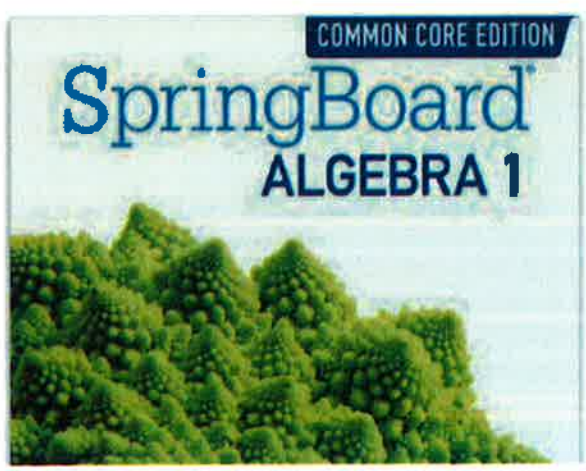
CollegeBoard

SpringB

Mathematics 2(

COMMON CORE ED

Algebra I Sampler
Unit 2



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Common Core Edition

Algebra 1 - Unit Sampler

The Pathway to Advanced Placement and College Readiness
SpringBoard provides a comprehensive and systematic approach to preparing ALL stud demands of rigorous AP courses, college classes, and other postsecondary experiences prepares students through sequential, scaffolded development of the prerequisite skill knowledge needed for success in AP Calculus and Statistics.

In each unit of study, explicit AP Connections are outlined in the Planning the Unit pag teacher editions and are reinforced as they appear in student activities. Through ongoi rigorous mathematics content and experience with the thinking processes needed to a and explain complex math problems, students exit SpringBoard equipped with the kinc order thinking skills, knowledge, and behaviors necessary to be successful in AP classes

For More Information on the SpringBoard Program visit www.Collegeboard.org/Spring

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Common Core Edition
Algebra 1 Unit Sampler

Additional materials in the back of this Sampler include

To the Teacher

Welcome to *SpringBoard Mathematics*, a highly engaging, student-centered instructional program. This revised edition of SpringBoard is based on the standards defined by the **Common Core State Standards for Mathematics** for each course. The program may be used as a core curriculum that will provide the instructional content that students need to be prepared for future mathematical courses.

Shifts in Mathematics Instruction

With an increased emphasis on better preparing students to understand and master mathematical concepts, mathematics instruction has become a major focus of attention. Efforts at improvement center around the following points:

Greater Focus on the Content of the Standards:

- Learn more about less by spending more time on fewer concepts.
- Focus on the essential learning that helps students **develop strong foundational knowledge** and deep conceptual understanding.

Coherence to Link Major Topics:

- Connect learning within a grade and build knowledge across grades.
- Focus on learning progressions so that teachers can continue counting on students' deep conceptual understanding of core content and build on it.

Rigor with Balance:

- Develop fluency in procedural skills.
- Promote depth and mastery by connecting concepts, practice, and independent application.
- Learn and apply the mathematical practices.

College and Career Readiness

The goal of this increased focus on standards and mathematical practices is, of course, helping students be prepared for the expectations of either college or a career, or both. Students who are prepared for college or career will be able to:

- **Build on content knowledge:** Students will have a base knowledge of math concepts on which to extend their learning.
- **Use mathematical models:** Students will be able to use a variety of mathematical representations to model what they know and to justify how they are using their knowledge.
- **Communicate mathematics:** Students will communicate verbally and in writing to explain their discoveries and understanding of mathematics and how it works theoretically and in the real world.

To the Teacher *continued*

The implications of these student expectations are that students will need to develop greater depth of knowledge, higher-level thinking skills, and effective communication skills. What they need less of will be memorization, drills and worksheets, and "one size fits all" content.

SpringBoard's Role in Preparing Students for College and Career Success

Based on the Common Core State Standards for Mathematics and current research on best instructional practices, SpringBoard uses a "back-mapping" instructional design that starts with the end in mind, namely, the skills and knowledge students need to use mathematics effectively and to demonstrate that ability through performance on various assessments.

The mathematics instruction follows a balanced approach in which concepts are presented based on the most effective instructional methods: ***directed*** for basic mathematics principles, including examples and practice; ***guided*** for concepts that need a combination of direct instruction and investigatory learning; and ***investigative*** activities that allow students to explore and discover mathematics concepts through a contextual setting.

Organization of the Content

Instructional content is organized into coherent units of mathematical concepts. Each unit contains multiple activities that are divided into shorter lessons. The units are structured as follows:

- Unit opener content sets the stage for what students will learn in the unit.
- Getting Ready helps teachers assess students' current skills and knowledge to determine whether they have the basic knowledge on which to build new content presented in the unit.
- Multiple lessons per activity.
- Worked-out examples as needed to help students learn and apply concepts.
- Frequent Check Your Understanding questions to help students assimilate and apply knowledge.
- Mathematical practices called out so students are reminded to apply them as they respond to problems and applications.
- Lesson Practice problems to provide the opportunity to practice new learning and to build fluency.
- Activity Practice provides additional problems for each lesson.
- Embedded Assessments give students new contexts for applying the concepts learned in the unit and give you the opportunity for regular formative assessment.

Integration of Mathematical Practices

- Make sense of and connect mathematics concepts to everyday life.
- Model with mathematics to solve problems, justify solutions and their reasonableness, and communicate mathematical ideas.
- Use appropriate tools, such as number lines, protractors, technology, or paper and pencil, strategically to help solve problems.
- Communicate abstract and quantitative reasoning both orally and in writing through viable arguments and critiques.
- Analyze mathematical relationships through structure and repeated reasoning to connect ideas.
- Attend to precision in both written and oral communication.

Engaging and Interactive Online Edition

With this new edition, SpringBoard introduces an all-new interactive online experience for both students and teachers. In addition to providing all content online, the new SpringBoard Digital program:

- Allows access at any time.
- Discerns the device you're using and adjusts content to fit screens—from desktops to laptops to tablets.
- Provides exciting tools such as text marking (highlight, underline, circle, and so on), online calculators, graphing and equation tools, and handwriting recognition for entering equations, note-taking, and uploading of student papers.
- Allows teachers to edit teaching commentary, personalizing by adding their own notes and comments about lessons.
- Includes online student and teacher resources such as graphic organizers, blackline masters, mini-lessons, and other content to support instruction.

New Assessment Options

The SpringBoard program now provides the option of using the ExamView test generator program for all grades. Teachers will have multiple options for choosing pre-made tests or making their own. Options include:

- Unit tests aligned to standards and the content in each unit.
- Test banks allowing teachers to choose items and create tests for multiple needs, including benchmark tests and quarter or semester tests.
- Expanded test item types, including short response and interactive simulations and manipulatives.

What Sets SpringBoard Apart from Other Mathematics Programs?

To the Teacher *continued*

Unique features of SpringBoard include:

- **Rigorous, standards-based instruction:** Instructional content organized around the **Common Core State Standards for Mathematics** to provide coherent topics that build knowledge and skills throughout each course and across grade levels.
- **Mathematical practices:** Integration of the Standards for Mathematical Practice that support student learning and higher level thinking.
- **Research-based instruction:** Back-mapped instructional design gives students a learning target and scaffolds activities to develop students' knowledge and skills and prepare them to demonstrate their learning on an Embedded Assessment.
- **Student-centered, interactive, collaborative activities and lessons:** Each course is organized into short, interactive activities that are further divided into focused lessons. Lessons engage students and aid learning by having students participate in class discussions, solve problems and justify solutions, and demonstrate learning through multiple means of evaluation.
- **Integrated teaching and learning strategies:** Suggested Learning Strategies in each lesson help students use methodical approaches to learning new content, helping students take control of their own learning by identifying which strategies work best for them. Teachers also use strategies for instruction that demands a reflective and metacognitive approach to teaching and learning.
- **Assessment for learning:** Multiple assessment opportunities provide a formative look at students' knowledge and skills: before starting a unit of instruction to assess prerequisite knowledge (Getting Ready), during instruction to monitor understanding (Check Your Understanding), and after instruction to evaluate knowledge of concepts and how to apply them in a variety of situations (Practice, Embedded Assessments, Unit Tests).
- **Professional development:** Unparalleled professional development builds teacher capacity to deliver challenging curriculum to meet the needs of all students while honoring the creativity and intelligence teachers bring to the classroom. Face-to-face training is supported by an online system featuring resources that include an interactive professional learning *Community* that allows peer-to-peer sharing and sustains successful teaching.

The Pathway to Advanced Placement and College Readiness

SpringBoard provides a comprehensive and systematic approach to preparing ALL students for the demands of rigorous AP courses, college classes, and other post-secondary experiences. SpringBoard prepares students through sequential, scaffolded development of the prerequisite skills and knowledge needed for success in AP Calculus and Statistics. In each unit of study, explicit AP

Grade 6 students learn to:

- Model functions in numerical, symbolic (equation), table, and graphical forms.
- Communicate mathematics in writing and verbally, justifying answers and clearly labeling charts and graphs.
- Explore and represent data in a variety of forms.
- Use multiple representations to communicate their mathematical understanding.

Grade 7 students continue to:

- Acquire an algebraic and graphical understanding of functions.
- Write, solve, and graph linear equations; recognize and verbalize patterns; and model slope as a rate of change.
- Communicate clearly to explain methods of problem solving and to interpret results.
- Investigate concepts presented visually and verbally.

Grade 8 students extend their knowledge by:

- Writing algebraic models from a variety of physical, numeric, and verbal descriptions.
- Solving equations using a variety of methods.
- Justifying answers using precise mathematical language.
- Relating constant rate of change to verbal, physical, and algebraic models.
- Using technology to solve problems.
- Reinforcing and extending the vocabulary of probability and statistics.

Algebra 1 students:

- Gain an understanding of the properties of real numbers.
- Formalize the language of functions.
- Explore the behavior of functions numerically, graphically, analytically, and verbally.
- Use technology to discover relationships, test conjectures, and solve problems.
- Write expressions, equations, and inequalities from physical models.
- Communicate mathematics understanding formally and informally.

Geometry students:

- Read, analyze, and solve right triangle and trigonometric functions within contextual situations.
- Develop area formulas necessary for determining volumes of rotational solids, solids with known cross sections, and area beneath a curve.
- Explain work clearly so that the reasoning process can be followed

To the Teacher *continued*

Algebra 2 students:

- Develop the algebra of functions.
- Read and analyze contextual situations involving exponential and logarithmic functions.
- Work with functions graphically, numerically, analytically, and verbally.
- Learn optimization problems.
- Compare the relative rate of change of linear and exponential functions.
- Learn the concept of infinite sum as a limit of partial sums.
- Work with statistics in numerical summaries, calculations using the normal curve, and the modeling of data.

Precalculus students:

- Gain an introductory understanding of convergence and divergence.
- Collect, analyze, and draw conclusions from data.
- Solve problems in contextual situations dealing with polynomial, rational, logarithmic, and trigonometric functions.
- Model motion using parametric equations and vectors.
- Develop an intuitive understanding of limits and continuity.

The SpringBoard Mathematics Classroom

A SpringBoard classroom is an environment that supports high expectations for all students.

Collaborative Groups

The **student-centered classroom** capitalizes on collaboration. Collaborative groups provide a setting in which students feel safe to explore ideas and learn effective communication skills. Collaborative groups allow learning to be active as students engage in discussions, make conjectures, question, and discover new ideas as they fulfill tasks within the group.

Debriefing/Reflections

Frequently in a mathematics classroom, students and teachers should engage in **debriefings**. The purpose of debriefing is to allow students to reflect on their learning, correct misconceptions, identify thinking processes used during an activity, summarize information, and process what they have learned.

Interactive Word Wall

The class **Word Wall** facilitates vocabulary development and provides a reference during class and group discussions. Creating and maintaining a Word Wall is an ongoing activity. It should be an instructional tool, not just a display.

Contents

To the Teacher

Instructional Units

Contents *continued*

Contents *continued*

Contents *continued*

Unit 2 Planning the Unit

In this unit, students study functions and function concepts, including domain, range, slope as rate of change, and intercepts. Students write linear functions given a point and a slope, two points, a table of values, an arithmetic sequence, or a graph. They collect and model data with linear, quadratic, or exponential functions.

Vocabulary Development

The key terms for this unit can be found on the Unit Opener page. These terms are divided into Academic Vocabulary and Math Terms. Academic Vocabulary includes terms that have additional meaning outside of math. These terms are listed separately to help students transition from their current understanding of a term to its meaning as a mathematics term. To help students learn new vocabulary:

- Have students discuss meaning and use graphic organizers to record their understanding of new words.
- Remind students to place their graphic organizers in their math notebooks and revisit their notes as their understanding of vocabulary grows.
- As needed, pronounce new words and place pronunciation guides and definitions on the class Word Wall.

Embedded Assessments

Embedded Assessments allow students to do the following:

- Demonstrate their understanding of new concepts.
- Integrate previous and new knowledge by solving real-world problems presented in new settings.

They also provide formative information to help you adjust instruction to meet your students' learning needs.

Prior to beginning instruction, have students unpack the first Embedded Assessment in the unit to identify the skills and knowledge necessary for successful completion

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AP / College Readine

Unit 2 continues to hon
function by:

- Formalizing the lar
- Making the connec
represents a consta
- Exploring function
their behavior in a
graphical, analytica
- Collecting data and
quadratic, or expon

Unpacking the Embedd

The following are the key sk
will need to know for each a

Embedded Assessmen

Representations of Fu
Bryce Canyon Hiking

- Identify functions and
- Interpret key features c

Embedded Assessmen

Linear Functions and
Text Message Plans

- Model with, write, and
- Identify a direct variati

Planning the Unit *continued*

Suggested Pacing

The following table provides suggestions for pacing using a 45-minute class period. Space is left for you to write your own pacing guidelines based on your experiences in using the materials.

	45-Minute Period	Your Comments on
Unit Overview/Getting Ready	1	
Activity 5	3	
Activity 6	3	
Activity 7	3	
Activity 8	2	
Embedded Assessment 1	1	
Activity 9	1	
Activity 10	3	
Activity 11	4	
Embedded Assessment 2	1	
Activity 12	4	
Activity 13	3	
Embedded Assessment 3	1	
Total 45-Minute Periods	30	

Additional Resources

Additional resources that you may find helpful for your instruction include the following, which may be found in the eBook Teacher Resources.

- Unit Practice (additional problems for each activity)
- Getting Ready Practice (additional lessons and practice problems for the prerequisite skills)
- Mini-Lessons (instructional support for concepts related to lesson content)

Functions

2

Unit Overview

In this unit, you will build linear models and use them to study functions, domain, and range. Linear models are the foundation for studying slope as a rate of change, intercepts, and direct variation. You will learn to write linear equations given varied information and express these equations in different forms.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- causation

Math Terms

- mapping
- relation
- vertical line test
- independent variable
- dependent variable
- continuous
- discrete
- y-intercept
- relative maximum
- relative minimum
- x-intercept
- parent function
- translation
- direct variation
- constant of variation
- indirect variation
- inverse function
- one-to-one
- arithmetic sequence
- explicit formula
- recursive formula
- slope-intercept form
- point-slope form
- standard form
- scatter plot
- trend line
- correlation
- line of best fit
- linear regression
- quadratic regression
- exponential regression

ESSENTIAL QUESTIONS

How can you show mathematical relationships?

Why are linear functions useful in real-world settings?

EMBEDDED ASSESSMENTS

This unit has three embedded assessments, following Activities 8, 11, and 13. They will give you an opportunity to demonstrate what you have learned.

Developing Math Language

As this unit progresses, help students make the transition from general words they may already know

As needed, pronounce new terms clearly and monitor students' use of words in their discussions to ensure

UNIT 2
Getting Ready

Use some or all of these exercises for formative evaluation of students' readiness for Unit 2 topics.

Prerequisite Skills

- Identify and extend patterns. (Item 1) 4.OA.C.5; 5.OA.B.3
- Solve and interpret inequalities. (Item 2) 6.EE.B.5
- Evaluate algebraic expressions. (Item 3) 6.EE.A.2c
- Graph points on the coordinate plane. (Items 4, 5 and 8) 6.NS.C.8
- Represent data using an equation. (Item 6) 8.F.B.4
- Solve linear equations. (Item 7) 7.EE.B.4a; 8.EE.C.7b

Answer Key

1. Column one, row six: 14; Column two, row four: 17
2. −3, −2, −1, 0, 1, 2, 3
3. a. 1
   b. 6
4. a. *S*
   b. *R*
   c. *T*
5. Sample answer: Begin at the origin. Count three units to the right and then 4 units down.
6. B
7. D
8. A

UNIT 2

Getting Ready

Write your answers on notebook paper. Show your work.

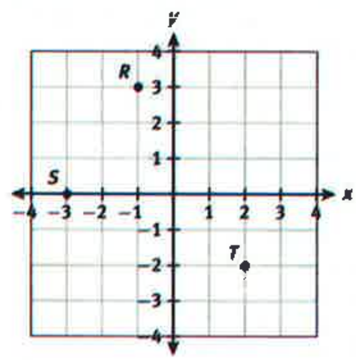
1. Copy and complete the table of values.

−1	−1
2	5
5	11
8	
11	23
	29

2. List the integers that make this statement true.
   $-3 \le x < 4$

3. Evaluate for $a = 3$ and $b = -2$.
   a. $2a - 5$ b. $3b + 4a$

4. Name the point for each ordered pair.
   a. (−3, 0) b. (−1, 3) c. (2, −2)



5. Explain how you would plot (3, −4) on a coordinate plane.

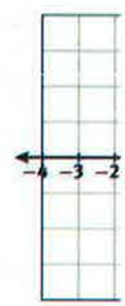
6. Which of the
   the data in the

x	
y	

   A. $y = 2x - 1$
   C. $y = x + 1$

7. If $2x + 6 = 2$,
   A. 4 B. 2

8. Which of the
   point on this



   A. (−1, 3)
   C. (−1, −3)

Getting Ready Practice

For students who may need additional instruction on one or more of the prerequisite skills for this unit,

Graphs of Functions

ACTIVITY 7

Experiment Experiences

Lesson 7-1 The Spring Experiment

My Notes

Learning Targets:

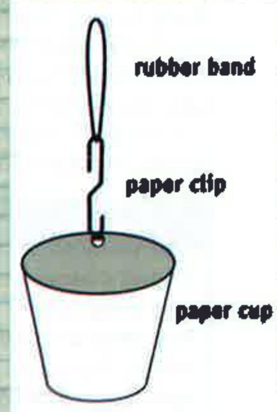
- Graph a function given a table.
- Write an equation for a function given a table or graph.

SUGGESTED LEARNING STRATEGIES: Discussion Groups, Look for a Pattern, Sharing and Responding, Think-Pair-Share, Create Representations, Construct an Argument

For the following experiment, you will need a paper cup, a rubber band, a paper clip, a measuring tape, and several washers.

A. Punch a small hole in the side of the paper cup, near the top rim.

B. Use the bent paper clip to attach the paper cup to the rubber band as shown in the diagram in the *My Notes* section.



1. What is the length of the rubber band?

Answers will vary.

Drop washers one at a time into the cup. Each time you add a washer, measure the length of the rubber band. Subtract the original length you recorded in Item 1 to find the distance that the rubber band has stretched.

2. Make a table of your data. **Check students' tables.**

Number of Washers x	Length of Stretch from Original Length y
1	
2	
3	
4	
5	

3. What patterns do you notice that might help you determine the relationship between the number of washers in the cup and the length of the rubber band stretch?

Answers will vary. In general, the length of the stretch should be approximately a constant multiple of the number of washers.

Common Core State Standards for Activity 7

HSA-REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions

ACTIVITY 7 Continued

1–7 (continued) Monitor the groups to make sure that all students are participating. Students may try to connect the points on their graphs and/or force the data to be perfectly linear. As group discussions are monitored, encourage students to think about whether their graphs should be discrete or continuous. Be sure to include a discussion of why the data is discrete in the whole-class Sharing and Responding after Item 7. In addition, students may need to be reminded that because their data may not be perfectly linear, they may need to generalize a little to write their equation. For example, the rate of change may be "about 1 inch per washer" instead of exactly 1 inch per washer. Include the fact that we are using an equation to "model" a real-world situation in the whole-class Sharing and Responding after Item 7.

Differentiating Instruction

Support students who have difficulty translating a pattern into an equation. Have students describe the pattern verbally. Have them consider which operation they would use as part of the pattern. Help them write a word equation using that operation. Then have them define the variables and translate the word equation into an algebraic equation. Have them test the equation with various ordered pairs in their table to verify that it describes the pattern.

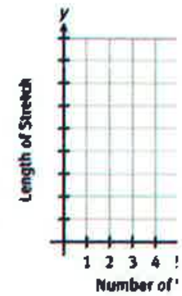
ACTIVITY 7
continued

My Notes

CONNECT TO SCIENCE

What you have revealed with your experiment is an example of Hooke's Law. Hooke's Law states that the distance *d* that a spring (in this case the rubber band) is stretched by a hanging object varies directly with the object's weight *w*.

4. Use your table to make a graph. Be s
the units on the *y*-axis. **Check stu**



5. Describe your graph.
The graph should resemble a line.

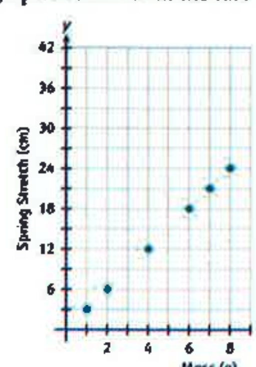
6. **Model with mathematics.** Use y
described in Item 3 to write an equa
between the number of washers and
Answers will vary.

7. Use your graph or your equation to
8 washers and for 10 washers.
Answers will vary; check students'

A group of students performed a similar
various masses. The data they collected i:

Mass (g)	Spring
2	
4	
6	
8	
10	
12	

8. Make a graph of the data in the tabl



Lesson 7-1
The Spring Experiment

My Notes

9. Reason quantitatively. How much does the spring stretch for each additional gram of mass added? Explain how you found your answer.
3 cm; you can see from the table and from the graph that each time the mass increases by 2 g, the stretch increases by 6 cm, which is equivalent to 3 cm for each additional gram.

10. Reason abstractly. Use the students' data to write an equation that gives the distance d that the spring will stretch in terms of the mass m. Explain your equation.
The stretch is always 3 times the mass, so $d = 3m$.

11. Use the equation or the graph to determine the length of the stretch for a mass of 1 gram. Graph the outcome on your graph.
3 cm; see graph above.

12. Use the equation or the graph to determine the length of the stretch for a mass of 7 grams. Graph the outcome on your graph.
21 cm; see graph above.

13. Use the equation or the graph to determine the length of the stretch for a mass of 13 grams. Graph the outcome on your graph.
39 cm; see graph above.

14. a. What do you notice about the points you graphed in Items 11–13?
They lie on the same line as the points graphed from the table.

b. How could you represent the set of all possible masses and corresponding stretches?
Connect all the points to form a line.

15. What is the y-intercept of the graph? What does it represent?
The y-intercept is (0, 0). It represents the length that the spring stretches with no mass attached to it.

16. What is the reasonable domain? Explain.
Domain: $\{m \geq 0\}$; m represents mass and mass cannot be negative.

ACTIVITY 7 Continued

17–19 Think-Pair-Share, Create Representations, Debriefing Students should notice the discrepancy in the change of weights (i.e., from 5 to 8, 3 ounces, and from 8 to 10, 2 ounces) they see in the table and explain how this will or will not affect how they answer each of the items that follow. It is important that students determine how far their spring stretches for each additional ounce of weight before attempting to write an equation. The debrief should include a discussion of any patterns students may have noticed relating the rate of change to the coefficient of the variable in each of the equations they have created in this activity. Students have not labeled the rate of change as of yet but should be beginning to see a connection.

Check Your Understanding

As you debrief with students, have them explain how they found the amount of change in the length of the spring for each 0.5- or 1-pound change in the weight. This lesson prepares students for the study of slope later in this unit.

Answers

20. 4 inches; The spring stretched 12 more inches when 3 pounds of weight were added, which is equivalent to 4 inches per pound.

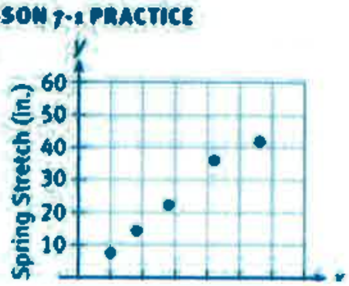
ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 7-1 PRACTICE

21.



22. Shanice, because her point is not on the line formed by the other points

23. (21, 31.5)

ACTIVITY 7
continued

My Notes

Mr. Hardiff's class conducts an experime
They record their data, but some of the ir

Weight (oz)	Sprin
5	
8	
10	
12	
15	
16	

17. How much does the spring stretch fo
2.5 inches

18. Describe how to use your answer to
data in the table.
The stretch is 2.5 inches per ounce

19. Use your equation from Item 18 to c
See above.

Check Your Understanding

20. A 4.5-pound weight stretches a spring
stretches the same spring 30 inches. H
each additional pound of weight? Exp

LESSON 7-1 PRACTICE

Jeremy and his classmates conduct an exp
spring. They record their results in the tabl

Student	Mass (lb)	S
Jeremy	5	
Adele	8	
Roberto	14	
Shanice	21	
Guillaume	28	

21. Make a graph of the data.

22. **Critique the reasoning of othe**
when taking their turn at the experi

23. If the mistake in Item 22 were corre
point be?

24. Write an equation to describe the st
data point you identified in Item 23.

Lesson 7-2
The Falling Object Experiment

ACTIVITY 7 *continued*

My Notes

Learning Target:

- Graph a function describing a real-world situation and identify and interpret key features of the graph.

SUGGESTED LEARNING STRATEGIES: Discussion Groups, Look for a Pattern, Construct an Argument, Think-Pair-Share, Summarizing, Sharing and Responding

1. The Empire State Building in New York City is 1454 feet tall. How long do you think it will take a penny dropped from the top of the Empire State Building to hit the ground?

Answers will vary.

In 1589, the mathematician and scientist Galileo conducted an experiment to answer a question much like the one in Item 1. Galileo dropped balls from the top of the Leaning Tower of Pisa in Italy and determined the time it took them to reach the ground. Galileo used several balls identical in shape but differing in mass. Because the balls all reached the ground in the same amount of time, he developed the theory that all objects fall at the same rate.

Galileo's findings can be represented with the equation $h(t) = 1600 - 16t^2$, where $h(t)$ represents the height in feet of an object t seconds after it has been dropped from a height of 1600 feet.

2. Make a table of values for Galileo's function $h(t) = 1600 - 16t^2$.

t (seconds)	$h(t)$ (feet)
0	1600
1	1584
2	1536
3	1456
4	1344
5	1200
6	1024
7	816
8	576
9	304
10	0

ACTIVITY 7 Continued

TEACHER to TEACHER

This lesson helps students review the features of functions in the context of a real-world situation. Ask questions that help students relate each response to the context of the problem. Emphasize that mathematics are a tool we use to understand and describe real-world phenomena.

ACTIVITY 7
continued

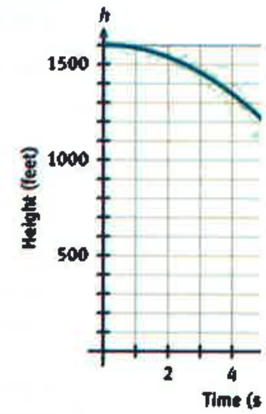
My Notes

MATH TERMS

The **x-intercept** is the point where a graph crosses the x-axis. The y-coordinate of the x-intercept is 0.

3. **Construct viable arguments.** W
not be appropriate in this context?
t represents the number of second
negative values do not make sense

4. Using your table of values, graph Ga



5. What is the reasonable domain of th
graph? What is the reasonable range
Domain: {0 ≤ t ≤ 10}; Range: {0 ≤ h

6. What is the y-intercept?
(0, 1600)

7. What does the y-intercept represent
The height of the object before it is

8. What is the **x-intercept**? What does
(10, 0); the number of seconds it tak
(a height of 0)

9. Identify any extrema of the function
extrema represent?
Maximum: 1600; the greatest height
minimum: 0; the least height of the

"Your homework assignment is to graph
says. She then points to the following fun

$f(x) = x^2$

In this case, the function is not limited by
it is important to use different types of de
to graph.

Lesson 7-2
The Falling Object Experiment

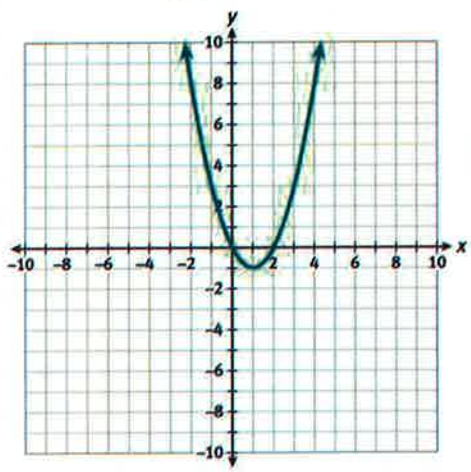
ACTIVITY 7 *continued*

My Notes

10. Using various values for x, make a table of values for $f(x) = x^2 - 2x$.
Answers will vary.

x	$f(x)$
−2	8
−1	3
0	0
1	−1
2	0
3	3

11. Using your table of values, graph the function.



12. Describe the differences between the domain of $f(x) = x^2 - 2x$ and the domain of Galileo's function.
For $f(x) = x^2 - 2x$, x can assume any values, including negative ones; therefore the domain is all real numbers. The domain of Galileo's function is limited to values of x that make sense in the real-world situation (non-negative numbers).

13. State the range of $f(x) = x^2 - 2x$.
$\{y: y \geq -1\}$

14. Identify the y-intercept of $f(x) = x^2 - 2x$.
[illegible]

15. What is the absolute maximum of $f(x) = x^2 - 2x$? What is the absolute minimum?
There is no absolute maximum value of $f(x)$ because the graph continues upward indefinitely; absolute minimum: −1

ACTIVITY 7 Continued

Check Your Understanding

Debrief this lesson by having students revisit the opening question in this lesson. As students offer their opinions, encourage them to refer to Galileo's equation and methods they used in this lesson.

Answers

16. Galileo's equation is for an object dropped from a height of 1600 feet. This is slightly greater than the height of the Empire State Building, so the amount of time it takes a penny dropped from the top of the Empire State Building to reach the ground will be close to, but less than, the amount of time Galileo's equation predicts, which is 10 seconds.

ASSESS

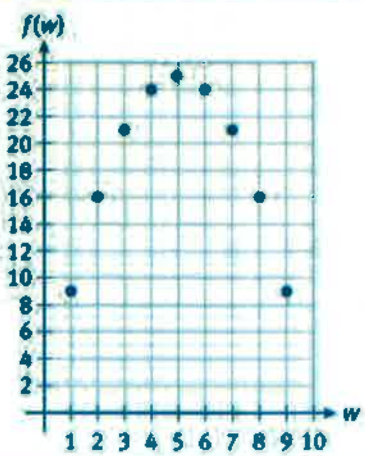
Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 7-2 PRACTICE

17.

w	1	2	3	4	5	6	7	8	9
$f(w)$	9	16	21	24	25	24	21	16	9



ADAPT

ACTIVITY 7
continued

My Notes

Check Your Understanding

16. Revisit your answer to Item 1 and re
long do you think it will take a penn
Empire State Building to hit the grou
equation to help you answer this que

LESSON 7-2 PRACTICE

The area of a rectangle with a perimeter of
where w is the width of the rectangle. Ass
this function to answer Items 17–20.

17. Make a table of values and a graph o

18. Attend to precision. Give a reaso
this context. Explain your answers.

19. Identify the y-intercept of the functi
represent within this context?

20. What is the absolute maximum of th
minimum?

For Items 21–23, use the function $f(x) =$:

21. Make a table of values and a graph o

22. What are the domain and range?

23. Identify the y-intercept, the absolute
minimum.

18. Domain: {all whole numbers w such that $1 \le w \le 9$}; w must be a whole number, and the width of a rectangle cannot be 0, so the least

21. Tables will vary.

Lesson 7-3
The Radioactive Decay Experiment

My Notes

Learning Targets:

- Given a verbal description of a function, make a table and a graph of the function.
- Graph a function and identify and interpret key features of the graph.

SUGGESTED LEARNING STRATEGIES: Discussion Groups, Look for a Pattern, Construct an Argument, Paraphrasing, Marking the Text, Think-Pair-Share

In the late nineteenth century, the scientist Marie Curie performed experiments that led to the discovery of radioactive substances.

A radioactive substance is a substance that gives off radiation as it decays. Scientists describe the rate at which a radioactive substance decays as its *half-life*. The half-life of a substance is the amount of time it takes for one-half of the substance to decay.

CONNECT TO SCIENCE

How much is half a life?
The half-life of a radioactive substance can be as little as 0.0018 seconds for Polonium-215 and as much as 4.5 billion years for Uranium-238.

1. Radium has a half-life of 1600 years. How much radium will be left from a 1000-gram sample after 1600 years?
500 grams

2. How much radium will be left after another 1600 years?
250 grams

3. Suppose a radioactive substance has a half-life of 1 second and you begin with a sample of 4 grams. Complete the table of values.

Time (seconds)	Amount Remaining (grams)
0	4
1	**2**
2	**1**
3	**0.5**
4	**0.25**
5	**0.125**

ACTIVITY 7 Continued

3–4 Look for a Pattern, Create Representations, Discussion Groups Monitor groups carefully to ensure students are creating graphs that do not intersect the x-axis. If groups are having trouble with this idea, encourage the addition of extra units of time to the table to identify a pattern.

5–6 Construct an Argument, Discussion Groups Students should use the real-world situation and the idea that a radioactive substance will never completely decay to explain why the domain of the function modeled by the graph includes 0 while the range cannot include 0.

7–8 Discussion Groups, Sharing and Responding Students connect the real-world situation to the mathematical model by interpreting the meaning of the y-intercept and maximum within the context. It is important that they make the connection between the range of the function and the fact that there can be no absolute minimum. This idea lays the foundation for the identification of horizontal asymptotes and end behavior.

Differentiating Instruction

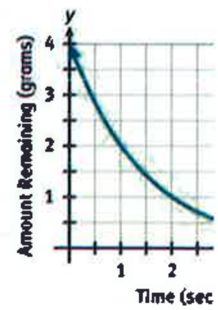
Extend student learning by asking students to research the half-lives of two different radioactive substances. Assuming they begin with 8 grams of each, students should create a table and a graph comparing the amount of each substance remaining over a period of time.

ACTIVITY 7
continued

Th

My Notes

4. Graph the data from the table on the



5. **Make use of structure.** Will the a remains ever reach 0? Explain.
   No; the amount divides in half every for which one-half of that amount is

6. What are the reasonable domain and in the graph? Explain.
   Domain: $\{x: x \geq 0\}$; x represents tim
   Range: $\{y: 0 < y \leq 4\}$; the greatest a the amount remaining gets smaller

7. What is the y-intercept and what do
   (0, 4); there are 4 grams of the subs

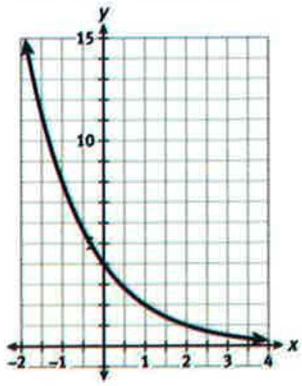
8. Identify the absolute maximum and represented in the graph, and tell wh
   Absolute maximum: 4; the greatest (at time = 0). There is no absolute m

Lesson 7-3
The Radioactive Decay Experiment

ACTIVITY 7 *continued*

My Notes

The function that describes the substance's decay is $f(x)=4\left(\frac{1}{2}\right)^x$. The graph of this function when it does not model a real-world situation is shown below.



9. What are the domain and range of the function?
[illegible]

10. How is this graph different from your graph in Item 4?
The graph in Item 4 appears in the first quadrant only, because *x* is restricted to nonnegative values. In the graph above, *x* can be any value, and so the graph continues on the left side of the *y*-axis.

11. How do the values of *y* change as the values of *x* increase?
The values of *y* decrease; they approach, but do not reach, 0.

12. How do the values of *y* change as the values of *x* decrease?
The values of *y* increase.

13. Identify the absolute maximum and absolute minimum of the function.
They do not exist.

ACTIVITY 7 Continued

Check Your Understanding

Debrief student answers to these items to determine their level of understanding of exponential functions. At this point, student understanding will be basic but should indicate an awareness of the similarities and differences between exponential and linear functions.

Answers

14. $\frac{g}{2}$, or $\frac{1}{2}g$
15. The function never reaches 0, so it cannot be the absolute minimum.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 7-3 PRACTICE

16. A value is reduced by half after every fixed time period.
17.

Time (years)	Value ($)
0	20,000
1	10,000
2	5,000
3	2,500
4	1,250
5	625

18. 6 years

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand exponential equations and are able to discuss the features of their graphs. Continue to expect students to use mathematical terminology when discussing the functions and their graphs.

ACTIVITY 7
continued

My Notes

Th

Check Your Understanding

14. A scientist has g grams of a radioacti that shows the amount of the substa half-life.
15. **Critique the reasoning of othe** $f(x) = 4\left(\frac{1}{2}\right)^x$ and said, "This functio the absolute minimum." Explain wh

LESSON 7-3 PRACTICE

Suppose the value of your new car is redu own it. You paid $20,000 for your new ca

16. Describe how this situation is simila substance.
17. Copy and complete the table below.

Time (years)
0
1
2
3
4
5

18. **Make sense of problems.** For in considered scrap when its value falls will your new car be considered scra

ACTIVITY 7 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 7-1

A weight of 15 ounces stretches a spring 10 inches. A weight of 24 ounces stretches the same spring 16 inches. Use this information to answer Items 1–4.

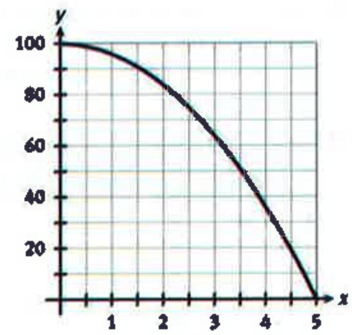
1. How many inches does the spring stretch per ounce of additional weight?
   A. $\frac{2}{3}$ inch
   B. $\frac{3}{2}$ inches
   C. 25 inches
   D. 150 inches
2. Write an equation to describe the relationship between the distance d that the spring stretches and the weight w that is attached to it.
3. How much will the spring stretch for a weight of 9 ounces?
4. The spring is stretched 14 inches. How many ounces is the weight that is attached to it?

A spring stretches 2.5 inches for each ounce of weight. Use this information for Items 5–7.

5. Determine a function that represents this situation.
6. If you were to graph the function represented by this situation, what would be the reasonable domain? Explain.
7. Which of the following data points would **not** lie on the graph representing this function?
   A. (0, 0)
   B. (1, 2.5)
   C. (2.5, 1)
   D. (10, 25)

Lesson 7-2

Suppose that the height of an object after x seconds is given by $f(x) = 100 - 4x^2$, as shown in the graph below.



Use the function or the graph for Items 8–14.

8. What is the reasonable domain of the function?
9. What is the reasonable range of the function?
10. Identify the y-intercept of the function.
11. What does the y-intercept represent?
12. Identify the x-intercept of the function.
13. What does the x-intercept represent?
14. Loni says that because of the negative sign in front of $4x^2$, the reasonable domain for this function is only negative values. Is her reasoning correct? Explain.

ACTIVITY 7 Continued

15. 4 hours (Note: after 3 hours, there is 12.5 oz left, so an additional hour is needed to reach 12.25 oz)

16. 7 grams; The graph shows that this is the amount when time = 0.

17. Domain: $\{x: x \geq 0\}$; range: $\{y: 0 < y \leq 7\}$

18. Absolute maximum: 7; the greatest amount of the substance (at time = 0); There is no absolute minimum value.

19.

Number of Cuts, x	Area of Remaining Piece, y
0	150
1	75
2	37.5
3	18.75
4	9.375

20. A value is repeatedly being reduced by one-half.

21. No; the number of cuts must be a whole number.

22. In this situation, the reasonable domain consists of whole numbers only because the domain values represent a number of cuts. In a radioactive decay situation, the reasonable domain consists of all nonnegative numbers, because the domain values represent time.

23. (0, 150); the area of the original piece of paper (after 0 cuts)

24. 150; the area of the original piece of paper, which is the piece that has the greatest area

25. Yes; Possible justification: Maude will spend her birthday money in 15 days, as shown in the table.

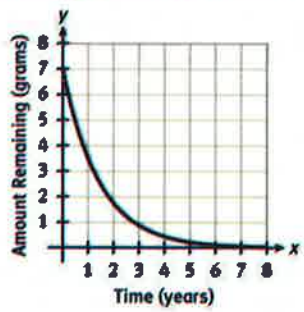
Day	Money Remaining
0	$100
1	$50
2	$25
3	$12.50
4	$6.25
5	$3.13
6	$1.57
7	$0.79
8	$0.40

ACTIVITY 7
continued

Lesson 7-3

15. The half-life of a radioactive substance is 1 hour. If you begin with 100 ounces of the substance, how many hours does it take for 12.25 ounces to remain?

The graph below represents a radioactive decay situation. Use this graph for Items 16–18.



16. What is the original amount of the radioactive substance? Explain how you know.

17. What are the reasonable domain and range?

18. Identify the absolute maximum and absolute minimum values of the function. What do these values represent?

Barry has a piece of paper whose area is 150 square inches. He cuts the paper in half and discards one of the pieces. He repeats this procedure several times. Use this information for Items 19–24.

19. Copy and con
area of the rer

Number Cuts,
0
1
2
3
4

20. Describe how
half-life of a r

21. If you were to
would you co

22. Describe how
situation is di
in a radioacti

23. Identify the y-

24. Identify the al
it represent?

MATHEMATICA
Construct Viabl
the Reasoning o

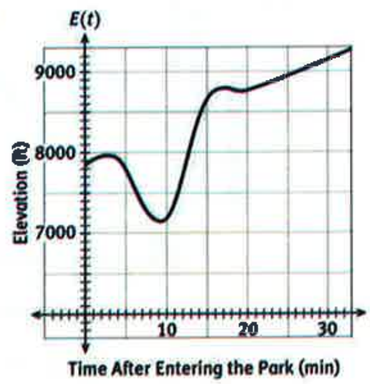
25. Maude receiv
going to spen
day until non
for her to beli
all of the mon

Because Maude cannot spend half cents, she will have to round as she spends half of her money each day (for example, on

Representations of Functions
BRYCE CANYON HIKING

Embedded Assessment 1
Use after Activity 8

While on vacation, Jorge and Jackie traveled to Bryce Canyon National Park in Utah. They were impressed by the differing elevations at the viewpoints along the road. The graph describes the elevations for several viewpoints in terms of the time since they entered the park.



1. The graph represents a function $E(t)$. Describe why the graph represents a function. Identify the domain and range of the function.
2. Is this discrete or continuous data? Explain.
3. What is the y-intercept? Interpret the meaning of the y-intercept in the context of the problem.
4. Identify a relative maximum of the function represented by the graph.
5. What is the absolute maximum of the function represented by the graph? What does it represent?
6. Identify a relative minimum of the function represented by the graph.
7. What is the absolute minimum of the function represented by the graph? What does it represent?

While at Bryce Canyon National Park, Jorge and Jackie hiked at an average speed of about 2 miles per hour.

8. Copy and complete the table below to show the distance hiked by a person whose constant speed is 2 miles per hour.

Time (hours)	Distance (miles)
0	0
1	2
2	
3	
4	
5	

9. Write a function $f(x)$ to describe the data in the table. What are the reasonable domain and range?
10. Create a graph of the function.
11. How long will it take this person to hike 5 miles? Justify your answer.
12. On the same coordinate grid that you used in Item 9, create a graph of another function by translating the graph 5 units up.
13. Write a function to describe the graph you created in Item 12. Explain how you determined your answer.

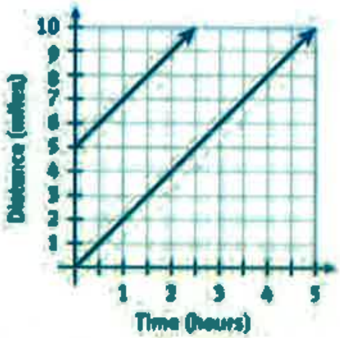
Common Core State Standards for Embedded Assessment 1

HSF-IF.A.1: Understand that a function from one set (called the domain) to another set (called the

Embedded Assessment 1

11. 2.5 hours; Students may use the table, graph or equation to justify their response.

12.



13. $g(x) = 2x + 5$; The graph is a vertical translation 5 units up from the original graph, so add 5 to the function rule.

Teacher to Teacher

You may wish to read through the scoring guide with students and discuss the differences in the expectations at each level. Check that students understand the terms used.

Unpacking Embedded Assessment 2

Once students have completed this Embedded Assessment, turn to Embedded Assessment 2 and unpack it with them. Use a graphic organizer to help students understand the concepts they will need to know to be successful on Embedded Assessment 2.

Embedded Assessment 1

Use after Activity 8

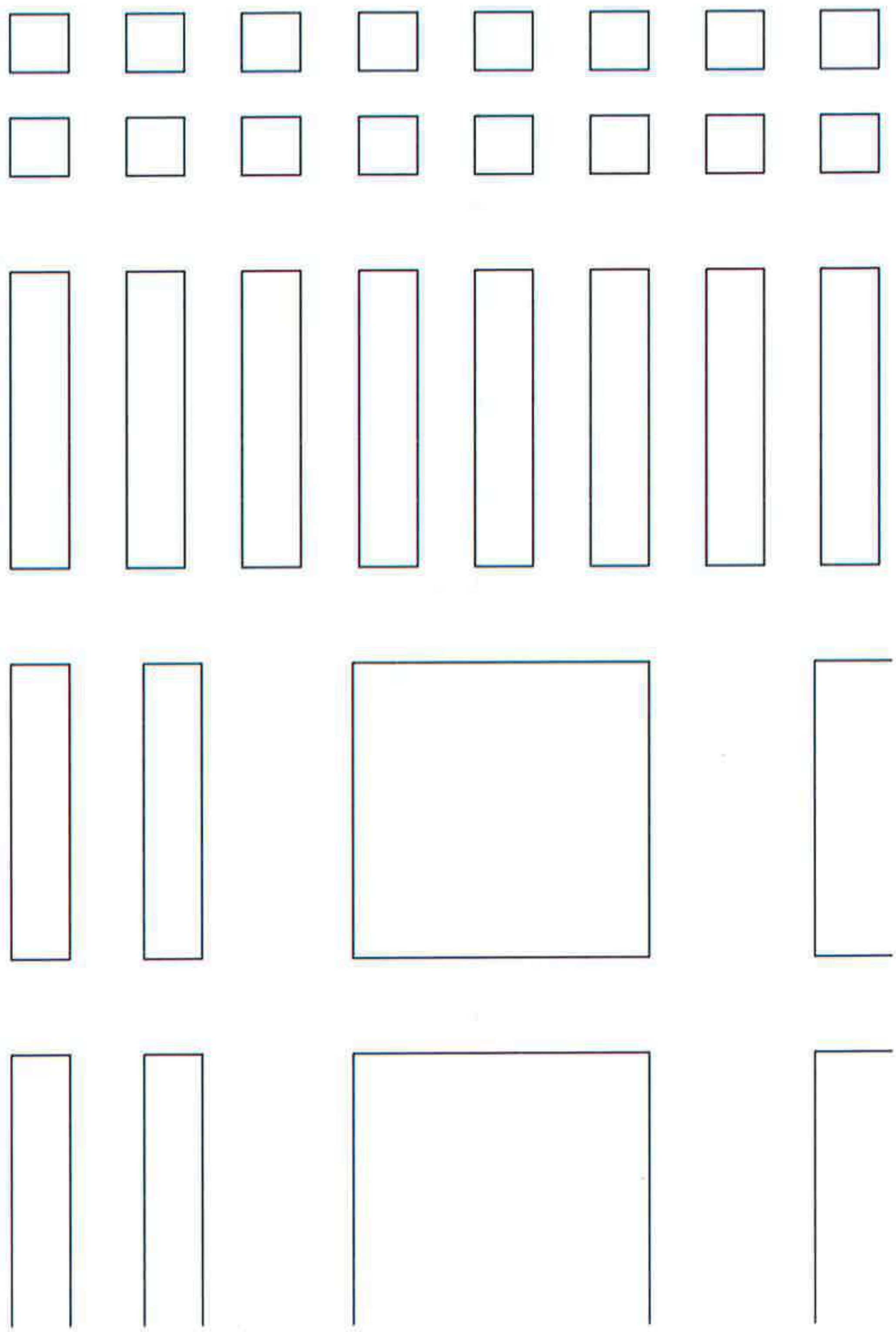
Representati

Scoring Guide	Exemplary	Proficient	Emer
	The solution demonstrates the following characteristics:		
Mathematics Knowledge and Thinking (Items 1, 3–7)	• Clear and accurate identification of key features of the function and its graph, including domain, range, y-intercept, maximums, and minimums	• Correct identification of most of the key features of the function and its graph, including domain, range, y-intercept, maximums, and minimums	• Partially corr identificatior key features and its graph domain, ranc maximums,
Problem Solving (Item 11)	• Appropriate and efficient strategy that results in a correct answer	• Strategy that may include unnecessary steps but results in a correct answer	• Strategy that some incorre
Mathematical Modeling / Representations (Items 8–10, 12, 13)	• Effective understanding of how to complete a table of real-world data, and how to write, graph, and interpret the associated function • Fluency in translating a graph and writing the associated function	• Largely correct understanding of how to complete a table of real-world data, and how to write, graph, and interpret the associated function • Little difficulty translating a graph and writing the associated function	• Partial under how to comp real-world d write, graph, the associate • Some difficul graph and w associated fu
Reasoning and Communication (Items 1–3, 5, 7, 13)	• Precise use of appropriate math terms and language to describe key features of a graph and to explain how a function rule was determined from a translated graph • Clear and accurate interpretations of the graph of a function	• Adequate description of key features of a graph • Reasonable interpretations of the graph of a function • Adequate explanation of how a function rule was determined from a translated graph	• Confusing de features of a • Partially corr interpretatio of a function • Confusing ex how a functi determined f translated gr

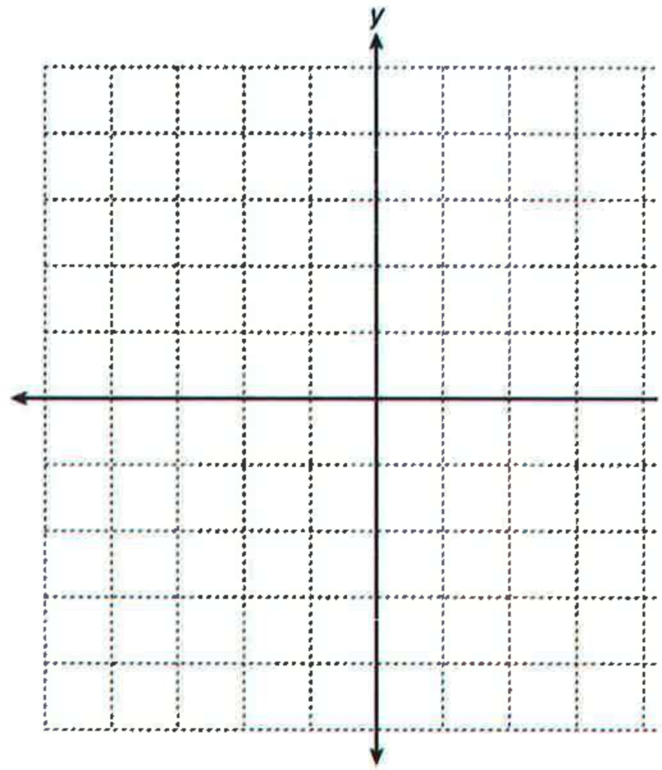
Common Core State Standards for Embedded Asse

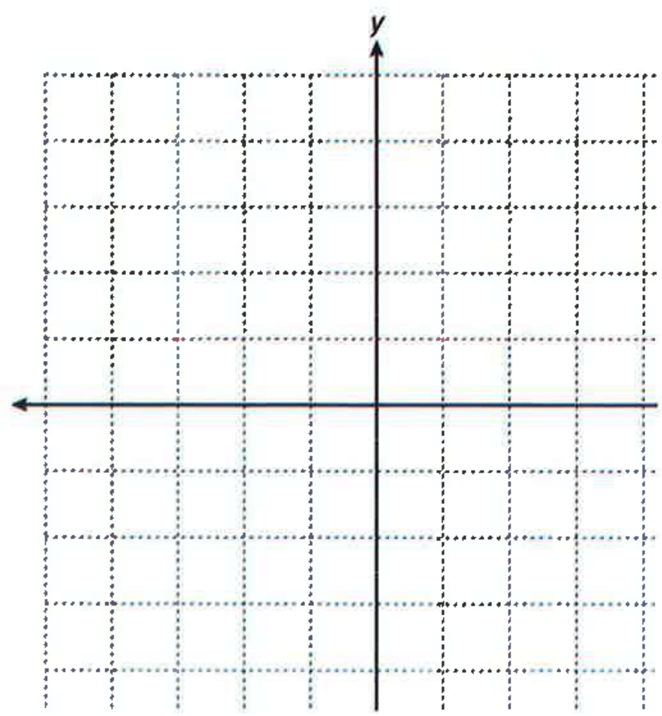
HSF-IF.B.5: Relate the domain of a function to its graph and, where ap

Algebra Tiles

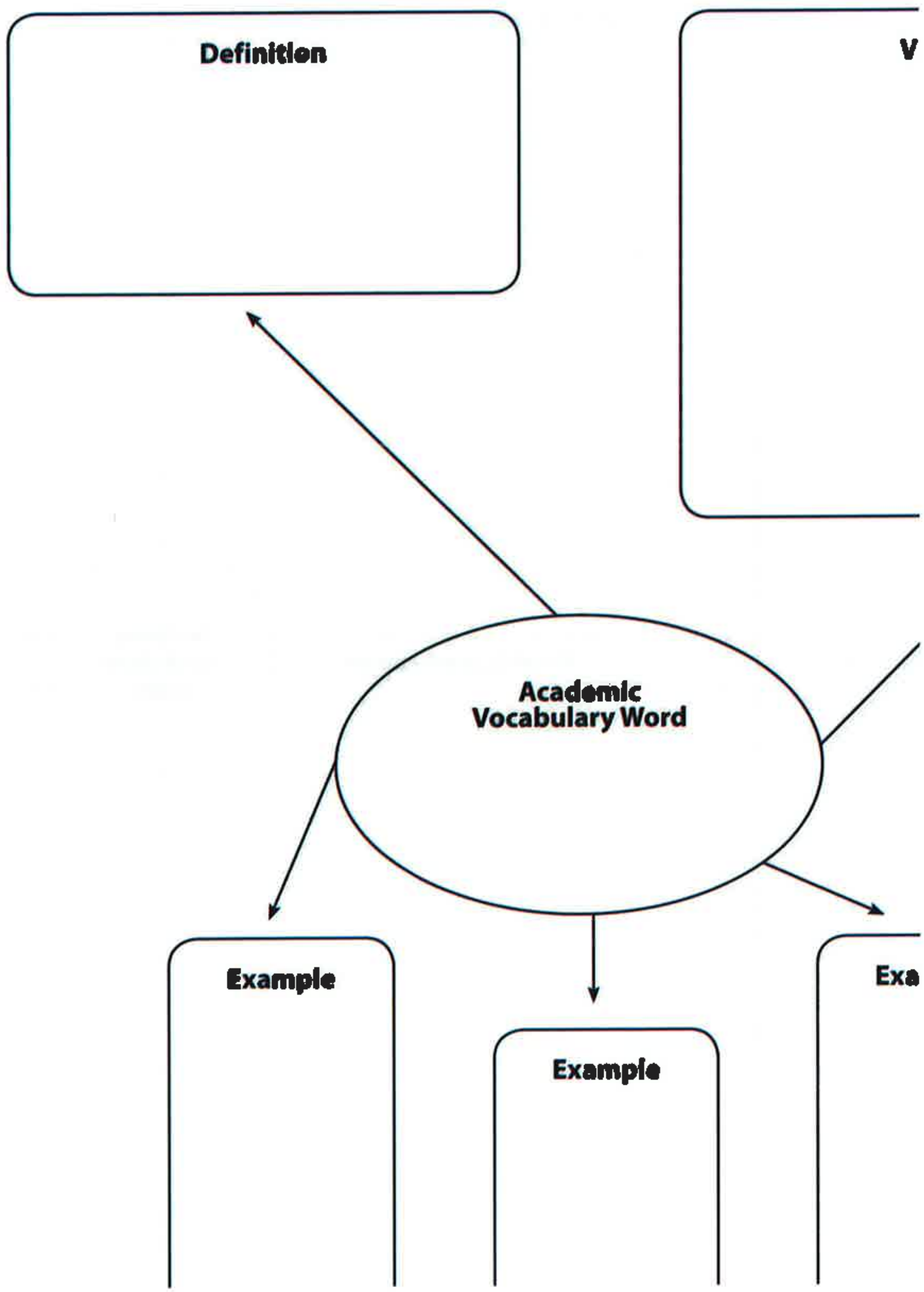


Tables and Coordinate Grids





Word Map


Definition
V
Academic Vocabulary Word
Example
Example
Exa

Verbal & Visual Word Association

Definition in Your Own Words

Important Elem

Academic Vocabulary Word

Visual Representation

Personal Associa