CollegeBoard

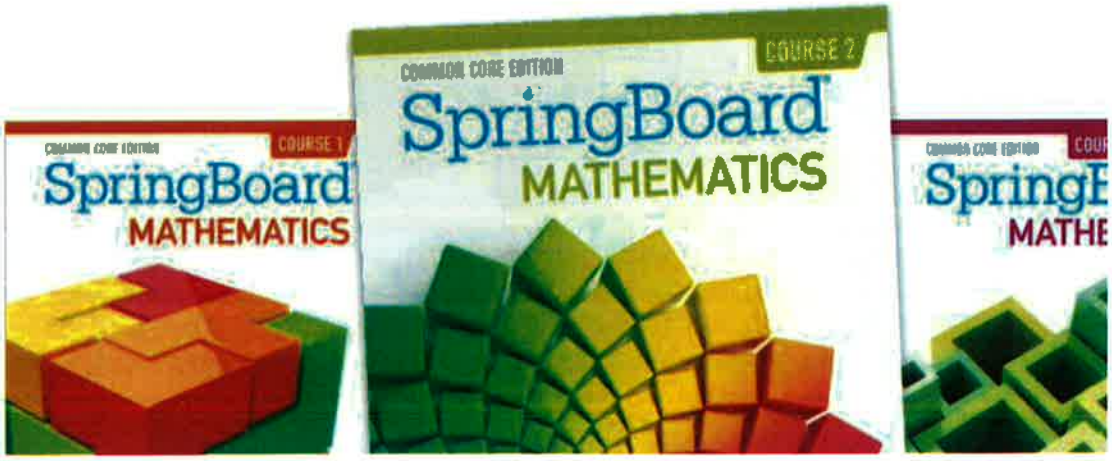
SpringB

Mathematics 2(

COMMON CORE ED

Middle School Sampler

Course 2 – Unit 3



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Common Core Edition

Course 2 - Unit Sampler

The Pathway to Advanced Placement and College Readiness

SpringBoard provides a comprehensive and systematic approach to preparing ALL stud demands of rigorous AP courses, college classes, and other postsecondary experiences prepares students through sequential, scaffolded development of the prerequisite skill knowledge needed for success in AP Calculus and Statistics.

In each unit of study, explicit AP Connections are outlined in the Planning the Unit pag teacher editions and are reinforced as they appear in student activities. Through ongoi rigorous mathematics content and experience with the thinking processes needed to a and explain complex math problems, students exit SpringBoard equipped with the kinc order thinking skills, knowledge, and behaviors necessary to be successful in AP classes

For More Information on the SpringBoard Program visit www.Collegeboard.org/Spring

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Common Core Edition
Course 2 Unit Sampler

Additional materials in the back of this Sampler include...

- Vertical Unit Connections

To the Teacher

Welcome to *SpringBoard Mathematics*, a highly engaging, student-centered instructional program. This revised edition of SpringBoard is based on the standards defined by the **Common Core State Standards for Mathematics** for each course. The program may be used as a core curriculum that will provide the instructional content that students need to be prepared for future mathematical courses. SpringBoard courses do the following:

- Expect students to practice applying mathematical ways of thinking to real-world issues and challenges.
- Require students to develop a depth of understanding and ability to apply mathematics to novel situations—as college students and employees regularly are called **to do**.
- Emphasize mathematical modeling and reasoning—using mathematics and statistics to analyze empirical situations, understand and make inferences about the situations, and improve decision making about how to solve problems and justify those solutions.

Shifts in Mathematics Instruction

With an increased emphasis on better preparing students to understand and master mathematical concepts, mathematics instruction has become a major focus of attention. Efforts at improvement center around the following points:

Greater Focus on the Content of the Standards:

- Learn more about less by spending more time on fewer concepts.
- Significantly narrow and deepen the scope of how time and energy are spent in the classroom.
- Focus on the essential learning that helps students **develop strong foundational knowledge** and deep conceptual understanding to enable them to transfer skills and knowledge across concepts and grades.

Coherence to Link Major Topics:

- Connect learning within a grade and build knowledge across grades.
- Spiral learning so that students can build new understanding on the foundations built in previous years.
- Focus on learning progressions so that teachers can continue counting on students' deep conceptual understanding of core content and build on it.

Rigor with Balance:

- Develop fluency in procedural skills—computation, application, understanding.
- **Promote depth and mastery by connecting concepts, practice, and independent application.**

career, or both. Students who are prepared for college or career will be able to do the following:

- **Build on content knowledge:** Students will have a base knowledge of number and quantity, algebra, functions, geometry, and statistics and probability on which to extend their learning.
- **Use mathematical models:** Students will be able to use a variety of mathematical representations to model what they know and to justify how they are using their knowledge.
- **Communicate mathematics:** Students will communicate verbally and in writing to explain their discoveries and understanding of mathematics and how it works theoretically and in the real world.
- **Collaborate with others:** Students will participate in discourse focused on discovery and problem solving, evaluate the contributions of others, and collaborate to present and defend viable solutions.
- **Use technology:** Students will use appropriate technology to enhance their understanding of mathematics and to gain greater precision in areas where technology is appropriate.

The implications of these student expectations are that students will need to develop greater depth of knowledge, higher-level thinking skills, and effective communication skills. What they need less of will be memorization, drills and worksheets, and "one size fits all" content.

SpringBoard's Role in Preparing Students for College and Career Success

With this revised edition, the SpringBoard program provides a roadmap for attaining the knowledge and skills students require for success in Advanced Placement courses, in college-level work, and in careers. Based on the Common Core State Standards for Mathematics and current research on best instructional practices, SpringBoard uses a "back-mapping" instructional design that starts with the end in mind, namely, the skills and knowledge students need to use mathematics effectively and to demonstrate that ability through performance on various assessments.

The mathematics instruction follows a balanced approach in which concepts are presented based on the most effective instructional methods: ***directed*** for basic mathematics principles, including examples and practice; ***guided*** for concepts that need a combination of direct instruction and investigatory learning; and ***investigative*** activities that allow students to explore and discover mathematics concepts through a contextual setting.

To the Teacher *continued*

- Multiple lessons per activity.
- Worked-out examples as needed to help students learn and apply concepts.
- Frequent Check Your Understanding questions to help students assimilate and apply knowledge.
- Mathematical practices called out so students are reminded to apply them as they respond to problems and applications.
- Lesson Practice problems to provide the opportunity to practice new learning and to build fluency.
- Activity Practice provides additional practice problems for each lesson in the activity.
- Embedded Assessments give students new contexts for applying the concepts learned in the unit and give you the opportunity for regular formative assessment.

Integration of Mathematical Practices

Through its instructional design, the SpringBoard mathematics program requires students to integrate effective mathematical practices into their learning. With its process of questioning students within a lesson and asking them to think through concepts and applications, SpringBoard reinforces the actions and practices that help students build knowledge and skills. SpringBoard requires students to:

- Make sense of and connect mathematics concepts to everyday life through problem contexts.
- Model with mathematics to solve problems, justify solutions and their reasonableness and communicate mathematical ideas.
- Use appropriate tools, such as number lines, protractors, technology, or paper and pencil, strategically to help solve problems.
- Communicate abstract and quantitative reasoning both orally and in writing through viable arguments and critiques.
- Analyze mathematical relationships through structure and repeated reasoning to connect ideas.
- Attend to precision in both written and oral communication of mathematical ideas.

Engaging and Interactive Online Edition

With this new edition, SpringBoard introduces an all-new interactive online experience for both students and teachers. In addition to providing all content online, the new SpringBoard Digital program:

- Allows access at any time.
- Discerns the device you're using and adjusts content to fit screens—from desktops to laptops to tablets

New Assessment Options

The SpringBoard program now provides the option of using the ExamView test generator program for all grades. Teachers will have multiple options for choosing premade tests or making their own. Options include:

- Unit tests aligned to standards and the content in each unit.
- Test banks allowing teachers to choose items and create tests for multiple needs, including benchmark tests and quarter or semester tests.
- Expanded test item types, including short response and interactive simulations and manipulatives.

What Sets SpringBoard Apart from Other Mathematics Programs?

Three key things set SpringBoard apart:

1. The expectation that students can do rigorous work with the **right** preparation.
2. Learning materials that reflect both **rigor** and the **expectations** about what students should know and be able to do.
3. Extensive teacher support through **professional development** and coaching services.

Unique features of SpringBoard include:

- **Rigorous, standards-based instruction:** Instructional content organized around the **Common Core State Standards for Mathematics** to provide coherent topics that build knowledge and skills throughout each course and across grade levels.
- **Mathematical practices:** Integration of the Standards for Mathematical Practice that support student learning and higher level thinking.
- **Research-based instruction:** Back-mapped instructional design gives students a learning target and scaffolds activities to develop students' knowledge and skills and prepare them to demonstrate their learning on an Embedded Assessment.
- **Student-centered, interactive, collaborative activities and lessons:** Each course is organized into short, interactive activities that are further divided into focused lessons. Lessons engage students and aid learning by having students participate in class discussions, solve problems and justify solutions, and demonstrate learning through multiple means of evaluation.
- **Integrated teaching and learning strategies:** Suggested Learning Strategies in each lesson help students use methodical approaches to learning new content, helping students take control of their own learning by identifying which strategies work best for them. Teachers also use

To the Teacher *continued*

- **Professional development:** Unparalleled professional development builds teacher capacity to deliver challenging curriculum to meet the needs of all students while honoring the creativity and intelligence teachers bring to the classroom. Face-to-face training is supported by an online system featuring resources that include an interactive professional learning *Community* that allows peer-to-peer sharing and sustains successful teaching.

The Pathway to Advanced Placement and College Readiness

SpringBoard provides a comprehensive and systematic approach to preparing ALL students for the demands of rigorous AP courses, college classes, and other postsecondary experiences. SpringBoard prepares students through sequential, scaffolded development of the prerequisite skills and knowledge needed for success in AP Calculus and Statistics. In each unit of study, explicit AP Connections are outlined in the Planning the Unit pages of the teacher editions and are reinforced as they appear in student activities. Through ongoing exposure to rigorous mathematics content and experience with the thinking processes needed to analyze, solve, and explain complex math problems, students exit SpringBoard equipped with the kind of higher-order thinking skills, knowledge, and behaviors necessary to be successful in AP classes and beyond.

From Pre-AP to AP and Beyond

Beginning in middle school, students are introduced to concepts and skills that are fundamental to success in AP mathematics and statistics courses.

Grade 6 students learn to:

- Model functions in numerical, symbolic (equation), table, and graphical forms.
- Communicate mathematics in writing and verbally, justifying answers and clearly labeling charts and graphs.
- Explore and represent data in a variety of forms.
- Use multiple representations to communicate their mathematical understanding.

Grade 7 students continue to:

- Acquire an algebraic and graphical understanding of functions.
- Write, solve, and graph linear equations; recognize and verbalize patterns; and model slope as a rate of change.
- Communicate clearly to explain methods of problem solving and to interpret results.
- Investigate concepts presented visually and verbally.

Algebra 1 students:

- Gain an understanding of the properties of real numbers.
- Formalize the language of functions.
- Explore the behavior of functions numerically, graphically, analytically, and verbally.
- Use technology to discover relationships, test conjectures, and solve problems.
- Write expressions, equations, and inequalities from physical models.
- Communicate mathematics understanding formally and informally.

Geometry students:

- Read, analyze, and solve right triangle and trigonometric functions within contextual situations.
- Develop area formulas necessary for determining volumes of rotational solids, solids with known cross sections, and area beneath a curve.
- Explain work clearly so that the reasoning process can be followed throughout the solution.

Algebra 2 students:

- Develop the algebra of functions through operations, composition, and inverses.
- Read and analyze contextual situations involving exponential and logarithmic functions.
- Work with functions graphically, numerically, analytically, and verbally.
- Learn optimization problems.
- Compare the relative rate of change of linear and exponential functions.
- Learn the concept of infinite sum as a limit of partial sums.
- Work with statistics in numerical summaries, calculations using the normal curve, and the modeling of data.

Precalculus students:

- Gain an introductory understanding of convergence and divergence.
- Collect, analyze, and draw conclusions from data.
- Solve problems in contextual situations dealing with polynomial, rational, logarithmic, and trigonometric functions.
- Model motion using parametric equations and vectors.
- Develop an intuitive understanding of limits and continuity.
- Justify their reasoning and understanding verbally, in writing, and with models.
- Use technology to explore and support conjectures.

The SpringBoard Mathematics Classroom

To the Teacher *continued*

safe to explore ideas and learn effective communication skills. Collaborative groups allow learning to be active as students engage in discussions, make conjectures, question, and discover new ideas as they fulfill tasks within the group. Group structure should encourage all members to work together to complete a given task while ensuring that each student is held accountable for equal participation in the assignment.

Debriefing/Reflections

Frequently in a mathematics classroom, students and teachers should engage in **debriefings**. The purpose of debriefing is to allow students to reflect on their learning, correct misconceptions, identify thinking processes used during an activity, summarize information, and process what they have learned. Debriefing can be accomplished in a variety of ways, including whole-class discussion, small-group discussion, group presentation, and individual writing.

Interactive Word Wall

The class **Word Wall** facilitates vocabulary development and provides a reference during class and group discussions. Creating and maintaining a Word Wall is an ongoing activity. It should be an instructional tool, not just a display. Designate a specific space (such as a bulletin board or blank wall space) in the classroom for the Word Wall. Words may be written on index cards, sentence strips, or blank paper by you or by your students.

- You or your students may add new words to the Word Wall.
- Spend time revisiting words on the Word Wall whenever possible.
- Emphasize the categorization of words to help students see the logic in language.
- Invite students to generate a list of potential categories to sort words (alphabetical order, mathematical concepts, problem solving techniques, etc.).
- Make the words into manipulatives by writing them on cards so that they can be shifted, added, and/or eliminated.
- Encourage students to use words from the Wall correctly in their class and group discussions, and specifically on the Embedded Assessments.

Math Notebook

Keeping a **Math Notebook** helps students learn and explore new vocabulary while also summarizing notes about math concepts and ideas. It is an intentional tool for students to expand their understanding of mathematics terms and concepts. The Math Notebook may be any type of notebook, or it may be an online tool for students who have regular access to SpringBoard Mathematics Online. As students are introduced to new vocabulary, they can explore terms and concepts and make notes about other math terms that are

Contents

Contents *continued*

Contents *continued*

Unit 3 Planning the Unit

In this unit students develop an understanding of and apply proportional relationships as they study ratios, unit rates, equations and the constant of proportionality. Students study percent and a wide variety of applications such as tax, commission, mark-up, discount, and percent increase/decrease and error. They study and apply scale drawings and solve related problems.

Vocabulary Development

The key terms for this unit can be found on the Unit Opener page. These terms are divided into Academic Vocabulary and Math Terms. Academic Vocabulary includes terms that have additional meaning outside of math. These terms are listed separately to help students transition from their current understanding of a term to its meaning as a mathematics term. To help students learn new vocabulary:

- Have students discuss meaning and use graphic organizers to record their understanding of new words.
- Remind students to place their graphic organizers in their math notebooks and revisit their notes as their understanding of vocabulary grows.
- As needed, pronounce new words and place pronunciation guides and definitions on the class Word Wall.

Embedded Assessments

Embedded Assessments allow students to do the following:

- Demonstrate their understanding of new concepts.
- Integrate previous and new knowledge by solving real-world problems presented in new settings.

They also provide formative information to help you adjust instruction to meet your students' learning needs.

Prior to beginning instruction, have students unpack the first Embedded Assessment in the unit to identify the skills and knowledge necessary for successful completion

CollegeBo

Algebra / AP / Colleg

This unit develops stude proportion by:

- Using real-world cc learning and to de concepts.
- Encouraging stude reasonableness of relative accuracy.
- Providing opportu problem solving st groups to analyze and communicate
- Asking students to using mathematica

Unpacking the Embedd

The following are the key sk will need to know for each

Embedded Assessmer

Ratios, Proportions, a Reasoning, Weighing

- Solve problems involvi
- Convert between meas rates and using propor
- Represent constant rat of the form $y = kx$
- Determine the constan table, graph, or equatic

Embedded Assessmer

Planning the Unit *continued*

Embedded Assessment 3

Percents and Proportions, *Socializing and Selling*

- Find the percent of a number
- Find the percent that one number is of another
- Given the percent and the whole, find the part
- Solve problems about sales tax, tips, and commissions
- Solve problems about percent increase, percent decrease, markups, and discounts
- Solve problems about interest and percent error

Suggested Pacing

The following table provides suggestions for pacing using a 45-minute class period. Space is left for you to write your own pacing guidelines based on your experiences in using the materials

	45-Minute Period	Your Comments on
Unit Overview/Getting Ready	1	
Activity 8	4	
Activity 9	4	
Embedded Assessment 1	1	
Activity 10	4	
Embedded Assessment 2	1	
Activity 11	2	
Activity 12	4	
Embedded Assessment 3	1	
Total 45-Minute Periods	22	

Additional Resources

Additional resources that you may find helpful for your instruction include the

Ratio and Proportion

3

Unit Overview

In this unit, you will use pictures, graphs, tables, and verbal descriptions to study unit rates, rate of change, and proportions. You will solve problems involving scale, percentage, and proportional relationships.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- tip

Math Terms

- ratio
- rate
- unit rate
- proportion
- cross products
- conversion factor
- constant of proportionality
- constant ratio
- constant rate of change
- relative size
- scale drawing
- percent
- percent equation
- discount
- markup
- interest
- percent error

ESSENTIAL QUESTIONS

- How are ratios, unit rates, and proportions used to describe and solve real-world problems?
- How can representations, numbers, words, tables, and graphs be used to solve problems?

EMBEDDED ASSESSMENTS

These assessments, following activities 9, 10, and 12, will give you an opportunity to demonstrate how you can use ratios and rates to solve mathematical and real-world problems involving proportional relationships.

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Developing Math Language

As this unit progresses, use support among peers to help student develop background knowledge need to

As needed, pronounce new terms clearly and monitor students' use of words in their discussions to ensure

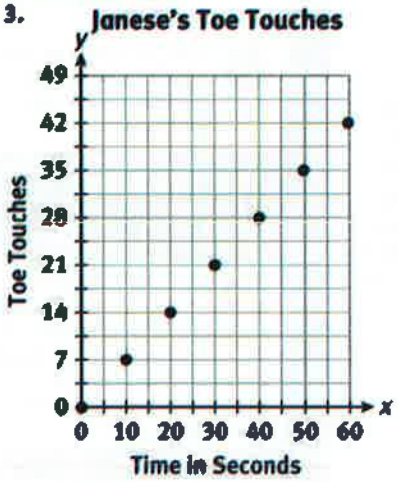
UNIT 3
Getting Ready

Use some or all of these exercises for formative evaluation of students' readiness for Unit 3 topics.

Prerequisite Skills

- Ratios, Tables and Graphs (Item 1, 2, 3) 6.RP.A.3, 6.RP.A.3a
- Expressions and Equations (Items 4, 5) 8.EE.C.7
- Fractions, Decimals and Percents (Items 6, 7, 8) 7.NS.A.2, 5.NF.B.4

Answer Key

1. $\frac{7}{10}$; 7:10; 7 to 10
2. See below table.
3. 

4. a. $\frac{\$106.25}{x}$ b. $\$3.67g$ c. $3n - 5$
5. a. $x = 1.5$ b. $x = 4$
6.

%	decimal	fraction
25%	0.25	$\frac{1}{4}$
50%	0.5	$\frac{1}{2}$
15%	0.15	$\frac{3}{20}$

7. a. 25% b. 40%
8. Answers will vary. To find $\frac{1}{3}$ of 60, multiply $\frac{1}{3} \times \frac{60}{1}$, which equals 20. To find 25% of 60, change 25% to a decimal .25 and multiply .25 × 60, which is 15. So, $\frac{1}{3}$ of 60 is larger, because 20 > 15.

UNIT 3

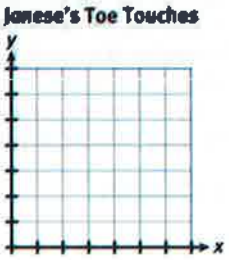
Getting Ready

Write your answers on notebook paper. Show your work.

1. Janese can complete 7 toe touches in 10 seconds. Write a ratio of Janese's toe touches to seconds in three ways.
2. Complete the following table representing Janese's toe touches.

Janese's Toe-Touching Record							
Time (in seconds)	0	10	20	30	40	50	60
Toe Touches							

3. Use the grid below to graph Janese's toe touches. Label the horizontal and vertical axes. Provide a scale on the horizontal and vertical axes.



4. Write an algebraic expression for each the following.
   a. The cost of each ticket, if x tickets cost $106.25
   b. The cost of g gallons of gas if each gallon costs $3.67
   c. Five less than 3 times a number

5. Solve each of
   a. $2x + 5 = 8$
   b. $16 + 3x =$
6. Copy and con
   equivalent val

%	Decir
25%	
	0.1

7. What percent
   a.
   b.
8. Explain how t
   following valu
   expression ha

Getting Ready Practice

For students who may need additional instruction on one or more of the prerequisite skills for this unit.

Proportional Reasoning

ACTIVITY 9

Scrutinizing Coins

Lesson 9-1 Equations Representing Proportional Relationships

Learning Targets:

- Given representations of proportional relationships, represent constant rates of change with equations of the form $y = kx$.
- Determine the meaning of points on a graph of a proportional relationship.
- Solve problems involving proportional relationships.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Marking the Text, Summarizing, Use Manipulatives, Look for a Pattern, Predict and Confirm, Discussion Groups

Ratios and proportions are used to solve all kinds of problems in the real world. For example, ratios and proportions are used in cooking to double recipes, by travelers to find distances on maps, and by architects to make scale models.

Work with your group to explore the proportional relationship between the number of pennies in a stack and their heights in millimeters. You will need a centimeter ruler and 25 pennies. As you work with your group, you may hear math terms or other words that are unfamiliar. Record words that are frequently used in your math notebook. Ask for clarification of their meaning and make notes to help you remember how they are used.

1. Without using your pennies or ruler, predict the height of a stack of 150 pennies, and explain why you made this prediction. Be sure to include units in your prediction.
   I thought 1 penny might be about 2 mm, so I multiplied 150 by 2 to get 300 mm.

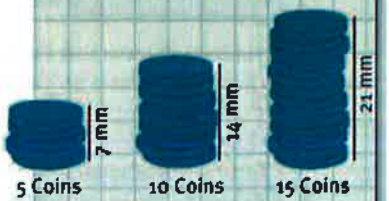
2. **Attend to precision.** Explore this finding by measuring and recording the height of a stack of each number of pennies in the table below. Sample answers are given.

Number of Pennies	10	15	20	25
Height of Stack (mm)	12	18	24	30

3. Write a ratio in fraction form that relates the number of pennies to the height of a stack. Answers will depend on results given in table.
   a. 10 pennies $\frac{10}{12}$ b. 15 pennies $\frac{15}{18}$
   c. 20 pennies $\frac{20}{24}$ d. 25 pennies $\frac{25}{30}$

4. Write a ratio that relates the number of pennies in each stack at the right to the height of the stack.
   The ratio is 5:7. For every 5 pennies added, the stack will increase by 7 mm.

My Notes



Common Core State Standards for Activity 9

7.RP.A.2 Recognize and represent proportional relationships between quantities.

ACTIVITY 9 Continued

7–8 Create Representations, Look for a Pattern Students will use proportional reasoning to complete a table using the ratio 5 : 7, which represents the relationship between the number of pennies in a stack and the height of the stack in millimeters. In Item 8, they write two equivalent ratios as an equation. Point out that since the ratios are equivalent, the equation is a proportion.

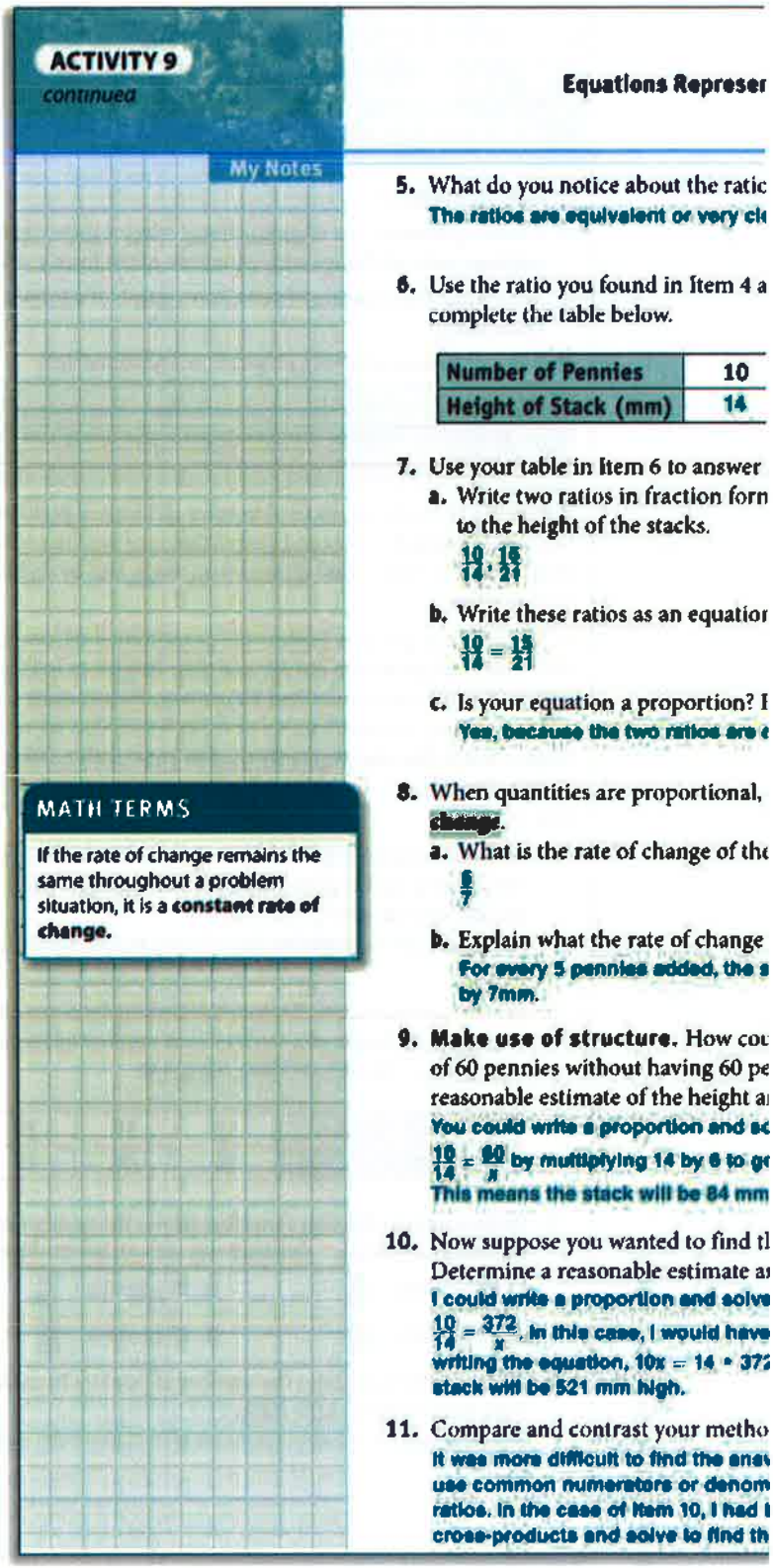
9–11 Debriefing In this set of Items, students should understand that they do not need to measure the height of a stack of 60 pennies in order to predict height. Instead they can use the proportion they wrote in Item 8 with the height of the pennies as a variable, and then solve the proportion for the value of the variable. They will need to recall how to solve proportions from Lesson 8–3.

TEACHER TO TEACHER

In Item 9, it is important that students first make a reasonable estimate of the height of 60 pennies before they solve the proportion. This will help them find errors when writing the proportion. It is important to debrief this part of the lesson to remind students that if the height of the stack is the denominator of the left-hand side of the proportion, then the unknown height of the stack is also in the denominator of the right-hand side of the proportion. In Item 11, you want students to contrast the two ways of solving a proportion.

CONNECT TO AP

Students observe that the height of a stack of pennies is proportional to the number of pennies in the stack. They will spend a great deal of time in future mathematics courses, such as calculus, writing proportions from written descriptions of proportional relationships. They need a solid understanding of this kind of proportional reasoning as a basis of work they will do with volume in



ACTIVITY 9
continued

Equations Represer

My Notes

5. What do you notice about the ratic
The ratios are equivalent or very cl

6. Use the ratio you found in Item 4 a complete the table below.

Number of Pennies	10
Height of Stack (mm)	14

7. Use your table in Item 6 to answer
   a. Write two ratios in fraction form to the height of the stacks.
   $\frac{10}{14}, \frac{15}{21}$
   b. Write these ratios as an equatior
   $\frac{10}{14} = \frac{15}{21}$
   c. Is your equation a proportion? I
   Yes, because the two ratios are

MATH TERMS

If the rate of change remains the same throughout a problem situation, it is a **constant rate of change.**

8. When quantities are proportional, change.
   a. What is the rate of change of the
   $\frac{5}{7}$
   b. Explain what the rate of change
   For every 5 pennies added, the s by 7mm.

9. **Make use of structure.** How cou of 60 pennies without having 60 pe reasonable estimate of the height a
You could write a proportion and so
$\frac{10}{14} = \frac{60}{n}$ by multiplying 14 by 6 to g
This means the stack will be 84 mm

10. Now suppose you wanted to find tl Determine a reasonable estimate a
I could write a proportion and solve
$\frac{10}{14} = \frac{372}{x}$, in this case, I would have writing the equation, $10x = 14 \cdot 37$ stack will be 521 mm high.

11. Compare and contrast your metho
It was more difficult to find the ansv use common numerators or denom ratios. In the case of Item 10, I had t cross-products and solve to find th

Lesson 9-1
Equations Representing Proportional Relationships

ACTIVITY 9
continued

My Notes

12. Why might the value you determined for height in Items 9 and 10 be different from the actual measured height of a stack of 60 pennies or 372 pennies?

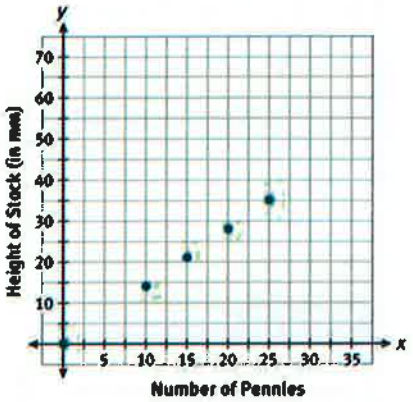
The ratio 5:7 gives an estimate only. Actual pennies may be worn or not stacked evenly or may have a number of other conditions that could influence the actual height.

13. Write and solve a proportion to determine the number of pennies, x, in a stack that is 100 mm high. Use numbers, words, or both to explain your method.

$\frac{5}{7} = \frac{x}{100\text{ mm}}$; Write the proportion as an equation using cross-products, $7x = 5 \cdot 100$. Next, solve the equation for x by dividing 500 by 7. The number of pennies in a 100 mm stack is about 71.

The proportional relationship between the number of pennies in a stack and the height of the stack that you recorded in the table in Item 7 can also be represented in a graph. The graph will help you predict the height of a stack of pennies.

14. Graph the data from Item 7 onto the graph.



15. What does a point (x, y) on the graph mean for this situation?

A stack of x pennies has a height of y mm.

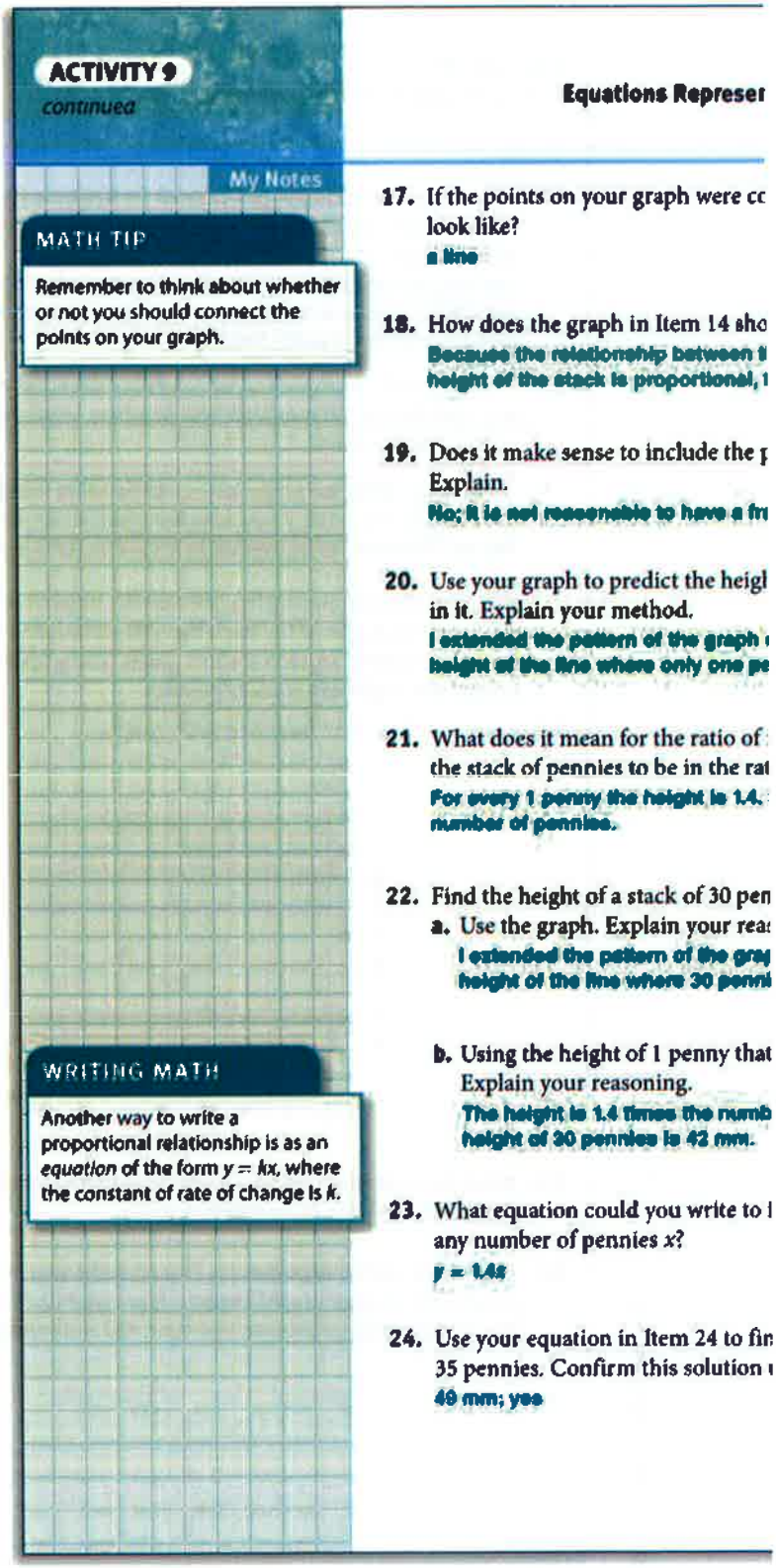
16. Construct viable arguments. Does it make sense to include the point (0, 0) on your graph? Explain. If yes, plot (0, 0) on your graph.

Yes; it represents no pennies in a stack that is 0 mm tall.

ACTIVITY 9 Continued

17–19 Visualization, Critique Reasoning After graphing the points, help students see that the points could be connected by a line through the origin. The graph represents a proportional relationship and can therefore be used to predict the height of a given stack of pennies. Discuss the meaning of each point (x, y) to help students understand the relationship between the number of pennies and the height of the stack of pennies. Have students explain why a point without whole-number coordinates does not make sense in this situation.

20–24 Look for a Pattern, Create Representations, Debriefing In this set of items, students use the graph they created in Item 14 to explore other possible points of the graph. If the other points lie along the same line, then they can be used to predict the height of a given number of pennies. Make sure you debrief these items to help students understand that this proportional relationship can also be written as an equation of the form $y = kx$, where k is the height of one penny. In later lessons, students will describe this type of equation as a *direct variation* equation.



ACTIVITY 9
continued

Equations Represer

My Notes

MATH TIP

Remember to think about whether or not you should connect the points on your graph.

17. If the points on your graph were cc look like?
a line

18. How does the graph in Item 14 sho
Because the relationship between t height of the stack is proportional, t

19. Does it make sense to include the p Explain.
No; it is not reasonable to have a fr

20. Use your graph to predict the heigl in it. Explain your method.
I extended the pattern of the graph height of the line where only one pe

21. What does it mean for the ratio of the stack of pennies to be in the rat
For every 1 penny the height is 1.4. number of pennies.

22. Find the height of a stack of 30 pen
a. Use the graph. Explain your rea
I extended the pattern of the gra height of the line where 30 penni

WRITING MATH

Another way to write a proportional relationship is as an *equation* of the form $y = kx$, where the constant of rate of change is k.

b. Using the height of 1 penny that Explain your reasoning.
The height is 1.4 times the numb height of 30 pennies is 42 mm.

23. What equation could you write to f any number of pennies x?
$y = 1.4x$

24. Use your equation in Item 24 to fin 35 pennies. Confirm this solution
49 mm; yes

MINI-LESSON: Equivalent Proportions

Students should be familiar with these proportions equivalent to

Check Your Understanding

25. **Model with mathematics.** Look back at your original prediction for the height of a stack of 150 pennies.
   a. Use a proportion to revise your original prediction. Explain your reasoning.
   b. Use the equation you wrote in Item 24 to revise your original prediction. Justify your reasoning.
   c. Explain how you could use your graph to revise your original prediction.

LESSON 9-1 PRACTICE

26. Solve the proportion $\frac{4}{5} = \frac{28}{x}$ using two different methods. Explain each method.

27. **Construct viable arguments.** Solve $\frac{x}{42} = \frac{3}{7}$ using two different strategies. Explain each strategy.

28. Is the ratio 4.2:1.5 proportional to the ratio12.6:4.5? Explain.

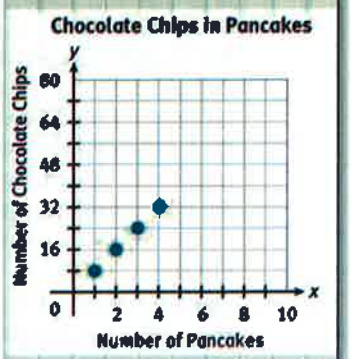
29. Is the ratio 35 to 10 proportional to the ratio 7 to 5? Explain.

30. At Lake Middle School, the average ratio of boys to girls in a classroom is 3:2. Use a proportion to predict the number of girls in a classroom that has 15 boys.

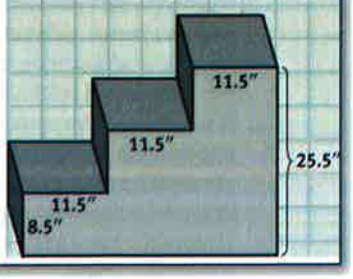
31. Complete the ratio table below to show ratios equivalent to 4:18.

48	160		8	
		20		90

32. Use the graph at the right.
   a. Predict the number of chocolate chips in nine pancakes. Explain.
   b. Predict the number of pancakes that would have 48 chocolate chips. Explain.
   c. What does the point (1, 8) mean in this situation?
   d. Which of the equations below represents this situation?
      A. $y = 16x$ B. $y = 8x$
      C. $y = x$ D. $y = 48x$



33. Three steps of a staircase are shown here.
   a. What is the ratio of the width of a step to its height? $\frac{11.5}{8.5}$, or $\frac{23}{17}$
   b. Explain why the staircase represents a constant rate of change.
      The ratio of the width to the height of each step is constant.
   c. What does the rate of change mean in the context of a staircase?
      For every width of 11.5 inches, there is a height of 8.5 inches.



LESSON 9-1 PRACTICE

26. $\frac{4}{5} = \frac{28}{x}$; One way to solve this is to multiply the denominator by 7. This gives an answer of 35. Another way to solve this proportion is to set the

31.

48	160	4.4	8	20
216	720	20	36	90

ACTIVITY 9 Continued

Lesson 9-2

PLAN

Pacing: 1 class period

Chunking the Lesson

#1–7 #8–14

Check Your Understanding

Lesson Practice

TEACH

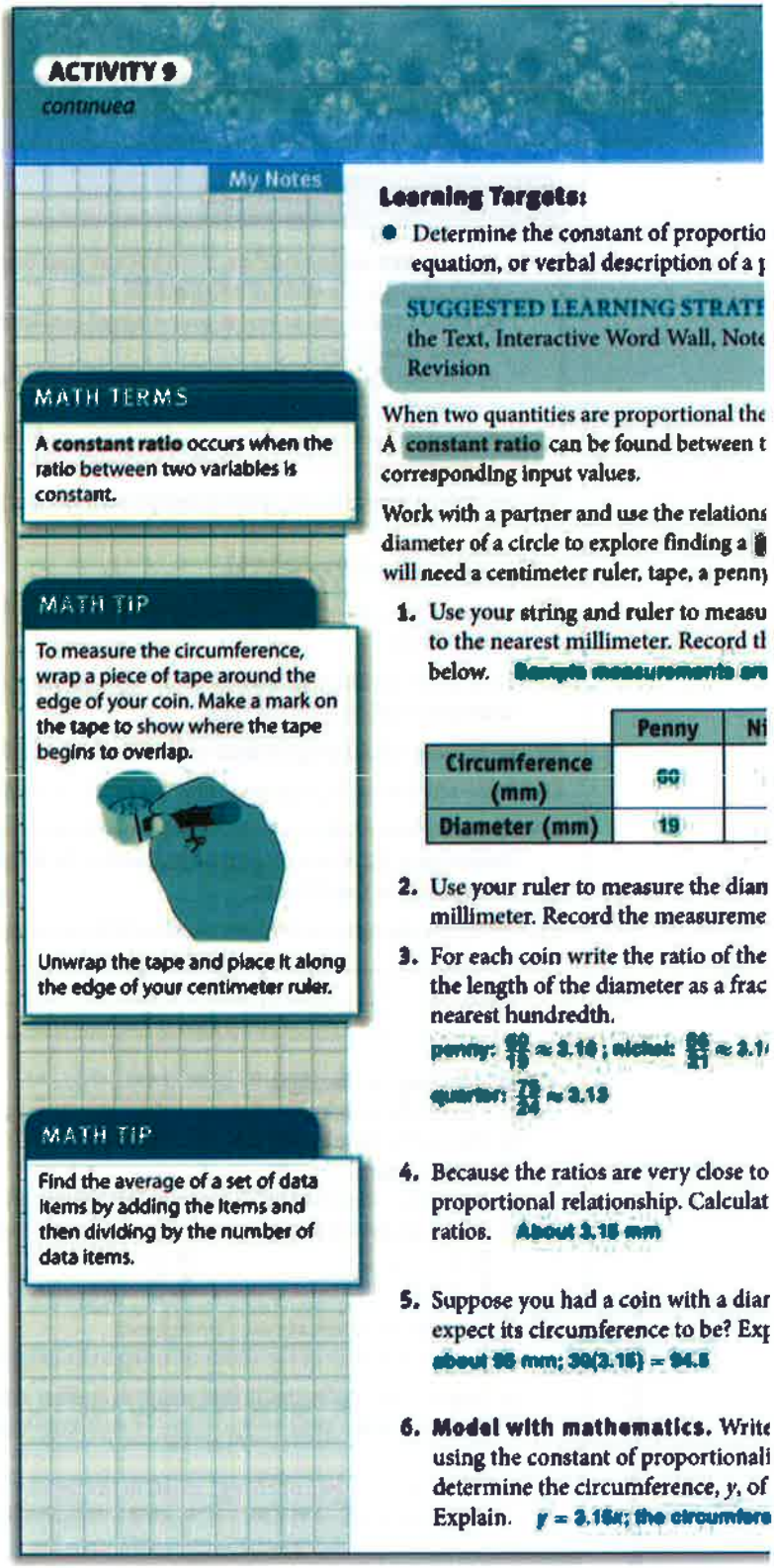
Bell-Ringer Activity

Have students determine the relationship between the number of students in the class and the number of chair legs in the class. Ask them to predict how the number of chair legs would change if 4 more students were added to the class. Discuss with students how they made their predictions.

Introduction Shared Reading Have students read the introductory paragraphs and highlight the important terms in cooperative learning groups. Remind students that the circumference of a circle is the length around the circle.

1–6 Create Representations, Look for a Pattern, Think-Pair-Share In these Items, students use tables to record the circumference and diameter of different circles (coins) to the nearest millimeter. They then write the ratios of the length of the circumference to the length of the diameter as a fraction. Point out that each ratio is equal to about 3.15, and that this is the approximate value of π (pi).

Have students recall the formula for the circumference C of a circle is $C = \pi d$, where d is the diameter. While students work on Item 6, point out that the formula is an equation of the form $y = kx$, where k is 3.15, y is C, and x is d. It is important for students to have the opportunity to discuss the model for circumference of a circle given the diameter before moving on to representing an equation of the form $y = kx$ in a graph.



ACTIVITY 9
continued

My Notes

MATH TERMS

A **constant ratio** occurs when the ratio between two variables is constant.

MATH TIP

To measure the circumference, wrap a piece of tape around the edge of your coin. Make a mark on the tape to show where the tape begins to overlap.

Unwrap the tape and place it along the edge of your centimeter ruler.

MATH TIP

Find the average of a set of data items by adding the items and then dividing by the number of data items.

Learning Targets:

- Determine the constant of proportio equation, or verbal description of a

SUGGESTED LEARNING STRATE the Text, Interactive Word Wall, Note Revision

When two quantities are proportional the A **constant ratio** can be found between t corresponding input values.

Work with a partner and use the relations diameter of a circle to explore finding a will need a centimeter ruler, tape, a penny

1. Use your string and ruler to measu to the nearest millimeter. Record tl below. Sample measurements are

	Penny	Ni
Circumference (mm)	60	
Diameter (mm)	19	

2. Use your ruler to measure the dian millimeter. Record the measureme
3. For each coin write the ratio of the the length of the diameter as a frac nearest hundredth.
   penny: $\frac{60}{19} \approx 3.16$; nickel: $\frac{66}{21} \approx 3.1$
   quarter: $\frac{79}{25} \approx 3.13$
4. Because the ratios are very close to proportional relationship. Calculat ratios. About 3.15 mm
5. Suppose you had a coin with a diar expect its circumference to be? Exp about 98 mm; 30(3.15) = 94.5
6. **Model with mathematics.** Write using the constant of proportionali determine the circumference, y, of Explain. $y = 3.15x$; the circumfere

Developing Math Language

Encourage students to express ideas using the appropriate academic

Have students add these words t math notebooks. As they move o

Lesson 9-2
Constants of Proportionality

ACTIVITY 9
continued

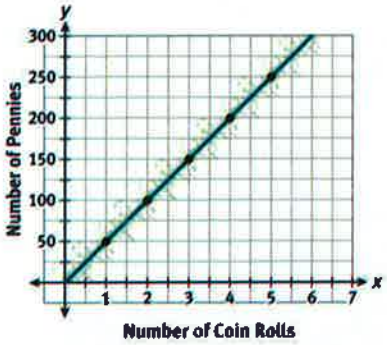
My Notes

The factor k that you multiplied by in Item 6 also represents the **constant rate of change** in the situation.

7. What is the constant rate of change in the equation you wrote?
   3.15

Graphs can also be used to find a constant of proportionality in proportional relationships.

The graph below shows the number of pennies in a number of standard coin rolls.



8. Plot a point at (0, 0) and connect the points with a line. What does the point (0, 0) represent?
   The point (0, 0) means that there are 0 pennies in 0 coin rolls.
9. Create a table showing this information in your My Notes column.
10. Why do the points in the graph lie on a straight line?
    It shows a proportional relationship and a constant rate of change.
11. What is the ratio of number of pennies to the number of coin rolls?
    $\frac{50}{1} = 50$
12. Define the variables and write an equation in the form $y = kx$ for this situation.
    x = the number of coin rolls, y = the number of pennies; $y = 50x$
13. What is the constant of proportionality in this situation?
    50
14. Describe what the constant of proportionality means in this situation.
    The constant of proportionality means that there are 50 pennies per coin roll.

Number of Coin Rolls	Number of Pennies
0	0
1	50
2	100
3	150
4	200
5	250

ACTIVITY 9 Continued

Check Your Understanding

Debrief students' answers to these items to ensure that students understand how a find the constant of proportionality for different representations.

Answers

15. Find the ratio of $y : x$.
16. Find the ratio of $y : x$.
17. Put equation in the form $y = kx$; k is the constant of proportionality.

ASSESS

Use the lesson practice to assess your students' understanding of how to complete a table and graph for a proportional relationship.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 9-2 PRACTICE

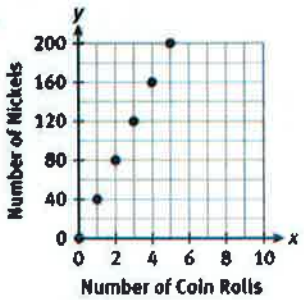
18. a. 40

b. x = the number of coin rolls, y = the number of nickels; $y = 40x$

c. Sample answer:

Number of Coin Rolls	Number of Nickels
0	0
1	40
2	80
3	120
4	160
5	200

d.



e. 320 nickels, I used the constant of proportionality and found $8(40) = 320$.

ADAPT

Check students' answers to the Lesson

ACTIVITY 9 continued

My Notes

Check Your Understanding

Describe how to find the constant of pr representation below.

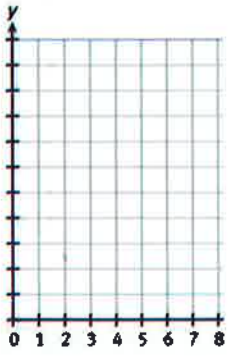
15. A ratio table
16. A graph of a proportional relations
17. The equation of a proportional rel

LESSON 9-2 PRACTICE

18. There are 40 nickels in every stand
    a. What is the constant of proporti
    b. **Model with mathematics.** De equation that can be used to sho
    c. Create a table of this informatio

Number of Coin Rolls			
Number of Nickels			

d. Represent this information in th



e. How many nickels are needed t determined your answer.

MINI-LESSON: Constant of Proportionality

ACTIVITY 9 PRACTICE

Write your answers on notebook paper. Show your work.

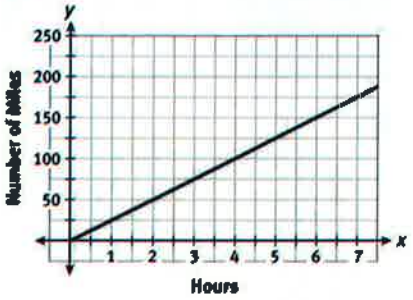
Lesson 9-1

1. Complete the ratio table to show ratios equivalent to 16:10.

2			8	
	36	9		72

2. Solve the proportion $\frac{3}{8} = \frac{21}{x}$ using two different methods. Explain each method.
3. Solve $\frac{x}{48} = \frac{5}{6}$ using two different strategies. Explain each strategy.
4. Is the ratio 25 to 16 proportional to the ratio 5 to 4? Explain.
5. Are the ratios 2.5:3.5 and 5:7 proportional? Explain.
6. Is the ratio 4.2:1.5 proportional to the ratio 12:5? Explain.
7. At the library, the average ratio of hardbound books to paperback books on a shelf is 5:3.
   a. Use a proportion to predict the number of hardbound books on a shelf that has 75 paperback books.
   b. Use a proportion to predict the number of paperback books on a shelf that has 75 hardbound books.

For Items 8–12, use the following graph to make predictions.



8. Use the graph to predict the number of miles driven in 8 hours. Choose the correct answer below.
   A. 150 miles
   B. 175 miles
   C. 200 miles
   D. 250 miles
9. Use the graph to predict the number of hours it would take to drive 162.5 miles. Choose the correct answer below.
   A. 15.5 hours
   B. 6 hours
   C. 6.5 hours
   D. 7 hours
10. What does the point (0, 0) mean in this situation?
11. What does the point (1, 25) mean in this situation?
12. Write an equation in $y = kx$ form to represent this situation.

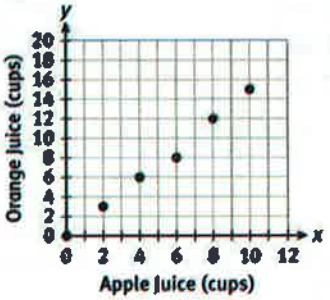
MINI-LESSON: **Constant of Proportionality *(continued)***

ACTIVITY 9 Continued

13. 1.5
14. x = cups of apple juice, y = cups of orange juice; $y = 1.5x$
15. Sample answer:

Apple Juice (cups), x	2	4	6	8	10
Orange Juice (cups), y	3	6	9	12	15

16.



17. 18 cups
18. $5\frac{1}{3}$ cups
19. (0, 0) means that if 0 cups of apple juice are used then 0 cups of orange juice should be used.
20. (1, 1.5) means that for every 1 cup of apple juice used 1.5 cups of orange juice should be used.
21. 1.6
22. $y = 1.6x$
23. $176
24. 1600

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

ACTIVITY 9 continued

Lesson 9-3

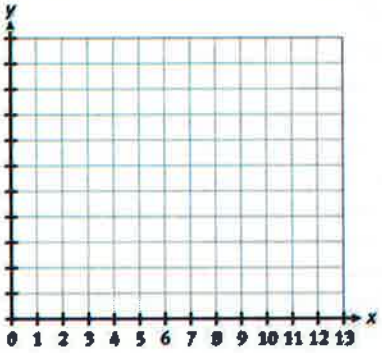
For Items 13–20, use the following information.

A fruit punch uses 1.5 cups of orange juice for every cup of apple juice.

13. What is the constant of proportionality used to find the number of cups of orange juice needed for any amount of apple juice?
14. Define the variables and write an equation that can be used to show this relationship.
15. Create a table of this information.

Apple Juice (cups), x					
Orange Juice (cups), y					

16. Represent this information in the graph below.



17. How many cups of orange juice are needed for 12 cups of apple juice?

18. How many cu
8 cups of oran
19. What does th
situation?
20. What does th
situation?
21. Therese is on
below to dete
U.S. dollars u
constant of p

British Pou	
U.S. Doll	

22. Use the table
convert Britis
23. Use your equ
number of U.
she bought a
pounds.

MATHEMATICA
Model with Mat

24. A tire maker
shipped. The
that 16 are de
20,000 tires w

Ratios, Proportions, and Proportional Reasoning

Embedded Assessment 1
Use after Activity 9

WEIGHING IN ON DIAMONDS

Write your answers on notebook paper. Show your work.

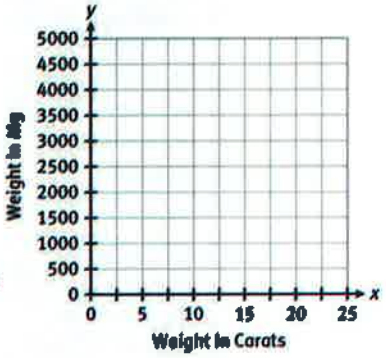
You may have had diamonds in your mouth before. Many dentists' drills are embedded with diamonds. In fact, 18% of your body is made up of carbon, and diamonds are also made of compressed carbon. That must mean you **are priceless!**

For Items 1–8, use the following information.

Diamonds are weighed in units called carats. Carat weight is based on the diamond's weight in milligrams. The table at the right shows the relationship between carats and milligrams.

Weight in Carats	Weight in Milligrams
$\frac{1}{2}$	100
2	400
4	800
6	1200
10	2000
25	

1. Write an equation to convert a diamond's weight in carats to its weight in milligrams. Be sure to define your variables.
2. What is the constant of proportionality represented in the table at the right?
3. Complete the last row of the table by using the constant of proportionality.
4. Use your equation to find the weight in milligrams of the Tiffany Yellow Diamond, which weighs 287.42 carats.
5. Create a graph of the information in the table.
6. Explain the meaning of the point (0, 0) on your graph.
7. Use your graph to determine the weight in milligrams of a diamond weighing 8 carats.
8. Give the ordered pair for the point on the graph that shows how many milligrams a 1-carat diamond weighs.



Solve.

9. The Cullinan is the largest rough gem-quality diamond ever found. It was 3,106.75 carats. It weighed about 0.62 kg uncut. Recall that 1 kg is equal to 2.2 pounds. What was the uncut Cullinan weight in pounds?
10. How many pounds would a 0.5 kg diamond weigh?
11. The ratio of a diamond's hardness to its specific gravity is 10:3.515, and the ratio of the hardness to specific gravity for a ruby is 9:4.05. Are these ratios in proportion? Explain your answer.

Common Core State Standards for Embedded Assessment 1

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and

Embedded Assessment 1

TEACHER to TEACHER

You may wish to read through the scoring guide with students and discuss the differences in the expectations at each level. Check that students understand the terms used.

Unpacking Embedded Assessment 2

Once students have completed this Embedded Assessment, turn to Embedded Assessment 2 and unpack it with them. Use a graphic organizer to help students understand the concepts they will need to know to be successful on Embedded Assessment 2.

Embedded Assessment 1 *Use after Activity 9*

Ratios, Propor

For Items 12–13, use the following inf

The largest diamond is thought to be Lu
Its weight is *10 billion trillion trillion car*
away from Earth. One light-year is abou
distance light travels through space in o

12. Use a proportion to determine how
Earth Lucy is.

13. Write an equation in $y = kx$ form t
equation to check your answer fror

Scoring Guide	Exemplary	Proficient	Eme
	The solution demonstrates these characteristics:		
Mathematics Knowledge and Thinking (Items 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13)	• Clear and accurate understanding of ratios, unit rates, and solving proportions.	• An understanding of ratios, unit rates, and solving proportions that usually results in correct answers.	• An understa unit rates, a proportions sometimes r correct answ
Problem Solving (Items 4, 7, 9, 10, 12)	• Accurate interpretation of the solution of a proportion to solve a problem.	• Interpretation of the solution of a proportion to solve a problem.	• Difficulty int solution of a solve a probl
Mathematical Modeling / Representations (Items 1, 5, 6, 7, 8, 13)	• Accurate representation of a problem situation with a proportional equation or a graph.	• A mostly correct representation of a problem situation with a proportional equation or a graph.	• Difficulty rep problem situ proportional graph.
Reasoning and Communication (Items 6, 11)	• Precise use of appropriate math terms and language to explain proportional relationships.	• An adequate explanation of solutions using proportional relationships.	• A misleading explanation using proper relationships

Common Core State Standards for Embedded Assess

7.RP.A.2c Represent proportional relationships by equations. *For example*

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Common Core Edition

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Unit 4 **Ratios**	Unit 3 **Ratio & Proportion**	
Unit 5 **Geometric Concepts**	Unit 4 **Geometry**	Unit 3 **Geomet**
		Unit 4 **Functio**
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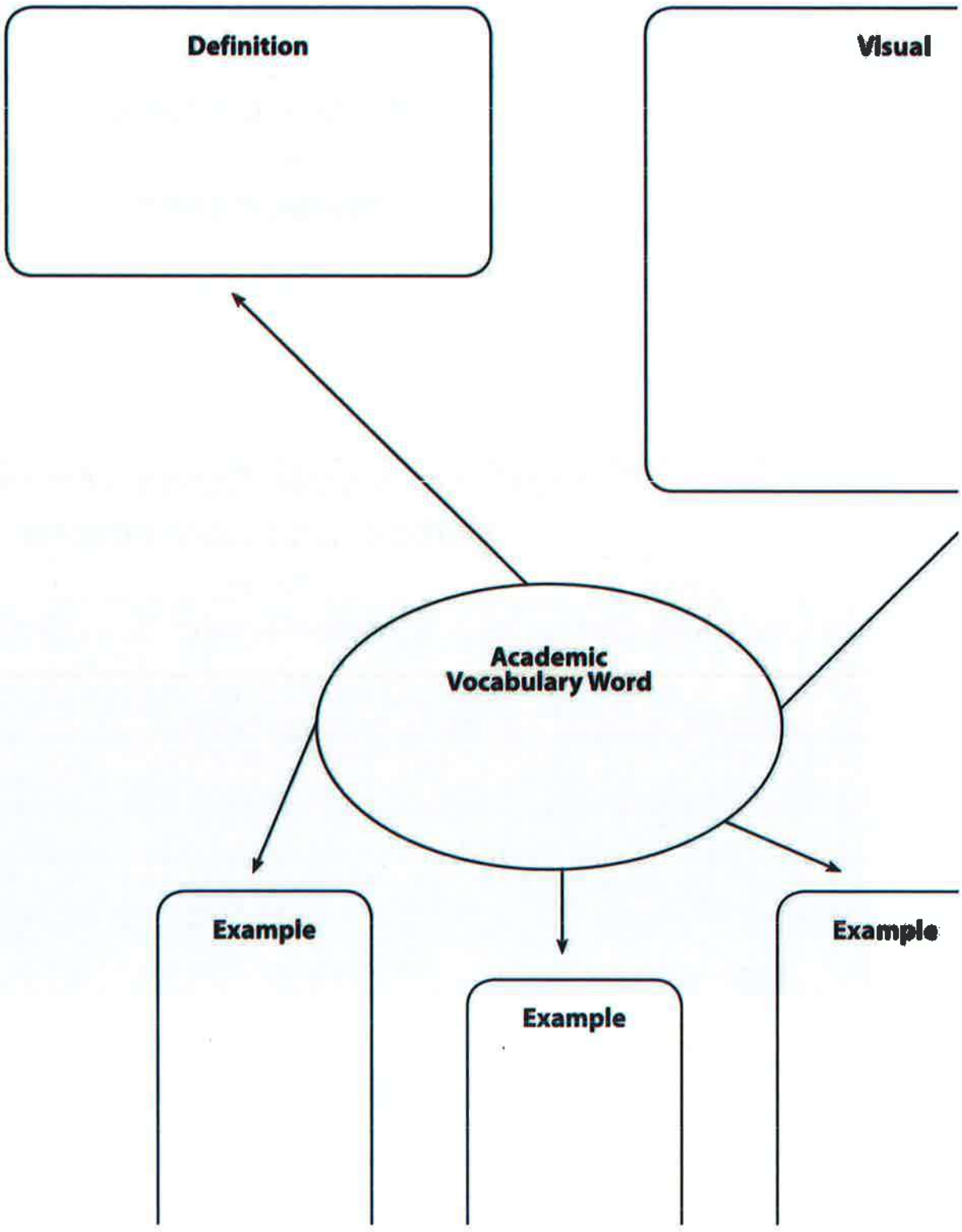
Unit 7 **Personal Financial Literacy**	Unit 7 **Personal Financial Literacy**	Unit 6 **Persona**

Word Map


Definition
Visual
Academic Vocabulary Word
Example
Example
Example

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SpringBoard Learning Strategies

READING STRATEGIES

STRATEGY	DEFINITION	PURPOS
Activating Prior Knowledge	Recalling what is known about a concept and using that information to make a connection to a new concept	Helps students establish c between what they alread that knowledge is related
Chunking the Activity	Grouping a set of items/questions for specific purposes	Provides an opportunity to and assess student unders moving on to a new conce
Close Reading	Reading text word for word, sentence by sentence, and line by line to make a detailed analysis of meaning	Assists in developing a cor understanding of the text
Graphic Organizer	Arranging information into maps and charts	Builds comprehension anc discussion by representinç visual form
Interactive Word Wall	Visually displaying vocabulary words to serve as a classroom reference of words and groups of words as they are introduced, used, and mastered over the course of a year	Provides a visual reference aids understanding for rea and builds word knowledç
KWL Chart (Know, Want to Know, Learn)	Activating prior knowledge by identifying what students know, determining what they want to learn, and having them reflect on what they learned	Assists in organizing inforr reflecting on learning to b knowledge and increase c
Marking the Text	Highlighting, underlining, and /or annotating text to focus on key information to help understand the text or solve the problem	Helps the reader identify i information in the text anc about the interpretation o and concepts to apply to r
Predict and Confirm	Making conjectures about what results will develop in an activity; confirming or modifying the conjectures based on outcomes	Stimulates thinking by ma and correcting predictions evidence from the outcor
Levels of Questions	Developing literal, interpretive, and universal questions about the text while reading the text	Focuses reading, helps in ç into the text by seeking ar prepares one for group an
Paraphrasing	Restating in your own words the essential information in a text or problem description	Assists with comprehensic information, and problem
Role Play	Assuming the role of a character in a scenario	Helps interpret and visuali a problem
Shared Reading	Reading the text aloud (usually by the teacher) as students follow along silently, or reading a text aloud by the teacher and students	Helps auditory learners do and analyze challenging te
Summarizing	Giving a brief statement of the main points in a text	Assists with comprehensic practice with identifying a information
Think Aloud	Talking through a difficult text or problem	Helps in comprehending t

SpringBoard Learning Strategies

COLLABORATIVE STRATEGIES

STRATEGY	DEFINITION	PUR
Critique Reasoning	Through collaborative discussion, respond to the arguments of others; question the use of mathematical terminology, assumptions, and conjectures to improve understanding and to justify and communicate conclusions	Helps students learn f as they make connect mathematical concep verbalize their unders their arguments with that make sense to pe
Debriefing T	Discussing the understanding of a concept to lead to consensus on its meaning	Helps clarify misconce understanding of con
Discussion Groups	Working within groups to discuss content, to create problem solutions, and to explain and justify a solution	Aids understanding th ideas, interpretation of of problem scenarios
Group Presentation	Presenting information as a collaborative group	Allows opportunities t collaborative solution responsibility for deliv an audience
Jigsaw	Reading different texts or passages, students become "experts" and then move to a new group to share their information; after sharing, students go back to the original group to share new knowledge	Provides opportunitie present information t that facilitates unders passage (or multiple t without having each s
Sharing and Responding	Communicating with another person or a small group of peers who respond to a piece of writing or proposed problem solution	Gives students the op their work with peers, for improvement to th and/or to receive appr feedback on their owr
Think-Pair-Share	Thinking through a problem alone, pairing with a partner to share ideas, and concluding by sharing results with the class	Enables the developm are then tested with a for revising ideas and larger group

WRITING STRATEGIES

Drafting	Writing a text in an initial form	Assists in getting first form and ready for rev
Note Taking	Creating a record of information while reading a text or listening to a speaker	Helps in organizing id information
Prewriting	Brainstorming, either alone or in groups, and refining thoughts and organizing ideas prior to writing	Provides a tool for beg process and determin writing
Quickwrite	Writing for a short, specific amount of time about a designated topic	Helps generate ideas i

SpringBoard Learning Strategies

PROBLEM-SOLVING STRATEGIES

Construct an Argument	Use mathematical reasoning to present assumptions about mathematical situations, support conjectures with mathematically relevant and accurate data, and provide a logical progression of ideas leading to a conclusion that makes sense	Helps develop the process mathematical information reasoning skills, and enhar communication skills in su conjectures and conclusio
Create a Plan	Analyzing the tasks in a problem and creating a process for completing the tasks by finding information needed for the tasks, interpreting data, choosing how to solve a problem, communicating the results, and verifying accuracy	Assists in breaking tasks in parts and identifying the s complete the entire task
Create Representations	Creating pictures, tables, graphs, lists, equations, models, and /or verbal expressions to interpret text or data	Helps organize informatio ways to present data and t question or show a proble
Guess and Check	Guessing the solution to a problem, and then checking that the guess fits the information in the problem and is an accurate solution	Allows exploration of diffe a problem; guess and chec when other strategies for s obvious
Identify a Subtask	Breaking a problem into smaller pieces whose outcomes lead to a solution	Helps to organize the piec problem and reach a comp
Look for a Pattern	Observing information or creating visual representations to find a trend	Helps to identify patterns t to make predictions
Simplify the Problem	Using "friendlier" numbers to solve a problem	Provides insight into the p strategies needed to solve
Work Backward	Tracing a possible answer back through the solution process to the starting point	Provides another way to cl answers for accuracy
Use Manipulatives	Using objects to examine relationships between the information given	Provides a visual represent supports comprehension c problem