \frac{3!}{3!(3-3)!} = \frac{3*2*1}{3!*0!} = \frac{6}{3*2*1*1} = \frac{6}{6} = 1$$

a) A pizza shop offers twelve different toppings. How many different three-topping pizzas can be formed with the twelve toppings?

Using your calculator: To compute a combination using your calculator, do the following:

b) Your English teacher has asked you to select 3 novels from a list of 10 to read as an independent project. In how many ways can you choose which books to read?

A restaurant serves omelets that can be ordered with any of the ingredients shown:

Omelets \$4

(plus \$0.50 for each ingredient)

Vegetarian	Meat
green pepper	ham
red pepper	bacon
onion	sausage
mushroom	steak
tomato	
cheese	

c) Suppose you want *exactly* 2 vegetarian ingredients and 1 meat ingredient in your omelet. How many different types of omelets can you order?

d) Suppose you can afford *at most* 3 ingredients in your omelet. How many different types of omelets can you order?

Additional Practice

Directions: Simplify each expression to a single number or fraction.

1. $\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} =$

2. $\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} =$

3. $3! =$

4. $6! =$

5. $\frac{4!}{3!} =$

6. $\frac{6!}{4!} =$

7. $\frac{10!}{99!} =$

8. ${}_5P_2 =$

9. ${}_5P_5 =$

10. $P(7,3) =$

11. ${}_6C_2 =$

12. $\binom{8}{3}$

Directions: Determine whether each is an example of a permutation or a combination.

13. The number of ways you can choose a group of 3 puppies to adopt from the animal shelter when there are 20 different puppies to choose from.

14. The number of ways you could award 1st, 2nd, and 3rd place medals for the science fair

15. The number of seven-digit phone numbers that can be made using the digits 0 – 9

16. The number of ways a committee of 3 could be chosen from a group of 20

17. The number of ways a president, vice-president, and treasurer could be chosen from a group of 20

Directions: Solve each problem.

18. Little Caesars is offering a special where you can buy a large pizza with one cheese, one vegetable, and one meat for \$7.99. There are 3 kinds of cheese, 9 vegetables, and 5 meats to choose from. How many different variations of the pizza special are possible?

19. If there are 11 people on a baseball team, determine how many different ways a pitcher and a catcher could be chosen.

20. There are eight seniors on the football team that are being considered as team captains. If there will be 3 team captains, how many different ways can 3 of these seniors be chosen as captains?

21. Nine people in your class want to be on a 5-person bowling team to represent the class. How many different teams can be chosen?

22. There are 5 people on a bowling team. How many different ways are there to arrange the order the people bowl in?

23. There are 5 people on a bowling team. How many ways can you choose your bowling team captain and team manager?

24. Determine how many ways a president, vice president, and treasurer can be chosen from a math club that has 7 members.

25. California license plates are: number, letter, letter, letter, number, number, number. For example: 3YNR975. How many possible license plate combinations are there in California?

26. There are 13 people on a softball team. How many ways are there to choose 10 players to take the field?

27. There are 13 people on a softball team. How many ways are there to assign them to play the 10 different positions on the field?

28. A standard deck of cards has 52 playing cards. How many different 5-card hands are possible?

29. You are eating dinner at a restaurant. The restaurant offers 6 appetizers, 12 main dishes, 6 side orders, and 8 desserts. If you order one of each of these, how many different dinners can you order?
30. A pizza parlor has a special on a three-topping pizza. How many different special pizzas can be ordered if the parlor has 8 toppings to choose from?
31. Find the number of possible committees of 4 people that could be chosen from a class of 30 students.
32. How many different 3-digit numbers can you make using the numbers 1, 2, 3, 4, and 5? Assume numbers can be repeated.
33. How many different seven-digit telephone numbers can be formed if the first digit cannot be zero or one?
34. How many different 5-digit zip codes are there if any of the digits 0 – 9 can be used?
35. How many ways can you arrange the letters JORDAN to create 6-letter “words”?
36. How many ways can you arrange the letters ILLINOIS to create 8-letter “words”?
37. A committee is to be formed with 5 girls and 5 boys. There are 8 girls to choose from and 12 boys. How many different committees can be formed?
38. You are buying a new car. There are 7 different colors to choose from and 10 different types of optional equipment you can buy. You can choose only 1 color for your car and can afford only 2 of the options. How many combinations are there for your car?
39. An amusement park has 20 different rides. You want to ride *at least* 15 of them. How many different combinations of rides can you go on?