Unit 3 Notes

N.CN.9

The Fundamental Theorem of Algebra

If $p(x)$ is a polynomial of degree $n \geq 1$, then $p(x)$ has exactly n real or complex roots (including any repeated roots).

Example: $3x^4 + 6x - 4$ has a degree of 4 and therefore has 4 roots

A.SSE.1

The **degree of a term** is the exponent on its variable.

To find the **degree of a polynomial**, find the term with the largest degree (exponent).

Terms are expressions or numbers that are added or subtracted.

A polynomial is **linear** if it has a degree of 1, **quadratic** if it has a degree of 2, and **cubic** if it has a degree of 3.

A polynomial is a **monomial** if it has 1 term, a **binomial** if it has 2 terms, and a **trinomial** if it has 3 terms.

An expression in **quadratic form** can be written as au^2+bu+c, where a, b, and c are numbers with $a \neq 0$, and u is some expression in x.

Example: $x^4 + 6x^2 + 9$ can be rewritten where $u = x^2$

$u^2 + 6u + 9$ so that it can be factored…

$(u + 3)(u + 3) = (u + 3)^2 = (x^2 + 3)^2$

Since the quadratic x^2+3 has no real roots, by the Factor Theorem, it does not have any linear factors with real coefficients.

Factor Theorem: The binomial $x–r$ is a linear factor of the polynomial $p(x)$ if and only if r is a zero (or root) of $p(x)$.

Example: $x^4 + 18x^2 + 81$ … Solve … Let $u = x^2$

$u^2 + 18u + 81 = (u + 9)^2 = (x^2 + 9)^2 = (x + 3)^2 (x – 3)^2$

Examples:

$f(x) = x^5 – 3x^3 – 10x$ $f(x) = x^4 + 11x^2 + 18$ $f(x) = x^3 – 49x$

SSE.2

Using graphing and synthetic division to find zeros (desmos.com)

Examples:

$x^3 – 6x^2 + 11x – 6$ $x^3 – 5x^2 + 8x – 4$ $x^3 – 3x^2 + 3x – 1$

Polynomial Identities
A.APR.4

$(a + b)^2 = a^2 + 2ab + b^2$ (Binomial expansion)

$(a + b)(c + d) = ac + ad + bc + bd$ (FOIL)

$a^2 – b^2 = (a + b)(a – b)$ (Difference of squares)

$ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{}$ (Quadratic Formula)

$a^3 + b^3 = (a + b)(a^2 – ab + b^2)$ (Sum of Cubes)

$a^3 – b^3 = (a – b)(a^2 + ab + b^2)$ (Difference of Cubes)

Examples:

$8x^3 + 125$ $125x^3 – 216y^3$ $81x^3 + 192y^3$