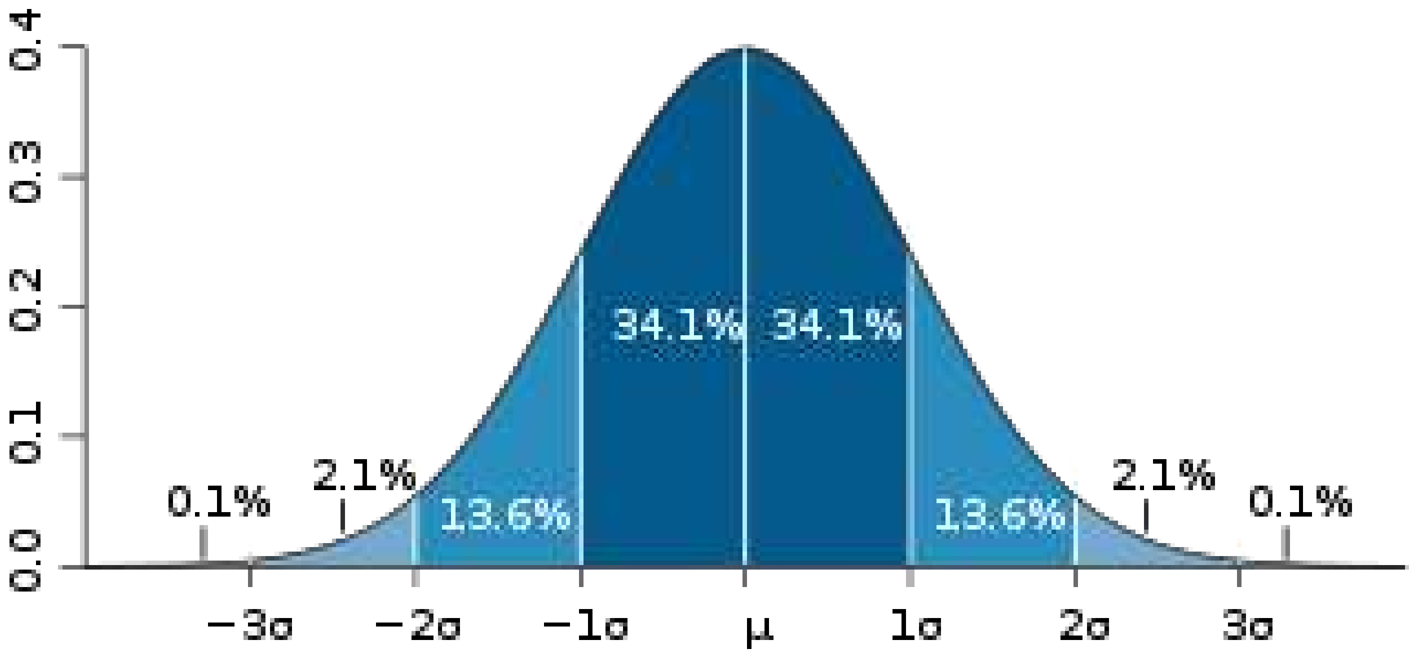
Normal Distribution

Normal Distribution Curve

A normal distribution curve is symmetrical, bell-shaped curve defined by the mean and standard deviation of a data set.

The normal curve is a probability distribution with a total area under the curve of 1.



One standard deviation away from the mean (μ) in either direction on the horizontal axis accounts for around **68 percent** of the data. Two standard deviations away from the mean accounts for roughly **95 percent** of the data with three standard deviations representing about **99.7 percent** of the data.

Standard Normal Distribution

The mean of the data in a standard normal distribution is 0 and the standard deviation is 1.

A standard normal distribution is the set of all z-scores.

z-scores

When a set of data values are normally distributed, we can standardize each score by converting it into a **z-score**.

z-scores make it easier to compare data values measured on different scales.

z-scores

A ***z*-score** reflects how many standard deviations above or below the mean a raw score is.

The ***z*-score** is positive if the data value lies above the mean and negative if the data value lies below the mean.

z-score formula

$$z = \frac{x - \mu}{\sigma}$$

Where x represents an element of the data set, the mean is represented by μ and standard deviation by σ.

Analyzing the data

Suppose SAT scores among college students are normally distributed with a mean of 500 and a standard deviation of 100. If a student scores a 700, what would be her z-score?

Answer Now

Analyzing the data

Suppose SAT scores among college students are normally distributed with a mean of 500 and a standard deviation of 100. If a student scores a 700, what would be her z-score?

$$z = \frac{700 - 500}{100} = 2$$

Her z-score would be 2 which means her score is two standard deviations above the mean.

Analyzing the data

- A set of math test scores has a mean of 70 and a standard deviation of 8.

- A set of English test scores has a mean of 74 and a standard deviation of 16.

For which test would a score of 78 have a higher standing?

Answer Now

Analyzing the data

A set of math test scores has a mean of 70 and a standard deviation of 8.
A set of English test scores has a mean of 74 and a standard deviation of 16.
For which test would a score of 78 have a higher standing?

To solve: Find the z-score for each test.

$$\text{math } z\text{-score} = \frac{78\text{-}70}{8} = 1$$

$$\text{English } z\text{-score} = \frac{76\text{-}74}{16} = .25$$

The math score would have the highest standing since it is 1 standard deviation above the mean while the English score is only .25 standard deviation above the mean.

Analyzing the data

What will be the miles per gallon for a Toyota Camry when the average mpg is 23, it has a z value of 1.5 and a standard deviation of 5?

Answer Now

Analyzing the data

What will be the miles per gallon for a Toyota Camry when the average mpg is 23, it has a z value of 1.5 and a standard deviation of 2?

Using the formula for z-scores: $z = \dfrac{x - \mu}{\sigma}$

$$1.5 = \frac{x - 23}{2} \qquad 3 = x - 23 \qquad x = 26$$

The Toyota Camry would be expected to use 26 mpg of gasoline.

Normal Distribution Probability

With a graphing calculator, we can calculate the probability of normal distribution data falling between two specific values using the mean and standard deviation of the data

Normal Distribution Probability

Example:

A Calculus exam is given to 500 students. The scores have a normal distribution with a mean of 78 and a standard deviation of 5. What percent of the students have scores between 82 and 90?

TI 83/84

Normal Distribution Probability

Example: A Calculus exam is given to 500 students. The scores have a normal distribution with a mean of 78 and a standard deviation of 5. What percent of the students have scores between 82 and 90?

TI 83/84 directions:

a. Press [2nd][VARS](DISTR) [2] (normalcdf)

b. Press [82] [,] [90] [,] [78] [,] [5] [)][Enter]

normalcdf(82,90,
78,5)
.2036578048

There is a 20.37% probability that a student scored between 82 and 90 on the Calculus exam.

Normal Distribution Probability

Extension: A Calculus exam is given to 500 students. The scores have a normal distribution with a mean of 78 and a standard deviation of 5.

How many students have scores between 82 and 90?

Using the probability previously found:

$$500 * .2037 = 101.85$$

There are about 102 students who scored between 82 and 90 on the Calculus exam.

Normal Distribution Probability

Practice:

A Calculus exam is given to 500 students. The scores have a normal distribution with a mean of 78 and a standard deviation of 5. What percent of the students have scores above 60?

Hint: Use 1E99 for upper limit;
E is [EXP] on Casio and
[2^{nd}][,] on T I

Answer Now

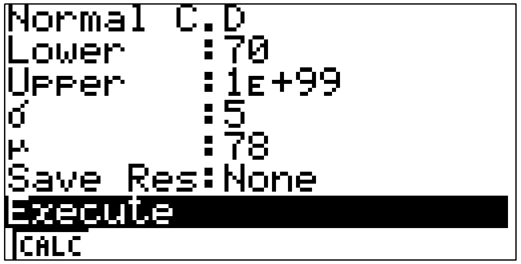
Normal Distribution Probability

Practice: A Calculus exam is given to 500 students. The scores have a normal distribution with a mean of 78 and a standard deviation of 5. How many students have scores above 70?

TI 84

Normalcdf(70,1E9
9,78,5)
.9452007106

Casio 9850



Normal C.D
prob=0.9452

500*.9452= 472.6

About 473 students have a score above 70 on the Calculus exam.

Normal Distribution Probability

Practice:

Find the probability of scoring below a 1400 on the SAT if the scores are normal distributed with a mean of 1500 and a standard deviation of 200.

Hint: Use -1E99 for lower limit;
E is [EXP] on Casio and
[2^{nd}][,] on T I

Answer Now

Normal Distribution Probability

Practice: Find the probability of scoring below a 1400 on the SAT if the scores are normal distributed with a mean of 1500 and a standard deviation of 200.

TI 84

Normalcdf(-1E99, 1400,1500,200)
.3085375322

Casio 9850

```
Normal C.D
Lower    :-1E+99
Upper    :1400
σ        :200
μ        :1500
Save Res:None
Execute
None LIST
```

```
Normal C.D
p      =0.30853753
```

There is a **30.85%** probability that a student will score below a 1400 on the SAT.