Calculus
Notes: One sided Limits

Limits: How you write a limit: How you say a limit:
$$\lim_{x \to c} f(x) = L$$

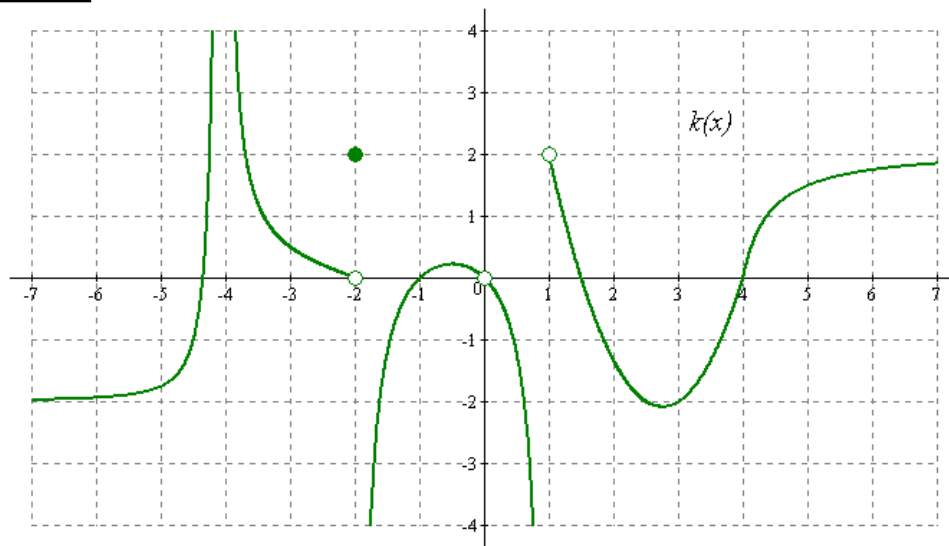   Right Handed Limit: Left Handed Limit:

For a limit to exist:

3 Ways to Evaluate Limits:
1.
2.
3.

Graphically

Example 1:   Use the graph of $k(x)$ below to evaluate the following expressions.

$\lim_{x \to -4^-} k(x) =$ $\lim_{x \to -4^+} k(x) =$ $\lim_{x \to -4} k(x) =$ $k(-4) =$

$\lim_{x \to -2^-} k(x) =$ $\lim_{x \to -2^+} k(x) =$ $\lim_{x \to -2} k(x) =$ $k(-2) =$

$\lim_{x \to 0^-} k(x) =$ $\lim_{x \to 0^+} k(x) =$ $\lim_{x \to 0} k(x) =$ $k(0) =$

$\lim_{x \to 1^-} k(x) =$ $\lim_{x \to 1^+} k(x) =$ $\lim_{x \to 1} k(x) =$ $k(1) =$

$\lim_{x \to 3^-} k(x) =$ $\lim_{x \to 3^+} k(x) =$ $\lim_{x \to 3} k(x) =$ $\lim_{x \to \infty} k(x) =$

$$\lim_{x\to 4^-} k(x) = \qquad\qquad \lim_{x\to 4^+} k(x) = \qquad\qquad \lim_{x\to 4} k(x) = \qquad\qquad \lim_{x\to -\infty} k(x) =$$

Example 2:

Given the graph of f to the right, find the following:

$$\lim_{x\to 0^-} f(x)$$

$$\lim_{x\to 0^+} f(x)$$
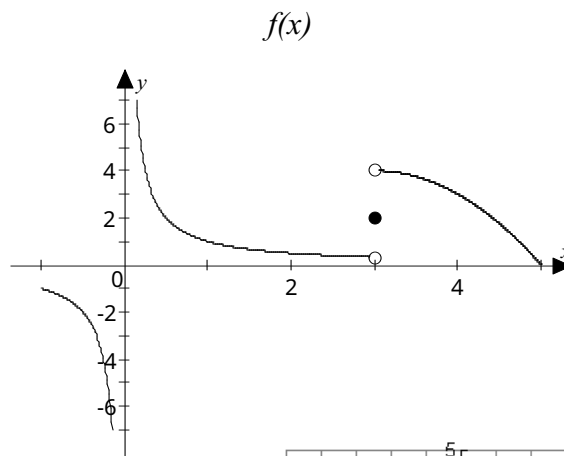
$$\lim_{x\to 0} f(x)$$

$$\lim_{x\to 3^-} f(x)$$

$$\lim_{x\to 3^+} f(x)$$

$$\lim_{x\to 3} f(x)$$

f(x)

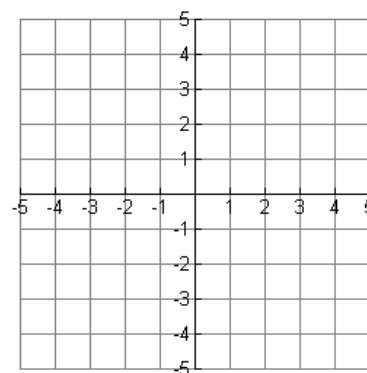**Example 3:**

Let $g(x)=\begin{cases} -x & \text{if } x\le -1 \\ 1-x^2 & \text{if } -1<x<1 \\ x-1 & \text{if } x>1 \end{cases}$.

Find $\lim_{x\to -1^-} g(x)$, $\lim_{x\to -1^+} g(x)$, $\lim_{x\to -1} g(x)$, $\lim_{x\to 1^-} g(x)$, $\lim_{x\to 1^+} g(x)$, and $\lim_{x\to 1} g(x)$.

Numerically

Example 4: Find the one-sided and two sided limits as x approaches -1 for the functions $h(x), p(x),$ and $r(x)$ given the following table of values.

x	−1.1	−1.003	−1.0001	−0.9999	−0.8762	−0.6522
h(x)	89	677	5009	2.003	2.088	2.113
p(x)	16.222	16.111	16.002	15.999	15.802	15.777
r(x)	−99	−999	−9999	−8853	−871	−86

$$\lim_{x\to -1} h(x) =$$

$$\lim_{x\to -1} p(x) =$$

$$\lim_{x\to -1} r(x) =$$

Example 5: $\lim_{x\to -1} \dfrac{x^2-2x-3}{x+1} =$

Example 6: $\lim_{x\to 0} \dfrac{\sqrt{x+2}-\sqrt{2}}{x} =$

| Example 7: | $\displaystyle\lim_{x \to 2} \ x^2 - x + 7 =$ |