Criterion Three: Mission, Goals, Educational Objectives

Mission

The New Maurice J. Moyer Academy's mission is to provide opportunities for students to achieve the highest academic and personal goals. The mission underscores the importance of its research-based curriculum, which offers traditional classroom instruction, K-12 world class technologies, and data-driven instruction while recognizing and meeting the unique, learning differences of each student.

Brief Summary of Modification

The Board of Directors for the New Moyer Academy is requesting a modification to the original charter application. For our high school classes, the requested modification specifically addresses the conversion from an online instructional program to a "traditional", teacher directed instructional program for our core courses (mathematics, English, history, science). For our elective classes, students will be engaged in "Blended Learning" classes. A blended learning environment will be created by students receiving instruction online and having onsite teachers provide the necessary support for students. In both electives and core courses, students will use the award-winning K12 curriculum. For elective classes, students are engaged through online lessons; whereas, for the core courses, teachers will use interactive whiteboards to deliver instruction. The structure of the middle school is not impacted by the requested modification.

Rationale Behind the Modification Request

The Board of Directors for the New Moyer Academy is requesting a modification to the original charter application due to the historical underperformance of Moyer students both behaviorally and academically. The Board of Directors felt that the lack of student engagement in a "whole school" blended learning program (high school only) resulted in high failure rates and an overall disorderly school environment. With this said, the Board of Directors understands the potential of exposing our students to "blended learning", but would like to limit its use to elective classes only.

Curriculum and Instruction

At The New Moyer Academy, our instructional program focuses on maximizing student achievement via instructional best practices and blended learning strategies. To support this goal, we set rigorous standards and provide high-quality curriculum and ongoing development. Our teachers regularly analyze student data to drive our daily instruction and long-term planning. We work to ensure that every student achieves.

Curriculum

Moyer will be able to offer more than 100 K12 elective courses designed to help students earn their high school diploma and find their own path to post-high school success- whether in college or the workforce. K12 will offer math, English, science, and history courses, directed by teachers using interactive whiteboards. To meet the needs of struggling learners, students can take remediation and credit recovery courses. Through the K12 curriculum/learning platform, students can take up to four years of a world language and choose from a variety of electives, including anthropology, Web design, Entrepreneurship, and Green Design & Technology. Unlike other programs, where a student must be in a particular "academic path", the K12 program provides students with more autonomy over charting their own academic course**.**

Our Instructional Program

Measurable, Standards-Aligned Classroom Objectives: Regardless as to receiving online or traditional styled instruction, every class has a measurable standards-aligned daily objective. Whether the objective is to analyze the characteristics of a novel or solve a quadratic equation, the objective drives the content in the lesson. Using multiple forms of assessments at various times during the instructional period, teachers track their students' mastery of the objective and progress toward their academic goals**.**

Modeled and guided practice: Teachers introduce the objective by clearly modeling a process to attack the day's objective. Then, the teacher leads the class into a guided practice of the objective, in which students model their own answers and thought processes, the goal of which is to ensure that every student is set up for sustained, successful independent practice.

Independent practice: Students spend the bulk of the lesson in a sustained, successful independent practice, with several attempts to practice the objective on their own. In other words, students do the "heavy lifting" of the task at hand, getting several chances to flex their academic muscles until they get it right.

The Use of the Interactive White Board: For core courses, the interactive whiteboard will serve as primary mode for delivering instruction. Through the K12 online curriculum, teachers have access to thousands of resources simply by "pointing and clicking" on the interactive whiteboard. As a part of instructional lessons, students will be engaged in virtual tours, pictures, artifacts, differentiated assignments, etc. While we believe teachers to be content area specialists, using interactive whiteboards as a resource strengthens the educational experience for all students**.**

Students Use of the Interactive White Board: We believe that students benefit from delivering instruction. At the New Moyer Academy, students receive training in their use of the interactive whiteboards. During Moyer's orientation process, students are trained in the appropriate use of the whiteboards. In addition, students are provided with an overview of the various tools and research power of the whiteboards**.**

Blended Learning: With the goals of capturing the educational interest for students, they are given the opportunity to select from over 100 online elective classes. Since this mode of learning is relatively new, students participate in a comprehensive training to orient them to the online learning platform. During the year, certified online teachers engage students in instructional lessons. Like in the "traditional classrooms", online teachers use objectives to drive their instructional lessons. During lessons, students are engaged in online "chats", discussion boards, information postings, and assignments with students from across the country**.**

Standards-aligned assessments, data-driven analysis and planning: At The New Moyer Academy, we will use four interim assessments to drive our instructional program. Each interim assessment will be used to determine the level of mastery of state standards by each student. This data provides teachers the information they need to edit their unit plans, target their instruction and schedule interventions for struggling students. This data-driven lens on instruction gives teachers the unique flexibility to cater the curriculum for each individual student and anticipate challenges.

Professional Development

At The New Moyer Academy, we recognize that teachers are the key to maximizing student achievement. Our teachers will participate in a comprehensive professional program in the summer. During this session, teachers will be oriented to high leverage instructional practices, the use of data to guide

instruction, and the daily routines and rituals to ensure the needs of all students are being met. Throughout the school year, teachers will continue to participate in ongoing professional development experiences to strengthen their instructional repertoire.

Healthy School Culture

A healthy School Culture is the foundation for the success of any school. Our school culture plan is designed to both systematically recognize students for both academic and behavior success and progressively discipline students who violate school policies.

At the New Moyer Academy, there is purposeful initiative to systematically recognize student success, both academically and behaviorally. It is our belief that recognizing students' success will result in additional students striving to modify their own behaviors to be recognized. Through our regular, systematic approach to recognizing both academic and behavioral success, we will be establishing a behavioral and academic standard in the school. The following activities will be conducted to recognize student success.

- Biweekly Community Meetings, Quarterly Awards Ceremonies, Student Recognition Postings, Bulletins, Special Monthly Activities

Interpretation of the Mission

The mission from the original charter has not changed. The original mission statement is completely applicable to the requested modifications to the new instructional model. For the sake of clarification, each area within the mission statement will be addressed. The New Moyer Academy will**….**

Use a Research-based curriculum: K12's curriculum is research based. In addition to teacher directed instruction using the K12 curriculum, students will also be taking online elective classes using "Virtual" teachers**.**

Offers traditional classroom instruction: Traditional (teacher delivered) instruction for core classes will occur in grades 6-12**.**

Use K-12 world class technologies: Students will use computer for both LMS and OLS platforms for their elective classes. Since K12 has developed the infrastructure to provide an entire curriculum online, this technology is only possible through K12. Some of the benefits to using K12 online classes are that students are able to track their academic performance, teachers and parents are provided with immediate data, and teachers can make ongoing/immediate decisions based on the data stream that is generated**.**

Use Data-driven instruction: Through both the traditional and online instructional models, teachers (onsite and online) use data to inform their instructional decisions**.**

Recognize and meet unique, learning differences of each student: We recognize unique student needs in a variety of ways. Instruction, both onsite and online, is differentiated based on students' instructional level. The degree to which we support students with special needs is detailed in Criterion 7. Through our online courses (electives), students have the ability to select from over 100 online classes, based on their unique, educational interests. Even though the new model utilizes teachers to deliver our instructional program, it is still very unique and utilizes the innovative nature of the K12 technological platform. During the tenure of our high school students, they can participate in up to 30% in a "blended learning" environment. Specifically, students will receive instruction in online classes and be supported by onsite staff. Due to the low number of high schools across the country that participate in a blended learning model, this is considered unique. Additionally, students have a robust number of elective courses from which to select (over 200). This is also a unique feature that supports innovation. In a traditional classroom/learning environment, students would be limited to taking only the courses that the

individual school offers. While participating in elective classes online, students will be engaged with students from across the country, not just students within Moyer or an individual classroom**.**

The modified instructional model was implemented at Moyer in August of 2012. We felt that the modification was needed to better meet academic and social needs of students. Based on academic and behavior data from SY 2011-2012, we were concerned with our DCAS and behavior data. Academically, students were not excelling in their core courses, as the online courses were intended to promote. Based on discipline data, students were not engaged in the online courses, resulting in a high number of student referrals and an unacceptable amount of instructional time being wasted.

By implementing a modified instructional program, we have experienced drastic improvements in both academic and behavior data. When comparing our DCAS scores from Spring of 2012 to spring of 2013, our students' scores improved in both mathematics and reading by over 20 percentage points, 41.6% and 53.8%, respectively. Socially, disciplinary referrals are approximately half of what they were last school year (4 per day)**.**

Purpose

The purpose of the mission is to develop, in students, a solid foundation in reading, writing, calculating, computing, and synthetic thinking so they visualize themselves as competent, confident learners and persons who are well prepared for postsecondary options in higher education, for the workplace, and for the responsibilities of good citizenship. The school's purpose is rooted in a belief that respect and responsibility are fundamental elements of the mission, where all Moyer stakeholders, students, teachers, parents, and governing board demonstrate respect for themselves and others, utilize respectful behaviors in all interactions, and take responsibility for their actions. Central within the definition of respect and responsibility are the following**:**

Students who are ready to learn and prepared to adhere to the Moyer Code of Conduct
Parents who support and promote the Moyer academic and behavior culture
Teachers who are committed to delivering differentiated and essential learning
A governing Board of Directors dedicated to upholding high academic and behavior standards
A school community built upon consistent communication and common goals
A learning environment where students have opportunities outside the classroom to deepen their knowledge

All of the information reported within the "Purpose" section will remain the same. The modification request does not impact the Purpose**.**

Goals

The mission and purpose of The New Moyer Academy define the framework for our goals which are considered in three broad areas listed below.

Educational Goals
Each student will achieve 80% mastery of all learning objectives measured by the teaching staff of The New Moyer Academy.
Each student will be able to make real world connections to their learning.
Each student will value their Moyer education as the first step in their progress as lifelong learners.
Each student will have the skills to measure their mastery of learning and their achievement.
The school and parents will support common goals of high academic achievement and a positive school culture.

The school will partner with parents to support and celebrate high academic achievement, improved effort, and hard work.

Social, Emotional, Health Goals

The New Moyer Academy will utilize a positive behavior support will be at the core of the school's culture. This will include activities such as honor roll, student and teacher recognitions, athletic events, and program related field trips.
Students will have opportunities for training, skill building, and participating in peer support activities.
The school will schedule at least four assemblies each year featuring distinguished speakers.
The school will promote special programs which respond to the diverse needs of the school's 6th grade, middle, and high school populations.
The school will promote special programs to include wellness which responds to the diverse needs of all students in grades 6-12.

Civic Responsibility Goals

Students will be required to participate in a school community service project at approved community locations.
Middle school and high school student government organizations will be given a role in school decision making and model the American tradition of representative government.
The school will build a unified, shared culture based upon the concepts of respect, responsibility, and integrity.
The code of conduct is administered in a fair and consistent manner.
Diversity of thought, belief, and background are valued.

All Goals will remain the same. The requested modifications will not impact the goals of The New Moyer Academy. The modification to our educational plan makes the goals more attainable**.**

Core Philosophy

The mission, goals, and educational objectives are consistent with the legislative intent of 14 Delaware Code, Section 501, and the restrictions on charter schools set forth in Section 506 as follows:

K12's curriculum is research-based and aligned to Delaware State Standards
The mission and goals provide a unique instructional approach combining both traditional classroom and online instruction.
Learners see, do, act, and interact through classroom activities and online programs. The diversity of learning modes provides students with many more opportunities for achieving proficiency.
Parents have access to the online curriculum.
Through information retrieved from data, students, parents, teachers, and the staff are able to respond readily to individual student needs and adjust learning to meet those needs thereby creating a solid foundation in learning.

Consistent with the requirements in 14 Delaware Section 506, The New Maurice J. Moyer Academy is an open enrollment charter school and will not discriminate in its admissions or school policies because of race, creed, color,, gender, handicap, or national origin, or because the students' school district of residence has a per student local expenditure lower than another student seeking admission. The school will not charge tuition, will not be home-based; and will not engage in any sectarian or religious practices in its educational program, admissions policies, employment policies or operations.

In accordance with 14 Delaware Code, Section 506(b), The New Moyer Academy will give enrollment preference to enrolled students; students who are siblings of a student enrolled at Moyer, students who are children of a Moyer employee employed on a permanent basis for at least 30 hours per week during the school year, and those who have a specific interest in online learning as provided in 14 Delaware Code, Section 5069b)(3)9c): "Students enrolling in a new (non converted) charter school may be given preference under the following circumstances as long as the school has described its preferences in the school's charter: (c) Students who have a specific interest in the school's teaching methods, philosophy, or educational focus**".**

Criterion Four: Goals for Student Performance

The New Moyer Academy will use the Charter Performance Framework to evaluate the school's academic progress. The chart below will serve as our academic target in the respective areas. With the State of Delaware transitioning to Smarter Balanced Assessments in SY 2014-2015, at this time, academic targets have not been established for SY 14-15.

DE Performance Framework Indicators	2012 Benchmark	2013 Target	2014 Target
DCAS Performance			
% of students meeting DCAS Math Standard	22.6%	32%	42%
% of students meeting DCAS Reading Standard	32.7%	42%	52%
% of African American students meeting DCAS Math Standard	22.1%	32%	32%
% of African American students meeting DCAS Reading Standard	30.4%	40%	50%
% of Sp Ed students meeting DCAS Math Standard	-	20%	40%
% of Sp Ed students meeting DCAS Reading Standard	-	20%	40%
% of Low SES students meeting DCAS Math Standard	21.3%	31%	41%
% of Low SES students meeting DCAS Reading Standard	31.5%	42%	52%
% of high school students meeting a score of 1550 or better on the SAT	0%	10%	20%
% of 12th grade students graduation from high school	50%	80%	90%

Performance Plus Assessment	2012 Baseline	2013 Target	2014 Target
% of students making "Student Progress Over Time" in Math	30%	40%	50%
% of students making "Student Progress Over Time" in Reading	36%	46%	56%
% of "lowest-performing students meeting fall and spring instructional scale scores in math	50%	60%	70%
% of "lowest-performing students meeting fall and spring instructional scale scores in reading	43%	53%	63%
% of students making annual growth to achieve proficiency status within 3 years in math (by 10th gr)	32%	42%	52%
% of students making annual growth to achieve proficiency status within 3 years in reading (by 10th gr)	49%	59%	69%

Our School Improvement Plan, which included these targets, was submitted on April 15, 2013. Prior to its submission, we were told that our academic targets need to remain consistent. This was a result of discrepancies between academic targets on our approved Focus School Grant and subsequent submissions of academic targets. Based on the benchmark data from 2012, we used a standard 10% increase for each subgroup/per year as our growth target. In the DE Performance Framework, AYP is used as a measurement. Historically, the reduction of 10% of non-proficient students in both whole groups and subgroups resulted in a school achieving AYP status via Safe Harbor. By using AYP as a standard, an annual growth of 10% on standardized tests, demonstrated significant growth. For SY 2012-2013, we have demonstrated significant gains in both mathematics and reading. In hindsight, we could have

achieved our academic targets even if they were higher; however, we based current targets on baseline data generated by historical DCAS performance trends and DCAS data from SY 2011-2012. On DCAS in both math and reading and all subgroups, Moyer demonstrated a poor record of academic performance over the past 3 years. As a result of our historical record, we intentionally set low academic targets in an attempt to establish the foundation to drive slow and sustained academic growth. Admittedly, it was an extremely conservative approach.

On DCAS for SY 2012-2013, our special education proficiency rates were 24% and 14% in reading and mathematics, respectively. In prior years, 0% of our students were proficient in reading or mathematics. Like our academic targets for the overall student population, we were conservative when establishing academic targets for students who receive special education services**.**

Our School Improvement Team meets 4 times per year to monitor our progress in all areas addressed in our local school plan. With the academic gains Moyer students have demonstrated during SY 2012-2013, our School Improvement Team will need to determine more rigorous academic targets for our overall population and each subgroup. The School Improvement Team will convene a meeting during the month of August to establish more rigorous instructional targets**.**

Criterion Five: Evaluating Student Performance

Based on the academic data from our current students at The New Moyer Academy, the vast majority of our new enrollees are significantly deficient in both mathematics and reading skills. This requires a variety of learning interventions to support students. These interventions include after school tutoring, individual or small group support from math and ELA specialists, computer based programs through K12 designed to support learning, Achieve 3000, and Compass Learning. Our plan is to understand students' needs and tailor an instructional program to address them.

We will evaluate students' academic performance in a variety of ways. At the start of each school year, students will be administered a Scantron Performance Assessment (K12 assessment). By determining a benchmark for measuring growth at the start of the school year, Moyer can continually measure the effect of our educational program as it relates to student growth. The Scantron Performance Assessment is also administered during the last month of the school year to measure the amount of student growth that occurred during the course of the year.

In addition to the Scantron Performance Assessment, students will participate in DCAS, Delaware's statewide standardized assessment. The students in grades 6-10 will participate in all rounds of DCAS testing. For example, if the DCAS testing window allows for 3 rounds of DCAS, the students of The New Moyer Academy will take 3 DCAS tests within a given school year. Students will also participate in DCAS Social Studies and DCAS Science tests. Having students participate in the DCAS testing program provides additional data points that drive our instructional program. After each DCAS assessment, teachers will conduct individual conferences with students to set academic goals for the subsequent assessment.

Starting SY 2013-2014, our students will be administered four interim assessments during the course of the school year. These assessments will be based on DE Learning and Common Core Standards. The use of the interim assessments to drive our instructional program is based on the research of Paul Bambrick-Santoyo. The use of interim assessments is a component of a data cycle that ultimately uses student data to drive the instructional program. Interim assessments also generate student friendly data in which students utilize to understand their strengths and weaknesses, maximizing their learning opportunities. Students in grades 6-8 will be administered assessments in both mathematics and reading.

Through the K12 curriculum, ongoing formative assessments will be administered to students. These assessments include activities to measure learning on a daily basis, as students complete their assignments, mini unit assessments (weekly), and major unit assessments administered between interim assessments. Teachers will work to ensure the alignment between all of the various assessments.

For ELA and mathematics, formative and official interim assessments will serve as the basis for ongoing instructional decisions. Formative (ie-daily exit slips, quizzes, and unit tests) and interim assessments (4 in both ELA and math) will both be aligned to DE Learning and Common Core Standards. Each of these assessments will be designed to assess specific learning standards. Regarding the formative assessments, students who demonstrate mastery in a particular standard will advance to the next standard; whereas, students who are deficient in certain standards, will be re-taught the standard, until mastery is achieved.

For the interim assessments, an 8 week data cycle is used to drive instructional decisions. Approximately every 8 weeks, teachers will administer interim assessments to students. These interims are standards based and completed aligned to DE teaching standards. The interim assessments generate specific data as to the mastery of each standard. As a part of the data cycle, teachers analyze trends related to correct/incorrect answers and the rationale behind incorrect/correct answer selections. In addition to their analysis, they conduct a self reflection as to their teaching strategy and how it promoted correct or incorrect answer selections. Teachers also participate in an exercise of estimating "proficiency" rates on the interims, comparing their expected understanding of student mastery to actual levels of student mastery. Finally, teachers dedicate a period time (4-5 days) where they re-teach standards that students demonstrated deficiencies. We expect interim assessments to improve students' performance on both DCAS and Scantron Assessments.

I have included an assessment calendar below.

Interim Assessment Calendar

Time Frame	Unit or Assessment	Notes:
8 Weeks (8/25-10/10)	Unit 1	Approximately 1 hour per assessment, Aligned State Test objectives for 8 weeks
3 Days	Interim Assessment #1	Re-teach based on test results analysis
1 Week	Re-Teach Objectives from Interim Assessment #1	
7 Weeks	Unit 2	
4 Days	Interim Assessment #2	Cumulative: All objectives from units 1 and 2
1 Week	Re-Teach Objectives from interim assessments 1 and 2	Re-teach based on test results analysis
6 Weeks	Unit 3	
4 Days	Interim Assessment #3	Cumulative: All objectives from units 1-3:

1 Week	Re-Teach Objectives from units 1-3	Re-teach based on test results analysis
6 Weeks	Unit 4	
1 Week	Interim Assessment #4	Cumulative: all objectives from units 1-4
2-3 Weeks	Re-Teach objectives from Units 1-4	
DCAS (State Test) (5/1-5/15)	DCAS State Test	
5 Weeks	Unit 5	
1 Week	Final Performance Task Preparation	
1 Week	Final Performance Tasks	Oral Presentations and large math projects

Teachers will use the following tool for effective planning (post interim assessment)

Interim Assessment Analysis Sheet and Instructional Plan

Assessment Number and Subject: _____

Class/ Grade:

Date:

Standard Analysis	Analysis of Why Students Did Not Learn Standard	Instructional Plan-What techniques will you use to address these standards?
What standards warrant more time for whole-class instruction, re-teaching and review		

Whole Class Instruction: • • • • • •		
Small Group Instruction:		Instructional Plan: How or when will you structure small group instruction?
What Standards warrant more time for small-group instruction and review?		
• • •		

Criterion Six: Educational Program

Curriculum:

In grades six through eight, K12 courses are categorized into four core courses: language arts/English, mathematics, science, and history (social studies). Students in grades 6-8 will also participate in health and physical education, world drumming, and art courses appropriate for the Delaware Content Standards at their grade level. The K12 curriculum includes all of the courses that students need to complete their 6th through 8th grade education. These courses focus on developing fundamental skills and teaching the key areas, meet state standards and complete more advanced coursework. The curriculum is mastery-based with assessments built into every lesson to ensure mastery and provide for remediation or enrichment where necessary. Students at Moyer will use the language arts/English, mathematics, history and courses from K12 and science curriculum for the Delaware Science Coalition. K12's language arts and mathematics curriculum are aligned with the Common Core Standards.

Mathematics

Building upon the success of long established program, K12 math balances mastery of fundamental skills with critical thinking and problem-solving. K12 math emphasizes an active, multi-sensory approach to ensure that students understand the concrete realities that underlie mathematical concepts. Regular practice and review ensures mastery of basic skills. Online educational games and animations motivate students and help illustrate concepts, while challenge problems help develop critical thinking skills. From helping younger students make the link between the concrete and abstract to introducing older students to Algebra, K12 Math provides a thorough mathematics foundation. K12's mathematics curriculum is aligned with the Common Core Standards.

Science

The New Moyer Academy has signed a Memorandum of Understanding with the Science Coalition. The Science Coalition has provided science curriculum units, materials, and assessments for all students and

ongoing professional development including instructional coaching for Moyer teachers. The Coalition instructional science coach will incorporate the use of technology in his/her communications with Moyer teachers. Moyer administration accepts the responsibility for releasing teachers for professional development, even if sessions occur during the instructional school day. The administrators at Moyer will fulfill the mandate of science teachers attending professional development sessions.

Language Arts/English

K12 language arts/English curriculum helps students develop important reading and writing skills, while also inspiring a love of literature. Combining Literature, Language Skills, and Spelling Lessons, the Language Arts/English program emphasizes classic works, teaches writing as a process, and prepares students for standardized tests in the areas of language skills and reading comprehension. Students develop literary analysis and comprehension skills by reading novels and nonfiction work.

History (Social Studies)

With integrated topics in geography and civics, K12 History opens students' minds and imaginations to far-off lands, distant times, and diverse cultures. K12 emphasizes the story in History, a story that includes not only great men and women but also everyday people.

Curriculum in Grades 9-12

The New Moyer Academy will offer more than 130 K12 high school courses designed to help students earn their high school diploma and find their own path to post-high school success-whether that's in college or in the workforce. At Moyer Academy, instructional content will be delivered using both traditional (teacher delivers lessons) and computer based (blended learning) modes of instruction. In the core courses (math, English, history, and science), teachers, certified by the State of DE (onsite) will deliver the K12 instruction, using interactive whiteboards. For their elective classes, including world languages, students will receive their instruction via computer based curriculum. In K12's blended learning environment, our high school students can select from more than 130 courses. In addition to the K12 online course offerings, students will receive onsite instruction from our world drumming, physical education, and art teachers.

<u>High School Courses by Grade Level</u>

Grade 9	Grade 10	Grade 11	Grade 12
Algebra I	Geometry	Algebra II	Math Elective
Literary Analysis I	Literary Analysis II	American Literature	British Literature
Physical Science	Biology	Chemistry	Science Elective
World History	US History	Social Studies Elective	
Career Pathways	Career Pathways	Career Pathways	
World Language	**World Language**	**World Language**	**World Language**
Physical Education	**Physical Education**	**Physical Education**	**Physical Education**
Health Education	**Health Education**	**Health Education**	**Health Education**

Elective	**Elective**	**Elective(s)**	**Elective(s)**

- Courses in Bold require less than 4 credits (for graduation) in their respective discipline, resulting in optional scheduling for each semester.

Special Material and Use of Technology

Every classroom in The New Moyer Academy will be equipped with Internet access, an interactive whiteboard, and a teacher laptop. Also, the facility will be equipped with two computer laboratories. For all core courses, onsite teachers will access lessons and instructional resources online. The lessons for students in grades 6-8 are housed in the K12 Online School (OLS), and teachers will use the laptop and interactive whiteboard to deliver these lessons to the entire class. The OLS and interactive whiteboard allow for increased visualization and interaction with the material compared to a traditional textbook.

The lessons for students in grades 6-12 are housed in the eCollege Learning Management System (LMS). Like in grades 6-8, core courses (English, Math, History, and Science) will be taught by using teachers to deliver lessons to the entire class via their laptop and interactive whiteboard. For elective classes, students will use the LMS to view their material and assignments for the week, submit assignments, and view their grades for the assignments. For elective classes, students in high school will receive instruction from Highly Qualified, Delaware-certified virtual teachers. The courses will be conducted in the computer lab, equipped with desktops and networked printers. Instruction will be given via Elluminate Live!, a virtual classroom that allows for delivery of instruction, student collaboration with peers, and students' demonstration of knowledge and skills.

Mobile laptop carts will be available for classroom use. In certain instances, some students may progress at different rates through a particular lesson(s), an individual laptops provide students with the means to work at their own pace. Students will also use the laptops to complete research papers and other projects. Students will have access to many technology courses as one of their electives, or as part of their Career Pathway. These classes will be conducted in the computer lab. All desktops in the lab (as well as all teacher laptops and mobile laptops) will be equipped with Microsoft Office Suite. During this time, as well as during daily use of technology during instructional time, students will be exposed to the International Society for Technology in Education's National Education Technology Standards (NETS). A deep understanding of the NETS will help to ensure that students are college and career-ready upon graduation.

High Risk Students: In the original charter application, it was stated that Moyer expected "85% of new students to be At Risk". Based on current enrollment trends, approximately 40% of our students are significantly deficient in mathematics and/or reading. We determine student's academic levels by analyzing a variety of assessments (DCAS, Scantron Performance Series), academic grades, and feedback from teachers. Within the first 6 weeks of school, teachers, school counselor, and administrator(s) conduct formal meetings to consider the various support systems for students who are identified as At Risk. After the initial meeting, the team of educators will meet with the student and parent to determine the appropriate supports needed to address the child's academic and social needs. The parent and student are encouraged to provide feedback as to the support of the student, and commit to participating in the various programs that the staff of Moyer will offer. These offerings may include but are not limited to Achieve 3000, Compass Learning, after school tutorials, and individual or small group instructional lessons with math and ELA specialists. K12 also offers a variety of computer based instructional support programs that may be utilized.

The academic and behavioral progress of students who were determined to be at risk will be regularly monitored by our school guidance counselor (weekly). On a quarterly basis, the student's teachers, counselor, parents (and additional relevant staff) will meet to discuss the student's progress. Depending on the report, the student's educational plan may include additional academic supports as listed in the prior paragraph. Based on findings, students may also be required to receive counseling, commit to behavior contracts, be assigned a tutor, etc. We will attempt to provide all appropriate supports to promote the social and academic growth of each student.

High School Graduation Requirements

Course	Required credits in SY 13-14	Required credits in SY 14-15
Math	4	4
English	4	4
Social Studies/History	3	3
Science	3	3
Foreign Language	0	2
Physical Education	1	1
Health	.5	.5
Electives	3.5	3.5
Career Pathways	3	3
Total Credits Required for Graduation	**22**	**24**

Student Teacher Ratio: The student to teacher ratio for Grade 6 is 20:1. The student teacher ratio for grades 7-12 is 25:1.

Unique features of the school:

School-Within-a School

Moyer's school design is based upon a research-based school-within-a school model. This model provides small learning communities called "Houses" within the larger school and promotes a greater sense of personal connections among students, faculty, and families.

The House Plan

Moyer will be led by the school principal who is the instructional leader of the school. As instructional leader, the principal is responsible for establishing and ensuring the school lives up to its vision and mission. The principal is responsible for maintaining high standards for instruction and learning, ensuring smooth daily operations, and overseeing the fiscal soundness of the school's operating plan. The principal also provides leadership for all external relations, such as school business partnerships, external community relationships, extracurricular activities, and school wide outreach.

In order to foster more direct and sustained connections, students and teachers are assigned to one of several smaller groups call Houses. Each section (based on grade/class) will be named to a House. To serve as a constant reminder as to the importance of higher education, each House will be named after a college or university. For each grade level, one staff member will serve as House leader. The House Leader (team leader) will serve as the lead contact for matters dealing with academic and social issues. While the House Leader deals with minor social issues, he/she does not serve in the role of the Dean of Students. The House Leader reports directly to the principal and is also expected to teach students throughout the school day. The support services (within the school) available to help students and staff are guidance counselor, school nurse, dean of students, assistant principal, and principal. The continuity of learning and counseling provided by the House system supports safety, security, and student achievement.

Students will remain in the same House from grades 6-12. Change of House may be considered at the request of a student, parent, teacher, or administrator. The principal has final decision making authority over such requests. Sixth grade students are self-contained classroom where core subject instruction is delivered by one classroom teacher. Beginning in grade seven, classes are departmentalized, resulting in students moving from class to class.

How does Moyer Deliver its Instructional Model in this Environment?

In our core courses (English, math, science, and history), students will receive their instruction in individual classrooms. Each classroom is equipped with a functional whiteboard connected to the Internet. Teachers and students will work through lessons using the interactive whiteboard as the central point of their lessons. Additional instructional materials, text books, novels, consumable materials will also be used to supplement our instructional program

For elective classes, students will be engaged in online classes, delivered in our state of the art computer lab. Students will use our online management system to take elective classes of their choice. The computer lab is staffed by both a certified teacher and instructional coach. Both are trained to support students through technical challenges along with instructional support. A full-time technology specialist is assigned to Moyer to ensure the online management system remains functional at all times.

High Quality Professional Development Calendar:

In SY 2012-2013, the teaching faculty at The New Moyer Academy were inexperienced. Specifically, over 60% of teachers have less than 2 years of teaching experience. For the sake of perspective, during SY 2011-2012, over 85% of teachers were first year teachers. While our teachers are becoming more seasoned, we recognize that they lack the repertoire of instructional strategies that support rigor and high expectations. As a teaching faculty, best practices need to be modeled, shared, and replicated to meet the instructional needs of students.

Several books will be used as the foundation for our instructional practices. Teach like a Champion (Doug Lemov) and Skillful Teacher (John Saphier) will be utilized to strengthen the instructional repertoire for teachers. Starting in March 2013, teachers began to participate in professional development sessions using Teach like a Champion. Several times per month, professional development sessions, led by the principal, will be conducted. During these sessions, teachers will be provided with 2-3 instructional strategies that support high level instructional practices.

Subsequent to the professional development sessions, the administrators are expected to provide short observations in each classroom to ensure that the instructional practices are being implemented. This practice of using brief observations (and providing targeted feedback) to drive instructional practices is supported by the research in Rethinking teacher supervision and evaluation (Kim Marshall) and The Principal's Cycle (Keith Stephenson).

During the summer of 2013, all teaching staff will participate in an extensive, one week professional development session. The intent of the session is to establish high expectations around instructional practices, review of the interim assessment cycle, standardize school routines, and provide an overview of school policies.

The teachers' master schedule will be designed to support collaboration. Whenever possible, teachers will share a common planning period with their interdisciplinary team members. While the teaching teams are small, interdisciplinary teams will plan collaboratively at least one planning period per week. In addition to planning together, teachers will conduct peer observations 3 times per year. These observations will serve to support collaboration within instructional teams.

Our comprehensive professional development plan places an emphasis on best practices for delivering instruction and collaboration within our staff. By utilizing research based instructional practices to drive our professional development initiatives, we will undoubtedly develop the pedagogical toolbox for both our instructional staff and administrators within the school.

Teacher Professional Development

Providing Staff with the appropriate professional development opportunities is critical in the growth of our staff. Our professional development program has the goal of driving our instructional program by the use of assessment data. Other than the broad overview of activities that our staff must received to establish the policies of the school, our professional development activities will be driven by student's performance on the interim assessments and the data cycle that supports student's growth. Staff strengths and weakness will be evaluated on a regular basis. If needed, additional staff development in other areas will be provided.

Month	Activity	Resource/Presenter	Audience

August (Pre-Planning)	The New Moyer Academy Orientation	Board President/Administrators/Teachers	All Staff
	Curriculum Overview	Director of Curriculum	All Teachers/Admin.
	Developing Effective Lesson Plans	Director of Curriculum	All Teachers
	Assessment Cycle	Director of Curriculum	All Teachers/Admin
	Differentiated Instruction	Director of Curriculum	All Teachers/Admin
	Mandatory Reporter Training	Nurse/Guidance Counselor	All Staff
	Planning for Summer Bridge	Principal	All Staff
	Interactive Whiteboard Training	K12 Representative	All Teaching Staff
	Anti- Bullying Training	Board Member	
Aug-June	Teachers will participate in professional development sessions using Teach Like a Champion (TLAC) techniques.	Principal/Director of Curriculum	All Staff
Aug-June	Participation in the Data Cycle Lesson Planning Administer Interim Assessment Access/analyze interim data Reflection Goal Setting	Director of Curriculum	All Staff

Principal Leadership

The principal of The New Moyer academy will participate in professional development opportunities. The experiences will work to strengthen the principal's repertoire of leadership strategies related to

instructional leadership, distributive leadership, collaboration, and operations. Many of the professional development experiences have already begun and will continue throughout SY 2013-2014

Time Period	Strategy	Resources
Beginning 3/1/13	Lead Professional Development sessions on TLAC instructional strategies	TLAC Book, Moyer Admin Team, Teaching faculty
Beginning 3/1/13	Participate in State of Delaware PD series related to curriculum and DPAS	DE DOE Professional Development Offerings
Beginning 4/1/13	Principal will participate in visits to high performing charter schools	K12 Inc, cooperating schools
Beginning 7/1/13	K12 supervisor/mentor will provide support to principal in designing master schedule, instructional teams, and assessment calendar	K12 Inc
Beginning 7/1/13	Regular Sessions with supervisor to calibrate his professional practices to meet the needs of the school	Moyer Admin Team
7/1/13 (ongoing)	Principal to participate in mandated teacher planning time	

Instructional Strategies/Formative and Summative Assessments:

For a detailed description of how Moyer staff and students will use both summative and formative assessments, please refer to Criterion Five: Evaluating Student Performance. I have listed both formative and summative assessments that Moyer will utilize throughout the year.

Formative Assessments:
Daily assessments created by teachers
Unit tests generated by teachers
4 Interim Assessments (reading and math)

Summative Assessments:
Scantron Performance Series
DCAS (reading, math, science, history)

WORK PLAN

Focal Point 1: Create a Data Driven Environment to Guide Instructional Decisions			
Year (Time Pd)	Strategies	Resources	Indicator(s)

5/15/13	Submit K12's version of Scantron Assessment to be used for Performance Plus Assessment to DOE for approval (DPAS Component 5 and DE Performance Framework Growth)	K12 Inc & DE DOE	-Timely submission and approval of K12's version of Scantron Test
5/15/13	Hire a Director of Curriculum to design interim assessments and data cycle support system	K12 Director of Curriculum working	Use K12 to Director of Curriculum
5/31/13	Submit completed Units of Instruction for K12 curriculum to DE DOE	K12 Curriculum Specialists and Moyer Staff	-Submit completed Units of Instruction to DE DOE by 5/31/13
6/15/13	Hire all new staff members for SY 2013-2014 Finalize Assessment Calendar for SY 2013-2014 (includes data cycle)	K12 Inc & Moyer Admin Team	-All Staff hired with Moyer Staff
7/1/13	Master schedule completed to include common planning periods for interdisciplinary teams	K12 Inc, & Moyer Admin Team	
7/15/13	Principal leads week long professional development session with Moyer Staff Bi weekly professional development sessions to implement "Teach Like a Champion" strategies	K12 Inc, & Moyer Admin Team	- Completion of Master Schedule by 7/15/13
8/11/13	Interim assessments are being administered and data cycle implemented	K12 Inc, & Moyer Admin Team	-Professional development completed by 8/18/13
Year 1	Achieve 3000 online instructional exercises are being utilized by students	Teach Like a Champion Book	- Scheduled PD sessions are occurring and strategies are implemented in class
Year 1	Moyer Staff conducts 2 data nights with parents of students in grades 6-10	Dir of Curriculum	- Dir of Curriculum is providing support around interim data and data cycle
Year 1		Achieve 3000	- regular use of Achieve 3000 software -Monitoring that the data nights occurred with

DE Performance Framework Indicators	2012 Benchmark	2013 Target	2014 Target
DCAS Performance			
% of students meeting DCAS Math Standard	22.6%	32%	42%
% of students meeting DCAS Reading Standard	32.7%	42%	52%
% of African American students meeting DCAS Math Standard	22.1%	32%	32%
% of African American students meeting DCAS Reading Standard	30.4%	40%	50%
% of Sp Ed students meeting DCAS Math Standard		5%	10%
% of Sp Ed students meeting DCAS Reading Standard		5%	10%
% of Low SES students meeting DCAS Math Standard	21.3%	31%	41%
% of Low SES students meeting DCAS Reading Standard	31.5%	42%	52%
% of high school students meeting a score of 1550 or better on the SAT	0%	10%	20%
% of 12th grade students graduation from high school	50%	80%	90%

** 2012 Benchmark Scores were those of the former Moyer Academy and not The New Moyer Academy

** In SY 2014-2015, State of DE will implement Smarter Balanced Assessment. Targets have not been established at this time.

Performance Plus Assessment	2012 Benchmark	2013 Target	2014 Target
% of students making "Student Progress Over Time" in Math	30%	37%	44%
% of students making "Student Progress Over Time" in Reading	36%	43%	50%
% of "lowest-performing students meeting fall and spring instructional scale scores in math	50%	57%	64%
% of "lowest-performing students meeting fall and spring instructional scale scores in reading	43%	50%	57%
% of students making annual growth to achieve proficiency status within 3 years in math (by 10th gr)	32%	39%	46%
% of students making annual growth to achieve proficiency status within 3 years in reading (by 10th gr)	49%	56%	63%

Criterion Seven: Students with Special Needs

All students, including students with disabilities, benefit from the mission and philosophy of The New Moyer Academy due to the particularly innovative blended learning model. All students are given the opportunity for academic success and earning a high school diploma through individualized support and education, with the goal that all students will earn a diploma. The Blended Model strategically allows students to receive more individualized support when they need it.

Students identified as Special Education are held to the same high standards and expectations for student growth and achievement. To ensure all students are obtaining substantial achievement and growth, progress monitoring and analysis of students receiving special education are essential. Through quarterly, targeted progress monitoring of specific students receiving special services Moyer Academy frequently assess, alter and adjust curriculum to ensure ALL students are showing high growth and achievement in all areas of learning, thus effectively preparing them for postsecondary opportunities.

Moyer Academy combines state-of-the-art online and face-to-face instruction to provide high-quality, innovative, and effective individualized public school education, offering a wide variety of technology and foreign language courses in addition to core courses. Moyer Academy uses the innovative K^{12} web-based curriculum, dynamic teaching tools, highly-qualified certified teachers, and powerful technology to deliver individualized and differentiated instruction to students. The strength of the parent/school partnership is integral to the success of the school. Parents are able to access the curriculum and follow their child's progress in real time, enabling them to contact the teacher and address any concerns almost as soon as they arise.

Child Find

Moyer Academy's mission is to locate, identify, evaluate and serve all students suspected of having a disability. Proactively the school's Child Find policy is publicly posted on the school's website and newly enrolled families are provided with a copy within their enrollment packet. Annually, all of Moyer Academy's stakeholders are provided professional development training on student identification procedures.

Continuum of Services

An exemplary special services program is a program that empowers teachers, parents, students, administrators, and the community through a dynamic framework grounded in a belief that all students can learn. The Moyer Academy is built on a continuum of services that supports students academically, functionally, and behaviorally regardless of their disability. The continuum provides the resources and options necessary to meet individual student needs both proactively and responsively. This continuum creates a seamless blend of services between general education and special education and brings together the collective expertise and resources of both systems to appropriately support students with special needs. Working closely with families, teachers and administration, Moyer Academy is committed to educating the student in the least restrictive environment. Based on students' individual needs and the IEP team agreement of services, students receive their primary instruction in the following ways:

General Education setting – Majority of the day (>80%) spent in the general education classroom utilizing the general education curriculum. Special education supports may be provided by teacher – consultation, indirect, pull-out, push-in or co-teaching services.

Resource Room- Students spend 79%-40% in the general education setting with the primary curriculum provided through general education. Students receive additional support for their areas of weakness

through explicit, direct remedial instruction provided through teacher-consultation, indirect, pull-out, push-in or co-teaching services.

Self-Contained Classroom – For students with extensive needs more than 60% of the school day student receive specialized instruction in a special education setting with the primary curriculum provided through an explicit, direct remedial and individualized approach.

Homebound and Hospitalization – Students with significant disabilities that need an alternate placement outside of the brick and mortar will have primary curriculum provided through individualized instruction tailored to meet their needs.

For students identified with more severe needs an alternate online adaptive curriculum purchased from an outside vendor (Conover Life skills) with heavy emphasis on transitional, independent living and workability skills.

Related Services

Additionally, as determined by the students' individual education plans, students related service needs are met through speech language services, occupational therapy, physical therapy, behavioral counseling, visual and auditory supports, and assistive technology based on individual needs. Contracts are provided and signed with premier therapy staffing and services providers offering highly qualified speech, occupational, physical, behavioral health, psychological and related therapy services to educational and healthcare organizations across the United States.

Special education and related service providers collaborate daily at Moyer Academy. Related service providers conduct onsite class observations and receive feedback on application of student skills regularly from general education teachers. Formal progress monitoring and appropriate goal analysis are completed quarterly and is shared and reviewed with the respective families.

Individualized Curriculum

Moyer Academy particularly lends itself to special education collaboration since the entire school model encourages teachers to individualize instruction and academic support. Annual training takes place training all stake holders on accommodations and modification including data collection and analysis. Since K12 uses cutting edge technology to deliver content, assistive technology is fluidly implemented to ensure access to grade level courses. Such assistive technology includes but is not limited to: new vocabulary words are introduced prior to each lesson and highlighted throughout the lesson, screen readers that read/highlight/define unknown online print material, word prediction software, and audio text. Individual accommodations and modifications as indicated on each student's IEP is implemented and monitored to determine their effectiveness in ensuring access to the curriculum and a "Free and Appropriate Education."

Progress Monitoring IEP Goals

Special Education staff receives continuous professional development throughout the school year on progress monitoring and using data to drive instruction. The Special Education Staff tracks and assesses student progress in a multitude of ways including but not limited to: student classroom performance including grades, daily behavior or social skills data collection forms, state assessments, formal assessments and benchmark assessments, work samples, classroom observations, related service documentation, and special education daily student work logs. Formal progress monitoring is documented within the IEP quarterly, a copy will be mailed home to the respective families and a second copy will be kept within the individual students' confidential file.

Criterion Eight: Economic Viability

This will be in included in a later submission.

Criterion 14: Management Companies

The Moyer Board of Directors has contracted with K12, Inc to serve as its Educational Management Organization (EMO). The relationship between the Moyer Board of Directors and K12, Inc is rooted in the pre-existing partnership between DE DOE and K12, Inc. Through the contract, K12, Inc is responsible for providing a range of products and services to the students of The New Moyer Academy.

The following evaluation will be utilized to determine the effectiveness of K12, Inc, the management company employed by the Moyer Board of Directors. This evaluation will be completed annually with evaluative checkpoints at least twice per year. The evaluation tool is based on a combination of compliance mandates established by DE DOE and areas covered by the Product and Service Agreement signed by the Board of Directors and K12, Inc.

This section was approved.

REFERENCES

Bambrick-Santoyo, P. (2010). Driven by data: A practical guide to improve instruction.

Cantor, L. (2009). Assertive discipline: Positive behavior management for today's classroom

Marshall, K. (2009). Rethinking teacher supervision and evaluation: How to work, build collaboration, and close the achievement gap

Stephenson, K. (2011). The principal's cycle: A blueprint for the inexperienced and experienced principal

	Charter School Application Budget Worksheet										Page 1
	State Local & Loan Revenue										
		FY 2013		**FY 2014**		**FY 2015**		**FY 2016**		**FY 2017**	
1	State Appropriations	$1,455,514		$1,635,265		$1,798,980		$1,973,675		$1,973,675	
2	School District Local Fund Transfers	$875,097		$937,744		$1,046,400		$1,148,542		$1,148,542	
3	Prior Year Carryover Funds	$247,205		$91,831		$12,000		$71,000		$207,300	
	**Unbudgeted Carryover					$88,000		$78,000			
	**Summer Pay Set Aside			$ 307,000							
	STATE LOCAL & LOANS REVENUE	**$2,577,816**		**$2,971,840**		**$2,945,380**		**$3,271,217**		**$3,329,517**	
	State Local & Loans Expenses										
		FY 2013		**FY 2014**		**FY 2015**		**FY 2016**		**FY 2017**	
	Personnel Salaries / Other Employer Costs		FTE		FTE		FTE		FTE		FTE
4	Classroom Teachers	$510,988	11.00	$510,988	11.00	$624,988	13.00	$681,988	14.00	$681,988	14.00
5	Special Education Teachers	$0	0.00	$0	0.00	$0	0.00	$0	0.00	$0	0.00
6	Special Teachers (phys Ed, Art, Music)	$152,571	4.00	$196,000	4.00	$196,000	4.00	$196,000	4.00	$196,000	4.00
7	Counselors	$0	0.00	$0	0.00	$0	0.00	$0	0.00	$0	0.00
8	Principal/Administrative	$49,209	1.00	$65,000	1.00	$65,000	1.00	$65,000	1.00	$65,000	1.00
9	Nurse	$40,061	1.00	$48,000	1.00	$48,000	1.00	$48,000	1.00	$48,000	1.00
10	Clerical	$26,869	2.84	$124,560	2.84	$124,560	2.84	$124,560	2.84	$124,560	2.84
11	Custodial	$58,417	2.50	$58,840	2.50	$58,840	2.50	$58,840	2.50	$58,840	2.50
12	Para Professionals	$57,355	2.00	$65,500	2.00	$94,000	3.00	$123,000	4.00	$123,000	4.00
13	Food Service -Other	$11,104	1.50	$11,104	1.50	$11,104	2.00	$11,104	2.00	$11,105	2.00
14	Other Employer Costs (29.85 % of Salaries)	$270,612		$328,750		$372,127		$398,305		$398,305	
15	Health Insurance	$96,768		$182,869		$207,324		$223,771		$223,771	
16	Other Benefits	$0		$0		$0		$0		$0	
	SUBTOTAL SALARIES / OTHER EMPLOYER COSTS	**$1,273,954**	**25.84**	**$1,591,611**	**25.84**	**$1,801,943**	**29.34**	**$1,930,568**	**31.34**	**$1,930,569**	**31.34**
	Student Support										
17	Transportation	$99,316		$152,761		$179,587		$207,159		$207,159	
18	Extra Curricular Transportation	$0		$0		$0		$0		$0	
19	Cafeteria	$13,021		$16,097		$18,959		$21,820		$21,820	
20	Extra Curricular	$0		$0		$0		$0		$0	
21	Supplies and Materials	$16,208		$20,037		$23,600		$27,162		$27,162	
22	Textbooks	$5,978		$6,000		$6,000		$6,000		$6,000	
23	Curriculum	$0		$0		$0		$0		$0	
24	Professional Development	$6,523		$7,000		$7,000		$7,000		$7,000	
25	Assessments	$0		$0		$0		$0		$0	
26	Other Educational Program	$0		$0		$0		$0		$0	
27	Therapists (Occupational, Speech)	$2,905		$3,600		$4,300		$4,900		$4,900	
28	Classroom Technology	$2,289		$0		$0		$0		$0	
29	School Climate	$0		$0		$0		$0		$0	
30	Computers	$14,572		$15,000		$15,000		$15,000		$15,000	
31	Contracted Services	$74,828		$80,000		$85,000		$90,000		$90,000	
32	Other	$0		$0		$0		$0		$0	
	SUBTOTAL STUDENT SUPPORT	**$235,640**		**$300,495**		**$339,446**		**$379,041**		**$379,041**	
	Operations and Maintenance of Facilities										
33	Insurance (Property/Liability)	$14,813		$15,000		$15,000		$15,000		$15,000	
34	Rent	$325,000		$300,000		$300,000		$300,000		$300,000	
35	Mortgage			$0		$0		$0		$0	
36	Utilities	$119,836		$120,000		$120,000		$120,000		$120,000	
37	Maintenance	$53,262		$54,000		$54,000		$54,000		$54,000	
38	Telephone/Communications	$10,240		$10,300		$10,300		$10,300		$10,300	
39	Construction	$0		$0		$0		$0		$0	
40	Renovation	$0		$0		$0		$0		$0	
41	Custodial Supplies	$5,267		$5,300		$5,300		$5,300		$5,300	
	SUBTOTAL OPERATIONS AND MAINTENANCE OF FACILITIES	**$528,417**		**$504,600**		**$504,600**		**$504,600**		**$504,600**	
	Administrative/Operations Support										
42	Equipment Lease/Maintenance	$50,190		$50,200		$50,200		$50,200		$50,200	
43	Equipment Purchase	$0		$0		$0		$0		$0	
44	Supplies and Materials	$11,937		$15,000		$17,400		$20,000		$20,000	
45	Printing and Copying	$36,007		$44,500		$52,400		$60,300		$60,300	
46	Postage and Shipping	$2,860		$3,500		$4,200		$4,800		$4,800	
47	Enrollment / Recruitment	$0		$0		$0		$0		$0	
48	Staffing (recruitment and assessment	$448		$500		$500		$500		$500	
50	Legal Services	$8,370		$8,400		$8,400		$8,400		$8,400	
50	Auditors	$31,160		$17,000		$17,000		$17,000		$17,000	
	SUBTOTAL ADMINISTRATIVE/OPERATIONS SUPPORT	**$140,973**		**$139,100**		**$150,100**		**$161,200**		**$161,200**	
	Management Company										
51	Fees	$0		$40,275		$0		$71,000		$207,300	
52	Salaries/Other Employee Costs	$0		$0		$0		$0		$0	
53	Curriculum	$0		$0		$0		$0		$0	
54	Accounting and Payroll	$0		$0		$0		$0		$0	
55	Other	$0		$0		$0		$0		$0	
	SUBTOTAL MANAGEMENT COMPANY	**$0**		**$40,275**		**$0**		**$71,000**		**$207,300**	
	STATE LOCAL & LOANS EXPENDITURES	**$2,178,985**		**$2,576,081**		**$2,796,089**		**$3,046,409**		**$3,182,710**	
56	**# Students**	182		225		265		305		305	
	REVENUE LESS EXPENDITURES	**$398,831**		**$395,759**		**$149,291**		**$224,808**		**$146,807**	
	2 % CONTINGENCY CHECK	$51,556.32		$59,436.80		$58,907.60		$65,424.34		$66,590.34	

FY	FUND	APPR	TY	APPROPRIATION DESCR	END DATE	AVAILABLE FUNDS	REQUISITIONS	ENCUMBRANCES	CURR YR EXPEN	PRIOR YR EXPEN	REMAIN. SPEND. AUTH.
13	100	00099	00	General Fund Revenue	06/30/13	0.00	0.00	0.00	0.00	0.00	0.00
				REVENUE - Type 00 Totals:		0.00	0.00	0.00	0.00	0.00	0.00
13	075	00100	01	Unfunded Payroll	06/30/13	0.00	0.00	0.00	0.00	0.00	0.00
13	175	00100	01	Unfunded Payroll	06/30/13	0.00	0.00	0.00	0.00	0.00	0.00
13	575	00100	01	Unfunded Payroll	06/30/13	0.00	0.00	0.00	0.00	0.00	0.00
13	775	00100	01	Unfunded Payroll	06/30/13	0.00	0.00	0.00	0.00	0.00	0.00
13	175	00556	01	Mentoring Pilot	06/30/13	7,500.00	0.00	0.00	7,500.00	0.00	0.00
13	175	05149	01	Homeless Transportatio	09/30/13	0.00	0.00	0.00	0.00	0.00	0.00
13	175	05213	01	Operations	06/30/14	1,343,518.00	0.00	0.00	1,343,518.00	0.00	0.00
13	175	05215	01	Educ Acontab	06/30/13	576.00	0.00	0.00	576.00	0.00	0.00
13	100	05244	01	School Improvement	07/02/12	0.00	0.00	0.00	0.00	0.00	0.00
13	175	05244	01	School Improvement	06/30/14	84,402.50	0.00	0.00	38,129.66	0.00	46,272.84
13	100	05289	01	Ed Sustainment Fund	07/02/12	0.00	0.00	0.00	0.00	0.00	0.00
13	175	05289	01	Ed Sustainment Fund	06/30/13	42,295.00	0.00	0.00	42,295.00	0.00	0.00
				MAIN - Type 01 Totals:		1,478,291.50	0.00	0.00	1,432,018.66	0.00	46,272.84
12	175	05149	03	Homeless Transportatio	09/30/12	319.00	0.00	0.00	0.00	319.00	0.00
12	175	05213	03	Operations	06/30/13	1,208,565.00	0.00	0.00	24,956.46	1,183,608.54	0.00
11	775	10017	03	Mci	06/30/13	0.00	0.00	0.00	0.00	0.00	0.00
				MAIN - Type 03 Totals:		1,208,884.00	0.00	0.00	24,956.46	1,183,927.54	0.00
13	075	91100	20	Cafeteria	06/30/13	84,067.01	0.00	0.00	0.00	0.00	84,067.01
13	075	98000	20	Local Funds	06/30/13	317,423.70	0.00	0.00	0.00	0.00	317,423.70
13	075	98037	20	Summer School	06/30/13	-307,000.00	0.00	0.00	0.00	0.00	-307,000.00
13	075	98220	20	Fundraising	06/30/13	13,736.46	0.00	0.00	0.00	0.00	13,736.46
				KK_NSF_REV - Type 20 Totals:		108,227.17	0.00	0.00	0.00	0.00	108,227.17
13	075	91100	20	Cafeteria	06/30/13	93,220.98	0.00	0.00	83,615.03	0.00	9,605.95
12	775	91198	20	MCI/State	06/30/14	14,741.00	0.00	0.00	14,741.00	0.00	0.00
13	075	98000	20	Local Funds	06/30/13	1,091,908.34	0.00	0.00	648,590.04	0.00	443,318.30
13	075	98037	20	Summer School	06/30/13	0.00	0.00	0.00	0.00	0.00	0.00
13	075	98041	20	CSCRP	06/30/13	1,276.40	0.00	0.00	1,276.40	0.00	0.00
13	075	98220	20	Fundraising	06/30/13	16,545.71	0.00	0.00	15,581.95	0.00	963.76
13	075	98237	20	Local Unique Alt	06/30/13	5,472.38	0.00	0.00	0.00	0.00	5,472.38
				KK_NSF_EXP - Type 20 Totals:		1,223,164.81	0.00	0.00	763,804.42	0.00	459,360.39
10	575	40108	40	ARRA State Fiscal Stab	12/15/11	38,963.63	0.00	0.00	0.00	0.00	38,963.63
10	575	40114	40	Iasa Title Ii: Prof D	12/01/12	0.00	0.00	0.00	0.00	0.00	0.00

STATE OF DELAWARE

DEPARTMENT OF FINANCE - DIVISION OF ACCOUNTING

DAILY VALIDITY REPORT

AS OF 06/30/13

Report ID: DGL060
Department ID: 957500 Maurice J. Moyer Academy
DEPARTMENT: 950000 Department of Education
DIVISION: 957500 Maurice J. Moyer Academy
BUDGET: 957500 Maurice J. Moyer Academy

Run Date:07/03/2013
Run Time:21:01:45

FY	FUND	APPR	TY	APPROPRIATION DESCR	END DATE	AVAILABLE FUNDS	REQUISITIONS	ENCUMBRANCES	CURR YR EXPEN	PRIOR YR EXPEN	REMAIN. SPEND. AUTH.
11	575	40114	40	Iasa Title Ii: Prof D	03/31/12	40,935.05	0.00	0.00	0.00	0.00	40,935.05
12	572	40114	40	Iasa Title Ii: Prof D	03/31/13	0.00	0.00	0.00	0.00	0.00	0.00
12	575	40114	40	Iasa Title Ii: Prof D	11/01/13	20,388.00	0.00	0.00	0.00	0.00	20,388.00
13	575	40114	40	Iasa Title Ii: Prof D	11/01/14	0.00	0.00	0.00	0.00	0.00	0.00
10	575	40192	40	ARRA Race to Top-State	06/13/14	91,301.61	0.00	0.00	0.00	0.00	91,301.61
10	575	40269	40	Title Ii Part D Enhanc	03/31/11	0.00	0.00	0.00	0.00	0.00	0.00
11	575	40360	40	Education Jobs Fund	12/15/12	109,891.05	0.00	0.00	0.00	0.00	109,891.05
10	575	40554	40	Title I	12/01/12	4,330.39	0.00	0.00	0.00	0.00	4,330.39
11	575	40554	40	Title I	03/31/12	253,511.98	0.00	0.00	0.00	0.00	253,511.98
12	575	40554	40	Title I	11/01/13	251,018.00	0.00	0.00	0.00	0.00	251,018.00
13	575	40554	40	Title I	11/01/14	272,660.74	0.00	0.00	0.00	0.00	272,660.74
10	575	40564	40	Idea-B	07/31/11	0.00	0.00	0.00	0.00	0.00	0.00
11	575	40564	40	Idea-B	03/31/12	91,426.18	0.00	0.00	0.00	0.00	91,426.18
12	575	40564	40	Idea-B	11/01/13	42,387.99	0.00	0.00	0.00	0.00	42,387.99
13	575	40564	40	Idea-B	11/01/14	14,556.29	0.00	0.00	0.00	0.00	14,556.29
08	575	40602	40	Charter School Dump	06/29/12	112,748.00	0.00	0.00	0.00	0.00	112,748.00
10	575	40930	40	Title Iv D/F	12/01/12	0.00	0.00	0.00	0.00	0.00	0.00
09	575	41212	40	Title I Arra	12/01/12	49,885.98	0.00	0.00	0.00	0.00	49,885.98
09	575	41213	40	Idea-B Arra	12/15/11	29,940.70	0.00	0.00	0.00	0.00	29,940.70
				GR_G_REV - Type 40 Totals:		1,423,945.59	0.00	0.00	0.00	0.00	1,423,945.59
10	575	40108	40	ARRA State Fiscal Stab	12/15/11	38,963.63	0.00	0.00	0.00	38,963.63	0.00
10	575	40114	40	Iasa Title Ii: Prof D	12/01/12	22,547.31	0.00	0.00	0.00	0.00	22,547.31
11	575	40114	40	Iasa Title Ii: Prof D	03/31/12	40,935.05	0.00	0.00	0.00	40,935.05	0.00
12	572	40114	40	Iasa Title Ii: Prof D	03/31/13	0.00	0.00	0.00	0.00	0.00	0.00
12	575	40114	40	Iasa Title Ii: Prof D	11/01/13	20,662.50	0.00	0.00	19,681.00	707.00	274.50
13	575	40114	40	Iasa Title Ii: Prof D	11/01/14	32,293.00	0.00	0.00	2,914.42	0.00	29,378.58
10	575	40192	40	ARRA Race to Top-State	09/13/14	174,959.00	0.00	0.00	11,921.07	86,555.00	76,482.93
10	575	40269	40	Title Ii Part D Enhanc	03/31/11	0.00	0.00	0.00	0.00	0.00	0.00
11	575	40360	40	Education Jobs Fund	12/15/12	109,891.05	0.00	0.00	22,373.02	87,518.03	0.00
10	575	40554	40	Title I	12/01/12	20,347.38	0.00	0.00	0.00	4,330.39	16,016.99
11	575	40554	40	Title I	03/31/12	253,511.98	0.00	0.00	0.00	253,511.98	0.00
12	575	40554	40	Title I	11/01/13	251,018.00	0.00	0.00	24,642.51	226,375.49	0.00
13	225	40554	40	Title I	12/31/13	0.00	0.00	0.00	0.00	0.00	0.00
13	575	40554	40	Title I	11/01/14	407,682.00	0.00	0.00	290,325.08	0.00	117,356.92
10	575	40564	40	Idea-B	07/31/11	3,944.25	0.00	0.00	0.00	0.00	3,944.25
11	575	40564	40	Idea-B	03/31/12	91,426.18	0.00	0.00	0.00	91,426.18	0.00
12	575	40564	40	Idea-B	11/01/13	42,443.90	0.00	0.00	13,852.29	28,535.70	55.91
13	575	40564	40	Idea-B	11/01/14	38,198.00	0.00	0.00	16,703.04	0.00	21,494.96
08	575	40602	40	Charter School Dump	06/29/12	112,748.00	0.00	0.00	0.00	112,748.00	0.00
10	575	40930	40	Title Iv D/F	12/01/12	4,962.43	0.00	0.00	0.00	0.00	4,962.43
09	575	41212	40	Title I Arra	12/01/12	50,098.30	0.00	0.00	0.00	49,885.98	212.32
09	575	41213	40	Idea-B Arra	12/15/11	29,940.70	0.00	0.00	0.00	29,940.70	0.00

STATE OF DELAWARE
DEPARTMENT OF FINANCE - DIVISION OF ACCOUNTING
DAILY VALIDITY REPORT
AS OF 06/30/13

FY	FUND	APPR	TY	APPROPRIATION DESCR	END DATE	AVAILABLE FUNDS	REQUISITIONS	ENCUMBRANCES	CURR YR EXPEN	PRIOR YR EXPEN	REMAIN. SPEND. AUTH.
				GR_DETAIL - Type 40 Totals:		1,746,572.66	0.00	0.00	402,412.43	1,051,433.13	292,727.10
10	775	50022	50	MCI	06/30/14	17,632.00	0.00	0.00	0.00	17,631.97	0.03
13	775	50022	50	MCI	06/30/15	14,320.00	0.00	0.00	14,320.00	0.00	0.00
11	775	50323	50	Mci Operations	06/30/13	0.00	0.00	0.00	0.00	0.00	0.00
				MAIN - Type 50 Totals:		31,952.00	0.00	0.00	14,320.00	17,631.97	0.03
				Dept 957500 Totals :		7,221,037.73	0.00	0.00	2,637,511.97	2,252,992.64	2,330,533.12
				REVENUE - Type 00		0.00	0.00	0.00	0.00	0.00	0.00
				MAIN - Type 01		1,478,291.50	0.00	0.00	1,432,018.66	0.00	46,272.84
				MAIN - Type 03		1,208,884.00	0.00	0.00	24,956.46	1,183,927.54	0.00
				KK_NSF_REV - Type 20		108,227.17	0.00	0.00	0.00	0.00	108,227.17
				KK_NSF_EXP - Type 20		1,223,164.81	0.00	0.00	763,804.42	0.00	459,360.39
				GR_G_REV - Type 40		1,423,945.59	0.00	0.00	0.00	0.00	1,423,945.59
				GR_DETAIL - Type 40		1,746,572.66	0.00	0.00	402,412.43	1,051,433.13	292,727.10
				MAIN - Type 50		31,952.00	0.00	0.00	14,320.00	17,631.97	0.03
				GENERAL FUNDS		2,687,175.50	0.00	0.00	1,456,975.12	1,183,927.54	46,272.84
				SPECIAL FUNDS		4,533,862.23	0.00	0.00	1,180,536.85	1,069,065.10	2,284,260.28

AGREEMENT OF SALE

THIS AGREEMENT OF SALE dated July ____, 2013, is made by and between ***The New Maurice J. Moyer Academy, Inc.*** *and/or assigns, ("Buyer") and* ***The Reinvestment Fund*** ("Seller").

1. PROPERTY. Seller agrees to sell and convey to Buyer, and Buyer agrees to purchase and pay for, the real property ("Property")consisting of two Tax Parcel Numbers, being approximately 1.75 acres, and the buildings erected thereon, known as:

A. 610 East 17th Street, Wilmington, DE Tax Parcel No. 26-029.40-028.
B. 611 East 17th Street, Wilmington, DE Tax Parcel No. 26-029.40-027.

2. PURCHASE PRICE. Buyer shall pay Two Million One Hundred Thousand Dollars ($2,100,000.00) for the Property, payable as follows: Two Hundred Fifty Thousand Dollars ($250,000.00) on signing of this Agreement("Initial Deposit"), said sum to be forwarded by Buyer to Rhodunda and Williams, LLC, ("Escrow Agent"), to be held by Escrow Agent and delivered and disbursed at settlement or sooner in accordance with the provisions of this Agreement. Purchaser shall pay a Second Deposit of Two Hundred Fifty Thousand Dollars ($250,000.00) ("Additional Deposit") shall be forwarded to the Escrow Agent within three business days after the end of Due Diligence. The balance of the Purchase Price, less the Initial and Additional deposits, One Million Six Hundred Thousand Dollars ($1,600,000.00) and shall be paid at the time of settlement. If Seller gives notice to Buyer at least five (5) banking days prior to settlement, the balance of the Purchase Price or the net proceeds of sale payable to Seller, or both, shall be paid by wire transfer or by cashier's or certified check as specified by Seller.

3. MORTGAGE CONTINGENCY. There is no Financing Contingency.

4. SETTLEMENT. Settlement shall be held in New Castle County, Delaware, within Thirty (30) business days following the end of the Due Diligence period, or within 10 days following Seller's obtaining title to the Property via the foreclosure process, whichever is later in time.

5. POSSESSION AND INSPECTION. Possession of the Property shall be delivered by Seller to Buyer at settlement, by delivery of all keys in Seller's possession or under Seller's control. If Buyer wishes to make a pre-settlement inspection of the Property, it is Buyer's responsibility to arrange for and make such inspection. Seller is responsible for insuring that utilities are on so that systems can be inspected. Seller will permit Buyer to have access to the Property within 48 hours prior to settlement for purposes of such inspection.

6. TRANSFER TAXES; PRO-RATED. Applicable transfer taxes shall be paid one-half by Buyer and one-half by Seller. Taxes, water, sewer and any other lienable charges imposed by the State of Delaware, any political subdivision thereof, any school district or any neighborhood association and any condominium common expenses shall be apportioned pro-rata

at the time of settlement, as shall rents and prepaid operating expenses if the Property is sold subject to a lease.

7. TITLE. Title to the Property is to be conveyed by deed of special warranty and is to be good, marketable, fee simple absolute title of record, free and clear of all liens and encumbrances of record and free and clear of zoning and governmental subdivision violations, but subject to all existing easements and restrictions of record. If Seller is unable to give a good and marketable title meeting the foregoing requirements, such as will be insured at regular rates by a title insurer duly authorized to transact insurance in Delaware, Buyer shall have the option of taking such title as Seller can give, without reduction of the Purchase Price, or of being repaid all deposit money, and this Agreement shall be null and void. Seller may use the purchase price payable to Seller at settlement to discharge liens and encumbrances of record in fixed and ascertainable amounts.

8. SELLER'S REPRESENTATIONS.

a. Seller shall provide Buyer a copy of all records, plans government approval documents and any other materials related to the Property in its possession within Ten (10) days of the execution of this Agreement

b. Seller represents that it is taking all commercially reasonable steps to take title to the Property via foreclosure. Seller's obligation to convey the Property is contingent on the Seller obtaining title via the foreclosure process.

9. DUE DILIGENCE. Buyer, at its own expense, for a period of Thirty (30) days following the execution of this Agreement, by both parties, shall have a Due Diligence period. The Buyer may inspect the entire Property, including, but not limited to the roof, HVAC system, electrical and plumbing systems, zoning, governmental violations, flood zone, wood destroying insect infestation and for any other tests, inspections and investigations that Buyer determines, in its sole opinion, are reasonably suitable and acceptable for Buyer's intended use provided Buyer immediately returns the Property and building to its previous condition ("Due Diligence Period"). In addition, Buyer may inquire upon public agencies, perform a title search and consult engineers to determine there are not right-of-ways, easements or restrictions that will impede the use of the Property by Buyer. Buyer hereby indemnifies and holds Seller harmless from and against any and all loss or expenses resulting from such activity and will restore the Property to its original condition if any damages occur during Due Diligence. Buyer shall require that all contractors that assist with the Due Diligence are bonded and insured to cover any work, studies of other activity at the Property. If not suitable and acceptable to Buyer, the Buyer may rescind this Agreement and recover its deposit monies upon notice. Failure to give notice within such Thirty (30) day period shall be deemed a waiver of this condition by Buyer. Time is of the essence of this provision. Buyer agrees that any information received as a result of such testing will not be disclosed to any governmental agency without first obtaining Seller's prior written consent, which consent shall not be unreasonably withheld. Seller shall provide Buyer a copy of all records, plans, government approval documents and any other materials related to the Property which are in the possession of the Seller to Buyer at no cost to Buyer within Ten (10) days of the execution of this Agreement. . In the event that settlement does not occur for any

reason, Buyer shall provide Seller a copy of all records, plans, government approval documents and any other materials related to the Property, which have been obtained by Buyer, at no cost to Seller.

10. OTHER CONTINGENCIES. This Agreement shall be contingent upon the following:

a. Buyer, at its sole expense, may procure a Phase I Environmental Audit for said Property during the Due Diligence Period. In the event the Audit is not satisfactory to Buyer, the Buyer Shall notify the Seller in writing prior to the end of the Due Diligence period and identify any issues that Buyer contends need to be addressed by Seller. Seller shall have the option of correcting any unsatisfactory findings. If Seller elects not to correct the Property, then this agreement shall be declared null and void and all deposit monies shall be returned to Buyer. Buyer may waive this contingency.

b. Seller represents that it is taking all commercially reasonable steps to take title to the Property via foreclosure. Seller's obligation to convey the Property is expressly contingent on the Seller obtaining title via the foreclosure process.

In the event that any of the aforesaid contingencies can not be met, then this Agreement shall become null and void and the Deposit shall be returned to Buyer.

11. RISK OF LOSS. Any loss or damage to the Property by fire, windstorm or other casualty prior to settlement shall be borne by Seller. Seller shall maintain any existing casualty insurance through the date of settlement.

12. NO REPRESENTATIONS; ENTIRE AGREEMENT. Buyer has inspected the Property and has agreed to purchase the Property in its present condition unless otherwise specified in this Agreement. Buyer and Seller agree that they have read and fully understand this Agreement, that it contains the entire agreement between them and that they do not rely on any written or oral representation or statement not expressly written in this Agreement. Without limiting the generality of the foregoing, Buyer acknowledges that Buyer is not purchasing the Property based on any representation or statement of fact or opinion contained in any advertisement, listing agreement, multi-list description or multi-list information sheet, or made by Seller, any broker, salesperson or any agent or employee of any of them. Furthermore, this Agreement shall not be amended except in writing signed by Buyer and Seller.

13. NO RECORDING. This Agreement shall not be recorded in the office of any recorder or in any other office or place of public record. If Buyer shall record this Agreement or

cause or permit it to be recorded, Seller may, at Seller's option, elect to treat such act as a breach of this Agreement.

14 NO REAL ESTATE COMMISSION. Neither Buyer nor Seller has engaged a real estate agent/broker to represent them in this matter. Therefore, no real estate commission shall be paid by either party.

15. Escrow Agent Indemnification. The Buyer and Seller agree to hold Escrow Agent harmless and indemnify Escrow Agent for and against any and all claims, demands, causes of action, actions and expenses, including attorneys' fees and costs, relating to or arising out of Escrow Agent acting as escrow holder hereunder.

15. TIME OF ESSENCE; DEFAULT OF BUYER; TENDER. Time is of the essence of this Agreement. If Buyer fails to make any payment as specified in Paragraph 2, knowingly furnishes false or incomplete information to Seller, Seller's broker, any agent or employee of Seller's broker or the lending institution concerning Buyer's legal or financial status, fails to make application or cooperate in the processing of the mortgage loan application, which act(s) would result in failure to obtain a mortgage financing commitment, or violates or fails to perform any of the terms or conditions of this Agreement, then Seller shall have the right and option to declare this Agreement null and void, to retain any deposit money as liquidated damages for such default by Buyer, or to exercise any legal or equitable right or remedy to which Seller may be entitled and in connection therewith to apply any deposit money either on account of the purchase price or on account of damages, as Seller may elect. Formal tender of deed and tender or purchase money are waived.

16. NOTICES; DELIVERY OF DOCUMENTS. Whenever a notice is to be given or a document is to be delivered to Seller hereunder, it shall be addressed or delivered to:

William J. Rhodunda, Jr., Esquire
Rhodunda & Williams, LLC
1220 Market Street, Suite 701
Wilmington, DE, 19801

and whenever a notice is to be given or a document is to be delivered to Buyer hereunder, it shall be addressed or delivered to:

Dr. C.T. Curry
Board President
New Moyer Board of Directors
1400 B Street
Wilmington, DE 19801

All notices shall be in writing. An original, facsimile or photocopy of a document shall be effective when sent or delivered. A facsimile or photocopy of a signed offer, addendum, counter offer or written notice shall constitute an original signed document. In the case of facsimiles or photocopies, the parties further agree to promptly deliver or mail the document with original signatures to the other party.

17. SUCCESSION. This Agreement shall benefit and bind the parties hereto, their respective heirs, personal representatives, successors and assigns. However, Buyer may not assign Buyer's rights and interest under this Agreement without Seller's prior written consent. Seller will unreasonably withhold written consent, unless the Buyer's assignee entity has the same ownership.

18. LIMITATION OF AUTHORITY. Buyer and Seller understand this Agreement is not binding until signed by and delivered to all parties.

19. MISCELLANEOUS.

a. Delaware law governs this Agreement.

b. The paragraph captions of this Agreement are inserted for purposes of convenient reference only and are not intended to limit or enlarge the substance of this Agreement.

c. The singular forms Buyer and Seller are used in this Agreement solely as a convenience and are intended to include, anytime they are used, all parties who are buyers or sellers.

20. LEGAL RIGHTS AFFECTED. This Agreement substantially affects important legal rights. Buyer and Seller are entitled to consult attorneys of their choice prior to signing this Agreement. Buyer and Seller acknowledge: (a) the availability of arbitration to resolve disputes pursuant to the arbitration agreement set forth below; and (b) receipt of written material explaining the arbitration system.

IN WITNESS WHEREOF, the parties hereto have hereunto set their hands and seal the day and year first above written.

BUYER:

The New Maurice J. Moyer Academy, Inc

_____ BY: _____

WITNESS

Name:
Title:

SELLER:

The Reinvestment Fund

_____ BY: _____

WITNESS

Name:
Title:

New Moyer Maurice J. Moyer Academy, Inc. ("New Moyer")

Debt Service

600 and 611 E. 17th Street, Wilmington, DE (the "Property")

- New Moyer has a lease (the "Lease") with Reinvestment II, LLC ("REII") to use the school building and related facilities located at the Property.
- The Base Rent under the Lease is $25,000.00 a month
- REII currently owns the Property subject to a mortgage in favor of The Reinvestment Fund, Inc. ("TRF")
- TRF brought mortgage foreclosure proceedings against the Property and a Sheriff's Sale was conducted on July 9, 2013
- There were no bidders at the Sheriff's Sale so TRF was deemed the successful bidder
- On August 19th, the Superior Court will confirm the Sheriff's Sale and a deed will be prepared and recorded transferring the Property to TRF
- All liens (i.e., mechanic's liens, mortgages, etc.) will be wiped away following the transfer of the Property to TRF. New Moyer will continue to lease the Property pursuant to the Lease.
- Once TRF owns the Property, free and clear of all liens, it will be able to sell the Property to New Moyer.
- New Moyer will purchase the Property with a loan from K12, Inc. in the amount of $2,100,000 (the "Loan").
- The term of the Loan is 5 years with monthly payments of approximately $14,150 based upon a 20 year amortization schedule. There is a balloon payment due at the end of the 5 year term.
- **MONTHLY DEBT SERVICE UNDER LEASE**
  - $25,000 (Monthly Lease Payment)
- **MONTHLY DEBT SERVICE UNDER LOAN**
  - $14,150 (Monthly Loan Payment)

FIRST AMENDMENT TO AMENDED AND RESTATED LEASE AGREEMENT

THIS FIRST AMENDMENT TO AMENDED AND RESTATED LEASE AGREEMENT (this "Agreement"), is made this 28th day of June, 2013, by and between THE NEW MAURICE J. MOYER ACADEMY, INC., a Delaware non-profit non-stock corporation (hereinafter called "Tenant"), and REINVESTMENT II, LLC, a Pennsylvania limited liability company (hereinafter called "Landlord").

RECITALS

WHEREAS, Landlord and Tenant are parties to a certain Amended and Restated Lease Agreement executed and delivered as of October 15, 2012, but having an effective date of August 1, 2012 (as amended, supplemented, restated or otherwise modified, the "Lease"), pursuant to which Tenant leases certain land located upon Tax Parcel Nos. 26-029.40-027 and 26-029.40-028, known as 600 and 611 E. 17th Street, Wilmington, Delaware, and the improvements thereon consisting of two school buildings and related facilities; and

WHEREAS, the Lease term expires June 30, 2013, and Landlord and Tenant desire to extend the term of said Lease for three (3) additional months, pursuant to all of the terms and conditions of the Lease as expressly modified hereby.

AGREEMENTS

NOW, THEREFORE, in consideration of the foregoing and of the mutual promises hereinafter contained, the parties, intending to be legally bound, agree with each other as follows:

1. The Recitals above are a part of this Agreement. Unless otherwise expressly defined in this Agreement, terms defined in the Lease shall have the same meaning under this Agreement.

2. Tenant hereby represents and warrants that (a) it is a non-profit corporation duly organized, existing and in good standing under the laws of the State of Delaware, (b) it has the power, authority and legal right to execute and deliver this Agreement and to engage in the transactions contemplated hereby, (c) this Agreement has been duly executed and delivered by Tenant, (d) the person signing this Agreement on behalf of Tenant has the necessary authority to do so, and (e) the execution and delivery of, and the carrying out of the transactions contemplated by, this Agreement and the Lease, and the performance and observance of the terms and conditions thereof, have been duly authorized by all necessary organizational action by and on behalf of Tenant.

3. The Tenant further represents and warrants that no Default or Event of Default and no event which, with notice, lapse of time or both would constitute an Event of Default, has occurred and is continuing under the Lease.

4. The Term of the Lease is hereby extended for a further period of three (3) months, commencing July 1, 2013, and expiring at 5:00 p.m. on September 30, 2013, unless the Lease is sooner terminated in accordance with its terms.

5. Except as expressly modified by this Agreement, the parties ratify and confirm each and every provision of the Lease all of which shall remain in full force and effect. The

parties agree that the execution of this Agreement is not intended to and shall not cause or result in a novation with regard to the Lease.

6. The provisions of this Agreement shall be construed, interpreted and enforced in accordance with the laws of the State of Delaware as the same may be in effect from time to time. This Agreement shall be binding upon and shall insure to the benefit of the parties hereto and their successors and assigns. This Agreement may be executed in any number of counterparts, and each such counterpart shall be deemed to be an original, and it shall not be necessary that the signature of, or on behalf of, each party, or that the signatures of the persons required to bind any party, appear on more than one counterpart.

7. The parties agree that their respective signatures to this Agreement may be delivered by fax or .pdf. Any party who chooses to deliver its signature by fax or .pdf agrees to provide a counterpart of this Agreement with its inked signature to the other party, but the failure to deliver any such inked original signature shall in no way affect the validity, effectiveness or enforceability of this Agreement.

[Signatures Follow on Next Page]

IN WITNESS WHEREOF, the parties hereto have executed this Agreement under seal as of the day and year first above written.

LANDLORD:

REINVESTMENT II, LLC

By: _____ (SEAL)
Name: Michael M. Crist
Title: Member

TENANT:

THE NEW MAURICE J. MOYER ACADEMY, INC.

By: _____ (SEAL)
~~Name:~~ Dr. Christopher Curry
Title: Board Chairman

2300 Corporate Park Drive Herndon, VA 20171

The New Moyer Academy
Agreement of Management Fees

State & Local Revenue	$2,341,570
Federal Revenue	$478,172
Cafeteria Funds	$74,737
Total Revenue	**$2,894,479**
	$289,448

10% Management Fee includes the following positions
Head of School, Technology, Director of Curriculum, and Business Manager

5% Technology Fee: $14,473

The calculation does not include carry-over funds.

Per the Service agreement, K12 will not be paid until all other school financial obligations are satisfied. Based upon end of year resources, K12 will discount its fee to The New Moyer Academy.

I Acknowledge how K12 fees are calculated and understand K12 fees can only be paid from State & Local Funds.

5/10/13
Dr. C.T. Curry, President of the Board of Directors, The New Moyer Academy

5/10/13
Dr. Keith Stephenson, Regional Vice President, K12, Inc

Modification Document for Middle School ELA Scope and Sequences

Grade 6

Unit 1: Writing a Memoir – 1 week
Unit 2: Writing a Research Paper – 2 weeks
Unit 3: Writing to a Prompt – 1 week
Unit 4: Writing an Editorial – 1 week
Unit 5: Writing a Speech – 1 week
Unit 6: Writing a Business Letter – 1 week
Unit 7: Writing a Compare and Contrast Essay – 2 weeks
Unit 8: Writing a Character Sketch – 1 week
Unit 9: Grammar, Usage, Mechanics – 1 week
Unit 10: Literature: Lessons Learned – 1 week
Unit 11: Literature: Mostly Heroes– 1 week
Unit 12: Literature: The Prince and the Pauper– 1 week
Unit 13: Literature: Seasonal Change– 1 week
Unit 14: Literature: Curious Creatures – 2 weeks
Unit 15: Literature: Select a Novel (Optional)
Unit 16: Literature: Stories from the Bible (Optional)
Unit 17: Literature: Early American Lives– 2 weeks
Unit 18: Literature: Stories of Washington Irving– 2 weeks
Unit 19: Literature: Passing Moments– 1 week
Unit 20: Literature: Select a Novel (Optional)
Unit 21: Literature: William Shakespeare (Optional)
Unit 22: Literature: No Matter Where it's Going– 1 week
Unit 23: Literature: Don Quixote– 1 week
Unit 24: Literature: I Didn't Know That– 2 weeks
Unit 25: Literature: Select a Novel (optional)
Unit 26: Literature: Sherlock Holmes– 1 week
Unit 27: Literature: American Themes– 2 weeks
Unit 28: Literature: Paddle to the Sea– 1 week
Unit 29: Literature: Life Stories of the Young and the Brave– 2 weeks
Unit 30: Literature: Little Women– 1 week
Unit 31: Literature: American Tall Tales– 1 week
Unit 32: Select a Novel (Optional)
Unit 33: Spelling– 1 week

Modification Document for Middle School ELA Scope and Sequences

Grade 7

Unit 1: Introduction to the Paragraph – 2 weeks
Unit 2: Personal Narrative – 2 weeks
Unit 3: Persuasive Essay– 2 weeks
Unit 4: Research Report– 3 weeks
Unit 5: Advertisement– 1 week
Unit 6: Grammar, Usage, and Mechanics– 1 week
Unit 7: Literature: Lessons Learned–2 weeks
Unit 8: Literature: Animal Adventures– 2 weeks
Unit 9: Literature: Select a Novel– – 2 weeks
Unit 10: Literature: Creative Lives– 1 week
Unit 11: Literature: Select a Novel– 2 weeks
Unit 12: Literature: Stories of Our Time– 3 weeks
Unit 13: Literature: To Everything there is a Season– 2 weeks
Unit 14: Literature: Select a Novel– 2 weeks
Unit 15: Literature: ~~Twelfth Night~~ – 2 weeks
Replace Twelfth Night with choice of:
Man from the Other Side by: Orlev, Uri
Broken Drum by: Edith Hemingway
Other Bells for Us to Ring by: Robert Cormier
Unit 16: Literature: Select a Novel– 2 weeks
Unit 17: Vocabulary– 2 weeks

Grade 8

Unit 1: Letter to the Editor– 2 weeks
Unit 2: Research Report– 2 weeks
Unit 3: Propagandist Essay– 2 weeks
Unit 4: Cause and Effect Essay– 3 weeks
Unit 5: Fictional Narrative– 2 weeks
Unit 6: Grammar, Usage, and Mechanics – 1 week
Unit 7: The Heart's Deep Core – 2 week
Unit 8: Literature: Select a Novel– 2 weeks
Unit 9: Literature: Narrative Poetry – 1 week
Unit 10: Literature: Scientists Life Stories– 2 weeks
Unit 11: Literature: Life Stories– 2 weeks
Unit 12: Literature: What's Important? – 2 weeks
Unit 13: Literature: The Language of Poetry – 1 week
Unit 14: Literature: Advice and Instruction– 2 weeks
Unit 15: Literature: Select a Novel – 2 week
Unit 16: Literature: City– 2 weeks
Unit 17: Julius Caesar– 2 weeks
Unit 18: Literature: Select a Novel – 2 weeks
Unit 19: Vocabulary – 1 week

**When able to Select a Novel, the teacher should select among Lexile level appropriate non-fiction texts.

Curriculum Scope & Sequence

School Moyer Academy Grade or Course Grade 6: English/Language Arts Teacher _____

Unit Order By unit title and/ or time	**Learning Targets** Content Standards, Grade Level Expectations, Proficiency Level Expectations, or Grade	**Theme or Big Idea**	**Enduring Understandings**	**Essential Questions**
COMPOSITION: Writing a Memoir	**Writing** 2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. • Introduce a topic; organize ideas, concepts, and information, using strategies such as definition, classification, comparison/ contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. • Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. • Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. • Use appropriate transitions to clarify the relationships among ideas and concepts. • Use precise language and domain-specific vocabulary to inform about or explain the	Write for a specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers understand. • A writer's word choice and syntax are characteristics of voice which help to personalize text.	• Why share personal experiences in writing? • What makes writing worth reading? • How do writers express their thoughts and feelings? • Where do ideas for writing come from? • What makes writing flow? • How do effective writers hook and hold their readers? • What makes writing easy to follow? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • What is a complete thought? • Why am I writing? • For whom? • What am I trying to achieve through my writing?

	presented. 3. Write narratives to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences. • Engage and orient the reader by establishing a context and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically. • Use narrative techniques, such as dialogue, pacing, and description, to develop experiences, events, and/or characters. • Use a variety of transition words, phrases, and clauses to convey sequence and signal shifts from one time frame or setting to another. • Use precise words and phrases, relevant descriptive details, and sensory language to convey experiences and events. • Provide a conclusion that follows from the narrated experiences or events. 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.) 5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.			• Why does a writer choose the form of writing he/she does? • What is the relationship between reader and writer? • How do writers communicate clearly? • To what extent do conventions of language impact communication?

	frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes			
COMPOSITION: Writing a Research Paper	**Writing** 2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. • Introduce a topic; organize ideas, concepts, and information, using strategies such as definition, classification, comparison/ contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. • Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. • Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. • Use appropriate transitions to clarify the relationships among ideas and concepts. • Use precise language and domain-specific vocabulary to inform about or explain the topic. • Provide a concluding statement or section that follows from the information or explanation presented. 3. Write narratives to develop real or	Write for a specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers understand.	• What makes writing worth reading? • How do effective writers hook and hold their readers? • What makes writing easy to follow? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • What is a complete thought? • Why am I writing? • For whom? • What am I trying to achieve through my writing? • Who will read my writing? • What will work best for my audience? • Why does a writer choose the form of writing he/she does? • What is the relationship between reader and writer? • How do writers communicate clearly?

	• Engage and orient the reader by establishing a context and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically. • Use narrative techniques, such as dialogue, pacing, and description, to develop experiences, events, and/or characters. • Use a variety of transition words, phrases, and clauses to convey sequence and signal shifts from one time frame or setting to another. • Use precise words and phrases, relevant descriptive details, and sensory language to convey experiences and events. • Provide a conclusion that follows from the narrated experiences or events. 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.) 5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach. 7. Conduct short research projects to answer a question, drawing on several sources and refocusing the inquiry when appropriate. 8. Gather relevant information from multiple print and digital sources; assess the credibility of each source; and quote			

	others while avoiding plagiarism and providing basic bibliographic information for sources. 9. Draw evidence from literary or informational texts to support analysis, reflection, and research. • Apply *grade 6 Reading standards* to literature (e.g., "Compare and contrast texts in different forms or genres [e.g., stories and poems; historical novels and fantasy stories] in terms of their approaches to similar themes and topics"). • Apply *grade 6 Reading standards* to literary nonfiction (e.g., "Trace and evaluate the argument and specific claims in a text, distinguishing claims that are supported by reasons and evidence from claims that are not"). 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes. **Language** Use knowledge of language and its conventions when writing, speaking, reading, or listening. • Vary sentence patterns for meaning, reader/listener interest, and style.* • Maintain consistency in style and tone.*			

	1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a central idea of a text and how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments. 3. Analyze in detail how a key individual, event, or idea is introduced, illustrated, and elaborated in a text (e.g., through examples or anecdotes). 7. Integrate information presented in different media or formats (e.g., visually, quantitatively) as well as in words to develop a coherent understanding of a topic or issue. 8. Trace and evaluate the argument and specific claims in a text, distinguishing claims that are supported by reasons and evidence from claims that are not. 9. Compare and contrast one author's presentation of events with that of another (e.g., a memoir written by and a biography on the same person). 10. By the end of the year, read and comprehend literary nonfiction in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.			
COMPOSITION: Writing to a Prompt	**Writing** 1. Write arguments to support claims with clear reasons and relevant evidence. • Introduce claim(s) and	Write for a specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice).	• How do writers express their thoughts and feelings? • Where do ideas for writing come from? • What makes writing flow?

	reasons and relevant evidence, using credible sources and demonstrating an understanding of the topic or text. • Use words, phrases, and clauses to clarify the relationships among claim(s) and reasons. • Establish and maintain a formal style. • Provide a concluding statement or section that follows from the argument presented. 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.) **Language** 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. • Use punctuation (commas, parentheses, dashes) to set off nonrestrictive/		• Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers through the text to help avoid confusion. • A writer selects a form based on his purpose. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers understand.	• How do effective writers hook and hold their readers? • What makes writing easy to follow? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • What is a complete thought? • Why am I writing? • For whom? • What am I trying to achieve through my writing? • Who will read my writing? • What will work best for my audience? • Why does a writer choose the form of writing he/she does? • How do writers
COMPOSITION: Writing an Editorial	**Writing** 2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. • Introduce a topic; organize ideas, concepts, and information, using strategies such as definition,	Write for a specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers	• What makes writing worth reading? • How do writers express their thoughts and feelings? • Where do ideas for writing come from? • What makes writing flow? • How do effective writers hook and hold

	cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. • Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. • Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. • Use appropriate transitions to clarify the relationships among ideas and concepts. • Use precise language and domain-specific vocabulary to inform about or explain the topic. • Provide a concluding statement or section that follows from the information or explanation presented. 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.) 5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach. 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes		confusion. • A writer selects a form based on his purpose. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers understand. • A writer's word choice and syntax are characteristics of voice which help to personalize text.	• What makes writing easy to follow? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • What is a complete thought? • Why am I writing? • For whom? • What am I trying to achieve through my writing? • Who will read my writing? • What will work best for my audience? • Why does a writer choose the form of writing he/she does? • What is the relationship between reader and writer? • How do writers communicate clearly? • To what extent do conventions of language impact communication? • What is the voice thing, anyway?

	Language 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. • Vary sentence patterns for meaning, reader/ listener interest, and style.* • Maintain consistency in			
COMPOSITION: Writing a Speech	**Writing** Write arguments to support claims with clear reasons and relevant evidence. • Introduce claim(s) and organize the reasons and evidence clearly. • Support claim(s) with clear reasons and relevant evidence, using credible sources and demonstrating an understanding of the topic or text. • Use words, phrases, and clauses to clarify the relationships among claim(s) and reasons. • Establish and maintain a formal style. • Provide a concluding statement or section that follows from the argument presented. 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	Write for a specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Writers do not always say what they mean. Indirect forms of expression (e.g., satire, irony) require readers to read between the lines to find the intended meaning. • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers	• Why write? • Why share personal experiences in writing? • What makes writing worth reading? • How do writers express their thoughts and feelings? • Where do ideas for writing come from? • What makes writing flow? • How do effective writers hook and hold their readers? • What makes writing easy to follow? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • What is a complete thought? • Why am I writing? • For whom?

	editing, rewriting, or trying a new approach. 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences. **Languag e** 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. • Use punctuation (commas, parentheses, dashes) to set off nonrestrictive/ parenthetical elements.* • Spell correctly. **Speaking & Listening** 4. Present claims and findings, sequencing ideas logically and using pertinent descriptions, facts, and details to accentuate main ideas or themes; use appropriate eye contact, adequate volume, and clear pronunciation. 5. Include multimedia components (e.g., graphics, images, music, sound) and visual displays in presentations to clarify information. 6. Adapt speech to a variety of contexts and tasks, demonstrating		which help to personalize text.	• Who will read my writing? • What will work best for my audience? • Why does a writer choose the form of writing he/she does? • What is the relationship between reader and writer? • How do writers communicate clearly? • To what extent do conventions of language impact communication?
COMPOSITION:	**Writin g**	Write for a specific audience and purpose.	• Audience and purpose	• What makes writing flow? • What makes writing easy

Writing a Business Letter	2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. • Introduce a topic; organize ideas, concepts, and information, using strategies such as definition, classification, comparison/ contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. • Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. • Use appropriate transitions to clarify the relationships among ideas and concepts. • Use precise language and domain-specific vocabulary to inform about or explain the topic. • Establish and maintain a formal style. • Provide a concluding statement or section that follows from the information or explanation presented. **Language** 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. • Vary sentence patterns for meaning, reader/listener interest, and style.*		persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers through the text to help avoid confusion. • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers understand. • A writer's word choice and syntax are characteristics of voice which help to personalize text.	to follow? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • What is a complete thought? • Why am I writing? • For whom? • What am I trying to achieve through my writing? • What will work best for my audience? • Why does a writer choose the form of writing he/she does? • What is the relationship between reader and writer? • How do writers communicate clearly? • To what extent do conventions of language impact communication?

COMPOSITION: Writing a Compare and Contrast Essay	**Writing** 2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. • Introduce a topic; organize ideas, concepts, and information, using strategies such as definition, classification, comparison/ contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. • Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. • Use appropriate transitions to clarify the relationships among ideas and concepts. • Use precise language and domain-specific vocabulary to inform about or explain the topic. • Establish and maintain a formal style. • Provide a concluding statement or section that follows from the information or explanation presented. 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	Write for a specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers through the text to help avoid confusion. • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers understand. • A writer's word choice and syntax are characteristics of voice which help to personalize text.	• What makes writing worth reading? • How do writers express their thoughts and feelings? • Where do ideas for writing come from? • What makes writing flow? • How do effective writers hook and hold their readers? • What makes writing easy to follow? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • What is a complete thought? • Why am I writing? • For whom? • What am I trying to achieve through my writing? • Who will read my writing? • What will work best for my audience? • Why does a writer choose the form of writing he/she does? • What is the relationship between reader and writer? • How do writers

	when writing. • Use punctuation (commas, parentheses, dashes) to set off nonrestrictive/ parenthetical elements.* • Spell correctly. **Reading - Informational Text** 1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a central idea of a text and how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments. 9. Compare and contrast one author's presentation of events with that of another (e.g., a memoir written by and a			
COMPOSITION: Writing a Character Sketch	**Writing** 2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. • Introduce a topic; organize ideas, concepts, and information, using strategies such as definition, classification, comparison/ contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. • Develop the topic with relevant facts, definitions	Write for a specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers through the text to help avoid confusion. • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience.	• Where do ideas for writing come from? • What makes writing flow? • How do effective writers hook and hold their readers? • What makes writing easy to follow? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • What is a complete thought? • Why am I writing?

	information and examples. • Use appropriate transitions to clarify the relationships among ideas and concepts. • Use precise language and domain-specific vocabulary to inform about or explain the topic. • Establish and maintain a formal style. • Provide a concluding statement or section that follows from the information or explanation presented. 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences. **Languag e** 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. • Use punctuation (commas, parentheses, dashes) to set off nonrestrictive/ parenthetical elements.*		help readers understand what is being communicate d. • The purposeful use and non-use of language conventions help readers understand. • A writer's word choice and syntax are characteristics of voice which help to personalize text.	achieve through my writing? • Who will read my writing? • What will work best for my audience? • Why does a writer choose the form of writing he/she does? • What is the relationship between reader and writer? • How do writers communicate clearly? • To what extent do conventions of language impact communication?

GRAMMAR, USAGE, and MECHANICS Punctuatio n Nouns Pronouns I Pronouns II Adjectives Verbs I Verbs II Verbs III Adverbs Prepositions, Conjunctions, and Interjections I Prepositions, Conjunctions, and Interjections II	**Languag e** 1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. • Ensure that pronouns are in the proper case (subjective, objective, possessive). • Use intensive pronouns (e.g., *myself, ourselves*). • Recognize and correct inappropriate shifts in pronoun number and person.* • Recognize and correct vague pronouns (i.e., ones with unclear or ambiguous antecedents).* • Recognize variations from standard English in their own and others' writing and speaking, and identify and use strategies to improve expression in conventional language.* 2. Demonstrate command of the conventions of standard English	Demonstrate command of proper grammar conventions.	• Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers through the text to help avoid confusion. • Conventions of language help readers understand what is being communicated • A writer's word choice and syntax are characteristics of voice which help to personalize text.	• To what extent do conventions of language impact communication? • How do writers communicate clearly? • Why do we need grammar?

VOCABULARY: Units 1-17	**Languag e** 4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grade 6 reading and content, choosing flexibly from a range of strategies. • Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. • Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., *audience, auditory, audible*). • Consult reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. • Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary). 5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. • Interpret figures of speech (e.g., personification) in context. • Use the relationship between particular words	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Sometimes the author makes his/her meaning plain; often, however, a reader must dig beneath the "surface" of the text to find the meaning. • Good readers employ strategies to help them understand text. • Good readers may use many strategies that work, and they quickly try another one when the one they are using doesn't work. They not only know many different strategies, but they never get stuck in persisting with one that isn't working	• What do you do when you do not understand everything in the text? • What do good readers do? • What do they do when they do not understand?

	• Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., *stingy, scrimping, economical, unwasteful, thrifty*). 6. Acquire and use accurately grade- appropriate general academic and domain-specific words and phrases; gather			
LITERATURE: Lessons Learned Students analyze, compare, and creatively respond to a variety of works. The emphasis is on literary nonfiction. It will include a blend of literature (fiction, poetry, and drama. Students will read works of nonfiction, as well as read five novels (selected from a long list, such as: (Roll of Thunder Hear My Cry, The People Could Fly, Sorry, Wrong Number, Narrative of the Life of Frederick Douglas an American Slave)	**Reading - Literature** 3. Describe how a particular story's or drama's plot unfolds in a series of episodes as well as how the characters respond or change as the plot moves toward a resolution. 5. Analyze how a particular sentence, chapter, scene, or stanza fits into the overall structure of a text and contributes to the development of the theme, setting, or plot. **Speaking & Listening** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly. • Review the key ideas expressed and demonstrate understanding of multiple perspectives through reflection and paraphrasing.	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context. Humans relate to universal themes through story-telling	• Different types of texts (e.g., narrative, mystery, biography, expository, persuasive) have different structures. Understanding a text's structure helps a reader better understand its meaning. • A good story has a pattern or plan. • Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find that meaning.	• What do good readers do? • What do they do when they do not understand? • How do texts differ? • How should I read different types of texts? • What lies beneath the surface of this text? (In fiction: symbol and theme; in nonfiction texts: assumptions, biases, preconceptions)

LITERATURE: Mostly Heroes Explore literary heroes around the world and describe their characteristics. Make inferences and draw conclusions based on evidence from the text	**Reading - Literature** 1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 6. Explain how an author develops the point of view of the narrator or speaker in a text. 9. Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics. **Speaking & Listening** 2. Interpret information presented in diverse media and formats (e.g., visually, quantitatively, orally) and	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context. Good triumphs over evil	• Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find that meaning.	• From whose viewpoint are we reading? • What is the author's angle or perspective? • What should we do when texts or authors disagree?
LITERATURE: Informational Text/Narrative of the Life of Frederick Douglas and American Slave	**Reading - Literature** 1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a theme or central idea of a text and how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments. 5. Analyze how a particular sentence, chapter, scene, or stanza fits into the overall structure of a text and contributes to the development of the theme, setting, or plot. 7. Compare and contrast the experience of reading a story, drama, or poem to listening to or viewing an audio, video	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find that meaning.	• From whose viewpoint are we reading? • What is the author's angle or perspective?

	contrasting what they "see" and "hear" when reading the text to what they perceive when they listen or watch			
LITERATURE: Poetry The Poetry of Robert Frost, Chicago Poems, I, Too, Sing America	**Reading - Literature** 4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of a specific word choice on meaning and tone. 9. Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range. **Languag e** 5. Demonstrate understanding of figurative language, word	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex stories in which the inner and outer lives of human beings are revealed.	• What makes a book or story great? • What is the relationship between popularity and greatness in literature? • Is a "good read" always a great book? What makes a poem a poem?
LITERATURE: Informational Text/The Great Fire	**Reading - Literature** 1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 9. Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find that meaning. • Great literature provides rich and timeless insights into the key	• From whose viewpoint are we reading? • What is the author's angle or perspective? • What should we do when texts or authors disagree? • What makes a book or story great? • Is a "good read" always a great book?

	10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range. **Reading - Informational Text** 8. Trace and evaluate the argument and specific claims in a text, distinguishing claims that are supported by reasons and evidence from claims that are not. 10. By the end of the year, read and comprehend literary nonfiction in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range. **Speaking & Listening** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics		They present complex stories in which the inner and outer lives of human beings are revealed.	
LITERATURE: Harriet Tubman: Conductor on the Underground Railroad	**Reading - Literature** 1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 9. Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find that meaning. • Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex	• From whose viewpoint are we reading? • What is the author's angle or perspective? • What makes a book or story great? • Is a "good read" always a great book?

	the range.			
LITERATURE : Early American Lives "Letter on Thomas Jefferson"	**Reading - Literature** 1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a theme or central idea of a text and how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments. 5. Analyze how a particular sentence, chapter, scene, or stanza fits into the overall structure of a text and contributes to the development of the theme, setting, or plot. 6. Explain how an author develops the point of view of the narrator or speaker	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find that meaning. • Different types of texts (e.g., narrative, mystery, biography, expository, persuasive) have different structures. Understanding a text's structure helps a reader better understand its meaning. • A good story has a pattern or	• From whose viewpoint are we reading? • What is the author's angle or perspective? • What do good readers do? • What do they do when they do not understand? • How do texts differ? • How should I read different types of texts?
	text and how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments. 5. Analyze how a particular sentence, chapter, scene, or stanza fits into the overall structure of a text and contributes to the development of the theme, setting, or plot. 6. Explain how an author develops the point of view of the narrator or speaker in a text.		(e.g., narrative, mystery, biography, expository, persuasive) have different structures. Understanding a **text's structure** helps a reader better understand its meaning. • A good story has a pattern or	they do not understand? • How do texts differ? • How should I read different types of texts?

LITERATURE : People Could Fly	**Reading - Literature** 1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a theme or central idea of a text and how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments. 5. Analyze how a particular sentence, chapter, scene, or stanza fits into the overall structure of a text and contributes to the development of the theme, setting, or plot. 6. Explain how an author develops the point of view of the narrator or speaker in a text. 9. Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics. 10. By the end of the year, read and	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find that meaning. • Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex stories in which the inner and outer lives of human beings are revealed.	• From whose viewpoint are we reading? • What is the author's angle or perspective? • What makes a book or story great? • Is a "good read" always a great book?

	complexity band proficiently, with scaffolding as needed at the high end of the range. **Speaking & Listening** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly. • Come to discussions prepared, having read or studied required material; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. • Follow rules for collegial discussions, set specific goals and deadlines, and define individual roles as needed. • Pose and respond to specific questions with elaboration and detail by making comments that contribute to			
LITERATURE : The Adventures of Tom Sawyer	**Reading - Literature** 1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 9. Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics.	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find that meaning. • Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that [illegible]	• From whose viewpoint are we reading? • What is the author's angle or perspective? • What makes a book or story great? • Is a "good read" always a great book?

	dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end		inner and outer lives of human beings are revealed.	
	Speaking & Listening 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly. • Come to discussions prepared, having read or studied required material; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. • Follow rules for collegial discussions, set specific goals and deadlines, and define individual roles as needed. • Pose and respond to specific questions with elaboration and detail by making comments that contribute to			

LITERATURE : Sherlock Holmes	**Reading - Literature** 1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 6. Explain how an author develops the point of view of the narrator or speaker in a text. 9. Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find that meaning. • Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex stories in which the inner and outer lives of human beings are revealed.	• From whose viewpoint are we reading? • What is the author's angle or perspective?
LITERATURE : Wrong Number	**Reading - Literature** 1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of a specific word choice on meaning and tone. 9. Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find that meaning. • Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex stories in which the inner and outer lives of human beings are revealed.	• From whose viewpoint are we reading? • What is the author's angle or perspective?

LITERATURE : *Little Women*	**Reading - Literature** 1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 3. Describe how a particular story's or drama's plot unfolds in a series of episodes as well as how the characters respond or change as the plot moves toward a resolution. 6. Explain how an author develops the point of view of the narrator or speaker in a text. 9. Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics. 10. By the end of the year, read and comprehend literature, including stories	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find that meaning. • Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex stories in which the inner and outer lives of human beings are revealed.	• From whose viewpoint are we reading? • What is the author's angle or perspective?
	scaffolding as needed at the high end of the range			

LITERATURE : American Tall Tales	**Reading - Literature** 1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a theme or central idea of a text and how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments. 3. Describe how a particular story's or drama's plot unfolds in a series of episodes as well as how the characters respond or change as the plot moves toward a resolution. 4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of a specific word choice on meaning and tone. 6. Explain how an author develops the point of view of the narrator or speaker in a text. 9. Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find that meaning. • Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex stories in which the inner and outer lives of human beings are revealed.	• From whose viewpoint are we reading? • What is the author's angle or perspective?
LITERATURE : The Dark is	**Reading - Literature**	Learn new vocabulary words to be used in writing as well as learn how to define	• Sometimes the author makes his/her meaning plain; often	• From whose viewpoint are we reading? • What is the author's angle

	of what the text says explicitly as well as inferences drawn from the text. 9. Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of	vocabulary words in context.	reader must dig beneath the "surface" of the text to find that meaning. • Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex stories in which the inner and outer lives of human	or perspective?
SPELLING: Units 1-36	**Languag e** 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Conventions of language help readers understand what is being communicated.	• How do writers communicate clearly?

Unit Title: Poetry

Delaware Recommended Curriculum Unit Template

Preface: This unit has been created as a model for teachers in their designing or redesigning of course curricula. It is by no means intended to be inclusive; rather it is meant to be a springboard for teacher thought and creativity. The information we have included represents one possibility for developing a unit based on the Delaware content standards and the Understanding by Design framework and philosophy.

Grade Level(s): 6th Grade

Subject/Topic Area: English Language Arts/Poetry

Reviewed by: **Date:** 07.09.13

Brief Summary of Unit (This should include a brief unit summary including a description of unit goals, rationale for the approach taken, and where it appears in the course of study.)

In this unit, students will not only read, but the students will analyze and understand poetry. Students will produce their own poetry in the styles of those read. While exploring poetry, students will also learn about the humans experience expressed within poetry and will have a chance to record their own experiences. This unit uses an interactive approach to teach 6th grade students poetry.

Stage 1: Desired Results
(Determine What Students Will Know, Do and Understand)

DE Common Core State Standards measured by the Transfer Task and Assessments (This should include a list describing which DE CCSS are measured in Stage 3. This could include standards, Grade level expectations, benchmarks and performance indicators.)

CCSS.ELA-Literacy.RL.6.1 Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text

CCSS.ELA-Literacy.RL.6.2 Determine a theme or central idea of a text and how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments

CCSS.ELA-Literacy.RL.6.4 Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings, meanings, analyze the impact of a specific words choice on meaning and tone

CCSS.ELA-Literacy.RL.6.9 Compare and contrast texts in different forms or genes(e.g. stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics

Big Ideas (This should include transferable core concepts, principles, theories, and processes that should serve as the focal point of curricula, instruction, and assessment. Ex: Manifest Destiny, fighting for peace, writing process.)

Students will understand that

Poetry can be interpreted in many ways

Poets make deliberate and thoughful decisions about style, tone, rhythm, and structure, and word choice

Unit Title: Poetry

Poetry can take many forms

Poetry is about the use of carefully chosen language

Unit Enduring Understanding(s)

Full-sentence, important statements or generalizations that specify what students should understand from the Big Ideas(s) and/or Content Standards and that are transferable to new situations

Poems have been written by all cultures and peoples throughout history

As societies changed over time so did poetry

Early poetry is also believed to be connected to music, chanting, and prayer

Most early poetry was thought to have grown from oral tradition of reciting or singing

Poetry is an important and valued literary genre

Poets use carefully chosen language to express various forms of poetry

Tone and meaning are conveyed through the poet's word choice

Rhyme and rhythm influence meaning

Unit Essential Question(s) (This should include open-ended questions designed to guide student inquiry and focus instruction for "uncovering" the important ideas of the content. Ex: What is healthful eating? What is the relationship between fiction and truth?)

What are your own ideas and assumptions about poetry?

What are your favorite poems?

How can you express yourself through poetry writing?

What is poetry?

What is a poem?

What strategies can we use to understand poetry?

How does word choice help readers experience poetry?

What devices do poets use?

How do we make meaning of poetry?

How are song lyrics and poetry alike and different?

How do poets express themes of self and identity in their work?

What are some different poetic forms and structures and how do they affect meaning?

What is the significance of poem's cultural and historical context?

What makes a poet's voice intense, unique, and memorable?

How does performance affect the meaning of the written word in a poem?

Unit Title: Poetry

What do all poems have in common?

How can I express myself through poetry?

Knowledge & Skills (This should include key knowledge and skills that students will acquire as a result of this unit? Ex: Factors affecting climate, The causes of World War II.)
It should also include what students will eventually be able to do as a result of such knowledge and skill Ex: take notes, complete a bent-arm pull, compare fiction to nonfiction.)

Students will know….

the different types of poems

literary terms associated with poetry

what makes a poem a poem

poetry can take many forms

Students will be able to….

read, discuss, perform and appreciate a variety of poems

analyze poems and identity different poetic forms and devices

create original poetry using various forms and devices

create original poetry using various forms and incorporate various poetic devices

research the cultural and historical context of poems/poets

know and understand the body of work and cultural/historical context of at least one poet

write a collection of original poems that reflect different forms

respond to poetry

compare, contrast, and evaluate poetry

list characteristics of all poems

define poetry

Stage 2: Assessment Evidence
(Design Assessments To Guide Instruction)

(This should include evidence that will be collected to determine whether or not the Desired Results identified in Stage One have been achieved? [Anchor the unit in performance tasks that require transfer, supplemented as needed by other evidence –quizzes, worksheets, observations, etc.]

Group Discussions

Class Discussions

Student's Responses

Student's Self-written poetry pieces

Other Assessment Evidence
(This could include tests, quizzes, prompts, student work samples, and observations used to collect diverse

evidence of student understanding.)
Oral presentations Poetry Booklet Poetry Unit Test
Student Self-Assessment and Reflection (This should include opportunities for students to monitor their own learning. Ex: reflection journals, learning logs, pre- and post-tests, editing own work.)

Stage 3: Learning Plan
(Design Learning Activities To Align with Goals and Assessments)

Key learning events (lessons) needed to achieve unit goals

Lesson One: Love That Dog: An Introduction to Poetry

Skill Objective(s)

Students will write a response to poems heard in class
Students will listen to poetry read aloud by the teacher and other students.
Students will respond to daily lessons with a group discussion
Students will recall the term poetry

Materials

Love that Dog by Sharon Creech
Students will need pen and paper

Key Vocabulary

Poetry-prose that resembles a poem in some respect

Procedures/Activities

Begin by asking students to express any thoughts that come to mind when you say the word "poetry." Write their responses on the board.

Have the students get out notebook paper and a pen. Explain to them that you will read them a selection of poems put together in story format, and that you will periodically pause to give them an opportunity to respond.

Read *Love That Dog* aloud to the class. Pause after November 6, December 16, February 7, March 26, May 14, and after the ending to give the students a minute or two to write a response.

Ask students to share one/some/all of their responses with the class.
Lead a classroom discussion about whether or not poetry has to be hard. Ask such questions as "Were these poems confusing? Hard to understand? Did these poems talk about something we can understand? From where did Jack get his inspiration?"

Assessment/Evaluation

Use the classroom discussion and student responses to ensure that students understand the idea that not all poetry is hard and confusing.

Inform the students that they will have a response assignment after every poem they read in class. For this lesson, assign a poem that is inspired by *Love That Dog*.

Lesson Two: It's Not a Crime to Mix a Lime with Rhyme

Skill Objective(s)

Students will write a response to poems heard in class.
Students will listen to poetry read aloud by the teacher and other students.
Students will respond to daily lessons with a group discussion.
Students will learn the term rhyme.
Students will use poems to find examples of rhyme and rhyme scheme.
Students will write a poem demonstrating rhyme

Materials

Appendix A: Poetry Packet

Key Vocabulary

Rhyme-words that have the same end sound
Rhyme Scheme-the pattern of rhyming in a poem

Procedures/Activities

Ask the students what the word rhyme means and to give examples. Review the definition if it is not revealed through class discussion.

Introduce rhyme scheme and ask the students if they have any guesses as to the meaning. Write the definition on the board and have students record it in

Appendix A: Poetic Terminology.
Have the students skim through all of the poems to look for rhyme scheme. They should write an "RS" at the top of the page of all poems that contain rhyme scheme. Give them 10-15 minutes for this activity. Make a list of poems containing rhyme scheme. See Appendix B: Teacher Aid: Terms & Poems.
Read Robert Frost's "Stopping by Woods on a Snowy Evening" aloud to the class.
S e e Appendix C: Reading Poetry.

Work with the students on identifying the pattern of rhyming words. Write the words vertically on

the board. Label the rhyme scheme accordingly (ABBA).

Discuss with the students what is happening in this poem, and the images/sounds that this poem creates.

Assessment/Evaluation

Collect the "Inspired by Love That Dog" poem.
Assign a rhyming poem, at least eight lines long, about nature. Students need to integrate a rhyme scheme.

Lesson Three: Iamb a Meter

Skill Objective(s)

Students will write a response to poems heard in class.
Students will listen to poetry read aloud by the teacher and other students.
Students will respond to daily lessons with a group discussion.
Students will learn the terms meter, iamb, foot, and pentameter.
Students will identify the terms meter, iamb, foot and pentameter in poems.
Students will write a poem in iamb.

Materials

Appendix A: Poetry Packet

Key Vocabulary

Meter—the rhythm of a poem

Accented Syllable—the syllable in a word that carries the accent.

Unaccented Syllable—the syllable in a word that is not accented.
Foot—a combination of accented and unaccented syllables.
Iamb—a form of meter where each foot is one unaccented syllable followed by one accented syllable.
Pentameter—five feet per line.

Procedures/Activities

Read the first stanza of the poem "Apostrophe to the Ocean" by George Gordon Byron. Exaggerate the meter of the poem.
After the stanza, ask students what they noticed about the reading. Guide them to discuss the rhythm of the stanza.
Write the definition of Meter on the board and have students record it in their packet. Ask for volunteers to share examples of poems/songs with meter (not including the poems in the unit). Some examples are "The Night Before Christmas,"
Write the word "pleasure" on the board. Ask students how many syllables this word have (2). Ask if they know which syllable is accented, or which syllable is receiving the emphasis (plea), and which syllable is unaccented (sure). Mark the word accordingly on the board. See Appendix B for tips on

marking meter.
Looking at the first line of the poem, have students mark the accented syllables.
Help them discover that the pattern is unaccented followed by accented in a repeating manner.
Explain to them that this is iamb. They should record this in their packet.
Explain the meaning of "foot" to the class. As a group, count the number of feet in the first line. Have students count the feet in subsequent lines by themselves. Allow five to ten minutes for this, depending on their understanding.
Finish reading the poem. Point out that while difficult, overemphasizing the rhythm can ruin the understanding of the poem. When you are finished reading, ask if there is a rhyme scheme. Students should figure out and record the rhyme scheme in their packets.
Discuss the meaning of the poem, what happens, and the images that are created.

Assessment/Evaluation

Collect the Rhyme Scheme Poem.
Assign an Iamb Poem written in the form of a one-sided conversation (in the same way of Byron). It should be at least 8 lines long.

Lesson Four: Simile: You're on Many Metaphors!

Skill Objective(s)

Students will write a response to poems heard in class.
Students will listen to poetry read aloud by the teacher and other students.
Students will respond to daily lessons with a group discussion.
Students will learn the terms simile and metaphor.
Students will write a poem using a simile or metaphor

Materials

Appendix A: Poetry Packet

Key Vocabulary

Simile—A comparison between two objects using like or as.
Metaphor—A comparison between two objects without using like or as.

Procedures/Activities

Review the meaning of simile and metaphor with the class. They should write the definitions in their packets.
Read Emily Dickinson's "There Is No Frigate Like a Book." Ask the students to identify any similes or metaphors as you read the poem. Discuss the meaning of the comparisons and how they relate.
Review previous vocabulary terms by asking students if there are any rhyming words, and if they are consistent enough to have a rhyme scheme.
With the students, check the rhythm of the poem. Ask the class if it is iambic (it is).
Discuss how the images in this poem are created and why they are effective.
Have a student read "Sympathy" by Paul Laurence Dunbar. Students need to label the rhyme

scheme, and underline any similes that they find.
Discuss why this poem is a metaphor, and to what does Dunbar compare a caged bird?

Assessment/Evaluation

Collect the rhyming poems.
Assign students a simile/metaphor poem of at least 8 lines in length.

Lesson Five: More On Metaphors

Skill Objective(s)

Students will write a response to poems heard in class.
Students will listen to poetry read aloud by the teacher and other students.
Students will respond to daily lessons with a group discussion.
Students will write a poem in iamb.
Students will learn the terms simile and metaphor.
Students will write a poem using a simile or metaphor.
Students will edit and revise their poem using simile or metaphor

Materials

Appendix A: Poetry Packet

Key Vocabulary

Rhyme Scheme

Procedures/Activities

Ask for a student volunteer to read Shakespeare's "All the World's a Stage." During the reading, remind students that they need to look for similes and metaphors during the reading. As usual, they should also check for iamb and rhyme scheme.
Discuss all of the metaphors in this poem (life's a stage, the seven acts and the characters involved, etc.).Read Wadsworth's "I Wandered Lonely as a Cloud." Students should look for rhyme scheme and label it as necessary. Lead a class discussion on the meaning of this poem.

Assessment/Evaluation

Give students time to revise their simile/metaphor poem. Collect it.

Lesson Six: Poetry Day 6

Skill Objective(s)

Students will write a response to poems heard in class.
Students will listen to poetry read aloud by the teacher and other students.
Students will respond to daily lessons with a group discussion.
Students will learn the term free verse.

Students will relate cultural poems to their historical significance.
Students will label poems with poetic terms.

Materials

Appendix A: Poetry Packet

Key Vocabulary

Free Verse—Poetry written without a specific rhyme scheme or meter.

Procedures/Activities

As a class read "Mother to Son" by Langston Hughes. Students should determine rhyme scheme, meter, and simile/metaphor.
Discuss with the class why this poem was written. Students should discover why the mother is pushing the son to continue on. It's also important to not the relation this poem has to slavery and the way it has been viewed in the United States.
Talk briefly about the rhyme scheme (or lack of one) and meter. Explain that this is considered free verse poetry. Students should write the definition in their packets.
As a class read "Lift Every Voice and Sing" by James Weldon Johnson. Students should be able to identify the rhyme scheme and any simile/metaphor.
Ask to students to volunteer ideas as to the type of feelings that this poem creates. Search for ideas as to why this poem would be considered the national anthem of African Americans.

Assessment/Evaluation

As a homework assignment, students should read Angelou's "Woman Work." They need to label rhyming words/rhyme scheme and any simile/metaphors. Students should then write a brief description of what is happening in the poem, as well as any ideas on to why the rhyming breaks halfway through the poem.

Lesson Seven: A Psalm of Life

Skill Objective(s)

Students will write a response to poems heard in class.
Students will listen to poetry read aloud by the teacher and other students.
Students will respond to daily lessons with a group discussion.
Students will label poems with poetic terms.
Students will listen to their peers read a poem in front of an audience.

Materials

Appendix A: Poetry Packet

Key Vocabulary

Stanza

Rhyme scheme

Procedures/Activities

Break students into groups of two or three. Have them read "A Psalm of Life" by Longfellow aloud to each other. They need to label rhyme scheme (if it exists), meter (including iamb), and any simile/metaphor.
Assign each group a stanza that they need to decipher. They will then explain the meaning of their stanza to the rest of the class.

Assessment/Evaluation

Evaluate understanding based off of oral presentation.
For homework. Students need to read Dickinson's "A Narrow Fellow in the Grass." As usual, they should look for rhyme, meter, and simile/metaphor. On a separate piece of paper, they need to write what they think the "narrow fellow" is along with examples from the poem that support their idea(s).

Lesson Eight: Simply Nonsense

Skill Objective(s)

Students will write a response to poems heard in class.
Students will listen to poetry read aloud by the teacher and other students.
Students will respond to daily lessons with a group discussion.
Students will learn the term nonsense, as it relates to poetry.
Students will make a list of nonsensical things.
Students will write a nonsense poem.

Materials

Appendix A: Poetry Packet

Key Vocabulary

Nonsense—poetry that isn't logical

Procedures/Activities

Read "The Walloping Window-blind" by Charles E. Carryl aloud to the class.
Discuss rhyme, meter, and simile/metaphor.
Ask the students if this poem makes logical sense. Have them record "Nonsense" into their packets as well as write down the definition.
Make a list of things that are nonsensical in the poem. Have students explain why they are nonsense.
Remind students that nonsense poetry should be more on the silly side.
Summarize the plot of this poem with the class.

Assessment/Evaluation

Write a nonsense poem at least 8 lines long.

Unit Title: Poetry

Lesson Nine: Heritage

Skill Objective(s)

Students will write a response to poems heard in class.
Students will listen to poetry read aloud by the teacher and other students.
Students will respond to daily lessons with a group discussion.
Students will have a discussion about their heritage.

Materials

Appendix A: Poetry Packet

Key Vocabulary

Meter

Procedures/Activities

Read "A Song of Greatness." Are there any rhyming words? Consistent meter?
Simile/Metaphor?
Discuss with the class the voice of the poem. What is the narrator longing for?
How do you know? What makes a hero or a great person?
Lead a discussion with the class about family histories/heirlooms/stories that are important to them.
Make a list on the board.

Assessment/Evaluation

Write a heritage poem at least 13 lines long. Students may decide if they want to use meter or rhyme. The only requirement (besides length) is that the poem needs to be at least remotely related to family history.

Resources & Intervention Strategies (Consider the two questions below when completing this section.)

Books:

Creech, Sharon. *Love that Dog.* New York, NY. Joanna Cotler Books, 2001. 0-06-029287-3.

Hirsh, E.D. *What Your 6th Grader Needs to Know*. New York, NY. Delta Trade Paperbacks, 1993. 0-385-31467-1.

Kennedy, X. J., et. Al. *Handbook of Literary Terms: Literature, Language, Theory.* Longman, 2004. 0321202074

Accommodation/Differentiation ideas and tips (This should include a list or description of ways that you will differentiate instruction according to students' needs. This can include any curricular adaptations that are needed to meet special needs students. Ex: using reading materials at varying readability levels, putting text materials on tape, using spelling or vocabulary lists at readiness levels of students, meeting with small groups to re-teach an idea or skill for struggling learners, or to extend the

thinking or skills of advanced learners.
Read aloud **Guided notes** **Modified assignments** **Extended time** **Visual cues** **Small-group instruction**

Appendix A-1: Poetry Packet

Poetry Terminology

1. poetry_____

_____

2. rhyme_____

_____

3. rhyme scheme_____

_____

4. couplet_____

_____

5. meter_____

_____

6. accented syllable_____

_____

7. unaccented syllable _____

_____

8. foot_____

_____

9. iamb_____

_____

Appendix A-2: Poetry Packet

10. pentameter _____

_____

11. simile _____

_____

12. metaphor _____

_____

13. free verse _____

_____

14. nonsense _____

_____

15. _____

_____

16. _____

_____

17. _____

_____

18. _____

_____

19. _____

_____

Appendix B: Teacher Information on How to Mark Poetry

In this unit, students will be marking their poems by labeling rhyme scheme, identifying simile/metaphors, and distinguishing iamb. This appendix is a teacher's aid for consistency.

Rhyme Scheme

For the sake of middle school simplicity, rhyme scheme is identified by looking at the end of each line of poetry. Let's use Robert Frost's "The Road Not Taken" as an example:

Two roads diverged in a yellow wood,
And sorry I could not travel both
And be one traveler, long I stood
And looked down one as far as I could
To where it bent in the undergrowth;

The words we are looking at are: wood, both, stood, could, and growth. We will label wood with an A (the first sound) and both with B (the second sound—if it rhymed with wood it would be labeled A). Does stood rhyme with wood? Yes! So it is labeled with an A also. What about could? And finally, does growth rhyme with could? No, but it rhymes with both so it gets a B. Therefore, the rhyme scheme in this stanza is ABAAB. On the next stanza, a new sound should be labeled C, etc. If any other line ends with a word that rhymes with wood, it will get an A label. This is the way that students should be marking rhyme scheme. If you were to do this same approach with "A Song of Greatness," none of the lines would form a pattern (you could stretch it, but there is no rhyme scheme in this poem!).

Meter/Iamb

When figuring out meter and iamb, students should identify accented syllables (/)and unaccented syllables (---) in the multi-syllabic words. When searching for Iamb, they should look for a duh-DUH duh-DUH type rhythm. For example, in "Stopping by Woods on a Snowy Evening":

--- / --- / -- / -- /
Whose woods these are I think I know.
/ ---
His house is in the village though;

In these two lines, the first line has a natural feeling of unaccented followed by accented. The second line may not be so obvious. So starting with village, mark the accented syllable with a "/" (vill) and the unaccented with a "---" (age). This word should be pronounced "VILLage" instead of "village." Helping students identify the proper emphasis on syllables is the first step to finding the meter. Students should mark their poems with accent marks to help them discover if there is a consistent meter, and the visual help aids in determining the existence of iamb.

Appendix C: Teacher Information on How to read Poetry

Often times the beauty of poetry is lost in the reading, both verbally and silently. For some reason, students and teachers are rarely taught how to read poetry. This can be compared to driving a race car in a school zone, or looking at the back of a canvas instead of what the artist painted. Poorly read poetry can sour students (or bore them terribly as they think of nursery rhymes) quickly, so it is important to learn how to properly read poetry.

The first step to reading poetry is to not pause after every line. A line break does not mean pause, so don't. Only pause when there is a comma, and pause for a longer time frame when there is a period. A dash is also not a time to pause. In poetry, dashes represent speeding up or interrupting the thought and they need to be read as so. Practice reading a poem without pausing. Then read the poem only pausing when the author says to pause. Finally read the poem pausing after every line. Can you tell a difference? Which way reads best?

The second step is to avoid being sing-songy. It's very easy to turn rhythmic poems into nursery rhymes. We naturally fall into over-emphasizing accented and unaccented syllables. While reading poetry, it is imperative to force your voice to speak normally, without singing the poem. Practice reading a poem with a monotonic voice, keeping your tone, pitch, and rate all at the same level. You may find yourself starting to change the rhythm of your voice, but press on.

The final step to reading poetry is to match your voice with the feeling of the poem. For example, some sections of "The Raven" should be read more quickly and with intensity, while other parts should be read slower and more softly.

Appendix D-1: Poetry Final Test

Name _____ Date _____

Vocabulary

Match the vocabulary words with the correct definition. Write the letter of the definition next to the word.

_____ 1. Simile

_____ 2. Metaphor

_____3. Rhyme

_____ 4. Rhyme Scheme

_____ 5. Iamb

_____ 6. Meter

A. A comparison that uses like or as.

B. The rhythm of a poem.

C. A rhyming pattern within a poem such as ABAB or ABBA.

D. A specific meter in which an unaccented syllable is followed by an accented syllable.

E. A comparison between two or more objects that does not use like or as.

F. When the last sound of two or more words is the same.

Read "If" by Rudyard Kipling.

7. At either the beginning or end of the lines, label the rhyme scheme for the entire poem.

8. Identify and **highlight** at least five lines that are written in iamb.

9. Does this poem contain metaphor or simile? If yes, give an example.

10. In three or more sentences, explain the theme of this poem. Use examples from the poem to support your ideas.

_____

_____

_____

_____

Appendix D-2: Poetry Final Test

IF

by Rudyard Kipling

If you can keep your head when all about you
Are losing theirs and blaming it on you;
If you can trust yourself when all men doubt you,
But make allowance for their doubting too;
If you can wait and not be tired by waiting;
Or, being lied about, don't deal in lies,
Or, being hated, don't give way to hating;
And yet don't look too good, nor talk too wise;

If you can dream—and not make dreams your master;
If you can think—and not make thoughts your aim;
If you can meet with triumph and disaster
And treat those two imposters just the same;
If you can bear to hear the truth you've spoken
Twisted by knaves to make a trap for fools,
Or watch the things you gave your life to broken,
And stoop and build 'em up with worn out tools;

If you can make one heap of all your winnings
And risk it on one turn of pitch-and-toss,
And lose, and start again at your beginnings
And never breathe a word about your loss;
If you can force your heart and nerve and sinew
To serve your turn long after they are gone,
And so hold on when there is nothing in you
Except the Will which says to them: "Hold on";

If you can talk with crowds and keep your virtue,
Or walk with kings—nor lose the common tough;
If neither foes nor loving friends can hurt you;
If all men count with you, but none too much;
If you can fill the unforgiving minute
With sixty seconds' worth of distance run—
Yours is the Earth and everything that's in it,
And—which is more—you'll be a Man, my son!

Name: _____ Class: _____ Date: _____

POETRY BOOKLET

Create a collection of poems (a different poem for each category) that meet the requirements listed below:

-a narrative poem
-a poem with only one stanza
-a poem containing a simile
-a poem containing a metaphor
-a poem using personification
-a poem containing alliteration
-a poem using onomatopoeia
-a poem that includes ABAB rhyme
-a poem you really, really like
-a poem that appeals to the sense of taste or smell
-a love poem
-an angry poem
-a lyric poem
-a poem you can relate to
-a poem that makes you laugh

Each page must include:

- The title of the poem and the author
- The category (or requirement description)
- The poem itself following the original line breaks and spacing
- Creative representation is your choice
- A paragraph that explains how the poem met the requirement, the theme of the poem and where you found the poem

Reminders:

The booklet needs a table of contents
Each poem may only be used for one category
You may not use any poems shared in class as examples
The poem must be classroom appropriate (words and theme)
The poem must be a published piece of work
Unpublished poems are only accepted if written by you!

SCORING

Organization/Neatness: Ten points
Each poem: Three points (15 x 3 = 45)
Each Paragraph: Three points (15 x 3 = 45)

An example is on the back of this sheet

FOG

by: Carl Sandburg

THE fog comes
on little cat feet.

It sits looking
over harbor and city
on silent haunches
and then moves on.

This poem meets the requirements of a poem containing a metaphor. The poet compares the fog to a cat. This is clear from the first line when he says that the fog approaches the city the way a cat would approach people or new territory. He also compares the fog to a cat when he says that the cat sits silently "looking" at the city. Cats have a way of observing things from the outside and then moving on to different places. The theme of the poem is that there is beauty in normal and simple things that most people overlook.

The poem was found at Bartleby.com

Poetry Analysis Worksheet # _____

Answer each of the following questions to the best of your ability.

1. The title of this poem is _____.

2. It was written by _____.

3. This poem is written in the _____ (1^{st} or 3^{rd}) person point of view.

4. Who is the speaker?

_____

5. What is the basic situation?

_____

6. What is the poem's setting?

_____

7. Are there conflicts in the poem? If so, what are they?

_____

_____

8. What kind(s) of imagery do you see most often in the poem? Give some examples.

_____

_____

9. Does the poem have meter? If so, what is it?

_____

10. Does your poem have a rhyme scheme? If so, what is it?

_____

11. What other sound devices (alliteration, assonance, onomatopoeia) have been included by the poet? Give examples of each.

_____

_____

12. What figures of speech are included (metaphor, simile, personification, hyperbole, metonymy, apostrophe, etc.)? Include examples and explain the effect each one has on your understanding and appreciation of the poem.

_____

_____

_____

13. What is the mood of this poem? Explain your answer.

14. Identify words which have a connotative meaning which help to clarify the author's tone. Explain each example.

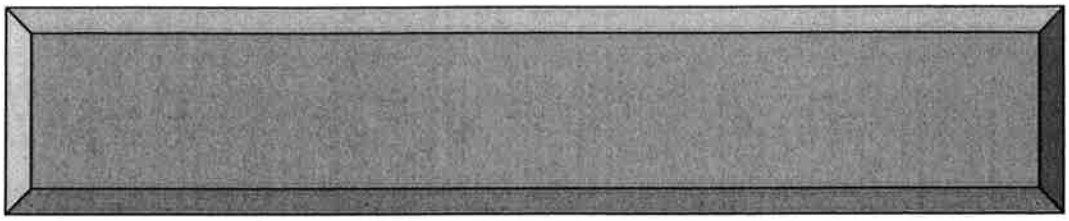
15. What is the author's tone (his or her attitude toward the subject?

16. Explain the significance of the poem's title.

17. Write a paragraph in which you briefly summarize the poem.

18. Based on your analysis, what do you think is the author's purpose in writing this poem? That is, what universal truth does he/she want to share with his/her readers (theme)?

Name: _____



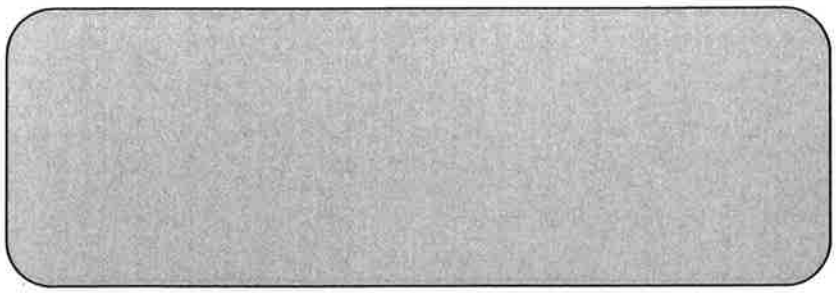
1 What is the overall message of the poem?

2 What does this poem make you think about?

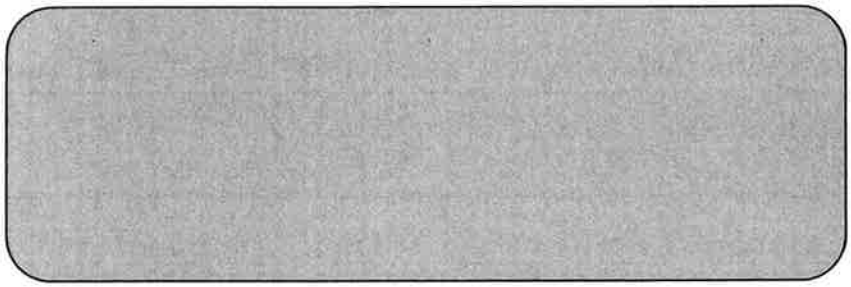
3 What are your personal reactions to the poem?

4 Why do you think the poet wrote this poem? What clues do you find that suggest this?

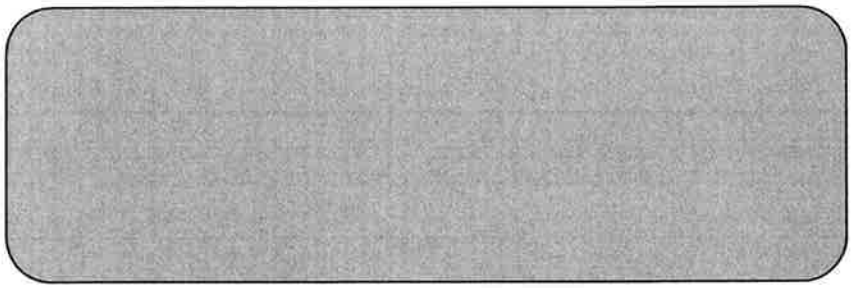
5 For what audience was the poem written? How do you know this?



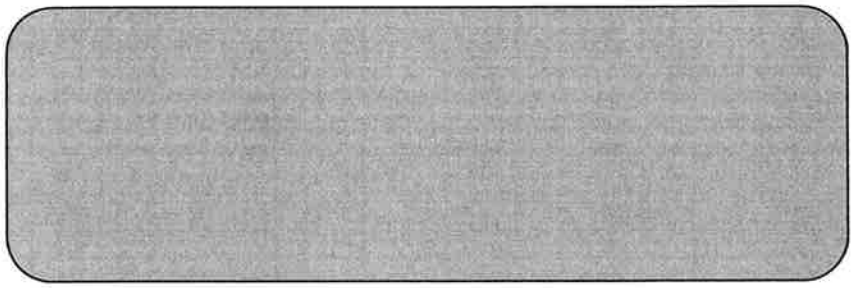
6 What does this poem tell you about what life was like during this period in history?



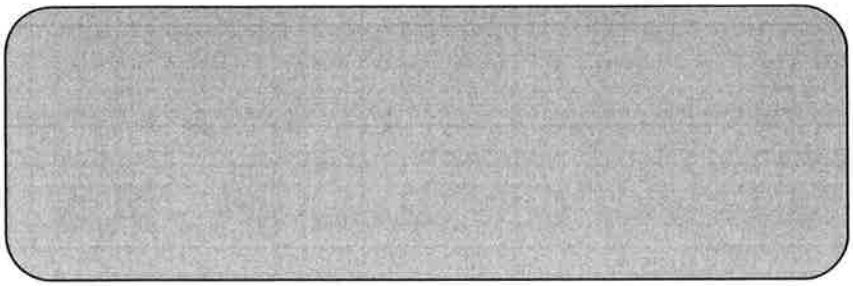
7 Identify expressive words or phrases. What about this language appeals to you?



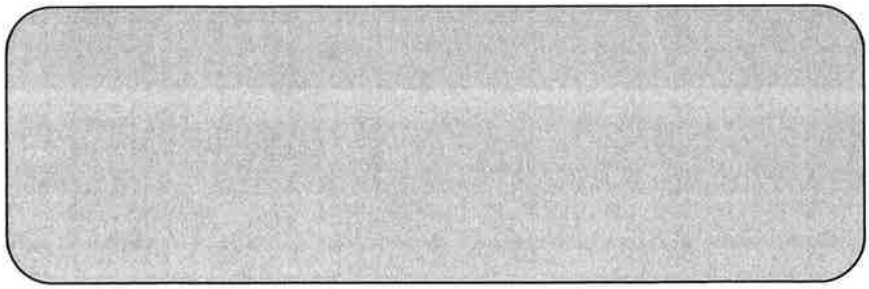
8 Identify the rhythm pattern



9 Identify a poetic device in this poem and discuss



10 Identify a poetic device in this poem and discuss



Name:	Reading
Date:	

Reading READathon: Poetry Assessment

Read each word and find the definition that correctly completes the poetic idea (5pts. ea.)

Vocabulary	Put the correct letter here.
1.Poetry	
2.Rhyme	
3. Line	
4. Theme	

Description: The meaning of the word.
a. The big idea the poem shares with the reader
b. When a words sound the same from one line to another
c. Anyone who writes poetry
d. Writing that shares an idea or emotion in fewer words than in fiction or non - fiction
e. A group of lines in a poem that are divided by space

Read the following poem and answer the following questions (# 5 – 8)

Sarah Cynthia Sylvia Stout Would Not Take the Garbage Out by Shel Silverstein

Sarah Cynthia Sylvia Stout would not take the garbage out!
She'd scour the pots and scrape the pans,
Candy the yams and spice the hams,
And though her daddy would scream and shout,
She simply would not take the garbage out.
And so it piled up to the ceilings:
Coffee grounds, potato peelings,
Brown bananas, rotten peas,
Chunks of sour cottage cheese.
It filled the can, it covered the floor,
It cracked the window and blocked the door
With bacon rinds and chicken bones,
Drippy ends of ice cream cones,
Prune pits, peach pits, orange peel,
Gloppy clumps of cold oatmeal,
Pizza crusts and withered greens,
Soggy beans and tangerines,
Crusts of black burned buttered toast,
Gristly bits of beefy roasts...
The garbage rolled on down the hall,
It raised the roof, it broke the wall...
Greasy napkins, cookie crumbs,
Globs of gooey bubble gum,
Cellophane from green baloney,
Rubbery blubbery macaroni,
Peanut butter, caked and dry,
Curdled milk and crusts of pie,
Moldy melons, dried-up mustard,
Eggshells mixed with lemon custard,
Cold french fries and rancid meat,
Yellow lumps of Cream of Wheat.
At last the garbage reached so high
That finally it touched the sky.
And all the neighbors moved away,
And none of her friends would come to play.
And finally Sarah Cynthia Stout said,
"OK, I'll take the garbage out!"
But then, of course, it was too late...
The garbage reached across the state,
From New York to the Golden Gate.
And there, in the garbage she did hate,
Poor Sarah met an awful fate,
That I cannot right now relate
Because the hour is much too late.
But children, remember Sarah Stout
And always take the garbage out!

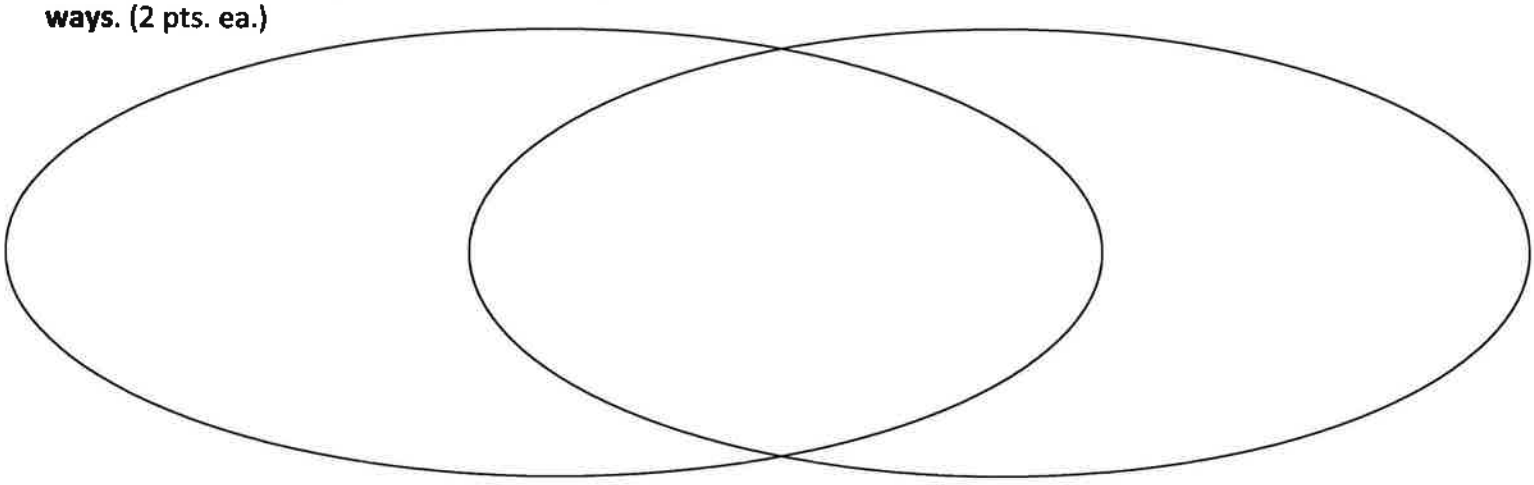
5. Write down one section of the poem where you can find a rhyme. (10 pts)

6 – 7. Underline another part of the poem (5 pts), and explain what is happening (10pts). Is it describing a feeling, an action or something else?

8. Summarize the poem in your own words. (20 pts)

9. What is the theme of the poem? (10 pts.)

10. Write down ways you know that poetry and fiction are the same or different. Make sure you include **6 ways**. (2 pts. ea.)



HS Poetry Unit – Culminating Transfer of Learning Tasks
Adapted from the NY Department of Education released Model CCSS Curriculum

The projects below should be introduced to students after the first week of the unit. In this way, students can work on their projects as they work through the unit. Provide each student with a folder. In this folder students will keep all their notes, drafts, poems, research, etc. that will be used for the final project.

The following culminating projects provide students with an opportunity to apply their learning in a variety of ways.

Option 1: For students who wish to write their own poetry (or song lyrics):

Create a poem anthology consisting of 8-12 original poems (or songs). Challenge the students to think about an overarching theme or strand (for example love poems, or poems about school, family, etc,) that relates to all the poems in some way. The Poetry anthology must include a title for the collection of poems, an introduction, dedication, a table of contents, and a short bio of the student- author.

***To be graded against the Narrative Writing Rubric**

Option 2: For students who wish to analyze and collect the poems of others

Select 8-12 poems to include in a poetry anthology. For this project, the students will act as editors. They will select the poems based on a specific theme or strand that appeals to them. Each poem that is selected for the anthology must be accompanied by an explanation as to why this poem is being included in the collection. The explanation should state why this poem is of particular appeal to the student and how it fits with the other poems in the collection. Students will need to determine a title for the collection, an introduction, dedication, table of contents, and a short bio of the student-editor. Students will select 2 poems from their anthology to compare and contrast the author's meaning and literary device choice, fully detailing the effect of author's choice on the meaning of the poem.

***To be graded against the Explanatory Writing Rubric**

Option 3: For students who wish to analyze a collection of poems by one poet Select 8-12 poems written by the same poet to include in an anthology. For this project the students will act as editors. Each poem that is selected for the anthology must be accompanied by an explanation as to why this poem is being included in the collection. The explanation should state why this poem is of particular appeal to the student and how it fits with the other poems in the collection. Students will need to determine a title for the collection, an introduction, dedication, table of contents, and a short bio of the student-editor. Students will select 2 poems from their anthology to compare and contrast the author's meaning and literary device choice, fully detailing the effect of author's choice on the meaning of the poem.

***To be graded against the Explanatory Writing Rubric**

Curriculum Scope & Sequence

School Moyer Academy Grade or Course: Grade 7: Intermediate Lit/Lang Skills A Teacher ____

Unit Order By unit title and/ or time frame	**Learning Targets** Content Standards, Grade Level Expectations, Proficiency Level Expectations, or Grade	**Theme or Big Idea**	**Enduring Understandings**	**Essential Questions**
COMPOSITION: Introduction to the Paragraph	**Writing** 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.) 5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. 6. Use technology, including the Internet, to produce and publish writing and link to and cite sources as well as to interact and collaborate with others, including linking to and citing sources. 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences. **Language**	Write for specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Writers do not always say what they mean. Indirect forms of expression (e.g., satire, irony) require readers to read between the lines to find the intended meaning. • Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers through the text to help avoid confusion. • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers understand. • A writer's word choice and syntax are characteristics of voice which help to personalize text.	• Why write? • What if writing didn't exist? • What makes writing worth reading? • How do writers express their thoughts and feelings? • Where do ideas for writing come from? • What makes writing flow? • How do effective writers hook and hold their readers? • What makes writing easy to follow? • What is the best beginning? • What is the best ending?

	conventions of standard English grammar and usage when writing or speaking. • Choose among simple, compound, complex, and compound-complex sentences to signal differing relationships among ideas. • Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.* 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. • Use a comma to separate coordinate adjectives (e.g., *It was a fascinating, enjoyable movie* but not *He wore an old[,] green shirt*). • Spell correctly. 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. • Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.* 6. Acquire and use accurately			• Who will read my writing? • What will work best for my audience? • Why does a writer choose the form of writing he/she does? • What is the relationship between reader and writer? • How do writers communicate clearly?

COMPOSITION: Personal Narrative	**Writing** 3. Write narratives to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences. • Engage and orient the reader by establishing a context and point of view and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically. • Use narrative techniques, such as dialogue, pacing, and description, to develop experiences, events, and/ or characters. • Use a variety of transition words, phrases, and clauses to convey sequence and signal shifts from one time frame or setting to another. • Use precise words and phrases, relevant descriptive details, and sensory language to capture the action and convey experiences and events. • Provide a conclusion that follows from and reflects on the narrated experiences or events. 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.) 5. With some guidance and support	Write for specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Writers do not always say what they mean. Indirect forms of expression (e.g., satire, irony) require readers to read between the lines to find the intended meaning. • Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers through the text to help avoid confusion. • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers understand. • A writer's word choice and syntax are characteristics of voice which help to personalize text.	• Why share personal experiences in writing? • How is written language different from spoken language? • What makes writing worth reading? • How do writers express their thoughts and feelings? • Where do ideas for writing come from? • What makes writing flow? • How do effective writers hook and hold their readers? • What makes writing easy to follow? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • What is a complete thought?

	editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. 6. Use technology, including the Internet, to produce and publish writing and link to and cite sources as well as to interact and collaborate with others, including linking to and citing sources. 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences. **Language** 1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. • Choose among simple, compound, complex, and compound-complex sentences to signal differing relationships among ideas. • Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.* 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.			• Why does a writer choose the form of writing he/she does? • What is the relationship between reader and writer? • How do writers communicate clearly? • To what extent do conventions of language impact communication ?

	old[,] green shirt). • Spell correctly. 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. • Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.* 6. Acquire and use accurately grade- appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when			
COMPOSITION: Persuasive Essay	**Writing** 1. Write arguments to support claims with clear reasons and relevant evidence. • Introduce claim(s), acknowledge alternate or opposing claims, and organize the reasons and evidence logically. • Support claim(s) with logical reasoning and relevant evidence, using accurate, credible sources and demonstrating an understanding of the topic or text. • Use words, phrases, and clauses to create cohesion and clarify the relationships among claim(s), reasons, and evidence. • Establish and maintain a	Write for specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Writers do not always say what they mean. Indirect forms of expression (e.g., satire, irony) require readers to read between the lines to find the intended meaning. • Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers through the text to help avoid confusion. • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being	• To what extent is the pen mightier than the sword? • How is written language different from spoken language? • What makes writing worth reading? • How do writers express their thoughts and feelings? • Where do ideas for writing come from? • What makes

	presented. 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.) 5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. 6. Use technology, including the Internet, to produce and publish writing and link to and cite sources as well as to interact and collaborate with others, including linking to and citing sources. 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences. **Languag e** 1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. • Choose among simple, compound, complex, and compound-complex sentences to signal		readers understand. • A writer's word choice and syntax are characteristics of voice which help to personalize text.	follow ? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • What is a complete thought? • Why am I writing? • For whom? • What am I trying to achieve through my writing? • Who will read my writing? • What will work best for my audience? • Why does a writer choose the form of writing he/she does? • What is the relationship between reader and writer? • How do writers communicate clearly?

	modifiers.* 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. • Use a comma to separate coordinate adjectives (e.g., *It was a fascinating, enjoyable movie* but not *He wore an old[,] green shirt*). • Spell correctly. 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. • Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.* 6. Acquire and use accurately grade- appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression. **Reading – Informational Text** 7. Compare and contrast a text to an audio, video, or multimedia version of the text, analyzing each medium's portrayal of the subject (e.g., how the delivery of a speech affects the impact of			

	claims. 9. Analyze how two or more authors writing about the same topic shape their presentations of key information by emphasizing different evidence or advancing different interpretations of facts. **Speaking & Listening** 3. Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and the relevance and sufficiency of the			
COMPOSITION: Research Report	**Writing** 2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. • Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information, using strategies such as definition, classification, comparison/ contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. • Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. • Use appropriate transitions to create cohesion and clarify the relationships among ideas and concepts	Write for specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Writers do not always say what they mean. Indirect forms of expression (e.g., satire, irony) require readers to read between the lines to find the intended meaning. • Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers through the text to help avoid confusion. • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers understand. • A writer's word choice and	• Where do ideas for writing come from? • What makes writing flow? • How do effective writers hook and hold their readers? • What makes writing easy to follow? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • What is a complete thought? • What am I trying to achieve through my

	inform about or explain the topic. • Establish and maintain a formal style. • Provide a concluding statement or section that follows from and supports the information or explanation presented. 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.) 5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. 6. Use technology, including the Internet, to produce and publish writing and link to and cite sources as well as to interact and collaborate with others, including linking to and citing sources. 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences. 7. Conduct short research projects to answer a question, drawing on several sources and generating additional related, focused questions for further research and investigation.		text.	writing he/she does? • How do writers communicate clearly?

	credibility and accuracy of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and following a standard format for citation. 9. Draw evidence from literary or informational texts to support analysis, reflection, and research. **Languag e** 1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. • Choose among simple, compound, complex, and compound-complex sentences to signal differing relationships among ideas. • Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.* 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. • Use a comma to separate coordinate adjectives (e.g., *It was a fascinating, enjoyable movie* but not *He wore an old[,] green shirt*). • Spell			

	reading, or listening. • Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.* 6. Acquire and use accurately grade- appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression. **Reading – Informational Text** 1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine two or more central ideas in a text and analyze their development over the course of the text; provide an objective summary of the text. 3. Analyze the interactions between individuals, events, and ideas in a text (e.g., how ideas influence individuals or events, or how individuals influence ideas or events). 4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the impact of a specific word choice on meaning and tone. 5. Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole			

	6. Determine an author's point of view or purpose in a text and analyze how the author distinguishes his or her position from that of others. 9. Analyze how two or more authors writing about the same topic shape their presentations of key information by emphasizing different evidence or advancing different interpretations of facts.			
COMPOSITION: Advertisement	**Writing** 2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. 2d. Use precise language and domain- specific vocabulary to inform about or explain the topic. 5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. 6. Use technology, including the Internet, to produce and publish writing and link to and cite sources as well as to interact and collaborate with others, including linking to and citing sources. 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. **Language**	Write for specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Writers do not always say what they mean. Indirect forms of expression (e.g., satire, irony) require readers to read between the lines to find the intended meaning. • Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers through the text to help avoid confusion. • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers understand. • A writer's word choice and syntax are characteristics of voice which help to personalize text.	• To what extent is the pen mightier than the sword? • How is written language different from spoken language? • Where do ideas for writing come from? • How do effective writers hook and hold their readers? • Why am I writing? • For whom? • What am I trying to achieve through my writing? • Who will read my writing? • What will work best for my audience? • Why does a writer choose the form of writing he/she does?

	and usage when writing or speaking. • Choose among simple, compound, complex, and compound-complex sentences to signal differing relationships among ideas. • Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.* 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. • Use a comma to separate coordinate adjectives (e.g., *It was a fascinating, enjoyable movie* but not *He wore an old[,] green shirt*). • Spell correctly. 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. • Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.* 5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings.			clearly? • To what extent do conventions of language impact communication ?

	respectful, polite, diplomatic, condescending). 6. Acquire and use accurately grade- appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression. **Speaking & Listening** 4. Present claims and findings, emphasizing salient points in a focused, coherent manner with pertinent descriptions, facts, details, and examples; use appropriate eye contact, adequate volume, and clear pronunciation. 5. Include multimedia components and visual displays in presentations to clarify claims and findings and emphasize salient points. 6. Adapt speech to a variety of contexts and tasks, demonstrating			
GRAMMAR, USAGE, and MECHANICS The Sentence Nouns and Pronouns Verbs and Complements Adjectives and Adverbs Other Parts of	**Languag e** 1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. • Explain the function of phrases and clauses in general and their function in specific sentences. • Choose among simple, compound, complex, and compound-complex sentences	Demonstrate command of proper grammar conventions.	• Conventions of language help readers understand what is being communicated • A writer's word choice and syntax are characteristics of voice which help to personalize text.	• How do writers communicate clearly? • To what extent do conventions of language impact communication ? • Why do we need grammar?

Speech and Review Phrases Sentence Structure	• Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.* 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. • Use a comma to separate coordinate adjectives (e.g., *It was a fascinating, enjoyable movie* but not *He wore an old[,] green shirt*). • Spell correctly.			
LITERATUR E UNIT: Informational text/ Travels with Charlie	**Reading – Literature** 1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a theme or central idea of a text and analyze its development over the course of the text; provide an objective summary of the text. 6. Analyze how an author develops and contrasts the points of view of different characters or narrators in a text. 10.By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range. **Writin**	Differentiate between different types of texts and demonstrate comprehension.	• Sometimes the author makes his/ her meaning plain; often however, a reader must dig beneath the "surface" of the text to find that meaning.	• From whose viewpoint are we reading? • What is the author's angle or perspective? • What is the author saying? • How do I know? • What is the gist? • What is the main idea? • How do I read between the lines? • How do I know I am getting the point and not merely imposing my views and experience?

	reflection, and research. • Apply *grade 7 Reading standards* to literature (e.g., "Compare and contrast a fictional portrayal of a time, place, or character and a historical account of the same period as a means of understanding how authors of fiction use or alter history"). **Speaking & Listening** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly. • Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. • Follow rules for collegial discussions, track progress toward specific goals and deadlines, and define individual roles as needed. • Pose questions that elicit elaboration and respond to others' questions and comments with relevant observations and ideas that bring the discussion back on topic as needed.			

	views. 2. Analyze the main ideas and supporting details presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how the ideas clarify a topic, text, or issue under study. 3. Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and the			
LITERATUR E UNIT: Freedom Walker: The Story of the Montgomery Bus Boycott	**Reading – Informational Text** 1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 5. Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to the development of the ideas. 5. Determine an author's point of view or purpose in a text and analyze how the author distinguishes his or her opinion from that of others. 10. By the end of the year, read and comprehend literary nonfiction in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range. **Writin g** 9. Draw evidence from literary or informational texts to support analysis, reflection, and research. • Apply *grade 7 Reading standards* to literary nonfiction (e.g. "Trace and evaluate the	Differentiate between different types of texts and demonstrate comprehension.	• Sometimes the author makes his/ her meaning plain; often however, a reader must dig beneath the "surface" of the text to find that meaning.	• From whose viewpoint are we reading? • What is the author's angle or perspective? • What should we do when texts or authors disagree? • What is the author saying? • How do I know? • What is the gist? • What is the main idea? • How do I read between the lines? • How do I know I am getting the point and not merely imposing my views and experience?

	reasoning is sound and the evidence is relevant and sufficient to support the claims"). **Speaking & Listening** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly. • Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. • Follow rules for collegial discussions, track progress toward specific goals and deadlines, and define individual roles as needed. • Pose questions that elicit elaboration and respond to others' questions and comments with relevant observations and ideas that bring the discussion back on topic as needed. • Acknowledge new information expressed by others and, when warranted, modify their own views. 2. Analyze the main ideas and supporting details presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how			

	specific claims, evaluating the soundness of the reasoning and the relevance and sufficiency of the evidence			
LITERATURE UNIT: Poetry/O Captain! My Captain, Paul Revere's Ride, The Complete Poems of Emily Dickinson	**Reading – Literature** 1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 10.By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	Differentiate between different types of texts and demonstrate comprehension.	• Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex stories in which the inner and outer lives of human beings are revealed.	• From whose viewpoint are we reading? • What is the author's angle or perspective? • What is the author saying? • What is the main idea? • How do I read between the
LITERATURE UNIT: Life Stories: Creative Lives	**Reading – Literature** 1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a theme or central idea of a text and analyze its development over the course of the text; provide an objective summary of the text. 7. Compare and contrast a written story, drama, or poem to its audio, filmed, staged, or multimedia version, analyzing the effects of techniques unique to each medium (e.g., lighting, sound, color, or camera focus and angles in film). 9. Compare and contrast a fictional portrayal of a time, place, or character and a historical account of the same period as a means of understanding how authors of fiction use or alter history. 10. By the end of the year, read and comprehend literary nonfiction in the grades 6–8 text complexity band	Differentiate between different types of texts and demonstrate comprehension.	• Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex stories in which the inner and outer lives of human beings are revealed.	• How do texts about other ages, genders, nationalities, races, religions, and disabilities tell experiences similar to mine?

	Speaking & Listening 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly. • Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. • Follow rules for collegial discussions, track progress toward specific goals and deadlines, and define individual roles as needed. • Pose questions that elicit elaboration and respond to others' questions and comments with relevant observations and ideas that bring the discussion back on topic as needed. • Acknowledge new information expressed by others and, when warranted, modify their own views. 2. Analyze the main ideas and supporting details presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how the ideas clarify a topic, text, or issue under study. 3. Delineate a speaker's argument and			

	5. Include multimedia components and visual displays in presentations to clarify claims and findings and emphasize salient points. 6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate. **Writing** 9. Draw evidence from literary or informational texts to support analysis, reflection, and research. • Apply *grade 7 Reading standards* to literature (e.g., "Compare and contrast a fictional portrayal of a time, place, or character and a historical account of the same period as a means of understanding how authors			
LITERATURE UNIT: T he Diary of Anne Frank	**Reading – Literature** 1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 10.By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	Differentiate between different types of texts and demonstrate comprehension.	• Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex stories in which the inner and outer lives of human beings are revealed.	• From whose viewpoint are we reading? • What is the author's angle or perspective? • What is the author saying? • What is the main idea? • How do I read between the
LITERATURE UNIT: Stories of Our Time	**Reading – Literature** 1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	Differentiate between different types of texts and demonstrate comprehension.	• Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex stories in which the inner and outer lives of human beings are revealed.	• From whose viewpoint are we reading? • What is the author's angle or perspective? • What is the author

	text and analyze its development over the course of the text; provide an objective summary of the text. 3. Analyze how particular elements of a story or drama interact (e.g., how setting shapes the characters or plot). 6. Analyze how an author develops and contrasts the points of view of different characters or narrators in a text. 10.By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range. **Writing** 9. Draw evidence from literary or informational texts to support analysis, reflection, and research. • Apply *grade 7 Reading standards* to literature (e.g., "Compare and contrast a fictional portrayal of a time, place, or character and a historical account of the same period as a means of understanding how authors of fiction use or alter history"). **Speaking & Listening** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others'			saying? • How do I know? • What is the gist? • What is the main idea? • How do I read between the lines? • How do I know I am getting the point and not merely imposing my views and experience?

	having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. • Follow rules for collegial discussions, track progress toward specific goals and deadlines, and define individual roles as needed. • Pose questions that elicit elaboration and respond to others' questions and comments with relevant observations and ideas that bring the discussion back on topic as needed. • Acknowledge new information expressed by others and, when warranted, modify their own views. 2. Analyze the main ideas and supporting details presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how the ideas clarify a topic, text, or issue under study.			
LITERATURE UNIT: To Everything There is a Season	**Reading – Literature** 1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of rhymes and other repetitions of	Differentiate between different types of texts and demonstrate comprehension.	• Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex stories in which the inner and outer lives of human beings are revealed.	• From whose viewpoint are we reading? • What is the author's angle or perspective? • What is the author saying? • How do I know? • What is the gist?

	alliteration) on a specific verse or stanza of a poem or section of a story or drama. 5. Analyze how a drama's or poem's form or structure (e.g., soliloquy, sonnet) contributes to its meaning. 7. Compare and contrast a written story, drama, or poem to its audio, filmed, staged, or multimedia version, analyzing the effects of techniques unique to each medium (e.g., lighting, sound, color, or camera focus and angles in film). 10.By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range. **Writing** 9. Draw evidence from literary or informational texts to support analysis, reflection, and research. • Apply *grade 7 Reading standards* to literature (e.g., "Compare and contrast a fictional portrayal of a time, place, or character and a historical account of the same period as a means of understanding how authors of fiction use or alter history"). **Language** 5. Demonstrate understanding of figurative language, word			• What is the main idea? • How do I read between the lines? • How do I know I am getting the point and not merely imposing my views and experience?

	Speaking & Listening 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly. • Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. • Follow rules for collegial discussions, track progress toward specific goals and deadlines, and define individual roles as needed. • Pose questions that elicit elaboration and respond to others' questions and comments with relevant observations and ideas that bring the discussion back on topic as needed. • Acknowledge new information expressed by others and, when warranted, modify their own views. 2. Analyze the main ideas and supporting details presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how the ideas clarify a topic, text, or issue under study. 3. Delineate a speaker's argument and			

LITERATURE UNIT: The Dragon Wakes	**Reading – Literature** 1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 10.By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	Differentiate between different types of texts and demonstrate comprehension.	• Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex stories in which the inner and outer lives of human beings are revealed.	• From whose viewpoint are we reading? • What is the author's angle or perspective? • What is the author saying? • What is the main idea? • How do I read between the
LITERATUR E UNIT: A Wrinkle in Time	**Reading – Literature** 1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. 3. Analyze how particular elements of a story or drama interact (e.g., how setting shapes the characters or plot). 5. Analyze how a drama's or poem's form or structure (e.g., soliloquy, sonnet) contributes to its meaning. 6. Analyze how an author develops and contrasts the points of view of different characters or narrators in a text. 7. Compare and contrast a written story, drama, or poem to its audio, filmed, staged, or multimedia version, analyzing the effects of techniques unique to each medium (e.g., lighting, sound, color, or camera focus and angles in a film). 10.By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8	Differentiate between different types of texts and demonstrate comprehension.	• Great literature provides rich and timeless insights into the key themes, dilemmas, and challenges that we face. They present complex stories in which the inner and outer lives of human beings are revealed.	• From whose viewpoint are we reading? • What is the author's angle or perspective? • What is the author saying? • How do I know? • What is the gist? • What is the main idea? • How do I read between the lines? • How do I know I am getting the point and not merely imposing my views and experience?

	Writing 9. Draw evidence from literary or informational texts to support analysis, reflection, and research. • Apply *grade 7 Reading standards* to literature (e.g., "Compare and contrast a fictional portrayal of a time, place, or character and a historical account of the same period as a means of understanding how authors of fiction use or alter history"). **Speaking & Listening** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly. • Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. • Follow rules for collegial discussions, track progress toward specific goals and deadlines, and define individual roles as needed. • Pose questions that elicit elaboration and respond to others' questions and			

	warranted, modify their own views. 2. Analyze the main ideas and supporting details presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how the ideas clarify a topic, text, or issue under study. 3. Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and the			
VOCABULARY Numbers All or Nothing More or Less Before and After Creativity Travel	**Languag e** 4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 7 reading and content*, choosing flexibly from a range of strategies. • Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. • Use common, grade-appropriate Greek or Latin	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Sometimes the author makes his/ her meaning plain; often, however, a reader must dig beneath the "surface" of the text to find the meaning. • Good readers employ strategies to help them understand text. • Good readers may use many strategies that work, and they quickly try another one when the one they are using doesn't work. They not only know many different strategies, but they never get stuck in persisting with one that isn't working	• What do you do when you do not understand everything in the text? • What do good readers do? • What do they do when they do not understand?

	(e.g., *belligerent, bellicose, rebel*). • Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. • Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary). 5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. • Use the relationship between particular words (e.g., synonym/antonym, analogy) to better understand each of the words. • Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., *refined, respectful, polite, diplomatic, condescending*). 6. Acquire and use accurately grade- appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when			

Curriculum Scope & Sequence

School_Moyer Academy_Grade or Course: 8th grade: Intermediate Lit/Lang Skills B Teacher _____

Unit Order By unit title and/ or time frame	**Learning Targets** Content Standards, Grade Level Expectations, Proficiency Level Expectations, or Grade Cluster Benchmarks	**Theme or Big Idea**	**Enduring Understandings**	**Essential Questions**
COMPOSITION: Letter to the Editor	**Writing** 1. Write arguments to support claims with clear reasons and relevant evidence. • Introduce claim(s), acknowledge and distinguish the claim(s) from alternate or opposing claims, and organize the reasons and evidence logically. • Support claim(s) with logical reasoning and relevant evidence, using accurate, credible sources and demonstrating an understanding of the topic or text. • Use words, phrases, and clauses to create cohesion and clarify the relationships among claim(s), counterclaims, reasons, and evidence. • Establish and maintain a formal style. • Provide a concluding statement or section that follows from and supports the argument presented. 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	Write for specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Writers do not always say what they mean. Indirect forms of expression (e.g., satire, irony) require readers to read between the lines to find the intended meaning. • Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers through the text to help avoid confusion. • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers understand. • A writer's word choice and syntax are characteristics of voice which help to personalize text.	• To what extent is the pen mightier than the sword? • How do writers express their thoughts and feelings? • Where do ideas for writing come from? • What makes writing flow? • How do effective writers hook and hold their readers? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • Why am I writing? • For whom? • What am I trying to achieve through my writing? • Who will read my writing? • What will work best for my

	editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. 6. Use technology, including the Internet, to produce and publish writing and present the relationships between information and ideas efficiently as well as to interact and collaborate with others. 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two). **Languag e** 1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. • Form and use verbs in the active and passive voice. • Form and use verbs in the indicative, imperative, interrogative, conditional, and subjunctive mood. • Recognize and correct inappropriate shifts in verb voice and mood.* 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.			

	• Spell correctly. 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. • Use verbs in the active and passive voice and in the conditional and subjunctive mood to achieve particular effects (e.g., emphasizing the actor or the action; expressing uncertainty or describing a state contrary to			
COMPOSITION: Research Report	**Writing** 2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. • Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. • Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. • Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts.	Write for specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Writers do not always say what they mean. Indirect forms of expression (e.g., satire, irony) require readers to read between the lines to find the intended meaning. • Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers through the text to help avoid confusion. • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being communicated.	• How do writers express their thoughts and feelings? • Where do ideas for writing come from? • What makes writing flow? • How do effective writers hook and hold their readers? • What makes writing easy to follow? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • What am I trying to achieve through my

	about or explain the topic. • Establish and maintain a formal style. • Provide a concluding statement or section that follows from and supports the information or explanation presented. 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.) 5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. 6. Use technology, including the Internet, to produce and publish writing and present the relationships between information and ideas efficiently as well as to interact and collaborate with others. 7. Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration. 8. Gather relevant information from multiple print and digital sources, using search terms effectively; assess the credibility and accuracy of each source; and quote or paraphrase the data and			

	for citation. 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two). **Language** 1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. • Form and use verbs in the active and passive voice. • Form and use verbs in the indicative, imperative, interrogative, conditional, and subjunctive mood. • Recognize and correct inappropriate shifts in verb voice and mood.* 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. • Use punctuation (comma, ellipsis, dash) to indicate a pause or break. • Use an ellipsis to indicate an omission. • Spell correctly. 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening.			

	emphasizing the actor or the action; expressing uncertainty or describing a state contrary to fact). **Reading – Informational Text** 1. Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a central idea of a text and analyze its development over the course of the text, including its relationship to supporting ideas; provide an objective summary of the text. 3. Analyze how a text makes connections among and distinctions between individuals, ideas, or events (e.g., through comparisons, analogies, or categories). 5. Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to the development of the ideas. 6. Determine an author's point of view or purpose in a text and analyze how the author distinguishes his or her position from that of others. 8. Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient; recognize when irrelevant evidence is introduced. 9. Analyze a case in which two or more texts provide conflicting information on the same topic and identify where the texts			

	2. Analyze the purpose of information presented in diverse media and formats (e.g., visually, quantitatively, orally) and evaluate the motives (e.g., social, commercial, political) behind its presentation.			
COMPOSITION: Propagandist Essay	**Writing** 1. Write arguments to support claims with clear reasons and relevant evidence. • Introduce claim(s), acknowledge and distinguish the claim(s) from alternate or opposing claims, and organize the reasons and evidence logically. • Support claim(s) with logical reasoning and relevant evidence, using accurate, credible sources and demonstrating an understanding of the topic or text. • Use words, phrases, and clauses to create cohesion and clarify the relationships among claim(s), counterclaims, reasons, and evidence. • Establish and maintain a formal style. • Provide a concluding statement or section that follows from and supports the argument presented. 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	Write for specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Writers do not always say what they mean. Indirect forms of expression (e.g., satire, irony) require readers to read between the lines to find the intended meaning. • Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers through the text to help avoid confusion. • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers understand. • A writer's word choice and syntax are characteristics of voice which help to personalize text.	• Why write? • What if writing didn't exist? • Why share personal experiences in writing? • To what extent is the pen mightier than the sword? • How is written language different from spoken language? • What makes writing worth reading? • How do writers express their thoughts and feelings? • Where do ideas for writing come from? • What makes writing flow? • How do effective writers hook and hold their readers? • What makes writing easy to follow? • What is the

	editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. 6. Use technology, including the Internet, to produce and publish writing and present the relationships between information and ideas efficiently as well as to interact and collaborate with others. 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two). **Language** 1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. • Form and use verbs in the active and passive voice. • Form and use verbs in the indicative, imperative, interrogative, conditional, and subjunctive mood. • Recognize and correct inappropriate shifts in verb voice and mood.* 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.			• For whom? • What am I trying to achieve through my writing? • Who will read my writing? • What will work best for my audience? • Why does a writer choose the form of writing he/she does? • What is the relationship between reader and writer? • How do writers communicate clearly? • To what extent do conventions of language impact communication? • What is the voice thing, anyway? • Why do we need grammar?

	• Spell correctly. 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. Use verbs in the active and passive voice and in the conditional and subjunctive mood to achieve particular effects (e.g., emphasizing the actor or the action; expressing uncertainty or describing a state contrary to fact). **Speaking & Listening** 3. Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and relevance and sufficiency of the evidence and identifying when irrelevant evidence is introduced. 4. Present claims and findings, emphasizing salient points in a focused, coherent manner with relevant evidence, sound valid reasoning, and well-chosen details; use appropriate eye contact, adequate volume, and clear pronunciation 5. Integrate multimedia and visual displays into presentations to clarify			
COMPOSITION: Cause and Effect Essay	**Writing** 2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. • Introduce a topic clearly, previewing what is to	Write for specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Writers do not always say what they mean. Indirect forms of expression (e.g., satire, irony) require readers to read between the lines to find the intended meaning. • Punctuation marks and	• How is written language different from spoken language? • What makes writing worth reading? • How do writers express their thoughts and

	information into broader categories; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. • Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. • Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. • Use precise language and domain- specific vocabulary to inform about or explain the topic. • Establish and maintain a formal style. • Provide a concluding statement or section that follows from and supports the information or explanation presented. 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.) 5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. 6. Use technology, including the Internet,		traffic signals. They guide readers through the text to help avoid confusion. • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers understand. • A writer's word choice and syntax are characteristics of voice which help to personalize text.	• What makes writing flow? • How do effective writers hook and hold their readers? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • What is a complete thought? • Why does a writer choose the form of writing he/she does? • What is the relationship between reader and writer? • How do writers communicate clearly?

	collaborate with others. 7. Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration. 8. Gather relevant information from multiple print and digital sources, using search terms effectively; assess the credibility and accuracy of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and following a standard format for citation. 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two). **Language** 1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. • Form and use verbs in the active and passive voice. • Form and use verbs in the indicative, imperative, interrogative, conditional, and subjunctive mood. • Recognize and correct inappropriate shifts in verb voice and mood.* 2. Demonstrate command of the			

	when writing. • Use punctuation (comma, ellipsis, dash) to indicate a pause or break. • Use an ellipsis to indicate an omission. • Spell correctly. 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. Use verbs in the active and passive voice and in the conditional and subjunctive mood to achieve particular effects (e.g., emphasizing the actor or the action; expressing uncertainty or describing a state contrary to fact). **Reading – Informational Text** 1. Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a central idea of a text and analyze its development over the course of the text, including its relationship to supporting ideas; provide an objective summary of the text. 3. Analyze how a text makes connections among and distinctions between individuals, ideas, or events (e.g., through comparisons, analogies, or categories). 5. Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole			

	author distinguishes his or her position from that of others. 8. Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient; recognize when irrelevant evidence is introduced. 9. Analyze a case in which two or more texts provide conflicting information on the same topic and identify where the texts disagree on matters of fact or interpretation. **Speaking & Listening** 2. Analyze the purpose of information presented in diverse media and formats (e.g., visually, quantitatively, orally) and evaluate the motives (e.g., social, commercial, political) behind its presentation. 3. Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and relevance and sufficiency of the evidence and identifying when irrelevant evidence is introduced. 4. Present claims and findings, emphasizing salient points in a focused, coherent manner with relevant evidence, sound valid reasoning, and well-chosen details; use appropriate eye contact, adequate volume, and clear pronunciation. 5. Integrate multimedia and visual displays into presentations to clarify information, strengthen claims and evidence, and add interest.			

	English when indicated or appropriate.			
COMPOSITION: Fictional Narrative	**Writing** 3. Write narratives to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences. • Engage and orient the reader by establishing a context and point of view and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically. • Use narrative techniques, such as dialogue, pacing, description, and reflection, to develop experiences, events, and/or characters. • Use a variety of transition words, phrases, and clauses to convey sequence, signal shifts from one time frame or setting to another, and show the relationships among experiences and events. • Use precise words and phrases, relevant descriptive details, and sensory language to capture the action and convey experiences and events. • Provide a conclusion that follows from and reflects on the narrated experiences or events. 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. 5. With some guidance and support	Write for specific audience and purpose.	• Audience and purpose (e.g., to inform, persuade, entertain) influence the use of literary techniques (e.g., style, tone, word choice). • Writers do not always say what they mean. Indirect forms of expression (e.g., satire, irony) require readers to read between the lines to find the intended meaning. • Punctuation marks and grammar rules are like highway signs and traffic signals. They guide readers through the text to help avoid confusion. • A writer selects a form based on his purpose. • A writer's point of view is influenced by his experience. • Conventions of language help readers understand what is being communicated. • The purposeful use and non-use of language conventions help readers understand. • A writer's word choice and syntax are characteristics of voice which help to personalize text.	• Why share personal experiences in writing? • How do writers express their thoughts and feelings? • Where do ideas for writing come from? • What makes writing flow? • How do effective writers hook and hold their readers? • What is the best beginning? • What is the best ending? • What is the best order (sequence)? • What is a complete thought? • Why am I writing? For whom? • What am I trying to achieve through my writing? • Who will read my writing? What will work best for my audience? • Why does a writer choose the form of

	approach, focusing on how well purpose and audience have been addressed. 6. Use technology, including the Internet, to produce and publish writing and present the relationships between information and ideas efficiently as well as to interact and collaborate with others. 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two). **Language** 1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. • Form and use verbs in the active and passive voice. • Form and use verbs in the indicative, imperative, interrogative, conditional, and subjunctive mood. • Recognize and correct inappropriate shifts in verb voice and mood.* 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. • Use punctuation (comma, ellipsis, dash) to indicate a pause or break. • Use an ellipsis to indicate			

	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. • Use verbs in the active and passive voice and in the conditional and subjunctive mood to achieve particular effects (e.g., emphasizing the actor or the action; expressing uncertainty or describing a state contrary to			
GRAMMAR, USAGE, and MECHANICS : Parts of Speech Kinds of Complements Phrases Verbals and Verbal Phrases	**Languag e** 1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. • Explain the function of verbals (gerunds, participles, infinitives) in general and their function in particular sentences. • Form and use verbs in the active and passive voice. • Form and use verbs in the indicative, imperative, interrogative, conditional, and subjunctive mood. • Recognize and correct inappropriate shifts in verb voice and mood.* 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. • Use punctuation (comma,	Demonstrate command of proper grammar conventions.	• Conventions of language help readers understand what is being communicated.	• How do writers communicate clearly? • To what extent do conventions of language impact communication ?

	• Spell correctly. 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. • Use verbs in the active and passive voice and in the conditional and subjunctive mood to achieve particular effects (e.g., emphasizing the actor or the action; expressing uncertainty or describing a state contrary to fact).			
LITERATURE: The Heart's Deep Core	**Reading – Literature** 1. Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a theme or central idea of a text and analyze its development over the course of the text, including its relationship to the characters, setting, and plot; provide an objective summary of the text. 3. Analyze how particular lines of dialogue or incidents in a story or drama propel the action, reveal aspects of a character, or provoke a decision. 4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts.	Differentiate between different types of texts and demonstrate comprehension .	• A good story has a pattern or plan. • Sometimes the author makes his/ her meaning plain; often however, a reader must dig beneath the "surface" of the text to find the meaning. • Different types of texts (e.g., narrative, mystery, biography, expository, persuasive) have different structures. Understanding a text's structure helps a reader better understand its meaning.	• What do good readers do? • What do they do when they do not understand? • How do texts differ?

	reader (e.g., created through the use of dramatic irony) create such effects as suspense or humor. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, at the high end of grades 6–8 text complexity band independently and proficiently. **Writing** 9. Draw evidence from literary or informational texts to support analysis, reflection, and research. • Apply *grade 8 Reading standards* to literature (e.g., "Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional stories, or religious works such as the Bible, including describing how the material is rendered new"). 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) **Speaking & Listening** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 8 topics, texts, and issues, building on others' ideas and expressing their own clear			

	draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. • Follow rules for collegial discussions and decision-making, track progress toward specific goals and deadlines, and define individual roles as needed. • Pose questions that connect the ideas of several speakers and respond to others' questions and comments with relevant evidence, observations, and ideas. • Acknowledge new information expressed by others, and, when warranted, qualify or justify their own views in light of the			
LITERATURE: Poetry/ Alice Through the Looking Glass, Twelfth Song of Thunder, The Road Not Taken, The Song of the Wandering Aengus, The Book of Questions	**Reading – Literature** 2. Determine a theme or central idea of a text and analyze its development over the course of the text, including its relationship to the characters, setting, and plot; provide an objective summary of the text. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, at the high end of grades 6–8 text complexity band	Differentiate between different types of texts and demonstrate comprehension .	• A good story has a pattern or plan. • Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find the meaning. • Different types of texts (e.g., narrative, mystery, biography, expository, persuasive) have different structures. Understanding a text's structure helps a reader better understand its meaning	• What do good readers do? • What do they do when they do not understand? • How do texts differ? • How should I read different types of texts?

	willful, firm, persistent, resolute). **Writing** 9. Draw evidence from literary or informational texts to support analysis, reflection, and research. • Apply *grade 8 Reading standards* to literature (e.g., "Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional stories, or religious works such as the Bible, including describing how the material is rendered new"). 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) **Speaking & Listening** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 8 topics, texts, and issues, building on others' ideas and expressing their own clear • Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under			

	individual roles as needed. • Pose questions that connect the ideas of several speakers and respond to others' questions and comments with relevant evidence, observations, and ideas. • Acknowledge new information expressed by others, and, when warranted, qualify or justify their own views in light of the evidence presented.			
LITERATURE: Scientists: Life Stories	**Reading – Literature** 1. Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a theme or central idea of a text and analyze its development over the course of the text, including its relationship to the characters, setting, and plot; provide an objective summary of the text. 3. Analyze how particular lines of dialogue or incidents in a story or drama propel the action, reveal aspects of a character, or provoke a decision. 4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts. 6. Analyze how differences in the points of view of the characters and the	Differentiate between different types of texts and demonstrate comprehension .	• A good story has a pattern or plan. • Sometimes the author makes his/ her meaning plain; often however, a reader must dig beneath the "surface" of the text to find the meaning. • Different types of texts (e.g., narrative, mystery, biography, expository, persuasive) have different structures. Understanding a text's structure helps a reader better understand its meaning.	• What do good readers do? • What do they do when they do not understand? • How do texts differ? • How should I read different types of texts?

	suspense or humor. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, at the high end of grades 6–8 text complexity band independently and proficiently. **Writin g** 9. Draw evidence from literary or informational texts to support analysis, reflection, and research. • Apply *grade 8 Reading standards* to literature (e.g., "Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional stories, or religious works such as the Bible, including describing how the material is rendered new"). 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) **Speaking & Listening** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 8 topics, texts, and issues, building on others' ideas and expressing their own clear • Come to discussions prepared, having read or researched			

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LITERATURE: Life Stories	**Reading – Literature** 1. Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a theme or central idea of a text and analyze its development over the course of the text, including its relationship to the characters, setting, and plot; provide an objective summary of the text. 3. Analyze how particular lines of dialogue or incidents in a story or drama propel the action, reveal aspects of a character, or provoke a decision. 4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of specific	Differentiate between different types of texts and demonstrate comprehension .	• A good story has a pattern or plan. • Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find the meaning. • Different types of texts (e.g., narrative, mystery, biography, expository, persuasive) have different structures. Understanding a text's structure helps a reader better understand its meaning.	• What do good readers do? • What do they do when they do not understand? • How do texts differ? • How should I read different types of texts? • What's new and what's old here? • Have we run across this idea before? • So what? What does it matter?

	or allusions to other texts. 6. Analyze how differences in the points of view of the characters and the audience or reader (e.g., created through the use of dramatic irony) create such effects as suspense or humor. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, at the high end of grades 6–8 text complexity band independently and proficiently. **Writin g** 3. Write narratives to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences. • Engage and orient the reader by establishing a context and point of view and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically. • Use narrative techniques, such as dialogue, pacing, description, and reflection, to develop experiences, events, and/or characters. • Use a variety of transition words, phrases, and clauses to convey sequence, signal shifts from one time frame or setting to another, and show the relationships among experiences and events. • Use precise words and phrases, relevant descriptive details, and			

	• Provide a conclusion that follows from and reflects on the narrated experiences or events. 9. Draw evidence from literary or informational texts to support analysis, reflection, and research. • Apply *grade 8 Reading standards* to literature (e.g., "Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional stories, or religious works such as the Bible, including describing how the material is rendered new"). 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) **Speaking & Listening** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 8 topics, texts, and issues, building on others' ideas and expressing their own clear • Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. • Follow rules for collegial discussions and decision-			

	individual roles as needed. • Pose questions that connect the ideas of several speakers and respond to others' questions and comments with relevant evidence, observations, and ideas. • Acknowledge new information expressed by others, and, when warranted, qualify or justify their own views in light of the evidence presented.			
LITERATURE: What's Important?	**Reading – Literature** 1. Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a theme or central idea of a text and analyze its development over the course of the text, including its relationship to the characters, setting, and plot; provide an objective summary of the text. 3. Analyze how particular lines of dialogue or incidents in a story or drama propel the action, reveal aspects of a character, or provoke a decision. 4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts. 6. Analyze how differences in the points of view of the characters and the	Differentiate between different types of texts and demonstrate comprehension .	• A good story has a pattern or plan. • Sometimes the author makes his/ her meaning plain; often however, a reader must dig beneath the "surface" of the text to find the meaning. • Different types of texts (e.g., narrative, mystery, biography, expository, persuasive) have different structures. Understanding a text's structure helps a reader better understand its meaning.	• What do good readers do? • What do they do when they do not understand? • How do texts differ? • How should I read different types of texts? • What's new and what's old here? • Have we run across this idea before? • So what? What does it matter?

	suspense or humor. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, at the high end of grades 6–8 text complexity band independently and proficiently. **Writin g** 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.) 5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. 6. Use technology, including the Internet, to produce and publish writing and present the relationships between information and ideas efficiently as well as to interact and collaborate with others. 9. Draw evidence from literary or informational texts to support analysis, reflection, and research. • Apply *grade 8 Reading standards* to literature (e.g., "Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional			

	rendered new"). 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) **Speaking & Listening** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 8 topics, texts, and issues, building on others' ideas and expressing their own clear • Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. • Follow rules for collegial discussions and decision-making, track progress toward specific goals and deadlines, and define individual roles as needed. • Pose questions that connect the ideas of several speakers and respond to others' questions and comments with relevant evidence, observations, and ideas. • Acknowledge new information expressed by others, and, when warranted, qualify or justify their own views in light of the			
LITERATURE: A Night to Remember	**Reading – Literature** 1. Cite the textual evidence that most strongly supports an analysis of what	Differentiate between different types of texts and	• Different types of texts (e.g., narrative, mystery, biography, expository, persuasive) have different structures.	• What do good readers do? • What do they

	text says explicitly as well as inferences drawn from the text. 2. Determine a theme or central idea of a text and analyze its development over the course of the text, including its relationship to the characters, setting, and plot; provide an objective summary of the text. 3. Analyze how particular lines of dialogue or incidents in a story or drama propel the action, reveal aspects of a character, or provoke a decision. 4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts. 6. Analyze how differences in the points of view of the characters and the audience or reader (e.g., created through the use of dramatic irony) create such effects as suspense or humor. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, at the high end of grades 6–8 text complexity band independently and proficiently. **Languag e** 5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings.	comprehension.	a text's structure helps a reader better understand its meaning. • A good story has a pattern or plan. • Sometimes the author makes his/ her meaning plain; often however, a reader must dig beneath the "surface" of the text to find the meaning.	understand? • How do texts differ? • How should I read different types of texts?

	particular words to better understand each of the words. • Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., *bullheaded, willful, firm, persistent, resolute*). **Writing** 9. Draw evidence from literary or informational texts to support analysis, reflection, and research. • Apply *grade 8 Reading standards* to literature (e.g., "Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional stories, or religious works such as the Bible, including describing how the material is rendered new"). 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) **Speaking & Listening** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 8 topics, texts, and issues, building on others' ideas and expressing their own clear			

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LITERATURE: Air Raid-Pearl Harbor	**Reading – Literature** 1. Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text. 2. Determine a theme or central idea of a text and analyze its development over the course of the text, including its relationship to the characters, setting, and plot; provide an objective summary of the text. 3. Analyze how particular lines of dialogue or incidents in a story or drama propel the action, reveal aspects of a character, or provoke a decision. 4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of specific	Differentiate between different types of texts and demonstrate comprehension .	• A good story has a pattern or plan. • Sometimes the author makes his/ he meaning plain; often however, a reader must dig beneath the "surface" of the text to find the meaning. • Different types of texts (e.g., narrative, mystery, biography, expository, persuasive) have different structures. Understanding a text's structure helps a reader better understand its meaning.	• What do good readers do? • What do they do when they do not understand? • How do texts differ? • How should I read different types of texts? • What's new and what's old here? • Have we run across this idea before? • So what? What does it matter?

	or allusions to other texts. 6. Analyze how differences in the points of view of the characters and the audience or reader (e.g., created through the use of dramatic irony) create such effects as suspense or humor. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, at the high end of grades 6–8 text complexity band independently and proficiently. **Writing** 9. Draw evidence from literary or informational texts to support analysis, reflection, and research. • Apply *grade 8 Reading standards* to literature (e.g., "Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional stories, or religious works such as the Bible, including describing how the material is rendered new"). 10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) **Speaking & Listening** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with			

	• Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. • Follow rules for collegial discussions and decision-making, track progress toward specific goals and deadlines, and define individual roles as needed. • Pose questions that connect the ideas of several speakers and respond to others' questions and comments with relevant evidence, observations, and ideas. • Acknowledge new information expressed by others, and, when warranted, qualify or justify their own views in light of the			
LITERATURE UNIT: And One for All	**Reading – Literature** 2. Determine a theme or central idea of a text and analyze its development over the course of the text, including its relationship to the characters, setting, and plot; provide an objective summary of the text. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, at the high end of grades 6–8 text complexity band	Differentiate between different types of texts and demonstrate comprehension.	• A good story has a pattern or plan. • Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find the meaning. • Different types of texts (e.g., narrative, mystery, biography, expository, persuasive) have different structures. Understanding a text's structure helps a reader better understand its meaning	• What do good readers do? • What do they do when they do not understand? • How do texts differ? • How should I read different types of texts?
LITERATURE: Underground Man	**Reading – Informational Text** 1. Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences	Differentiate between different types of texts and demonstrate comprehension.	• A good story has a pattern or plan. • Sometimes the author makes his/her meaning plain; often however, a reader must dig beneath the "surface" of the text to find the meaning.	• What do good readers do? • What do they do when they do not

	drawn from the text. 2. Determine a central idea of a text and analyze its development over the course of the text, including its relationship to supporting ideas; provide an objective summary of the text. 3. Analyze how a text makes connections among and distinctions between individuals, ideas, or events (e.g., through comparisons, analogies, or categories). 4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts. 5. Analyze in detail the structure of a specific paragraph in a text, including the role of particular sentences in developing and refining a key concept. 6. Determine an author's point of view or purpose in a text and analyze how the author acknowledges and responds to conflicting evidence or viewpoints. 10. By the end of the year, read and comprehend literary nonfiction at the high end of the grades 6–8 text		• Different types of texts (e.g., narrative, mystery, biography, expository, persuasive) have different structures. Understanding a text's structure helps a reader better understand its meaning.	• How should I read different types of texts? • What's new and what's old here? • Have we run across this idea before? • So what? What does it matter?
LITERATURE: Flowers for Algernan	**Reading – Literature** 3. Analyze how particular lines of dialogue or incidents in a story or drama propel the action, reveal aspects of a character, or provoke a decision. 4. Determine the meaning of words and	Differentiate between different types of texts and demonstrate comprehension .	• A good story has a pattern or plan. • Sometimes the author makes his/ her meaning plain; often however, a reader must dig beneath the "surface" of the text to find the meaning. • Different types of texts (e.g., narrative, mystery,	• What do good readers do? • What do they do when they do not understand? • How do texts differ?

	phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts. 5. Compare and contrast the structure of two or more texts and analyze how the differing structure of each text contributes to its meaning and style. 6.Analyze how differences in the points of view of the characters and the audience or reader (e.g., created through the use of dramatic irony) create such effects as suspense or humor. 7. Analyze the extent to which a filmed or live production of a story or drama stays faithful to or departs from the text or script, evaluating the choices made by the director or actors. 9. Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional stories, or religious works such as the Bible, including describing how the material is rendered new. 10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, at the high end of grades 6–8 text complexity band		expository, persuasive) have different structures. Understanding a text's structure helps a reader better understand its meaning.	texts? • What's new and what's old here? • Have we run across this idea before? • So what? What does it matter?
LITERATURE: The Dark is Rising	**Reading – Literature** 2. Determine a theme or central idea of a text and analyze its development over the course of the text, including its relationship to the characters, setting, and plot; provide an objective summary of the text.	Differentiate between different types of texts and demonstrate comprehension .	• Different types of texts (e.g., narrative, mystery, biography, expository, persuasive) have different structures. Understanding a text's structure helps a reader better understand its meaning. • A good story has a pattern or plan.	• What do good readers do? • What do they do when they do not understand? • How do texts differ?

	10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, at the high end of grades 6–8 text complexity band independently and proficiently.		meaning plain; often however, a reader must dig beneath the "surface" of the text to find the meaning.	texts?
VOCABULARY STRAND Unit 1: Motion Unit 2: Position Unit 3: Joining Unit 4: Separation	**Language** 4. Determine or clarify the meaning of unknown and multiple-meaning words or phrases based on *grade 8 reading and content*, choosing flexibly from a range of strategies. • Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. • Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., *precede, recede, secede*). • Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. • Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary). 5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. • Interpret figures of speech	Learn new vocabulary words to be used in writing as well as learn how to define vocabulary words in context.	• Good readers may use many strategies that work, and they quickly try another one when the one they are using doesn't work. They not only know many different strategies, but they never get stuck in persisting with one that isn't working • Sometimes the author makes his/ her meaning plain; often, however, a reader must dig beneath the "surface" of the text to find the meaning. • Good readers employ strategies to help them understand text. • A writer's word choice and syntax are characteristics of voice which help to personalize text.	• What do good readers do? • What do they do when they do not understand? • What do you do when you do not understand everything in the text?

	particular words to better understand each of the words. • Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., *bullheaded, willful, firm, persistent, resolute*). 6. Acquire and use accurately grade-appropriate general academic and domain- specific words and phrases; gather vocabulary knowledge when considering a word or phrase important			

Unit Title: Nonfiction

Delaware Recommended Curriculum Unit Template

Preface: This unit has been created as a model for teachers in their designing or redesigning of course curricula. It is by no means intended to be inclusive; rather it is meant to be a springboard for teacher thought and creativity. The information we have included represents one possibility for developing a unit based on the Delaware content standards and the Understanding by Design framework and philosophy.

Grade Level(s): 8th Grade

Subject/Topic: Nonfiction

Designed By: Ms. Mateen **Time Frame:** 7 days

Reviewed by: **Date:** 07.09.13

Brief Summary of Unit (This should include a brief unit summary including a description of unit goals, rationale for the approach taken, and where it appears in the course of study.)

The understanding for the unit on nonfiction is for the students to be able to read this type of literature and explore different literary concepts within the topic of nonfiction while comprehending the readings. By the end of this unit, the students should be able to identify vocabulary terms associated with the nonfiction subject and identify those within their nonfiction readings.

Stage 1: Desired Results
(Determine What Students Will Know, Do and Understand)

DE Common Core State Standards measured by the Transfer Task and Assessments (This should include a list describing which DE CCSS are measured in Stage 3. This could include standards, Grade level expectations, benchmarks and performance indicators.)

CCSS.ELA-Literacy.RL.8.1 Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text

CCSS.ELA-Literacy.RL.8.2 Determine a central idea of a text and analyze its development over the course of a text, including its relationship to supporting ideas; provide an objective summary of the text

CCSS.ELA-Literacy.RL.8.3 Analyze how a text makes connections among and distinctions between individuals, ideas, or events

Big Ideas (This should include transferable core concepts, principles, theories, and processes that should serve as the focal point of curricula, instruction, and assessment. Ex: Manifest Destiny, fighting for peace, writing process.)

Students will understand that…

A theme is the underlying message in a piece of writing

Figurative language is creative language that is used to create an effect or a feeling

Unit Title: Nonfiction

Nonfiction text structure is organized different than fiction text

Nonfiction text contains factual information

Nonfiction text consists of biographies, autobiographies, newspaper articles, etc…..

Unit Enduring Understanding(s)

Full-sentence, important statements or generalizations that specify what students should understand from the Big Ideas(s) and/or Content Standards and that are transferable to new situations

Reading is thinking
Nonfiction text features help you to understand information in a structured way
read nonfiction differently than fiction

Unit Essential Question(s) (This should include open-ended questions designed to guide student inquiry and focus instruction for "uncovering" the important ideas of the content. Ex: What is healthful eating? What is the relationship between fiction and truth?)

Can students compare and contrast various points of view and explain how voice affects literary text?
What makes nonfiction different?
How can we respond to nonfiction?

Why do you think the author chose to write about this particular nonfiction piece?

What comment do you think the author is making about this nonfiction piece?

Can students explain literary devices and how the devices affect the tone and meaning?

Knowledge & Skills (This should include key knowledge and skills that students will acquire as a result of this unit? Ex: Factors affecting climate, The causes of World War II.)
It should also include what students will eventually be able to do as a result of such knowledge and skill Ex: take notes, complete a bent-arm pull, compare fiction to nonfiction.)

Students will know…

the elements of a nonfiction piece
how to identify the theme in a nonfiction piece
how to distinguish figurative language and literary devices

Students will be able to…

identify key vocabulary related to nonfiction print and pick out examples of those terms in readings
to analyze a reading and determine the tone, audience, point of view, and the influence of bias

identify key events that occur in a nonfiction piece

identify elements of the genre that are different from other genres they have read so far

identify the elements of a nonfiction piece

will respond to a nonfiction piece

identify the author's purpose in writing

to listen, read a nonfiction piece and discuss their responses

Unit Title: Nonfiction

analyze nonfiction pieces through using a variety of reading comprehension strategies

create their own nonfiction writing

Stage 2: Assessment Evidence
(Design Assessments To Guide Instruction)

(This should include evidence that will be collected to determine whether or not the Desired Results identified in Stage One have been achieved? [Anchor the unit in performance tasks that require transfer, supplemented as needed by other evidence –quizzes, worksheets, observations, etc

Guiding questions for analyzing nonfiction
Pretest on vocabulary and paraphrasing
Class discussions
Group discussions
Writing assignments

Other Assessment Evidence
(This could include tests, quizzes, prompts, student work samples, and observations used to collect diverse evidence of student understanding.

Small-group informal data

Student Self-Assessment and Reflection (This should include opportunities for students to monitor their own learning. Ex: reflection journals, learning logs, pre- and post-tests, editing own work.)

Stage 3: Learning Plan
(Design Learning Activities To Align with Goals and Assessments)

Key learning events (lessons) needed to achieve unit goals

Lesson 1

Objective:

Students will examine different themes in non-fiction by looking at a variety of forms of expression. The students will be able to reflect and gain comprehension from the

readings. The students will be expected to paraphrase after reading an article and summarize what they learned.

Materials/Technology:

Power point, paper, pencil, smart board, worksheets-guided notes, KWL chart, RAP prompts, response cards, and markers

Focus/review:

The teacher will show a picture on the power point from the Holocaust and ask the students what they think it is. After the students have a short discussion about the picture, the teacher will introduce the unit. The teacher will state that it is important to be able to pick out the crucial information when reading. Some important things to look for when reading are the 5 W's,: who, what, when, where and why. Next, the teacher will hand out the unit organizer. Students will look over the organizer. The teacher will explain what the unit organizer is and how it can be used as a tool. The teacher will also go over what will be studied and when.

Teacher input:

We will start instruction by handing out a pretest for vocabulary and paraphrasing. The paraphrasing pretest will be used as data for those being taught the RAP technique. After the pretest is done, students will be given guided notes. The teacher will briefly explain the guided notes. In the guided notes there will be a KWL chart. The
students will then fill out what they know and what they want to know about the Holocaust. We will go through the guided notes learning key vocabulary dealing with nonfiction. Examples and non examples of the vocabulary words will be provided. The power point will include a background history of the holocaust. The co-teacher will support the class during instruction, providing prompts for filling out the guided notes. The SLANT strategy will be introduced to the students.

Guided practice:

The student will be given response cards. They will be asked a variety of
questions about the vocabulary and history of the holocaust. The teacher will show 1

questions first and answer it. Then the teacher and students will together answer a question on response cards. Finally the students will answer the subsequent questions. The co-teacher will float in the class and support students helping with the response cards.
Some students will be given preprinted response cards. The co-teacher will help demonstrate how to use these cards.

Independent practice:

The students will finish the KWL chart and fill in the final column of what they learned. They will be instructed to write a paragraph explaining what they learned and how they feel about the topic. The students will also be given a matching worksheet with the vocabulary words. At this time, the teacher will work with a small group of students (2-3). She or he will help with the assignment and present the RAP strategy. The teacher will also introduce the SLANT strategy to the every students and explain why both are important and how they can help.

Closure:

"Today we learned new vocabulary words and practiced summarizing and paraphrasing. We learned all about the history of the holocaust. Does anyone have and questions from today or need anything explained again? Next class we will be exploring readings from the holocaust. We will look at newspaper articles."

Evaluation:

The KWL chart and paragraph will be collected and read by teacher to give feed back on the writing. The vocabulary worksheet will be graded for accuracy. The pre-test will be used to compare to post tests.
Guided notes will be used for all students. Verbal prompting will be used to cue some students when to fill in the notes. Students will be allowed to type, write or record their paragraph. There will be audio, visual, visual audio representations through the power point and use of response cards. In the assignment, some students will be given an alternative assignment with fewer choices for matching.

Lesson 2

Objective:

Students will be able to identify and define different parts of a newspaper which are listed as vocabulary and literary terms. Students will be able to comprehend passages by paraphrasing the information.

Materials/Technology:

Power point, paper, pencil, smart board, worksheets-guided notes and unit organizer, newspaper articles, Pictures, Propaganda Ads, poster board, Markers

Focus/review:

Students will be shown a newspaper. Together the class will discuss what is interesting about newspapers and what you can find in it. Using response cards, students will review vocabulary from previous lesson focusing on vocab associated with newspapers. Students will review RAP and SLANT strategies as appropriate. Students will also be instructed to get out the unit organizer to look over what they will be doing today.

Teach input:

We will start out by looking through the power point. In the power point, there will be pictures and articles from newspapers, editorials that will be read aloud, propaganda. We will discuss each part of an article and newspaper. Students will be provided with example and non-examples of various newspaper related items.

Guided practice:

On the power point, the students will be shown a newspaper and articles. They will be asked to identify the parts by using response cards. The teacher will read 3 different articles; students will take notes on each. After students will discuss and analyze the similarities and differences. We will use the my turn, together, your turn strategy to practice the independent practice. The teacher will show an example of a newspaper

article with headline and caption that she created. Next the class will do an example together; the class will come up with a caption of the photo, a title and create a paragraph long article.

Independent practice:

Each student will be given a picture. They will be asked to come up with a caption for the photo, a title(headline) and write a shore article pertaining to the picture. Students will also be given a propaganda picture and be asked to explain what they think it means. Students will also be given a newspaper article and be asked to paraphrase it. After each is finished, they will share each part. The will be given posterpoards and markers to use if they wish.

Closure:

We will review the vocabulary in which pertains to newspaper. The class will review different holocaust newspaper articles that were read. The teacher will check for understanding by asking if they have any questions.

Evaluation:

The newspaper writings and paraphrasing activity will be evaluated to check for understanding of lesson.
Students will be allowed to type, write , or draw their independent practice assignment. There will be visual input in the power point and handed out to each student. Students will be engaged with choral responding. There will be modified directions for a few students who need extra support.

Lesson 3

Objective:

Students will be able to reflect on nonfiction readings using personal feelings and show an

awareness of how others might responds differently. The students will also be able to describe the difference between diaries and memoirs. In addition students will be able to identify whether they are reading a diary or memoir.

Materials/Technology:

Power point, paper, pencil, smart board, worksheets-guided notes, lesson organizer diary and memoir readings, posters, markers

Focus/review:

Students will be asked to write down a memory from something that happened in the past (a few years ago). Then they will be asked to write a diary of their day so far. The students will be asked differences. Students will then review vocabulary, newspapers, RAP and SLANT strategies. Students will have a "warm-up" activity to practice paraphrasing.

Teach input:

The teacher will go over diaries and memoirs. The class will read an example of a diary and memoir and discuss it. Power point will be used. Selections from *The Diary of Ann Frank* and *Night* will be read. Numbered Heads Together will be used after reading to discuss and answer comprehension questions.

Guided practice:

In groups, students will be given an excerpt from a diary/ memoir to read. The group will construct a wordle of words that describe the person who wrote it. Think of how they are similar/different from the author and present to class.
The class will return to their seats, they will then be given handout a passage from a diary or memoir to each student and each will read it. They will then go to the side of the room dedicated for memoirs or diaries. When in the group, they will make a poster of the characteristics of a diary or memoir. The two groups will compare and contrast.

Independent practice:

Using the diary/memoir from previous activity students will then use it to write an editorial article in a US newspaper describing to the US public exposing what they read, including a headline, newspaper name and author

Closure:

Students will review the difference in memoir and diary. The teacher will check for understanding and make sure no student is confused. Students will be told that they will be discussing documentaries tomorrow.

Evaluation:

The teacher will check the group work for understanding. The teacher will also grade the editorial written during independent practice. The teacher will grade the writing for grammar and spelling errors and also for comprehension of the article.

Students will be able to type, write or record the written assignment. Some students will be given a template for the editorial and an example to help. Students will be shown a power point, handed examples printed from internet and books.

Lesson 4

Objective:

Students will examine biographies and autobiographies. They will write a short biography and create a book cover. Students will be able to distinguish between biographies and autobiographies.

Materials/Technology:

Powerpoint, paper, pencil, smartboard, worksheets-guided notes, picture, markers,

computer

Focus/review:

Students will be asked what they would name a book about themselves.
They will then share the title and why they chose it. After this activity students will be reminded of the previous lesson on diary and memoirs. Students will review vocabulary

Teacher input:

Teacher will introduce biographies and autobiographies and explain their differences. Examples of each will be explored. Students will look at examples by reading parts of a biography and autobiography aloud in class.

Guided practice:

Teacher will display titles and authors of biographies and autobiographies. Students will respond using response cards to indicate which it is. Teacher will lead first example. Teacher and students will together do second example. Students follow for remaining examples.

Independent practice:

Students will research a person from the holocaust and create a cover for a biography. They will then use their research to write a short biography of the person highlighting the important accomplishments in his/her life. Each student will share why they created.

Closure: We will review biography and autobiography. Teacher will ask if there are any question or comment about the materials.

Evaluation: Students will be evaluated on their short biography. It will be graded for their ability to paraphrase and use correct grammar.

Students will be able to type, write or record the written assignment. Some students will be given a

template for the biography and an example to help. Students will be shown a power point, handed examples printed from internet and books. Students needing more supports will receive a list of sources to use to gain the information for the biography.

Lesson 5

Objective:

Students will be able to follow along with a documentary video. They will be able to summarize the vital pieces presented and answer comprehension questions.

Materials/Technology:

Computer, Video, Paper, Pencil, speakers, projector, Worksheet

Focus/review:

Students are going to review vocabulary words. A discussion about what it was like to experience the Holocaust will be started.

Teach input:

Teacher will explain what the students will be watching. Teacher will tell students to take note of important information.

Guided practice:

Teacher and students will discuss the comprehension questions.

Independent practice:

Students will write a reflection of the video.

Closure:

Today we watched a documentary on the holocaust. Lets discuss what you felt when you

saw the video.

Evaluation: The student's reflection will be graded along with the comprehension questions from the video.

The documentary will be closed captioned to help some of the students who do better with reading. There will be an outline of the video passed out to help students follow along

Resources & Intervention Strategies (Consider the two questions below when completing this section.)

Small-group instruction

Accommodation/Differentiation ideas and tips (This should include a list or description of ways that you will differentiate instruction according to students' needs. This can include any curricular adaptations that are needed to meet special needs students. Ex: using reading materials at varying readability levels, putting text materials on tape, using spelling or vocabulary lists at readiness levels of students, meeting with small groups to re-teach an idea or skill for struggling learners, or to extend the thinking or skills of advanced learners.

Read aloud
Guided notes
Modified assignments
Extended time
Graphic organizers
Visual cues
Small-group instruction

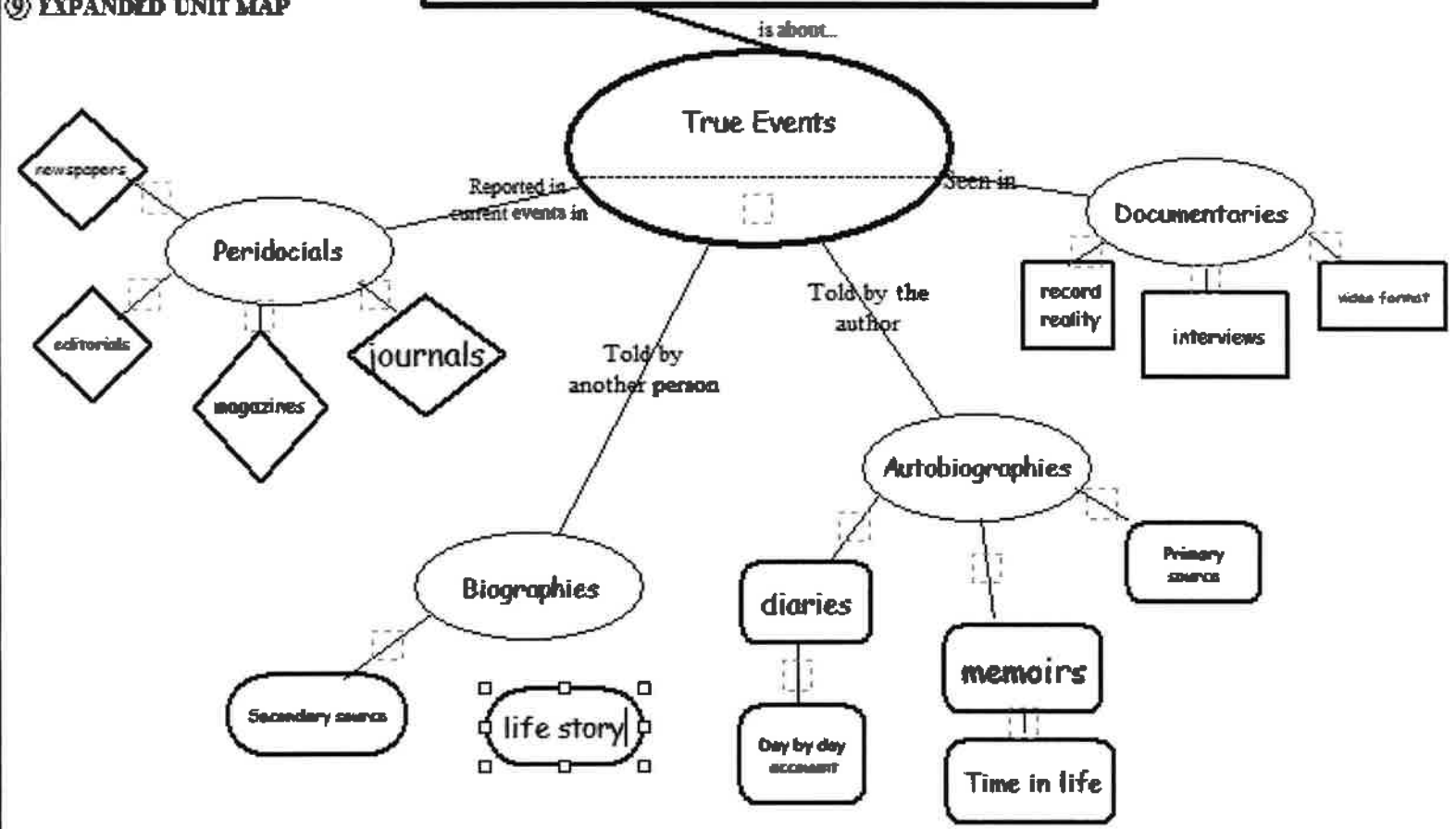
The Unit Organizer

NAME _____

DATE _____

Nonfiction

⑨ EXPANDED UNIT MAP



⑩ NEW UNIT SELF-TEST QUESTIONS

1. How do you think true events may guide fiction writings?

The Unit Organizer

NAME _____

DATE _____

④ BIGGER PICTURE

② LAST UNIT/Experience

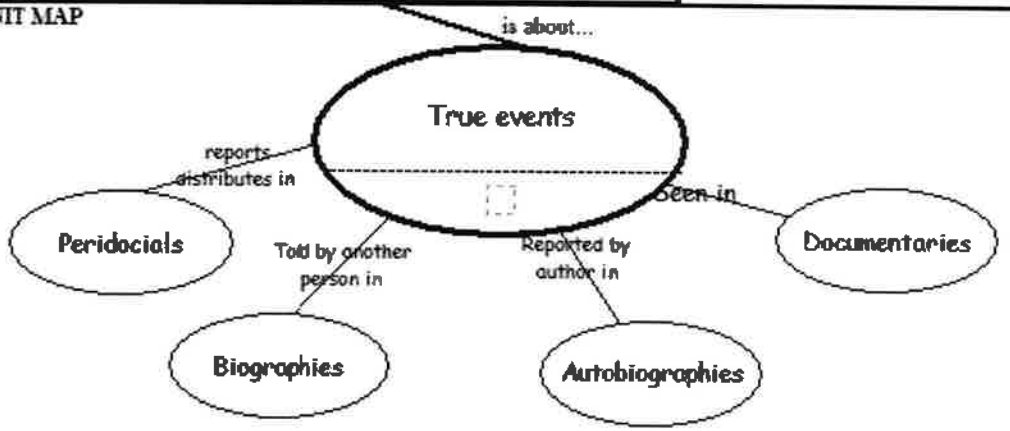
① CURRENT UNIT

③ NEXT UNIT/Experience

⑧ UNIT SCHEDULE

Intro to Nonfiction	
Background of Holocaust	
KWHL Chart	
Exploring News Paper Articles	
Propaganda	
Biographies V Autobiographies	
Ann Frank and Elie Wiesel	
Computer Lab activity	
Diaries and Memoirs	
Writing activity	
Documentary (video)	
Computer lab activity	
Test	

⑤ UNIT MAP



⑦ UNIT SELF-TEST QUESTIONS

1. What is nonfiction? Compare and contrast fiction and nonfiction.
2. How do biographies and autobiographies differ?
3. Name and be able to label different parts of a newspaper.
4. How are diaries and memoirs similar and different? Compare and contrast these with autobiographies.
5. How are documentaries made? Why are documentaries used?
6. Be able to pick describe the tone, audience and point of view from readings.

⑥ UNIT RELATIONSHIPS

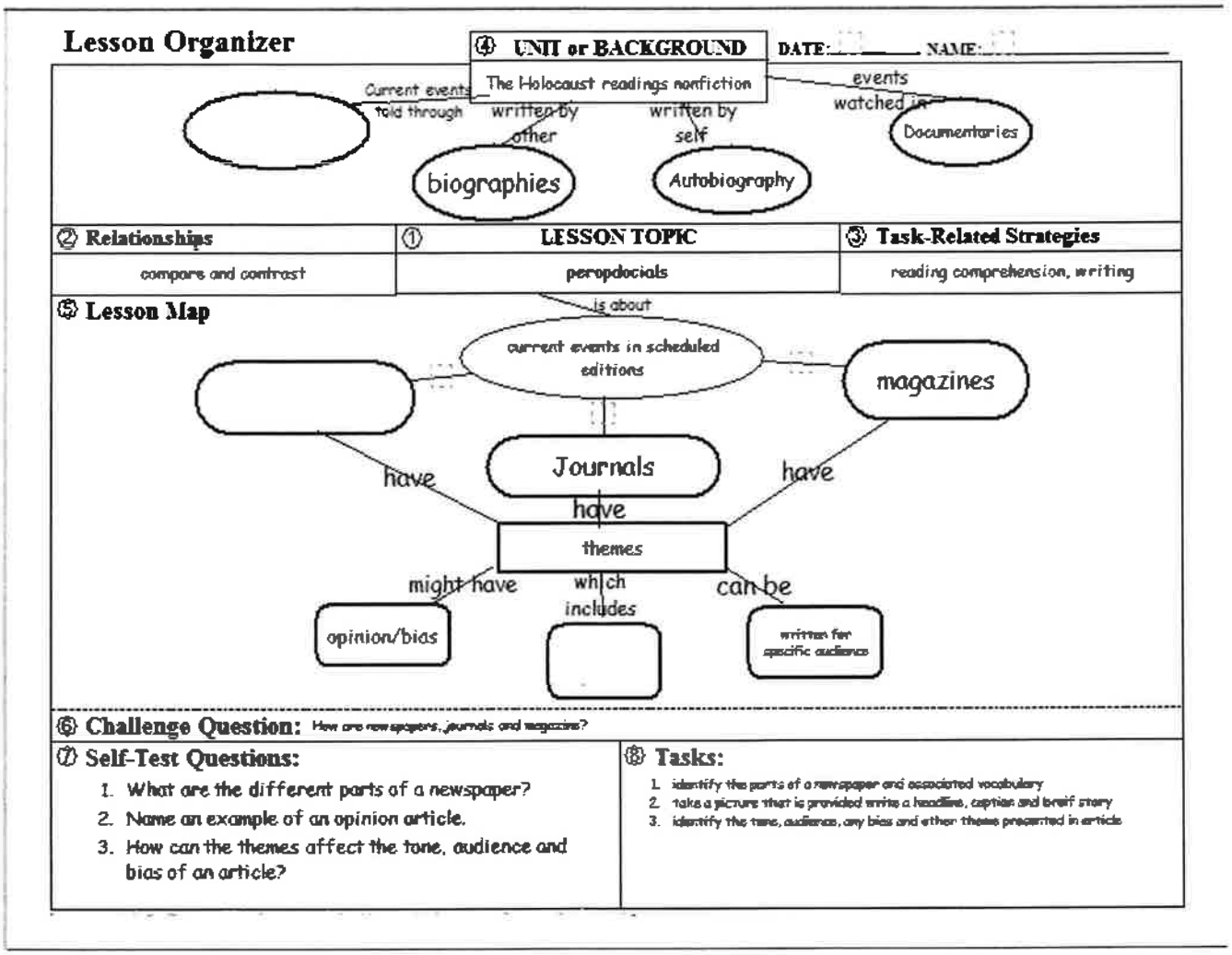
Compare and contrast
Characteristics/ descriptions
Explanations

Appendix A

Other Handouts and Materials

KWL Chart was given to students in the beginning of the unit to fill out the first two columns. The chart was then revisited at the end so students could share all of what they learned.

What do you know about the holocaust?	What do you want to know?	What did you learn?



Guided Notes were given to the students to increase active student responding. These notes consisted of all the vocabulary words from the unit. This handout was given on the first day of class.

Vocabulary

- Nonfiction- writing based on _____
- Biography- A type of writing in which the author writes about _____
- _____-author writes about his/her own personal life experiences.
- Diary-a _____account of a person's life

- Memoir- a book a person writes about a _____in their life.
- Primary source-document which was written or created _____ under study and offer an inside view of an event. First hand.
- _____- interprets and analyzes primary sources. These sources are one or more steps removed from the event
- Subjective-influenced by _____, biased
- Objective-_____ personal opinion, based on facts
- Point of view-The _____ from which a story is told
- Cause and effect-A relationship where an event leads to an outcome.
- Tone- writer's attitude toward subject and audience
- Audience- _____for whom the author writes
- Biased-favoring one viewpoint over another, influence and prejudice against someone or something
- _____-reference to a famous person, place, object, previous work of art, or historical event as a comparison
  Ex. He had Herculean strength
- Figure of Speech- not meant to be taken _____. This is symbolic language. Many times, authors use similes and metaphors to create figurative language.
  Ex. He ran as fast as lightening
  It was raining cats and dogs
- Summarize-restate in a _____
- _____-restating another person's idea in your own words.
- Documentary-a _____that provides a factual record or report
- Interview-questioning of a person to gain information
- Narrative essay-tells a _____
- Persuasive essay-A type of writing with the main objective of trying to affect the way the reader _____about a particular subject.

- Expository essay-Any type of writing that has the primary purpose of providing_____to the reader
- Descriptive essay- a writing that describes something.
- Newspaper- a scheduled publication of current events and informative articles.
- Headline- title of article usually in larger print to catch attention
- Editorial- article that expresses the _____of the editor
- Caption-describes a photograph.
- _____- Information, ideas, or rumors deliberately spread widely to help or harm a person, group, movement, institution or nation.

NONFICTION PYRAMID

_____

One word describing one major idea

_____ _____

Two words describing a supporting detail

_____ _____ _____

Three words describing another major idea

_____ _____ _____ _____

Four words describing another supporting detail

_____ _____ _____ _____ _____

Five words describing the author's purpose

_____ _____ _____ _____ _____ _____

Six important vocabulary words

_____ _____ _____ _____ _____ _____ _____

Seven words describing important reader's aids

_____ _____ _____ _____ _____ _____ _____ _____

Eight words telling what you learned

Delaware Department of Education

Argumentation/Opinion Text-Based Writing Rubric
Grades 9–10

	Score of 4	Score of 3	Score of 2	Score of 1
Reading/Research 2 × ___ = ___	The writing – ▪ makes effective use of available resources ▪ skillfully/effectively supports an opinion with relevant and sufficient facts and details from resources with accuracy ▪ uses credible sources*	The writing – ▪ makes adequate use of available resources ▪ supports an opinion with relevant and sufficient facts and details from resources with accuracy ▪ uses credible sources*	The writing – ▪ makes limited use of available resources ▪ inconsistently supports an opinion with relevant and sufficient facts and details from resources with accuracy ▪ inconsistently uses credible sources*	The writing – ▪ makes inadequate use of available resources ▪ fails to support an opinion with relevant and sufficient facts and details from resources with accuracy ▪ attempts to use credible sources*
Development 3 × ___ = ___	The writing – ▪ addresses all aspects of the writing task with a tightly focused response ▪ skillfully develops the claim(s) and counterclaims fairly, supplying sufficient and relevant evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level and concerns	The writing – ▪ addresses the writing task with a focused response ▪ develops the claim(s) and counterclaims fairly, supplying sufficient and relevant evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level and concerns	The writing – ▪ addresses the writing task with an inconsistent focus ▪ inconsistently develops the claim(s) and counterclaims fairly, supplying sufficient and relevant evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level and concerns	The writing – ▪ attempts to address the writing task but lacks focus ▪ attempts to establish a claim or proposal ▪ supports claim(s) using evidence that is insufficient and/or irrelevant
Organization 2 × ___ = ___	The writing – ▪ effectively introduces precise claim(s); distinguishes the claim(s) from alternate or opposing claims ▪ effectively creates an organization that establishes clear relationships among claim(s), counterclaim(s), reasons, and evidence ▪ skillfully uses words, phrases, and/or clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims ▪ provides an effective concluding statement or section that follows from and skillfully supports the argument presented	The writing – ▪ introduces precise claim(s); distinguishes the claim(s) from alternate or opposing claims ▪ creates an organization that establishes clear relationships among claim(s), counterclaim(s), reasons, and evidence ▪ uses words, phrases, and/or clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims ▪ provides a concluding statement or section that follows from and supports the argument presented	The writing – ▪ introduces the claim(s); however, may fail to distinguish the claim(s) from alternate or opposing claim(s) ▪ has a progression of ideas that may lack cohesion (ideas may be rambling and/or repetitive) ▪ inconsistently uses words, phrases, and/or clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims ▪ provides a sense of closure	The writing – ▪ identifies the claim(s) ▪ has little or no evidence of purposeful organization

	Score of 4	Score of 3	Score of 2	Score of 1
Language/Conventions **1 × ___ = ___**	The writing – ▪ demonstrates an exemplary command of standard English conventions ▪ skillfully employs language and tone appropriate to audience and purpose ▪ has sentences that are skillfully constructed with appropriate variety in length and structure ▪ follows standard format for citation with few errors*	The writing – ▪ demonstrates a command of standard English conventions; errors do not interfere with understanding ▪ employs language and tone appropriate to audience and purpose ▪ has sentences that are generally complete with sufficient variety in length and structure ▪ follows standard format for citation with few errors*	The writing – ▪ demonstrates a limited and/or inconsistent command of standard English conventions; errors may interfere with understanding ▪ inconsistently employs language and tone appropriate to audience and purpose ▪ has some sentence formation errors and/or a lack of sentence variety ▪ follows standard format for citation with several errors*	The writing – ▪ demonstrates a weak command of standard English conventions; errors interfere with understanding ▪ employs language and tone that are inappropriate to audience and purpose ▪ has frequent and severe sentence formation errors and/or a lack of sentence variety ▪ follows standard format for citation with significant errors*

* If applicable

Delaware Department of Education

Informational or Explanatory Text-Based Writing Rubric
Grades 9–10

	Score of 4	Score of 3	Score of 2	Score of 1
Reading/ Research 2 × ___ = ___	The writing – ▪ makes effective use of available resources ▪ effectively uses relevant and sufficient text support from the resources with accuracy ▪ effectively uses credible sources*	The writing – ▪ makes adequate use of available resources ▪ uses relevant and sufficient text support from the resources with accuracy ▪ uses credible sources*	The writing – ▪ makes limited use of available resources ▪ inconsistently uses relevant and sufficient text support from the resources with accuracy ▪ inconsistently uses credible sources*	The writing – ▪ makes inadequate use of available resources ▪ fails to use relevant and sufficient text support from the resources with accuracy ▪ attempts to use credible sources*
Development 3 × ___ = ___	The writing – ▪ addresses all aspects of the writing task with a tightly focused and detailed response ▪ skillfully develops the topic using well-chosen, relevant, and sufficient facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic	The writing – ▪ addresses the writing task with a focused response ▪ develops the topic using well-chosen, relevant, and sufficient facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic	The writing – ▪ addresses the writing task with an inconsistent focus ▪ inconsistently develops the topic using well-chosen, relevant, and sufficient facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic	The writing – ▪ attempts to address the writing task but lacks focus ▪ develops the topic using facts, definitions, concrete details, quotations, or other information and examples that are irrelevant and/or insufficient
Organization 2 × ___ = ___	The writing – ▪ effectively introduces the topic ▪ effectively organizes complex ideas, concepts, and information to make important connections and distinctions ▪ effectively uses appropriate and varied transitions to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts ▪ provides an effective concluding statement or a section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic)	The writing – ▪ introduces the topic ▪ organizes complex ideas, concepts, and information to make important connections and distinctions ▪ uses appropriate and varied transitions to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts ▪ provides a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic)	The writing – ▪ introduces the topic ▪ organizes ideas, concepts, and information in a manner that may lack cohesion (ideas may be rambling and/ or repetitive) ▪ inconsistently uses appropriate and varied transitions to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts ▪ provides a concluding statement or section	The writing – ▪ identifies the topic ▪ has little or no evidence of purposeful organization

	Score of 4	Score of 3	Score of 2	Score of 1
Language/Conventions **1 × ___ = ___**	The writing – ▪ demonstrates an exemplary command of standard English conventions ▪ skillfully employs language and tone appropriate to audience and purpose ▪ has sentences that are skillfully constructed with appropriate variety in length and structure ▪ follows standard format for citation with few errors*	The writing – ▪ demonstrates a command of standard English conventions; errors do not interfere with understanding ▪ employs language and tone appropriate to audience and purpose ▪ has sentences that are generally complete with sufficient variety in length and structure ▪ follows standard format for citation with few errors*	The writing – ▪ demonstrates a limited and/or inconsistent command of standard English conventions; errors may interfere with understanding ▪ inconsistently employs language and tone appropriate to audience and purpose ▪ has some sentence formation errors and/or a lack of sentence variety ▪ follows standard format for citation with several errors*	The writing – ▪ demonstrates a weak command of standard English conventions; errors interfere with understanding ▪ employs language and tone that are inappropriate to audience and purpose ▪ has frequent and severe sentence formation errors and/or a lack of sentence variety ▪ follows standard format for citation with significant errors*

* If applicable

Delaware Department of Education

Narrative Text-Based Writing Rubric
Grades 9–10

	Score of 4	Score of 3	Score of 2	Score of 1
Reading/ Research 2 × ___ = ___	The writing – ▪ makes effective use of available resources ▪ effectively uses relevant and sufficient text support from the resources with accuracy ▪ effectively uses credible sources*	The writing – ▪ makes adequate use of available resources ▪ uses relevant and sufficient text support from the resources with accuracy ▪ uses credible sources*	The writing – ▪ makes limited use of available resources ▪ inconsistently uses relevant and sufficient text support from the resources with accuracy ▪ inconsistently uses credible sources*	The writing – ▪ makes inadequate use of available resources ▪ fails to use relevant and sufficient text support from the resources with accuracy ▪ attempts to use credible sources*
Development 3 × ___ = ___	The writing – ▪ skillfully develops real or imagined experiences or events using well-chosen details ▪ skillfully uses narrative techniques (such as dialogue, pacing, description, reflection, and multiple plot lines) to develop experiences, events, and/or characters ▪ skillfully uses precise words and phrases, concrete details, and sensory language to convey a vivid picture of the experiences, events, setting, and/ or characters	The writing – ▪ develops real or imagined experiences or events using well-chosen details ▪ uses narrative techniques (such as dialogue, pacing, description, reflection, and multiple plot lines) to develop experiences, events, and/or characters ▪ uses precise words and phrases, concrete details, and sensory language to convey a vivid picture of the experiences, events, setting, and/ or characters	The writing – ▪ develops real or imagined experiences or events using insufficient details ▪ inconsistently uses narrative techniques (such as dialogue, pacing, description, reflection, and multiple plot lines) to develop experiences, events, and/or characters ▪ inconsistently uses precise words and phrases, concrete details, and sensory language to convey a vivid picture of the experiences, events, setting, and/or characters	The writing – ▪ develops real or imagined experiences or events using little or no detail ▪ uses few, if any, narrative techniques (such as dialogue, pacing, description, reflection. and multiple plot lines) to develop experiences, events, and/or characters ▪ uses few, if any, precise words and phrases, concrete details, and sensory language to convey a vivid picture of the experiences, events, setting, and/or characters
Organization 2 × ___ = ___	The writing – ▪ skillfully engages and orients the reader by setting out a situation or observation, establishing one or multiple point(s) of view, and, when appropriate, introducing a narrator and/or characters ▪ skillfully uses a variety of techniques to sequence events so that they build on one another to create a coherent whole ▪ skillfully creates a smooth progression of experiences or events ▪ skillfully provides a conclusion that follows from and reflects on what is experienced, observed, or resolved over the course of the narrative	The writing – ▪ engages and orients the reader by setting out a situation or observation, establishing one or multiple point(s) of view, and, when appropriate, introducing a narrator and/or characters ▪ uses a variety of techniques to sequence events so that they build on one another to create a coherent whole ▪ creates a smooth progression of experiences or events ▪ provides a conclusion that follows from and reflects on what is experienced, observed, or resolved over the course of the narrative	The writing – ▪ inconsistently engages and orients the reader by ineffectively setting out a situation or observation, establishing one or multiple point(s) of view, and, when appropriate, introducing a narrator and/or characters ▪ inconsistently uses a variety of techniques to sequence events so that they build on one another to create a coherent whole ▪ has a progression of experiences or events that may lack cohesion ▪ provides a conclusion that ineffectively follows from and reflects on what is experienced, observed, or resolved over the course of the narrative	The writing – ▪ fails to engage and orient the reader by ineffectively setting out a situation or observation, establishing one or multiple point(s) of view, and, when appropriate, introducing a narrator and/or characters ▪ fails to sequence events so that they build on one another to create a coherent whole ▪ may lack a clear progression of experiences or events ▪ may lack a conclusion that follows from and reflects on what is experienced, observed, or resolved over the course of the narrative

	Score of 4	Score of 3	Score of 2	Score of 1
Language/Conventions **1 × ___ = ___**	The writing – ▪ demonstrates an exemplary command of standard English conventions ▪ skillfully employs language and tone appropriate to audience and purpose ▪ has sentences that are skillfully constructed with appropriate variety in length and structure ▪ follows standard format for citation with few errors*	The writing – ▪ demonstrates a command of standard English conventions; errors do not interfere with understanding ▪ employs language and tone appropriate to audience and purpose ▪ has sentences that are generally complete with sufficient variety in length and structure ▪ follows standard format for citation with few errors*	The writing – ▪ demonstrates a limited and/or inconsistent command of standard English conventions; errors may interfere with understanding ▪ inconsistently employs language and tone appropriate to audience and purpose ▪ has some sentence formation errors and/or a lack of sentence variety ▪ follows standard format for citation with several errors*	The writing – ▪ demonstrates a weak command of standard English conventions; errors interfere with understanding ▪ employs language and tone that are inappropriate to audience and purpose ▪ has frequent and severe sentence formation errors and/or a lack of sentence variety ▪ follows standard format for citation with significant errors*

* If applicable

HS Fiction Literature Unit – Culminating Transfer of Learning Tasks

Adapted from the NY Department of Education released Model CCSS Curriculum

A Day's Wait by Ernest Hemingway

He came into the room to shut the windows while we were still in bed and I saw he looked ill. He was shivering, his face was white, and he walked slowly as though it ached to move.

"What's the matter, Schatz (a German term of endearment)?

"I've got a headache."

"You better go back to bed."

"I'm all right."

When the doctor came he took the boy's temperature.

"What is it?" I asked him.

"One hundred and two."

Downstairs, the doctor left three different medicines in different colored capsules with instructions for giving them. One was to bring down the fever, another a pain killer, the third to overcome an acid condition. The germs of influenza only exist in an acid condition, he explained. He seemed to know all about influenza and said there was nothing to worry about if the fever did not go above one hundred and four degrees. This was a light strain of flu and there was no danger if you avoided pneumonia.

Back in the room I wrote the boy's temperature down and made a note of the time to give the various capsules.

"Do you want me to read to you?"

"All right. If you want to," said the boy. His face was very white and there were dark areas under his eyes. He lay still in the bed and seemed very detached from what was going on.

I read aloud from Howard Pyle's *Book of Pirates*; but I could see he was not following what I was reading.

"How do you feel, Schatz?" I asked him.

"Just the same, so far," he said.

I sat at the foot of the bed and read to myself while I waited for it to be time to give another capsule. It would have been natural for him to go to sleep, but when I looked up he was looking at the foot of the bed, looking very strangely.

"Why don't you try to go to sleep? I'll wake you up for the medicine."

"I'd rather stay awake." After awhile he said to me, "You don't have to stay in here with me, Papa, if it bothers you."

“It doesn’t bother me.”

“No. I mean you don’t have to stay if it’s going to bother you.”

I thought perhaps he was a little light-headed and after giving him the prescribed capsules at eleven o’clock I went out for awhile. It was a bright, cold day, the ground covered with a sleet that had frozen so that it seemed as if all the bare trees, the bushes, the cut brush and all the grass and the bare ground had been varnished with ice. I took the young Irish setter for a little walk up the road and along a frozen creek, but it was difficult to stand or walk on the glassy surface and the red dog slipped and slithered and I fell twice, hard, once dropping my gun and having it slide away over the ice.

We flushed a covey of quail under a high clay bank with overhanging brush and I killed two as they went out of sight over the top of the bank. Some of the covey lit in trees but most of them scattered into brush piles and it was necessary to jump on the ice-coated mounds of brush several times before they would flush. Coming out while you were poised unsteadily on the icy, springy brush they made difficult shooting, and I killed two, missed five, and started back pleased to have found a covey close to the house and happy there were so many left to find on another day.

At the house they said the boy had refused to let anyone come into the room.

“You can’t come in,” he said. “You mustn’t get what I have.”

I went up to him and found him in exactly the position I had left him, white-faced, but with the tops of his cheeks flushed by the fever, staring still, as he had stared at the foot of the bed.

I took his temperature.

“What is it?”

“Something like a hundred,” I said. It was one hundred and two and four tenths.

“It was a hundred and two,” he said.

“Who said so?”

“The doctor.”

“Your temperature is all right,” I said. “It’s nothing to worry about.”

“I don’t worry,” he said, “but I can’t keep from thinking.”

“Don’t think, I said. “Just take it easy.”

“I’m taking it easy,” he said and looked straight ahead. He was evidently holding tight on to himself about something.

“Take this with water.”

“Do you think it will do any good?”

“Of course it will.”

I sat down and opened the *Pirate* book and commenced to read, but I could see he was not following, so I stopped.

"About what time do you think I'm going to die?" he asked.

"What?"

"About how long will it be before I die?"

"You aren't going to die. What's the matter with you?"

"Oh, yes, I am. I heard him say a hundred and two."

"People don't die with a fever of one hundred and two. That's a silly way to talk.

"I know they do. At school in France the boys told me you can't live with forty-four degrees. I've got a hundred and two."

He had been waiting to die all day, ever since nine o'clock in the morning.

"You poor Schatz," I said. "Poor old Schatz. It's like miles and kilometers. You aren't going to die. They use a different thermometer. On that thermometer, thirty-seven is normal. They measure heat using Celsius. We use Fahrenheit. On this kind it's ninety-eight."

"Are you sure?"

"Absolutely," I said. "It's like miles and kilometers. You know, like how many kilometers we make when we do seventy miles in a car?"

"Oh," he said.

But his gaze at the foot of the bed relaxed slowly. The hold over himself relaxed too, finally, and the next day it was very slack and he cried very easily at little things that were of no importance.

Understanding the Story

1. This story is most likely –
    a. Realistic Fiction
    b. Science Fiction
    c. Fairy Tale
    d. Fantasy
2. I know this is the genre because: _____
_____

3. Who is the narrator?
    a. the boy
    b. the boy's father
    c. third-person
    d. third-person omniscient

4. What is the boy's temperature when the doctor takes it?
    a. 98.6 degrees
    b. 102 degrees
    c. 104 degrees
    d. 105 degrees

5. To what does the father compare the different temperature scales?
    a. miles and kilometers
    b. yards and meters
    c. grams and pounds
    d. feet and inches

6. By the father calling the boy Schatz, we can tell
    a. The father gets angry often at the boy
    b. He is making a play on words and calling him "Schatz" because he's sick and needs shots
    c. Schatz must be a shortened version of the boy's full name
    d. He is very affectionate toward the boy

7. The doctor's reaction to the boy's condition can best be described as
    a. unconcerned
    b. annoyed
    c. slightly worried
    d. panicked

8. The boy's concern about his temperature indicates that he thinks that 102 degrees is
    a. far below normal
    b. far above normal
    c. normal
    d. slightly above normal

9. The reactions of the boy and the father to the illness are different in that
    a. The father is extremely worried and the boy is relaxed
    b. The father is not worried at all and the boy is slightly worried
    c. The father is slightly worried and the boy is slightly worried
    d. The father is slightly worried and the boy is extremely worried

10. Which of these ***best*** describe what the boy feels during this day?
    a. sick and sleepy
    b. happy and content
    c. rejected and hurt
    d. fear and anxiety

11. The fact that the father leaves the boy and goes hunting shows that the father
    a. does not think his son's condition is serious
    b. is a self-absorbed and neglectful parent
    c. cannot stand to be in the boy's presence
    d. is more attached to his dog than to his son

12. Why is it important that the boy wouldn't let anyone in to see him?
    a. It shows that the illness is making him angry and mean
    b. It shows that he believes his sickness is more serious than it is
    c. It shows that he is feeling sorry for himself and doesn't want people to see him cry
    d. It shows that he is embarrassed from his sickness

13. When the father tells the boy to take his medicine, the boy replies, "Do you think it will help?" What does this tell us about the boy?
    a. He doesn't like to take the medicine
    b. He doesn't believe that he can be cured
    c. He doesn't think that he is sick
    d. He is untrusting of the doctor and his medicine

14. How can the father's reaction to his son's concern best be described?
    a. impatient
    b. angry
    c. frustrated
    d. understanding

15. What can we infer from the last paragraph of the story?
    a. The boy's health got better but he was still upset about the thought of dying
    b. The boy's health continued to get worse
    c. The boy's health got better and he was in a much better mood
    d. The boy's health only got a little better, but he was still really sick

16. The author probably wrote this story to
    a. Explain how to take care of someone when they are sick
    b. Show the impact that influenza can have on a family
    c. Persuade readers not to jump to conclusions
    d. Display the bravery of a boy who thought he was much sicker than he really was

Understanding Story Elements

17. The exposition of a story introduces the reader to 2-3 parts of the story. What are these three things that you can find in the exposition?

_____ _____ _____

18. How does the setting of the story add to the **OVERALL** mood of the story?
    a. The sunny day emphasizes happiness and new life
    b. The sunny day emphasizes pain
    c. Winter emphasizes cold and death
    d. Winter emphasizes freshness and rebirth

19. What is the **MAIN** internal conflict in this story?
    a. The boy and the father disagree as to how sick the boy is
    b. The father is struggling to understand his son's behavior when he's sick
    c. The boy is trying to fight off influenza
    d. The father disagrees with the doctor's diagnosis and recommendation

20. Which type of conflicts are each of the examples below?
    a. The boy is afraid of death:
        -Intrinsic Conflict OR Extrinsic Conflict
        -Man vs. Man OR Man vs. Self OR Man vs. Nature

    b. The boy is trying to fight off influenza
        -Intrinsic Conflict OR Extrinsic Conflict
        -Man vs. Man OR Man vs. Self OR Man vs. Nature

    c. The father disagrees with the doctor's diagnosis and recommendation
        -Intrinsic Conflict OR Extrinsic Conflict
        -Man vs. Man OR Man vs. Self OR Man vs. Nature

21. How does the personality of the boy affect the conflict of the story?
    a. His openness allows him and his father to get along
    b. His anger and resentment prevent him and his father from getting along
    c. His passion allows him to battle influenza
    d. His strong will prevents his father from knowing why he's so upset

22. When the boy says he'd rather stay awake and that his father didn't have to stay in the bedroom with him if it bothered him, this is a good example of rising action because:
    a. It builds the tension up around the main conflict
    b. It shows the reader what the main conflict is
    c. It introduces the characters to the reader
    d. It lets the reader know how the conflict will probably be resolved

23. When the boy asks his father, "What time am I going to die?" This is most likely part of the:
    a. Rising action
    b. Climax
    c. Falling action
    d. Resolution

23b. I know this because: _____
_____

Refer to the marked-off part of the story for question # 24

24. What is the mood of this part of the story?
    a. Anger
    b. Amusement
    c. Fear
    d. Sadness

25. What is the mood during the resolution of the story?
    a. Relief
    b. Somber
    c. Anger
    d. Excitement

26. What is a theme you could learn from what the father experienced?
    a. You should always listen to your parents because they take care of you
    b. You should always trust your instincts
    c. You shouldn't believe everything you hear from your friends
    d. Just because people say they're fine doesn't mean they're really okay

27. How was this theme learned in the story?
    a. A misunderstanding between main characters
    b. A falling out between the main characters changes their relationship
    c. The main character changes after a major life event
    d. The main character makes a huge mistake and gets into trouble

Analyzing and Interpreting the Story

28. How did the point-of-view of this story make it more interesting?
    a. It gave the readers a better understanding of the boy's emotions
    b. It allowed us to see what everyone was doing and thinking
    c. It made it more suspenseful for the readers and allowed the end to be a surprise
    d. It allowed the reader to visualize the setting and what was going on

30. How does the boy show bravery?
    a. he tells his father that he is afraid of dying
    b. he never says that he is afraid of dying
    c. he says he doesn't want to take the medicine
    d. he takes the medicine even though he doesn't like it

31. What is the best example of irony from this story?
    a. When the boy explains that he thinks he's going to die
    b. When the father ends up finding a covey of quail
    c. When the doctor explains that the boy's temperature is 102
    d. When the boy refuses to let anyone into his room

32. What type of irony is displayed in the question above?
    a. verbal
    b. situational
    c. dramatic

33. Why didn’t the boy tell his father what he was thinking until the very end?
    a. He wanted to be strong in front of his father
    b. He and his father didn’t get along very well
    c. He was scared that his father would be angry with him
    d. He didn’t want his father to know that he was dying

34. How will this experience most likely affect the relationship between the father and son?
    a. The boy will not trust his father as much
    b. The father will not believe his son as often
    c. The father will have someone else take care of the boy when he’s sick
    d. The boy will open up more to his father

35. Which of these text-to-self connections would **MOST** help you understand the story better?
    a. Thinking of a time when you were sick and someone took care of you
    b. Thinking of a time when you were really scared and it turned out to be for no reason
    c. Thinking of a time when you went outside in the winter and slipped on some ice
    d. Thinking of a time when you made a mistake and got teased for it

Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title: Operations with Whole Numbers, 2-3 weeks

Designed by: Laura Mayer
District:Moyer Academy

Content Area:Math
Grade Level(s):7th

Summary of Unit

This unit builds upon the prior knowledge of positive numbers, negative numbers, and absolute value, and expands their understanding of integers to include both single step and mutli-step integer operations (addition, subtraction, multiplication, and division). Students will learn, apply, and justify the rules of the four integer operations; students will use real world applications to understand and explain integer operations and solve problems involving all integer operations, including positive and negative values, in single step and multi-step problems, as well as word problems that demonstrate a conceptual understanding. Students will experiment with a variety of applications for integer operations (including the chip method, money examples, number lines, altitude, good vs. bad; temperature, gains and losses, etc.) As a final assessment, students will take on the role of a financial advisor and provide suggestions to a client on how to balance a budget.

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

- Include those addressed in Stage 3 and assessed in Stage 2.

Mathematical Practice 1. Make sense of problems and persevere in solving them.

Mathematical Practice 4. Model with mathematics.

Mathematical Practice 5. Use appropriate tools strategically.

Mathematical Practice 8. Look for and express regularity in repeated reasoning.

(REVIEW) 6.NS.C.7c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of –30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

Big Idea(s)

- Transferable core concepts, principles, theories, and processes from the Content Standards

-There are many ways to visually represent numerical values.

-Laws, properties and theorems justify the correctness in which we solve problems.

-Understanding and demonstrating a process is often more important than obtaining a final answer.

-Real-world problems can be solved using mathematical models

Unit Enduring Understanding(s)

- Full-sentence, important statements or generalizations that specify what students should understand from the Big Ideas (s) and/or Content Standards and that are transferable to new situations

-Number systems can be visually represented to show relationships. (A number line is a visual representation of positive and negative values in the real number system and the way they relate to each other.)

-When problems contain multiple operations, we must follow a specific order when solving them.

-The rules of integer operations can be applied to real world scenarios such as those involving money, temperature, altitude, travel, etc.

Unit Essential Questions(s)

- Open-ended questions designed to guide student inquiry and learning

-Does the quantity and placement of digits relate to the value of a number?

-How much choice do you have when solving math problems?

-When is less more? In other words, when are negative values a positive thing?

Knowledge and Skills

- Needed to meet Content Standards addressed in Stage 3 and assessed in Stage 2

Students will know...

-The absolute value of a number is its distance away from 0 on a number line

-A number line is a visual representation of numbers and how they relate to each other

-Rules/patterns for operations with integers (all types of operations and combinations of numbers)

-The order of operations

Students will be able to...

-Define absolute value integer, positive number, negative number, number line, order of operations

-Find the absolute value of a number

-Visually represent absolute value (on a number line)

- Add and subtract two integers

-Visually represent addition and subtraction of integers on a number line

-Multiply and divide two integers

-Apply the Order of Operations to evaluate expressions

-Explain and justify answers to integer operation problems (using real world examples when possible)

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

Suggested Performance/Transfer Task(s)

- Performance/Transfer tasks as evidence of student proficiency

An effective assessment for ALL students should be designed to include:

*Complex, real-world, authentic applications

*Assessment(s) for student understanding of the Stage 1 elements (Enduring Understandings, Essential Questions, Big Ideas) found in the Content Standards

*Demonstration of high-level thinking with one or more facets of understanding (e.g., explain, interpret, apply, empathize, have perspective, self-knowledge)

H & R BLOCK ASSIGNMENT

As adults, we grapple with the constant struggle of keeping a balanced budget each month or payment cycle. The most responsible of us plan ahead, taking into account the earnings and expenses we foresee for next month in order to make sure we do not overdraw our credit cards, run out money, or fall into debt.

In order to prepare yourself to become a responsible adult (or start balancing your budget and saving your money right now!), you will become a Financial Advisor. You will use your knowledge of positive and negative numbers and how they compute with each other to help your client balance her budget.

[The diary of expenses and earnings will require students to make calculations with all four operations: earnings for addition, expenses for subtraction; earnings that occur multiple times (such as a weekly payment of $15 every Sunday) for multiplication of positives; expenses that occur multiple times (such as buying a $2 coffee every weekday morning before work) for multiplication of negatives; earnings that need to be divided (such as a gift from her mom that she has to split with her 2 sisters) for division of positives; shared expenses (monthly rent or electricity bill divided between 4 roommates).

First, you'll look at all her expenses and earnings from last month. You'll create a bank statement for her, showing her balance after each transaction and the calculations you did to reach them. Along with the bank statement you create, you'll write an analysis for your client of her spending and earning habits from the month. This should explain why your client ended up "in the red."

Second, you'll give your client some advice about how she can balance her budget next month. What were her biggest expenses? What expenses can she reduce or cut out to ensure that she ends up "in the black"? Make some recommendations about where to cut back and by how much, including some specific examples about how the cutbacks you're recommending would affect her balance.

Finally, suggest an overall strategy that your client can use to help her keep track of her balance throughout the month. This part will require you to explain how to add, subtract, multiply, and divide both expenses (negative numbers) and earnings (positive numbers).

*Would you use this strategy in your own life, in handling your own expenses? How? Evaluation Criteria:

Rubric(s)

- Scoring guide to evaluate performance/transfer tasks used as evidence of student proficiency

An effective scoring guide should:

*Measure what is appropriate for the Content Standard that is assessed.

*Provide opportunities for differentiation of the performance/transfer tasks used as evidence of student proficiency.

Evaluation Criteria:

Part 1:

-Balance statement shows all transactions from diary, with each one correlating to the correct operation

-Balance statement is neat and organized and presents calculations in an orderly, clear way. A reader can easily follow the work of the advisor and know what steps were taken

-Final balance is accurate and correct

-Analysis of bank statement identifies reasons for being in debt and explains how the client's earning and spending behavior led to the determined result

-Explanation shows use of unit's vocabulary

Part 2:

-Explanation correctly identifies spending behaviors that can be changed

-Explanation includes evidence in the form of calculations that prove how suggested changes would balance the budget

-Calculations are clear and accurate, showing all steps so that reader can easily follow the process that led to the answers

Part 3:

-Advisor suggests viable strategy to help client balance budget in the future that demonstrates understanding of the relationship between positive and negative integers

-Explanation of strategy is clear and easy to understand

-Includes explanation of why this strategy will be successful (including step-by-step instructions of how to add, subtract, multiply, and divide integers)

Overall Criteria

Accuracy: Are all calculations completed accurately? Did the student arrive at the correct amount of debt? Are the conclusions and recommendations made financially sound (i.e. will they get her out of debt next month)?

Explanation: Does the explanation accurately identify the reasons why the client went into debt? Does it employ unit vocabulary and key terms correctly to explain reasoning? Does the explanation provide proof of conclusions in the form of calculations, and refer to those calculations as evidence? Has the student provided a clear explanation of a strategy for balancing the budget in the future?

Strategies: Has the student selected the correct operations to represent the real world transactions and applied the rules of each operation appropriately? Has the student chosen a logical strategy for help in balancing the client's budget in the future, a strategy that demonstrates and understanding of multiply strategies (learned throughout the course of the unit)?

Presentation: Is the balance statement logical, readable, and clearly organized? Are the calculations neat and well organized, so that a reader can follow all steps taken? Is the explanation in the student's own words, logical, and clearly written?

Other Evidence

- Varied evidence that checks for understanding (e.g., tests, quizzes, prompts, student work samples, observations and supplements the evidence provided by the task)

Formative Assessments:

- Instructional Activity 1_Equal and Opposing Exit Ticket: This quick formative assesses whether students can provide a real-world example from the lesson. (7.NS.1a/2)
- Formative Assessment 1_Additive Inverse – This formative assess a student's ability to provide the additive inverse of a number and to state the sum of a number and its additive inverse equals 0. Short answer. (7.NS.1a/1, 7.NS.1b/1)
- Formative Assessment 2_Modeling Addition of Integers – This formative assesses student's ability to add integers using Algebra Squares/chips, plus/minuses, and number lines. (7.NS.1a/2)
- Formative Assessment 3_Exit Ticket: Subtracting Integers – This formative assesses student's understanding subtracting signed integers. (7.NS.1c/1)
- Formative Assessment 4_Exit Ticket: Distance Between Numbers – This formative assesses student's understanding of finding the distance between two numbers. (7.NS.1b/2)
- Formative Assessment 5_Using Properties to Add and Subtract Rationals – This formative assesses students ability to apply properties of operations to signed rationals. (7.NS.1d/2)

Student Self-Assessment and Reflection

- Opportunities for self-monitoring learning (e.g., reflection journals, learning logs, pre- and post-tests, self-editing – based on ongoing formative assessments)

-Formative Observations:

Throughout the unit, students will be observed on their application of mathmatical practices highlighted in the unit:

Mathematical Practices 1. Make sense of problems and persevere in solving them.

Mathematical Practice 4. Model with mathematics.

Mathematical Practice 5. Use appropriate tools strategically.

The observations will be shared with the students and parents throughout the unit. These conferences will be centered around the student's progressing score on the Mathematical Practices Rubric (Appendix A). Students will be prompted to reflect on their progress towards using these practices throughout the unit.

-After completing each Formative assessment performance task, students will be given the opportunity to reflect on their work. Students will then do a Think-Pair-Share with a peer and explain their work, defending both their problem solving method and the answers they reached.

**All performance based task work will be kept inside a reflective log that students will add to throughout the year. The teacher will assist students in creating the log and upkeep throughout the year, but it will be the student's responsibility to maintain. At the completion of each task, the students will reflect in writing on the relationship between the task and the learning objective of the lesson.

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

- Instructional activities and learning experiences needed to align with Stage 1 and Stage 2 expectations

Include these instructional elements when designing an effective and engaging learning plan for ALL students:

*Align with expectations of Stage 1 and Stage 2

*Scaffold in order to acquire information, construct meaning, and practice transfer of understanding

*Include a wide range of research-based, effective, and engaging strategies

*Differentiate and personalize content, process, and product for diverse learners

*Provide ongoing opportunities for self-monitoring and self-evaluation

The direct instruction resources needed to implement this course can be found within the k12 Pre-Algebra course, Semester A. Each day of instruction is accompanied by a powerpoint presentation. Use these presentations to guide your information delivery during Direct Instruction, for additional resources when working with small groups, or as a building block for student guided notes. Guided Practice Examples and additional Independent Practice activities are also embedded within the presentation materials (See example presentation in Appendix M).

-Before Day 1 of the unit, assign scholars homework that asks them to interview a parent, guardian, teacher, or other adult in their lives (put yourself out there as an option) about how they approach budgeting their money. Scholars will ask about expenses and earnings and the challenges of managing it all. Scholars will bring this information to class to use in our hook and preview of the unit on Integer Operations.

-In the preview, we will have a class discussion about what it means to have a balanced budget, what a financial advisor does, and our end performance assessment. This discussion will include the real life application: why is it important to balance our budgets? What happens when we don't? What are the dangers of not being able to manage our money? We will tell scholars that, by the end of the unit, they will have the skills that enable them to advise their families on how to keep a balanced budget. This will include a discussion of the standards, skills, essential questions and enduring understandings.

-Major learning experiences include exploration with physical models (i.e. human numbers line and chips), a simulation involving a trip to the bank (handling checks and bills), and an exploration around why it is important to follow the order of operations (experimenting with following different orders). These experiences will provide a deeper conceptual understanding through application of integers in the real world. Additionally, students will track the patterns they see as they progress through each activity using the attached Patterns Worksheet in accordance with Math Practice 8 (Appendix B).

Learning Experiences:

• Human number line (Appendix C)

o walking on the number line to demonstrate how absolute value is a distance

o Adding and subtracting on a number line: how all combos by using people: pick values out of a hat, show addition of 2 positives or 2 negatives or 1 of each by walking in negative and positive directions

• Chip method – practicing and experimenting with black and red chips/stickers to represent positive and negative values (early on)

• Checks and bills - practice with adding and subtracting positive and negative integers, practice understanding the mathematical representation of checks and bills in an account

• Order of Operations: scholars are given 1 multi-step expression and asked to simplify. Each scholar in the group completes the steps in a different order. Not all scholars end up with the same answer, but some do, despite differences in their order of steps. Scholars must discuss the following:

o Is there only one correct answer (and one correct way to get there?)

o Are all answers correct?

o Why did you arrive at different answers?

o Why are some of the answers the same?

**This will include a discussion of language, and how math is a language. In any given language, conventions, grammar rules, and meanings of words are agreed upon by all who use it: what would happen if everyone who spoke English disagreed on the order of words in a sentence? For example, in Spanish, we say the noun before the adjective, but in English, we say the adjective before the noun:

-chica rubia vs. blonde girl

Lesson Set 1: 1-2 days

Math: (1) define absolute value; (2) find the absolute value of rational numbers (positive and negative)

Problem Solving: use the concept of absolute value to solve real life problems (e.g. number line, coordinate plane, money, temperature, altitude, distance)

Use the Equal and Opposing scenarios worksheet to guide this lesson. Begin class with a discussion around the following question: Can you ever add two things together and have the answer be zero?

Allow students to do a Think-Pair-Share with a partner to get some ideas for the class discussion (3-5 mins).

Open the class up to a discussion around the question, prompting students to critique each other's thinking in a collaborative manner. The teacher's role at this stage should be to guide students in a general discussion, not to give specific answers.

Continue the class investigation using Newton's First Law of Motion (refer back to Appendix D). Discuss the Khan Academy video on Newton's First Law and the examples in the worksheet.

Have the students work together to complete the activities for the "Golf" and "Temperature" scenarios. After each video, break the students into pairs or small groups. Once each group has had time to collaborate, bring the class together for a discussion, again prompting students to critique each other's work.

Formative Assessment 1: Equal and Opposing "Elevation" and "Money" (Appendix D)

Have the students independently complete the final two scenarios, submitting these for evidence of concept mastery.

Lesson Set 2: 1 day

Direct Instruction: Work with students and guide them through the notes on Adding Signed Integers.

Problem Solving: Allow students to use a variety of methods to model/conceptualize addition of integers, such as number lines and physical manipulatives. Students should have access to these resources throughout the lesson and be given the chance to select any resources they believe will help them understand the problems. The teacher should explicitly teach the benefits and drawbacks of each resource and push the students to utilize appropriate resources when problem solving. If a student is struggling, remind them that these resources are available and help them select some to try.

Formative Assessment 2: Adding Signed Integers (Appendix E)

Lesson Set 3: 1 day

Math: Practice adding positive and negative integers (-20 to 20)

Instructional Activity 3: Adding Real World Sums (Appendix F)

Guide students to work with groups in a competition style format to complete the activity. Students should be prompted frequently to justify their situations and explain their reasoning to the class.

Lesson Set 4: 1-2 days

Direct Instruction: Work with students and guide them through the notes on subtracting positive and negative integers (-20 to 20)

Problem Solving: use number line and /or chip method to model subtraction of integers (Is adding a negative the same thing as subtracting a positive? Prompt the students to prove it using whatever resources they need. Informally check students' work during this time, noting students that are struggling with this key concept.)

Instructional Activity 4: Additive Inverse (Appendix G)

Review - Instructional Activity 5: Equal and Opposite (Appendix H)

Formative Assessment: Subtracting Integers (Appendix I)

Lesson Set 5: 2-3 days

Math: Solve integer addition and subtraction problems in context problems – money, temperature, altitude, distance, etc

Problem Solving: cumulative review of Days 1-4. Guide students to complete the group project in which students explore real world applications of adding and subtracting integers. Again, to reinforce Math Practice 5, allow students to select tools and strategies to solve the problems with as little assistance as possible. If students are struggling with the activity, pull them into a small breakout session and spend a few minutes reviewing key concepts before integrating them back into the group.

Group Instructional Activity: Real World Distance Problems (Appendix J)

Lesson Set 6: 2-3 days

Assessment: Absolute value, Adding and subtracting positive and negative integers (Appendix K)

Problem Solving: **Introduce Performance Assessment and rubric

Lesson Set 7: 1 day

Math: multiply and divide positive and negative integers

Problem Solving: Ask students clarifying questions about the Performance Assessment; begin working on projects in class

Math: Evaluate expressions using order of operations (not including absolute value or negative integers); *review from 6th grade with focus on common misconceptions

Lesson Set 8: 2-3 days

Math: identify the criteria for top-quality work presentation

Problem Solving: evaluate students work samples using criteria for top-quality work presentation

Instructional Activity: Students work on and complete Unit Summative Performance Task

Lesson Set 9: 1-2 days

Math: evaluate expressions including AV and negative integers

Problem Solving: identify the components of a successful presentation; give peer feedback to a presenter

Resources and Teaching Tips

- A variety of resources are included (texts, print, media, web links)
- Help in identifying and correcting student misunderstandings and weaknesses

Common Student Misunderstandings:

-PEMDAS leads to the misconceptions that addition always precedes subtraction and multiplication always precedes division.

-The larger the number, the greater the value (for both negative and positive numbers)

-Confusing the rules for the different types of operations can lead to misconceptions: such as when adding integers, if the sign is the same, the answer is always positive; when multiplying two negative numbers, the product is always negative; when multiplying or dividing a positive number times a negative number, you must find the absolute values of the number first

-We can apply the commutative property to subtraction just like we do for addition. Example: 4 – 8 = 4

Clarifications to make to address weaknesses in student performance:

-Add or subtract, in order from left to right; if subtraction comes before addition in order from left to right, subtract before you add. Multiply or divide, in order from left to right. If division comes before multiplication in order from left to right, divide before you multiply

-This is only true for positive numbers; with negative numbers, as the number increases, the value decreases (because it becomes MORE negative)

-The rules for addition/subtraction are different than and separate from the rules for multiplication/division. Examples: when adding integers, if both addends are positive, the sum is positive; if both addends are negative, the sum is negative; when multiplying two negative numbers, the product is always positive; when multiplying and dividing integers, you do not need to find the absolute values of the numbers

-Subtracting an integer is the same operation as adding its inverse. 4 – 8 = 4 + (-8)

Throughout the unit, students have access to a variety of resources.

Textbook

Smartboard and Projector

Graphic Organizer Paper

Reference Sheets

Additionally, each formative check for understanding provides an opportunity for the students to identify their own misunderstandings and weaknesses in mastering the content. Guide the students in understanding how to use these formative checks to enhance their learning through the use of rubrics, conferences, and remediation or acceleration plans designed with student input (Appendix L). While working towards mastery of grade level content standards, continuously refer students back to the standards for mathematical practices. These practice standards are the attitudes and skills that will allow students to achieve mastery of content standards, as well as deep understanding of mathematics.

Differentiation

- Stage 2 and 3 allow students to demonstrate understanding with choices, options, and/or variety in the products and performances without compromising the expectations of the Content Standards.
- Instruction is varied to address differences in readiness, interest, and/or learning profiles.
- Accommodations and differentiation strategies are incorporated in the design of Stage 2 and 3.

☐ Refer to the IEPs of your special education students

☐ Allow students to have tests, quizzes, worksheets, etc read to them.

☐ Allow the use of a calculator

☐ Communicate with the special education teacher about allowing testing in the special education room

☐ Pair and group students with differing ability levels.

☐ Use manipulatives as needed

☐ Allow for extra time on tests, quizzes, and performance tasks

☐ Allow for a word bank and a formula sheet on tests

☐ Allow students to choose to demonstrate understand in written or verbal format as needed.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.
- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning –** the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

(Briefly explain how design principle(s) are embedded within the unit design.)

In alignment with the Common Core State Standards, this unit pushes students to develop 21st Century Skills to meet the learning demands. Students come together as a learning community to analyze real-world applications of the math they are learning, think critically and reflect on the work of their peers as well as their own, and constantly work to apply their knowledge to new, more complex situations. This push towards using developed skills in new ways is an accurate reflection of the higher order thinking required by 21st Century jobs. Students should become excited and engaged by the "hands-off" approach of the teacher. Rather than giving students the formulas and a calculator and having them compute numbers and spit out answers, the teacher should present numbers and problems in a real world context, allowing the students to select the tools, resources, and knowledge needed to arrive at a reasonable answer. This push towards applying mathematical ways of

thinking to real world Issues will prepare them for the type of work they will encounter in high school, college, and in their careers.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- **8th Grade Technology Literacy** - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century(SETDA, 2003).

In alignment with Mathematical Practice 5 - Use appropriate tools strategically, students will have access to calculators as needed to complete the Performance Tasks. Additionally, when sharing their answers to the Performance Tasks, the teacher may opt to have the students construct a powerpoint presentation as an additional layer of the project.

Content Connections

Content Standards integrated within instructional strategies

Connections to other content areas are made both explicitly and implicitly throughout this unit. Students will watch videos, outlining the connection between elements of science (deep sea diving, forces, etc) and math, and be given a performance task that makes this connections clear. Additionally, the performance tasks at the end of each lesson connect the math content to music, economics, and personal finance. Additional connections are encouraged at the teacher's discretion. For example, when assigning the students additional practice work, make the work meaningful by aligning the examples with content currently being covered in other classes such as Science, English, and Social Studies.

Student Name_____ Evaluation Date_____ Overall Score_____

Mathematical Practice	Novice (1)	Apprentice (2)	Practitioner (3)	Expert (4)
1. Make sense of problems & persevere in solving them	I couldn't get started. I don't know how to begin. I may be feeling frustrated and wanting to give up.	I am stuck. I have part of the solution, but now I don't know what to do, I'm not sure my answer is right and I could use some help. I may be feeling frustrated.	I planned out how to solve this problem and I worked all parts of the problem. I may have felt challenged, but I didn't want to give up-I kept going until the problem was solved.	My solution is effective and inventive! I planned out how to solve this problem and I even showed you other ways to solve it. I checked to make sure my answer was right – I enjoyed this challenge!
2. Reason abstractly & quantitatively	I can't explain how I solved this problem. I don't know what quantities or equations to use. My work on this problem is built on misunderstandings of mathematical ideas.	I'm trying to explain how I solved this problem. My work on this problem may be built on misunderstandings of mathematical ideas. . I'm trying to take numbers out of the problem to work with them. I'm trying to check my work.	I explained how to solve this problem using formal math language. I took numbers out of the problem to work with them, and periodically put them back into the context of the problem to see if my work makes sense.	I clearly explained how to solve this problem using formal math language. I took quantities out of the context of the problem to work with them, and frequently put them back into the context of the problem to see if my work made sense.
3. Construct viable arguments & critique the reasoning of others	I can't describe how I solved the problem. I can't give feedback on someone else's reasoning OR my feedback in unclear, unhelpful, and/or incorrect.	I'm trying to describe how I solved the problem. I may or may not include pictures, tables, graphs, and/or models. I gave feedback on someone else's reasoning, but my feedback may have been unclear, unhelpful, or I may have been unable to back it up with my own work.	I described how I solved the problem using formal math language, pictures, tables, graphs, and/or models. I gave clear, helpful feedback to someone else's reasoning and backed up my feedback with my own work on the problem.	I clearly described how I solved the problem. I included every step so you don't have to guess what I did. I used formal math language, pictures, tables, graphs, and/or models. I gave clear, helpful feedback on someone else's reasoning and backed up my feedback with my own work on the problem.
4. Model with mathematics	I can't use mathematical representations, I don't know what to use for this problem and I can't describe how I would solve this problem.	I'm trying to use mathematical representations to solve the problem and show my solutions, but they may be inaccurate or inappropriate. I'm sharing my ideas, but I'm struggling to use formal math language.	I used appropriate & accurate mathematical representations to solve the problem and show my solutions. I used formal math language to share my ideas.	I used a variety appropriate, accurate, and detailed mathematical representations to solve the problem and show my solutions. I used formal math language to share my ideas.
5. Use appropriate	I don't know what tool to use to solve this problem and/or I	I picked an appropriate tool to use, but I may be struggling to	I picked the correct tool to use based on the needs of the	I selected and used a variety of appropriate tools

Student Name_____ Evaluation Date_____ Overall Score_____

tools strategically	didn't use the tool I picked in a valuable way.	use it in a valuable way to solve the problem.	problem. I planned and monitored my use of the tool.	collaboratively to solve the problem. I planned my use of the tools and monitored my progress.
6. Attend to precision	I can't get the correct answer and/or I can't communicate my answer in a way that is appropriate to the problem. I can't use correct units, labels, or definitions.	I may be getting the correct answer and/or I may be able to communicate my answer is a way that is appropriate to the problem. I may not use correct units, labels, or definitions.	I got the correct answer and I can communicate it using the correct labels, units, and definitions. The precision of my answer is appropriate for the problem.	I got the correct answer and I communicated it using correct labels, units, and definitions. I clearly stated the meaning of the symbols I chose and my answer is complete appropriate for the problem.
7. Look for & make use of structure	I don't see any structure in the problem. I can't make sense of any useful structures/patterns to help me solve the problem.	I may be able to find for structures/patterns in the problem, but I am struggling to use them to help me find a solution.	I found and used structures/patterns in the problem to help me find a solution. I noted these structures/patterns as I found them.	I found, noted, and used structures/patterns in the problem to help me find a solution. I was able to step back from the problem and analyze different perspectives.
8. Look for & express regularity in repeated reasoning	I don't see any regularity or any shortcuts or generalizations I can use to get a solution.	I may notice that some of my calculations are repeated but I may not be able to generalize that observation to help me get a solution.	I noticed some of my calculations were repeating and was able to make a generalization that helped me solve the problem. I evaluated the reasonableness of my work as I progressed.	I noticed some of my calculations were repeating and was able to make a generalization or a formula that helped me solve the problem. I continuously evaluated the reasonableness of my work as I progressed.

Please write patterns you see as a result of the work you have done in this activity.

Instructional Activity 4: A Human Number Line

Grade 7/Unit 2

In general, students will take positions on a number line and act out sums of the form ***p + q***. See document ***Whole-Group Agree/Disagree Signals*** for ideas for discussion ideas.

1.) Present students with a sum, for example, 6 + (-3). Write the following labels on the board to serve as a chart of the work.

Sum *or* p + q **Resulting Sum** **Distance from First Addend**

2.) Have a student go to the origin and "walk" the first addend, in this case, 6.

3.) The rest of the class needs to indicate whether they agree or disagree with the student's acting out and resulting position of the student. (See document "Agree/Disagree Signals) If the students don't agree, they discuss whether there needs to be change. Be a final judge if students cannot come to a correct conclusion.

4.) Then, have a second student go to the same position as the first student and walk the rest of the problem, in this case, + (-3). Ask for agreement/disagreement and discuss if necessary.

5.) Ask students, "How far from the first addend is the second?"

6.) Repeat this process for several examples, alternating the signs for a variety of situations.

7.) After several iterations, ask students to look at the third column of the chart. "What is the question I have asked every time to fill in this column?" ("How far ...") "What is another name for this quantity? Or equivalently, What is another label I can make for this column?" (Absolute Value, or |q|) Have students discuss what happened here physically; lead them with questioning to the objective/main discussion point. The main discussion point is: **In p + q, the result is always |q| from p, in the direction of the sign of q.**

EQUAL and OPPOSING

Grade 7/Unit 2

Below are real-world situations that illustrate the principle of opposite quantities combining to make 0. Use the collaborative structure of giving each scenario to a small group, have them discuss in the small group and answer questions, then report out to large group. Write the following guiding question on the board for when the students walk in the classroom. NOTE: The videos are not necessary for the understanding but can greatly enhance it.

What are some specific examples from the real world to illustrate the following principle?

Opposite quantities combine to make 0.

In general, students are going to investigate and then use a specific example to illustrate the following principle: **Opposite quantities combine to make 0.** They will be asked to:

1. Summarize the situation, telling the specific, equal and opposite quantity.
2. Write an equation illustrating the situation.

1. Newton's First Law of Motion

1. **Play the video** in this link from Khan Academy: http://www.5min.com/Video/Learn-about-Newtons-First-Law-of-Motion-99171587.

2. **Discuss the video by answering the following**: What is one thing you found interesting in this video?

3. **Read with your group this specific example**: An example is your hand holding an apple 3 feet above the ground. Let's say the force you are applying in a straight line up to keep the apple from falling to the ground is x. The apple is exerting a force against your hand as well, and since the apple remains at 3 feet above the ground and does not fall, the movement of the apple may be modeled with the following equation: $x + (-x) = 0$.

4. **Answer these questions as a group:** New Scenario: A straight pin is holding a tail on a donkey's end on a wall.
   1. If the tail does not fall to the ground, describe the equal and opposite forces affecting the movement of the tail.
   2. Write an equation to illustrate the movement of the tail.

2. Golf

1. **Play two videos** provided by these links on scoring in golf: 1.) http://www.youtube.com/watch?v=qktGvobBfDc&feature=youtube_gdata_player&edufilter=4flL6p8CLoghnIDbIAI_YQ, and 2.) http://www.youtube.com/watch?v=LTEjCjgbe3k&feature=youtube_gdata_player

2. **Discuss the video by answering these questions with your group:**
   1.) What is the integer for these golf scores?
   Eagle = ____
   Birdie = ____
   Par = _____
   Bogey = ___
   Double Bogey = ____
   Triple Bogey = ____

3. **Read with your group this specific example**: Going in to the 17th hole, Shawn's score was an even par. On Hole 17, he scored an eagle, and on Hole 18 he scored a double bogey. What was his final score? An eagle is 2 under par and a double bogey is 2 over par, so the equation would look like this: (-2) + 2 = 0. His score is even par.

4. **Answer these questions as a group:** New scenario: A new person golfing.
   1. Create a new story where the golfer scores an even par.
   2. Write an equation to illustrate how he/she arrived at the even par score.

3. Temperature

1. Play this video on an interesting thermometer.
http://www.youtube.com/watch?v=plQ8Y3XAKvA&feature=youtube_gdata_player

2. Discuss this video by answering these questions with your group: At first, this thermometer appears to be non-functioning. What was unique about this thermometer? How, in general, must it work?

3. Read with your group this specific example: A certain compound measured – 18 ° F. Placing it on a heat source, the temp rose 18°. The result was 0° F. Equation: (-18) + 18 = 0.

4. Answer these questions as a group: New scenario: temperature anywhere.

1.) Create a new scenario where opposite temps result in 0 degrees.

2.) Write an equation to represent the situation.

4. Elevation

1. Play this video at the link http://www.youtube.com/watch?v=m3oOpSBAeGI&feature=youtube_gdata_player.

2. Discuss this video by answering these questions with your group: What is one real-life activity the video use to represent a positive altitude? What is one real-life activity the video used to demonstrate a negative depth?

3. Read with your group this real-life example: Mr. Rios was flying his jet at 15,000 ft elevation. Later, he descended the 15,000 feet to land. My equation: 15,000 + (-15,000) = 0. 0 represents the elevation of the ground, which is not only where the plane would have started (the origin) but also where zero elevation would be.

4. Answer these questions with your group:

1.) Create a new scenario where opposite altitudes result in a zero altitude.

2.) Write an equation to represent the situation.

4. Money

1. Play this video at the link http://www.youtube.com/watch?v=m3oOpSBAeGI&feature=youtube_gdata_player.

2. Discuss this video by answering these questions with your group: What can happen when you write a check for more money than you have in your account?

3. Read with your group this real-life example: Dan made $350 on the weekend. On Monday, Dan bought a guitar for $350, which put him back to his beginning balance. An equation to represent the situation: 350 – 350 = 0. 0 represents the beginning balance, his starting point.

4. Answer these questions with your group:

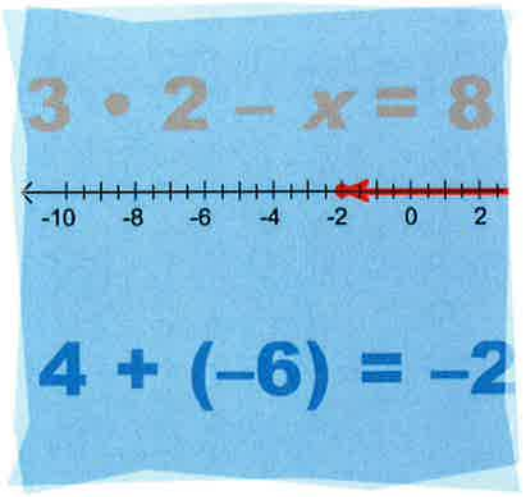
1.) Create a new scenario where opposite transactions result in a zero balance.

2.) Write an equation to represent the situation.

Unit 2

Lesson 3: Adding Integers

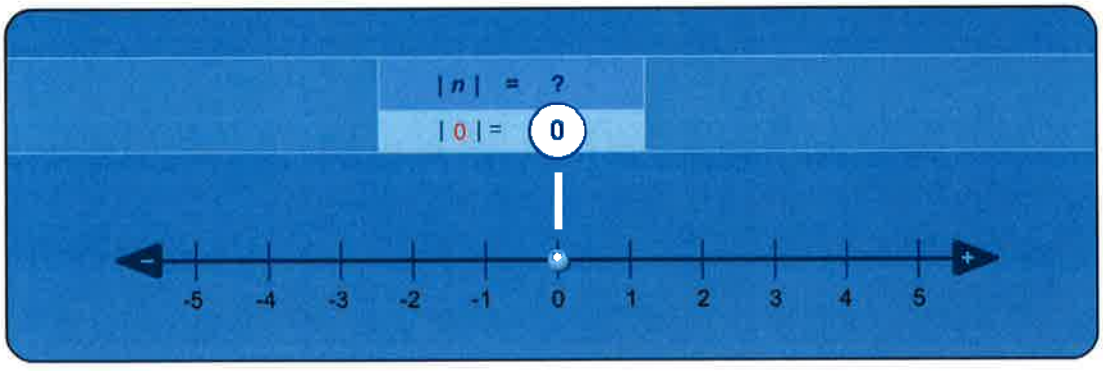
Think about the person who delivers mail in a tall office building. All day long she goes up and down on an elevator. Suppose she begins at the mail center on floor 5 of the building and goes up 7 floors and then down 10 floors. Where is she now in relation to floor 5? If you think of the elevator as a number line, you can find the answer. In this lesson you will learn to add integers.



Unit 2 | Lesson 3

GROUNDWORK: Preparing for the Lesson

Click to select a number below. Then select its opposite to compare the absolute value of the two numbers. Try numbers close to zero and numbers far from zero. Notice that the absolute value increases as the distance from zero increases.



LEARN: Using Number Lines to Add Integers

One way to add integers is by using a number line. The number line is a good way to visualize how positive and negative integers are added.

Read and follow this plan to add positive and negative integers on a number line.

Plan for Adding Integers Using a Number Line

1. Draw a number line.
2. Begin at zero and move the indicated number of units to the left or right.
   - Move to the right if you are adding a positive integer.
   - Move to the left if you are adding a negative integer.
3. Note where you end up after all your moves—that is the sum.

Unit 2 | Lesson 3

LEARN: Using Number Lines to Add Integers

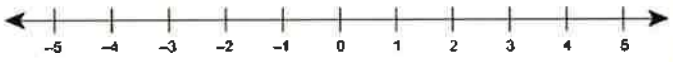
Now that you've seen the plan for adding positive and negative numbers on a number line, check out how the plan works.

Look at this example.

Add:

$4 + (-6)$

Draw a number line.



1 of 3 Next

Remember

LEARN: Using Rules to Add Integers

Drawing a number line is not always convenient when adding integers—think about how long your number line would have to be if you wanted to add 434 + 871

The screen at right shows you rules you can use to add positive and negative integers.

Rules for Addition of Integers

Rule 1
If a and b are both positive, then:

$$a + b = |a| + |b|$$

Example:

If $a = 4$ and $b = 2$, then:

$$4 + 2 = |4| + |2|$$
$$= 6$$

1 of 4 Next

Think About It

LEARN: Evaluating Expressions

To evaluate a variable expression, substitute the given values for the variables and then simplify.

Use the appropriate rule for addition of integers to simplify the expression.

Evaluate $b + c$ when $b = 4$, and $c = -15$.

Substitute the values for the variables in the expression.

$b + c = 4 + (-15)$	Substitute the given values for the variables.

Simplify.

$4 + (-15) = -(\|-15\| - \|4\|)$ $= -(15 - 4)$ $= -11$	Use the rules for the addition of integers.

Print

Big Idea

WORKED EXAMPLES: Adding Integers

Try some examples on your own. Remember to start by determining the correct rule for addition of integers.

Work the examples yourself; then check your own answers against the answers in the examples.

Find the sum.

3 + (–6)

Answer

1 of 4 Next

SUMMARY: Adding Integers

A number line is a good way to visualize how to add positive and negative integers.

Plan for Adding Integers Using a Number Line

1. Draw a number line.
2. Begin at zero and move the indicated number of units to the left or right.
   - Move to the right if you are adding a positive integer.
   - Move to the left if you are adding a negative integer.
3. Note where you end up after all your moves—that is the sum.

SUMMARY: Adding Integers

You don't always need the help of a number line to add positive and negative integers. In fact, there are certain rules you can use that make adding positive and negative easier.

The rules for addition of integers are summarized in the table you see here.

positive + positive = positive	negative + negative = negative

The sum of a positive number and a negative number depends on which number has the greater absolute value.

If the absolute value of the positive number is greater than the absolute value of the negative number, the sum is positive.

|positive| > |negative| = positive

If the absolute value of the negative number is greater than the absolute value of the positive number, the sum is negative.

|negative| > |positive| = negative

If the absolute value of the positive number equals the absolute value of the negative number, the sum is zero.

|positive| = |negative| = 0

Remember

OFFLINE LEARNING: Adding Integers

Now go offline to learn more about adding integers and to practice your skills:

- Review pages 45–48 in the Reference Guide, through the Application: Banking example.
- Complete Problems 1–11 odd and 19–29 odd on pages 48–49 of the Reference Guide.
- Complete Problems 1–12 even and 20–30 even on pages 48–49 of the Reference Guide for extra practice. (*optional*)

Materials

Adding Integers Solutions

Pre-Algebra: A Reference Guide and Problem Sets - pages 45-49

Optional

calculator

Instructional Activity 5_Adding Signed Integers Using Number Lines

Grade 7/Unit 2

Model and solve with number lines.

1.) 6 + (-5) =_____

2.) 7 + (-4) = _____

3.) (-3) + 8 = _____

4.) -6 + 5 = _____

5.) -7 + 4 =_____

6.) 3 + -8 = _____

7.) -3 + -2 = _____

8.) (-4) +(-3) = _____

9.) -3 + -3 = _____

10.) 3 + 2 = _____

11.) 4 + 3 = _____

12.) 3 + 3 = _____

Instructional Activity 6_Real-World Sums

Grade 7/Unit 2

An Example:

Model the real-world applications. The following is an example:

Write 5 + -2 on the board. Say, "Here is a sum. I will model how I want you to apply a real-world context to a sum. This sum represents a certain football players yardage gained on plays. He has done two plays, the first of which he gained 5 yards as seen by the +5. The second play was a loss of 2 yards, -2. His net yardage is the resulting sum of 3 yards, meaning overall he has gained 3 yards."

Give students three different examples like the one above. Be sure to include both the real-world representation as well as the integer representation ("a loss of 2 yards, -2") so that students will do the same.

A Competition:

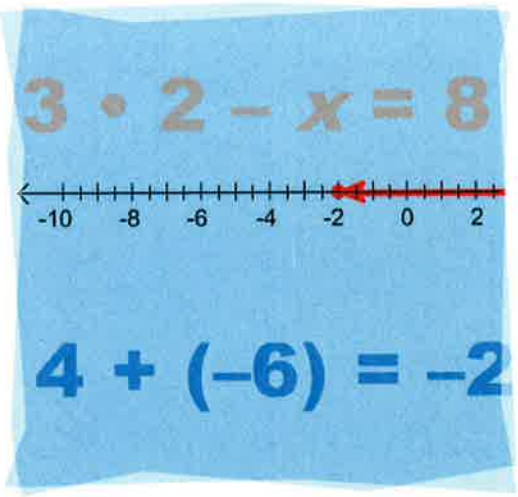
Small groups will be given an addition problem where the positive is bigger than the negative and 5 minutes to produce and write down as many real-world situations for the sum as the group can. Warn the students that you may call on anyone in the group to "Justify the Situation" which will mean the student will have to name the integer and what quantity it represents in the real-world situation. At the end of the 5 minutes, or whatever time you deem appropriate, groups will share out their examples. Randomly choose at least one situation per group that have to be "justified." The group with the most examples wins the round.

Repeat for two more rounds, using different examples (negative integer bigger than the positive, both negative.) On the last round, you may decrease the time limit slightly as middle-level students tend to get bored without an additional challenge.

Unit 2

Lesson 4: Subtracting Integers

The opposite of up is down and the opposite of cold is hot. Similarly, the opposite of a positive number is a negative number and the opposite of subtraction is addition. In this lesson, you will use the concept of opposites to subtract integers.



Unit 2
Lesson 4: Subtracting Integers

GOALS FOR THIS LESSON:

- Subtract integers.
- Evaluate expressions involving a difference of integers.

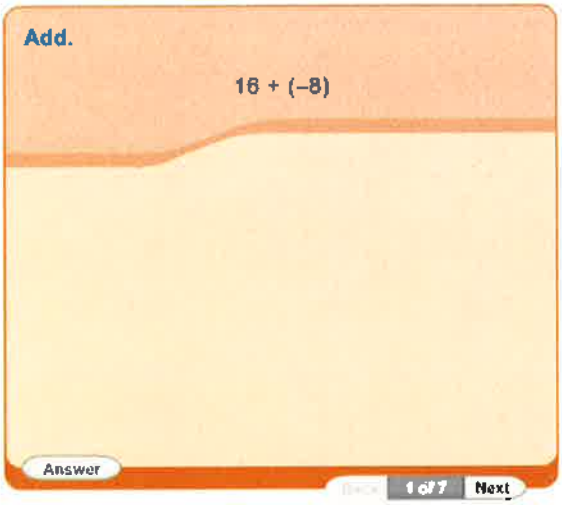
Materials

Student Guide

GROUNDWORK: Preparing for the Lesson

You now know how to add integers. This is an important step toward knowing how to subtract integers.

Begin this lesson with a brief review of adding integers. Study the examples to the right.



Activity Resources

LEARN: Subtracting Integers

The expressions "stay dry" and "don't get wet" demonstrate a general principle. You can sometimes accomplish something by not doing its opposite. This idea is true in math also: You can subtract a number by "un subtracting", that is, by adding its opposites.

There's more than one way to stay dry.

Activity Resources

LEARN: Subtracting Integers

Subtraction of integers can always be written as addition of integers. Instead of subtracting an integer, you can add its opposite.

Take a look at this example.

Suppose you want to subtract –5 from –11. How would you do it?

–11 – (–5)

–11 + 5 or you could **ADD** the opposite of –5 to –11.

Back Start Over

Remember

LEARN: Subtracting Integers

Once you turn subtraction into addition by adding the opposite, you can use the rules for addition of integers to find the answer.

Subtracting Integers
Subtracting an integer is the same as adding its opposite.

Example 1:

$$3 - 9 = 3 + (-9)$$
$$= -6$$

Example 2:

$$-11 - (-12) = -11 + (+12)$$
$$= -11 + 12$$
$$= 1$$

LEARN: Evaluating Expres

Some variable expressions involve the subtraction of integers. Use the following steps to evaluate that type of expression.

- Substitute the value into the expression for the variable.
- Rewrite the expression using addition.
- Simplify.

Apply those steps in this example.

Evaluate $d - 6$ when $d = -8$.

Substitute the value for the variable in the expression.

$d - 6 = -8 - 6$

Rewrite using addition.

$-8 - 6 = -8 + (-6)$

Simplify.

$-8 + (-6) = -14$

Print

Remember

WORKED EXAMPLES: Subtracting Integers

Here are some examples you can use to practice your skills.

Try each example on your own, then check your answer.

Find the difference.

$3 - 12$

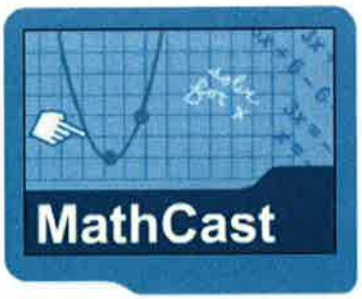
Answer

1 of 5 Next

MATHCAST: Value of the Expression

Now watch how it's done. Click Value of the Expression to see how to solve a typical problem.

This activity has audio. Please use speakers or headphones.



Value of the Expression

↑ Activity Resources

SUMMARY: Subtracting Integers

In this lesson, you have learned how to simplify expressions involving the subtraction of integers by rewriting the expression using addition.

Subtracting Integers
Subtracting an integer is the same as adding its opposite.

Example 1:

$$3 - 9 = 3 + (-9)$$
$$= -6$$

Example 2:

$$-11 - (-12) = -11 + (+12)$$
$$= -11 + 12$$
$$= 1$$

↑ Activity Resources

SUMMARY: Subtracting Integers

You have also learned how to evaluate and simplify variable expressions involving a difference of integers.

Evaluating Variable Expression

Evaluate $-m - n$ when $m = -3$ and $n = -5$.

$$\begin{aligned} -m - n &= -(-3) - (-5) \\ &= 3 + 5 \\ &= 8 \end{aligned}$$

OFFLINE LEARNING: Subtracting Integers

Now go offline to learn more about subtracting integers and to practice your skills:

- Review pages 50–51 in the Reference Guide.
- Complete Problems 1-29 on page 52 of the Reference Guide.
- For additional practice, see Problems 15 and 25 on page 53 of the Reference Guide.

Materials

Subtracting Integers Solutions

Pre-Algebra: A Reference Guide and Problem Sets

Optional

calculator

Name:

Formative Assessment 1: Additive Inverse

Grade 7/Unit 2

Provide the additive inverse of the following:

1.) -10 → ___

2.) 21 → ___

3.) −x → ___

4.) $|-8|$ → ___

Fill in the blank:

The sum of a number and its additive inverse is always _____.

KEY Formative 1: Additive Inverse

Grade 7/Unit 2

(7.NS.1a/1, 7.NS.1b/1)

Provide the additive inverse of the following:

1.) $-10 \rightarrow$ _10__

2.) $21 \rightarrow$ _-21_

3.) $-x \rightarrow$ _x_

4.) $|-8| \rightarrow$ _-8_

Fill in the blank:

The sum of a number and its additive inverse is always __zero___.

EQUAL and OPPOSING: A Practice

Grade 7/Unit 2

1.) Choose three of the following areas and write scenarios of real-life applications of the following principle:

Opposite quantities combine to make 0.

2.) Write an equation to model each scenario.

Areas for Real-Life Scenarios

Forces and Acceleration	**Banking**	**Football**
Atoms and Charged Particles	**Conservation of Energy**	
Golf	**Travel and Distances**	**Weights and /or Balance**
Weight	**Stocks and the Market**	
Deep Sea Diving	**Sonar and/or Radar**	**Assets and Debits**

Formative Assessment 3: Exit Ticket: Subtracting Integers

EXIT TICKET:

Fill in the blanks.

1.) $5 - 7 = 5 +$ _____

2.) Subtracting means "adding the _____."

3.) $3 - (-4) = 3$ ___ $4 =$ _____

Name:

EXIT TICKET:

Fill in the blanks.

1.) $5 - 7 = 5 +$ _____

2.) Subtracting means "adding the _____."

3.) $3 - (-4) = 3$ ___ $4 =$ _____

Instructional Activity 9_Real-World Distance Problems

Grade 7/Unit 2

1.) Sally is trying to change her eating and exercising habits. Below is the record that she kept of her weights over the last six months.

Month	February	March	April	May	June	July
Weight (lbs)	145.4	147.6	140.1	137.0	138.5	135.3

How many pounds are there between her highest weight and her lowest weight? Did that represent a loss or a gain of weight?

2.) Brad is investing in the stock market. The stock that earned him the most money this year was also the most volatile of his stocks. If the high was +3 ½ points and the low was -1/4 points, then how many points were there between the high point and the low point?

3.) On a certain map, two cities lie in a straight line from Bill's house. If Los Donas is at a position 7.5 miles from Bill's house, and San Bernado is 15.87 miles from Bill's place, then how far apart are the two cities?

4.) Holly is hanging art on her newly-painted walls. If the center of one piece is 12 ¾ feet from the corner, and the center of another is 15 3/8 feet from the corner, then how far apart are the centers of the two pieces?

5.) On a new board game, the spots on which pieces can land go in a circle, and pieces may move either direction. There are over 100 total spots on which to land. The "Win $1000" spot is 27 moves from the "Start," and the "Lose Your Flying Vehicle" is -14 spots from the "Start" spot. How many spots apart are the "Win $1000" and the "Lose Your Flying Vehicle" spots?

Signed Rationals

Grade 7/Unit 2

1. Describe a situation in each of the suggested areas below in which opposite quantities combine to make 0. Use specific details, like numbers, in your example. Finally, circle the two opposite quantities.

   Golf_____
   _____
   _____

   Forces_____
   _____
   _____

   Money_____
   _____
   _____

   Choose one of your examples above and write an equation that represents the situation.

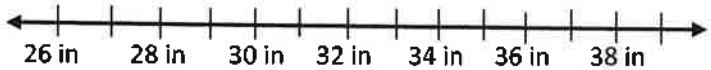
   Circle which example you are using: Golf Temperature Money

   Write your equation here: _____

At the Carnival

Donna, Jim, and Jim's little cousin, Derek, are attending a carnival; answer the following questions about their experiences.

2. In Toddler Town, the rides each have a sign that displays minimum allowable heights for adults to use to ensure the safety of little ones. The Kookie Kars sign tells Donna and Jim that 28 inches is the smallest height allowed. Derek is 6 inches taller than the lowest allowable height. Show on the number line below how to determine how tall Derek must be.



Derek must be _____ inches tall.

3. Which of the following are acceptable descriptions of the sum of -14 + -5 ? Choose all that apply.
   - ❒ a number |5| away from 14, in the negative direction
   - ❒ a number |5| away from -14, in the negative direction
   - ❒ the equivalent of -14 – 5
   - ❒ the equivalent of 14 + 5
   - ❒ a number 14 away from 0
   - ❒ the equivalent of -14 + 5
   - ❒ the equivalent of |14 + 5|
   - ❒ the equivalent of -|14 + 5|

Adding with Signed integers

Model the following addition problems by drawing the two indicated methods—on a number line and with Plus-Minuses. Include the answer on the blank given.

Problem	Number Line	Plus-Minuses

4. 10 + (-6) = _____

Problem	Number Line	Plus-Minuses

5. 4 + (-7) = _____

Problem	Number Line	Plus-Minuses

6. (-3) + (5) = _____

Problem Number Line Plus-Minuses

7. (-2) + (-6) = _____

8. Which of the following best describes the sum of 4 + (-10)?

   a.) A number that is 14 less than 4.
   b.) A number that is 4 less than 10.
   c.) A number that is |6| units from 4.
   d.) A number that is |10| units from 4.

9. a. Fill in the blank to make a correct statement:

   A number and it opposite have a sum of _____.

   b. Write a specific number sentence example that models the statement above.

   Write your equation here: _____

   c. Justify your statement by modeling the equation below. You may use a number line, Algebra Squares/Plus-Minuses, or a model of your choice.

d. Which of the following will correctly complete both of the following statements?

The opposite of a number is also called its _____.

A number and its _____ *always equal zero.*

i.) Negative reciprocal
ii.) Multiplicative inverse
iii.) Additive identity
iv.) Additive inverse

Tell Me a Story

First, find the indicated sum. Then, describe a specific situation in the area indicated that could apply to the given sum. Be sure to include specific details, like the numbers and what they represent, in your description. An example is given below:

5 + -3 = ___ Candy

Example Answer: 5 + (-3) = **2** Sandy has 5 pieces of candy. She shares 3 pieces with Dirk, which leaves her with 2 pieces.

10. 24 + (-15 ½) = _____ Area: Sewing

11. (-27.01) + (32.5) = _____ Area: Banking

12. 32,000 + (-10,000) = _____ Area: Elevation

13. 15.25 + (-6.75) = _____ Area: Track and field

14. -3 + (-10) = _____ Area: Temperature

Subtraction of Rationals

15. Rewrite the following subtraction problem as an addition problem:

10 – 17 = _____

16. Check all answers that make the following statement true:

Subtracting a number is the same as adding the _____ of the number.

- ❒opposite
- ❒absolute value
- ❒additive inverse
- ❒reciprocal
- ❒multiplicative inverse
- ❒negative reciprocal

17. Show that the distance between -7 and 3 on a number line is the same as the absolute value of their difference:

Number Line Model

Absolute Value of Their Difference

Answer from Number Line Model = Answer from Absolute Value of their Difference

18. The low point of the Japanese Trench in the Pacific Ocean is –10,372 meters. The low point of the Puerto Rico Trench in the Atlantic Ocean is -9,200 meters.

a. If these depths were graphed on a vertical number line, what is the distance between them? Show how you arrived at your answer.

b. Find the absolute value of the difference in the two depths. Compare your answer to the answer in part a. Show your work and comparison below.

19. The highest temperature in Fargo, North Dakota, in the first week in January was **- 6°** , and the week's low was **- 40°.**

In the box below, show that the **distance between the low and the high on a number line** is the same as **the absolute value of the difference**.

20. Which property would make the problem below easiest to compute? Explain why that property makes it easier. Then, solve the problem.

$$(17\frac{1}{2} + 7\frac{3}{4}) + 12\frac{1}{4}$$

21. Which property would make the problem below easiest to compute? Explain why that property makes it easier. Then, solve the problem.

$$-3.2 + 4.5 + (-1.4)$$

Signed Rationals

Grade 7/Unit 2

1. (Obj 1/7.NS1a/2)
   Description of Situations Scoring Guide
   ____Description of a golfing example with specific opposite quantities that combine to equal 0. (1 pt)
   ____Two opposites quantities from golf example are circled. (1 pt)
   ____Description of a forces example with specific opposite quantities that combine to equal 0. (1 pt)
   ____Two opposites quantities from forces example are circled. (1 pt)
   ____Description of a money example with specific opposite quantities that combine to equal 0. (1 pt)
   ____Two opposites quantities from money example are circled. (1 pt)
   ____Equation correctly represents the situation chosen. (1 pt) (Obj 3/7.NS1b/2)

At the Carnival

2. Scoring Guide (Obj 2/7.NS1b/1)

   ____Indication of starting at 28 on the number line.(1 pt)

   ____Indication of going up 6 from 28. (1 pt)

   ____An answer of 34 inches. (1 pt)

3. Correct Answers: NYYN NNNY (1 pt. each) (Obj 3/7.NS1b/2. Obj 5/7.NS.1c/2)

Adding Signed Integers

(Obj 2/7.NS1b/2) (1 pt each for each correct answer, each number line model, and each Algebra Squares/Plus-Minuses model)

4. $10 + (-6) = 4$ # Line: Start at 0, go to 10, go left(down) 6, land on 4 Alg. Squares/Plus-Minuses: Start with 10 +'s and 6 –'s. Then, 6 –'s cancel with 6 +'s, leaving 4 +'s.

5. $4 + (-7) = -3$ # Line: Go to 4, go down(left) 7, land on -3 Alg. Squares/Plus-Minuses: Start with 4 +'s and 7 –'s. Then, 4 –'s cancel with 4 +'s, leaving 3 -'s.

6. $(-3) + (5) = 2$ # Line: Go to -3, go up 5, land on 2 Alg. Squares/Plus-Minuses: Start with 5 +'s and 3 –'s. Then, 3 –'s cancel with 3 +'s, leaving 2 +'s.

7. $(-2) + (-6) = -8$ # Line: Go to -2, go down (left) 6, land on -8 Alg. Squares/Plus-Minuses: Start with 2 -'s and add 6 more –'s. Nothing cancels since there are no +'s, leaving all 8 -'s.

8. Answer: d.) A number that is $|10|$ units from 4. (1 pt) (Obj 2/7.NS1b/2)

9. a. Correct Answer: 0 (1 pt) (Obj 3/7.NS1b/1)

b. Answers Will Vary. Exemplary response: 7 + (-7) = 0 (1 pt) (Obj 3/7.NS1b/2)

c. Answers Will Vary. An exemplary response is a correct depiction of the equation given in part b. Whatever model is chosen, the model must show a number, it's opposite, and a sum of 0. Note: a model can be a written explanation as long as it contains the requisite parts. (1 pt) (Obj 3/7.NS1b/2)

d. Correct answer: iv.) Additive inverse (1 pt) (Obj 3/7.NS1b/1)

Tell Me a Story

(Obj 4/7.NS1b/2)

Scoring Guide for #'s 10 – 14: (1 pt for each correct sum, and 1 pt for each correct specific situation which must include what each addend and sum stand for in the context of the situation.)

Answers Will Vary. Possible acceptable answers given below.+

10. 24 + (-15 ½) = 8 ½ . One piece of cloth is 24 inches wide. Sam cuts off 15 ½ inches, leaving her with 8 ½ inches of cloth.
11. (-27.01) + 32.5 = 5.49 Sam was $27.01 in the hole in her bank account. She deposited $32.50, which leaves her with $5.49 in her account.
12. 32,000 + (-10,000) = 22,000 Sam was flying in her Iron Woman suit at an elevation of 32,000 feet. Then, she dropped 10,000 feet to fly at 22,000 feet.
13. 15.25 + (-6.75) = 8.50 Last year, Sam ran the mile in 15.25 minutes. This year, she improved that time by 6.75 minutes. Now she can run it at 8.5 minutes.
14. (-3) + (-10) = (-13) The temperature on a certain planet was -3 degrees. It dropped 10 more degrees. It is now -13 degrees.

15. Correct Answer: 10 + (-17) or acceptable equivalent. (1 pt) (Obj 5/7.NS1c/1)

16. Correct Answer: YNYNNN (1 pt each) (Obj 5/7.NS1c/1)

17. Scoring Guide: (Obj 6/7.NS1c/2)
    ____Correct Number line model (1 pt)
    ____Correctly represented absolute value of their difference, |-7 - 3| or | -7 + -3| (1 pt)
    ____Correct answer of 10 (1 pt)

18. a. Scoring Guide: (Obj 7/7.NS1c/2)
    ____Correct answer of 1,172 meters (1 pt)
    ____Correct depiction on number line (1 pt)

b. Scoring Guide: (Obj 7/7.NS1c/2)
____Correct answer of 1,172 meters (1 pt)
____Correct absolute value of their difference (1 pt)

19. a. Scoring Guide: (Obj 7/7.NS1c/2)
____Correct answer of 30 degrees (1 pt)
____Correct depiction on number line (1 pt)

b. Scoring Guide: (Obj 7/7.NS1c/2)
____Correct answer of 30 degrees (1 pt)
____Correct absolute value of their difference (1 pt)

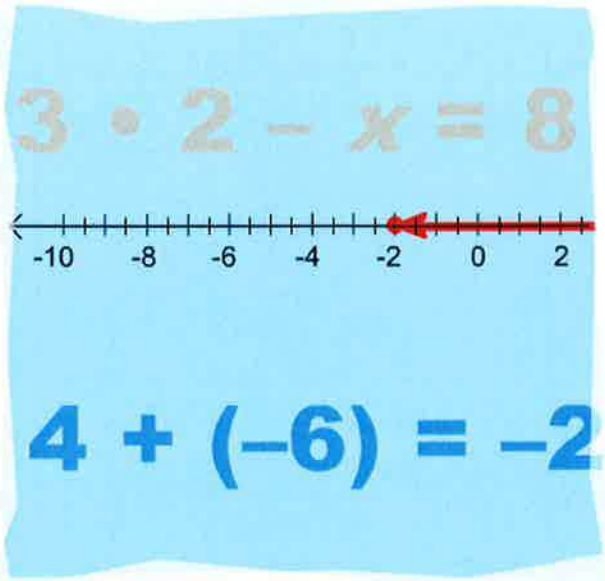
20. a. Scoring Guide: (Obj 8/7.NS1d/2)
____Correct answer of "Associative Property of Addition" (1 pt)
____Correct explanation of how the associative property would bring the fractions with common denominators (or the fractions that add to one) together so they can be added first (1 pt)
____Correct solution of 37 ½ (1 pt)

21. a. Scoring Guide: (Obj 8/7.NS1d/2)
____Correct answer of "Commutative Property off Addition" (1 pt)
____Correct explanation of how the commutative property would bring the negatives together so they can be added first (1 pt)
____Correct solution of -0.1 (1 pt)

Student Name	**Teacher Name**
Progress Report Starting Grade:_____ Progress 1: Date_____: Grade:_____ Progress 2: Date_____: Grade:_____ Progress 3: Date_____: Grade:_____	**Parent Contact Log – Contact if student is not complying with Improvement Plan** Phone #_____ Date, Reason, Outcome 1 _____ 2._____ 3. _____ 4.._____ 5.._____
Requirements of Your Improvement Plan: □ Attend afterschool tutoring on: Tues Weds □ Make-up Missing: □ Test □ Quiz □ Classwork □ Homework □ Other:_____	**Agreement to follow this Improvement Plan** Student Signature_____ Teacher Signature_____ Additional Comments (Student or Teacher):

Student Name	**Teacher Name**
Progress Report Starting Grade:_____ Progress 1: Date_____: Grade:_____ Progress 2: Date_____: Grade:_____ Progress 3: Date_____: Grade:_____	**Parent Contact Log** Phone #_____ Date, Reason, Outcome 1 _____ 2._____ 3. _____ 4.._____ 5.._____
Requirements of Your Acceleration Plan:	**Agreement to follow this Acceleration Plan** Student Signature_____ Teacher Signature_____ Additional Comments (Student or Teacher):

Unit 2: Addition and Subtraction



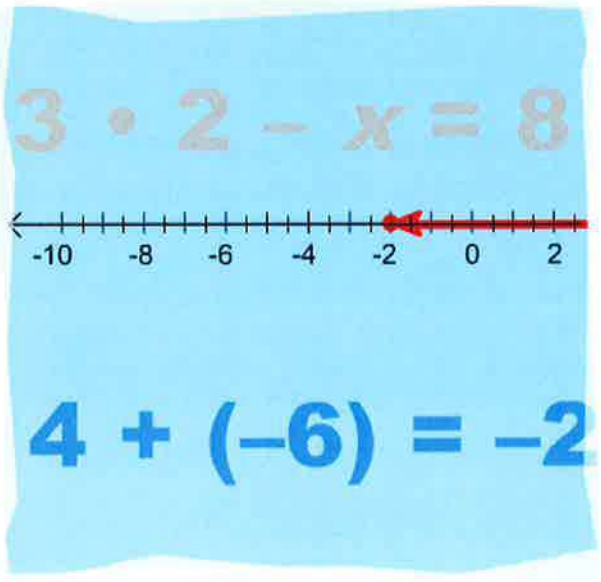
If you have two oranges and a friend gives you three oranges, how many do you have? If you then give four oranges to your friend, how many are you left with? This sort of addition and subtraction problem with passing fruit back and forth is the type of simple math you have done since you were very young. When you expand your addition and subtraction skills to negative numbers and decimals, you can solve many more complicated problems.

Unit Resources | Advance Preparation | Safety

Unit 2

Lesson 1: Integers on a Number Line, Part 1

Positive and negative integers can be used to measure temperatures, elevations, and much more. In this lesson you will learn to identify and graph positive and negative integers on a number line. You will also learn to compare and order positive and negative integers.



Lesson Resources | Advance Preparation | Safety

Unit 2

Lesson 1: Integers on a Number Line, Part 1

GOALS FOR THIS LESSON:

- Find the opposite of an integer.
- Determine the integer coordinate of a point on a number line.
- Graph positive and negative integers on a number line.
- Compare positive and negative integers.

Materials

Student Guide

Lesson Resources | Advance Preparation | Safety

GROUNDWORK: Preparing for the Lesson

Many real-world situations can be represented by integers. Review some of these situations and practice using integers.

Which positive or negative integer could be used to represent each situation?

Losing 3 pounds

Answer

1 of 6 Next

Activity Resources

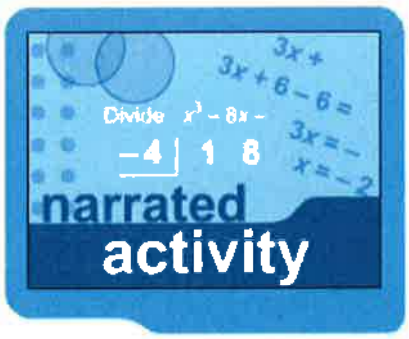
LEARN: Integers on a Number Line

A number line will help you visualize positive and negative integers.

On a number line, the integers to the left of zero are negative, and the integers to the right of zero are positive.

This activity has audio. Please use headphones or speakers.

Click Number Lines to begin.



Number Lines

Big Idea

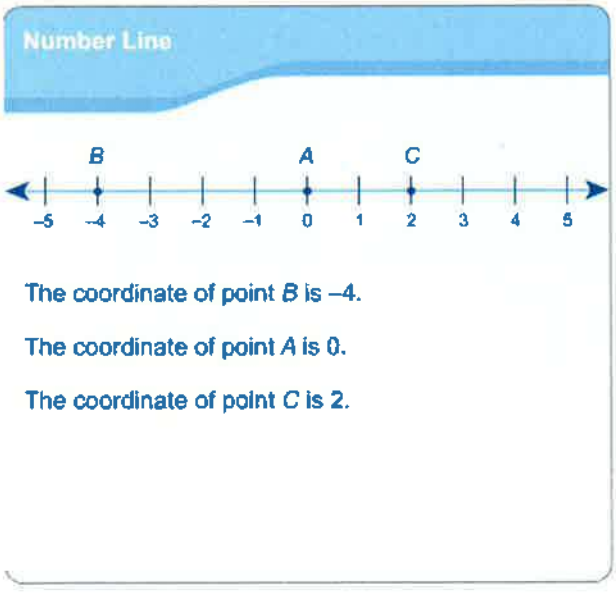
Activity Resources

Unit 2 | Lesson 1

LEARN: Integers on a Number Line

Every point on the number line has a number associated with it called a coordinate. Coordinates tell us exactly where a point can be found on a number line.

Look at the number line and see the coordinates of points A, B, and C. Which point is at the origin?



Remember

Activity Resources

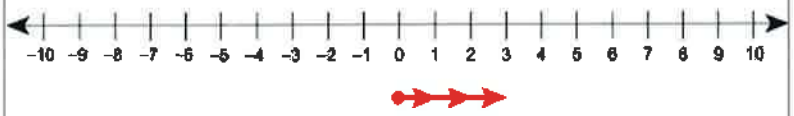
LEARN: Integers on a Number Line

If you want to graph an integer on a number line, start at the origin.

- If the integer is positive, move the correct number of units to the right of the origin.
- If the integer is negative, move the correct number of units to the left of the origin.

Graphing Integers on a Number Line

To graph the integer 3, start at 0 and move 3 units to the right.



1 of 2 Next

Activity Resources

LEARN: Comparing Integers

You can use math symbols to compare integers.

- The integer a is less than *b* if *a* is to the left of b on a number line. Using symbols, this would be written as $a < b$ or $b > a$.
- The integer b is greater than *a* if *b* is to the right of a on a number line. Using symbols, this would be written as $b > a$ or $a < b$.

Compare Integers on a Number Line

If *a* is to the left of *b*, then

$a < b$

or

$b > a$

Remember

Activity Resources

LEARN: Comparing Integers

Now it's your turn to compare integers using the integers on the number line.

Using the points on the graph below, one student wrote the following inequality:

$3 > 1$

Now write your own inequality statements using the points on the graph.

statements: ? ? ?

? ? ?

Remember

↑ Activity Resources

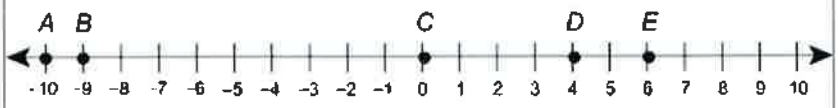
WORKED EXAMPLES: Integers on a Number Line, Part 1

Here are some examples.

Answer each question on your own first. Then check your answer.

Give the coordinate of the point on the number line.

Point *A*.



Answer

1 of 7 Next

↑ Activity Resources

Unit 2 | Lesson 1

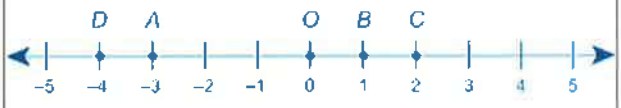
SUMMARY: Integers on a Number Line, Part 1

The set of integers is the set of all positive and negative whole numbers including zero: {... –3, –2, –1, 0, 1, 2, 3 ...}.

On a number line, zero is the origin. Positive numbers are to the right of the origin and negative numbers are to the left of the origin.

Opposites are two numbers that are the same distance from zero on a number line. Zero is neither positive nor negative, so it does not have an opposite.

On a number line, numbers increase in value as you move from left to right and decrease in value as you move from right to left.



A is a negative number (–3).

O is the origin (0).

B is a positive number (1).

C is the opposite of –2.

D is the opposite of 4.

↑ Activity Resources

Unit 2 | Lesson 1

OFFLINE LEARNING: Integers on a Number Line, Part 1

Now go offline to learn more about absolute values and to practice your skills:

- Review pages 39–40 in the Reference Guide.
- Complete Problems 1–27 odd and 30–32 on pages 43–44 of the Reference Guide.

Materials

Integers on a Number Line Solutions

Pre-Algebra: A Reference Guide and Problem Sets - pages 39-44

Optional

calculator

↑ Activity Resources

Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title: Using Variables (Properties of Operations), 1-2 weeks

Designed by: Laura Mayer
District:Moyer Academy

Content Area:Math
Grade Level(s):7th

Summary of Unit
Students will extend their knowledge of operations to generate equivalent expressions. Expressions in context will show how the quantities are related and will shed light on the problem and the importance of the use of properties. Students will continue to use variables to represent real-world situations, adding to that the use of the properties of operations to generate equivalent expressions for these situations.

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

- Include those addressed in Stage 3 and assessed in Stage 2.

Mathematical Practice 1. Make sense of problems and persevere in solving them.

Mathematical Practice 4. Model with mathematics.

Mathematical Practice 5. Use appropriate tools strategically.

Mathematical Practice 8. Look for and express regularity in repeated reasoning.

7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."

Big Idea(s)

- Transferable core concepts, principles, theories, and processes from the Content Standards

Equations and expressions can be rearranged, rewritten, expanded, or simplified in order to provide different perspectives on a real-life problem.

Unit Enduring Understanding(s)

- Full-sentence, important statements or generalizations that specify what students should understand from the Big Ideas (s) and/or Content Standards and that are transferable to new situations

Flexible methods of computation involve strategically grouping, and expanding, numbers and expressions.

Rules of arithmetic and algebraic properties can be used together with notions of equivalence to transform equations so solutions can be found.

The facts and choice of words in a word problem can offer clues as to which operations are used to solve it.

Unit Essential Questions(s)

- Open-ended questions designed to guide student inquiry and learning

- What are the properties and why do we need properties?

- How do properties make calculations easier?

- Why do we need different forms of equations and how does knowing the properties help explain these different forms?

- How do expressions in different forms help explain different aspects of the situation?

Knowledge and Skills

- Needed to meet Content Standards addressed in Stage 3 and assessed in Stage 2

Students will know...

- The distributive, commutative, identity, associative, and zero properties.

- How to identify like terms.

- That variable expressions can be simplified using these properties.

- That linear expressions can be broken down into factors.

- That algebraic expressions, equations, and inequalities can be represented in different ways depending on the situation.

- That expressions can be written to model a given situation.

Students will be able to...

- Students will use concrete objects and representations to illustrate the distributive property.
- Students will apply the understanding of the distributive, commutative, and associative properties to make calculations easier.
- Students will simplify variable expressions using properties.
- Students will factor and expand linear expressions including its variables and rational coefficients.
- Students will translate among different representations of algebraic expressions, equations, and inequalities.
- Students will write and simplify an expression in terms of a contextual situation.

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

Suggested Performance/Transfer Task(s)

- Performance/Transfer tasks as evidence of student proficiency

An effective assessment for ALL students should be designed to include:

*Complex, real-world, authentic applications

*Assessment(s) for student understanding of the Stage 1 elements (Enduring Understandings, Essential Questions, Big Ideas) found in the Content Standards

*Demonstration of high-level thinking with one or more facets of understanding (e.g., explain, interpret, apply, empathize, have perspective, self-knowledge)

Performance Task:

Ilustrative Mathematics - 7.EE Writing Expressions (Appendix A)

Write an expression for the sequence of operations.

a. Add 3 to , subtract the result from 1, then double what you have.

b. Add 3 to , double what you have, then subtract 1 from the result.

Rubric(s)

- Scoring guide to evaluate performance/transfer tasks used as evidence of student proficiency

An effective scoring guide should:

*Measure what is appropriate for the Content Standard that is assessed.

*Provide opportunities for differentiation of the performance/transfer tasks used as evidence of student proficiency.

Scoring guide for performance task (Appendix A)

Other Evidence

- Varied evidence that checks for understanding (e.g., tests, quizzes, prompts, student work samples, observations and supplements the evidence provided by the task)

-Summative Aessment (Appendix B)

-Students will be provided with opportunities to complete varied performance tasks throughout the unit as means of formative assessment to prepare them for the summative performance task.

-Class Discussions

Student Self-Assessment and Reflection

- Opportunities for self-monitoring learning (e.g., reflection journals, learning logs, pre- and post-tests, self-editing – based on ongoing formative assessments)

-Formative Observations:

Throughout the unit, students will be observed on their application of mathmatical practices highlighted in the unit:

Mathematical Practices 1. Make sense of problems and persevere in solving them.

Mathematical Practice 4. Model with mathematics.

Mathematical Practice 5. Use appropriate tools strategically.

The observations will be shared with the students and parents throughout the unit. These conferences will be centered around the student's progressing score on the

Mathematical Practices Rubric (Appendix C). Students will be prompted to reflect on their progress towards using these practices throughout the unit.

-After completing each Formative assessment performance task, students will be given the opportunity to reflect on their work. Students will then do a Think-Pair-Share with a peer and explain their work, defending both their problem solving method and the answers they reached.

**All performance based task work will be kept inside a reflective log that students will add to throughout the year. The teacher will assist students in creating the log and upkeep throughout the year, but it will be the student's responsibility to maintain. At the completion of each task, the students will reflect in writing on the relationship between the task and the learning objective of the lesson.

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

- Instructional activities and learning experiences needed to align with Stage 1 and Stage 2 expectations

Include these instructional elements when designing an effective and engaging learning plan for ALL students:

*Align with expectations of Stage 1 and Stage 2

*Scaffold in order to acquire information, construct meaning, and practice transfer of understanding

*Include a wide range of research-based, effective, and engaging strategies

*Differentiate and personalize content, process, and product for diverse learners

*Provide ongoing opportunities for self-monitoring and self-evaluation

The direct instruction resources needed to implement this course can be found within the k12 Pre-Algebra course, Semester A. Each day of instruction is accompanied by a powerpoint presentation. Use these presentations to guide your information delivery during Direct Instruction, for additional resources when working with small groups, or as a building block for student guided notes. Use these presentations to guide your information delivery during Direct Instruction. Guided Practice Examples and additional Independent Practice activities are also embedded within the presentation materials (See example presentation in Appendix V).

Lesson 1: Review of Integer operations (1 day)

Students should work in pairs to complete the Adding integers (Appendix D) and Subtracting Integers (Appendix E) review using algebra tiles as needed.

Lesson 2: Student investigation of Properties (1-2 days)

Put students into groups of 3-4 and distribute the Properties Investigation worksheet (Appendix F). Students should work together to complete the worksheet, using manipulatives as needed.

Once students have completed the investigation to question #5, stop and have them summarize their work into Key Point #1. Point out to students that they have already uncovered Key Point #1 during their investigation.

Allow students to continue working through the second page, again stopping them to summarize their investigation into Key Point #2.

Hand out Summary: Properties of Operations (Appendix G). Fill in the first Number Example for students. Allow them to guide you in filling out the rest of the examples (be sure to correct any misunderstandings at this stage, do not let students write down inaccurate examples on their summary sheet).

Formative Assessment 1 (Appendix H)

Lesson 3: The Distributive Property (1-2 days)

Direct Instruction with Guided Notes (Appendix I)

Independent Practice (Appendix J)

At this point, students have had 2-3 days immersion in the properties of operations. Open the class up to a discussion on these properties using the Class Discussions on Properties Guide (Appendix K).

Sorting Game: Pair students up and give them a deck of precut Properties of Operations cards (Appendix L). Students can work together or race to see who can finish first to match the card with the example. Run through the cards as many times as the students need to reinforce quick recall of the properties.

Lesson 4: Expressions with Geometric Shapes (1-2 days)

Have students work independently to find the perimeter and area of a few example rectangles on the board.

Think-Pair-Share: Pair students up and have them collaborate on their problem solving method for finding perimeter and area. Prompt them with questions about their method, what equation they may have used, and how quickly they were able to find the answers.

Distribute Instructional Activity: Expressions with Geometric Shapes (Appendix M).

Have students work with their partner to complete the exercise. Be sure to reinforce visually that $L+W+L+W = L+L+W+W = 2L+2W = 2(L+W)$ and have the students agree on which property of operations is being shown with each form of the expression.

Formative Assessment: Properties of Operations (Appendix N)

Use the data from this large formative assessment to guide instruction over the next few days. The teacher may need to go back and reteach, group and differentiate, or may be able to continue with instruction.

Instructional Activity: Property Sorting Cards (Appendix O)

These Flash Cards practice identifying the commutative, associative, and distributive properties. (Put the answers on the back of the flashcards so that the students can get instant feedback.) The second page contains examples of false statements, longer property statements, and expanded statements that can be added to the deck to make this activity harder for the more advanced students.

Lesson 5: Percentage Expressions in Context (2 days)

Give students the following problem: Find the total cost of a game that costs $58.00 with a tax rate of 6.5%.

Allow students to solve the problem on their own.

Class Discussion: Open the class up to a discussion: What method did they try? Did it work? Was it the most efficient?

Show the students the two methods outlined on the Percentage in Expressions in Context Worksheet (Appendix P).

Walk through the second example, paying particularly close attention to the last two, inaccurate, calculations. Have the students record in their journal why these last two methods did not accurately solve the problem. Partner the students up for a Think-Pair-Share on these inaccurate methods.

Allow students to practice independently with the rest of the work.

Group work: Matching Situations to Equations (Appendix Q)

Formative Assessment: Expressions in Context (Appendix R)

Lesson 7: Preparing for the Summative Assessment (1-2 days)

Gallery Walk: Students answers to the Discussion Questions (Appendix S) in groups on large paper. Then the student walk around the classroom rating the answer to these questions written by other groups. 1 information missing, 2 good answer, 3 I learned something by reading this answer.

Independent Practice Performance Task: Miles to Kilometers (Appendix T)

Summative Assessment and Performance Task (Appendices A and B)

Resources and Teaching Tips

- A variety of resources are included (texts, print, media, web links)
- Help in identifying and correcting student misunderstandings and weaknesses

Throughout the unit, students have access to a variety of resources.

Textbook

Smartboard and Projector

Graphic Organizer Paper

Reference Sheets

Additionally, each formative check for understanding provides an opportunity for the students to identify their own misunderstandings and weaknesses in mastering the content. Guide the students in understanding how to use these formative checks to enhance their learning through the use of rubrics, conferences, and remediation or acceleration plans designed with student input (Appendix U). While working towards mastery of grade level content standards, continuously refer students back to the standards for mathematical practices. These practice standards are the attitudes and skills that will allow students to achieve mastery of content standards, as well as deep understanding of mathematics.

Differentiation

- Stage 2 and 3 allow students to demonstrate understanding with choices, options, and/or variety in the products and performances without compromising the expectations of the Content Standards.
- Instruction is varied to address differences in readiness, interest, and/or learning profiles.
- Accommodations and differentiation strategies are incorporated in the design of Stage 2 and 3.

☐ Refer to the IEPs of your special education students

☐ Allow students to have tests, quizzes, worksheets, etc read to them.

☐ Allow the use of a calculator

☐ Communicate with the special education teacher about allowing testing in the special education room

☐ Pair and group students with differing ability levels.

☐ Use manipulatives as needed

☐ Allow for extra time on tests, quizzes, and performance tasks

☐ Allow for a word bank and a formula sheet on tests

☐ Allow students to choose to demonstrate understand in written or verbal format as needed.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Educatlon** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.

- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning –** the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

(Briefly explain how design principle(s) are embedded within the unit design.)

In alignment with the Common Core State Standards, this unit pushes students to develop 21st Century Skills to meet the learning demands. Students come together as a learning community to analyze real-world applications of the math they are learning, think critically and reflect on the work of their peers as well as their own, and constantly work to apply their knowledge to new, more complex situations. This push towards using developed skills in new ways is an accurate reflection of the higher order thinking required by 21st Century jobs. Students should become excited and engaged by the "hands-off" approach of the teacher. Rather than giving students the formulas and a calculator and having them compute numbers and spit out answers, the teacher should present numbers and problems in a real world context, allowing the students to select the tools, resources, and knowledge needed to arrive at a reasonable answer. This push towards applying mathematical ways of thinking to real world issues will prepare them for the type of work they will encounter in high school, college, and in their careers.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- **8th Grade Technology Literacy** - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century(SETDA, 2003).

In alignment with Mathematical Practice 5 - Use appropriate tools strategically, students will have access to calculators as needed to complete the Performance Tasks. Additionally, when sharing their answers to the Performance Tasks, the teacher may opt to have the students construct a powerpoint presentation as an additional layer of the project.

Content Connections

Content Standards integrated within instructional strategies

Connections to other content areas are made both explicitly and implicitly throughout this unit. Additionally, the performance tasks included connect the math content to science, finance, and consumer science. Additional connections are encouraged at the teacher's discretion. For example, when assigning the students additional

practice work, make the work meaningful by aligning the examples with content currently being covered in other classes such as Science, English, and Social Studies.

A

Illustrative Mathematics

7.EE Writing Expressions

Alignment 1: 7.EE.A.1

Write an expression for the sequence of operations.

a. Add 3 to x, subtract the result from 1, then double what you have.

b. Add 3 to x, double what you have, then subtract 1 from the result.

A

Commentary:
The instructions for the two expressions sound very similar, however, the order in which the different operations are performed and the exact wording make a big difference in the final expression. Students have to pay close attention to the wording: "subtract the result from 1" and "subtract 1 from the result" are very different.

Solution: Solution

a. This problem can be done step-by-step. We first add 3 to x:

$$x + 3.$$

Then we subtract the result that we just got from 1:

$$1 - (x + 3).$$

We then double, meaning we multiply this entire expression by 2:

$$2(1 - (x + 3)).$$

If we choose to simplify this expression, we use the distributive, commutative and associative properties in the following way:

$$\begin{aligned} 2(1 - (x + 3)) &= 2(1 - x - 3) && \text{distribute the -} \\ &= 2(-x - 2) && \text{combining like terms} \\ &= -2x - 4 && \text{distribute the 2} \end{aligned}$$

b. Again, we add 3 to x:

$$x + 3$$

This time, next we double, meaning multiplying this expression by 2:

$$2(x + 3).$$

Then we subtract 1 from the result and we have:

$$2(x + 3) - 1.$$

If we choose to simplify this expression, we use the distributive and associative properties in the following way:

$$\begin{aligned} 2(x + 3) - 1 &= (2x + 6) - 1 && \text{distribute the 2} \\ &= 2x + 5 && \text{combining like terms} \end{aligned}$$

Notice that the final expressions are very different, even though the instructions sounded very similar.

B

Summative Assessment Properties of Operations Name _____

1. Which of these expressions is equivalent to $3(x+4)$? (1pt)
   a. $3x + 4$ b. $3x + 12$ c. $4(x + 3)$ d. $4x + 12$

2. Which of these expressions is equivalent to $-14(3 - 2x) - 6x$? (1pt)
   a. $42 + 22x$ b. $-42 - 34x$ c. $-42 + 22x$ d. $-42 + 34x$

3. Identify the property used in each step. (7 pts)

$5(2x + 8) + 3(3x - 2) = 10x + 40 + 9x - 6$ _____

$10x + 9x + 40 - 6$ _____

$(10x + 9x) + 40 - 6$ _____

$(10 + 9)x + 40 - 6$ _____

$19x + 40 - 6$ _____

$19x + (40 - 6)$ _____

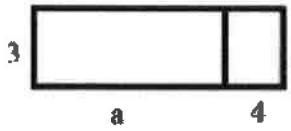
$19x + 34$ _____

4. Larry thinks the two expressions $4a^3 + 10$ and $4 + a + a + a + 7 + 3$ are equivalent? Is he correct? Explain why or why not? Substitute in any number for the variable to show their equality or nonequivalence. (1pt answer, 2 pts substitution)

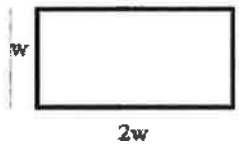
5. Suzanne thinks the two expressions $2(3a-2)+4a$ and $10a-4$ are equivalent? Is she correct? Explain why or why not? Substitute in any number for the variable to show their equality or nonequivalence. Use properties of numbers to explain their equality or nonequivalence. (1pt answer, 2 pts substitution, 2 pts properties)

6. Write an equivalent expression for $3a+12$ by finding the area of the whole shape. (2 pts)

B



7. A rectangle is twice as long as wide. Write an expression for finding the perimeter in two ways. (2 pts one for each expression)



8. The sales tax on the computer is 6.75%. If the computer cost $1,024, what is the final cost of the computer? Sam wrote the following expressions. Explain each expression (3 pts, one for each expression)

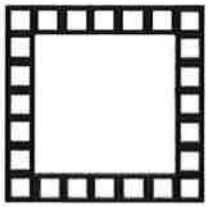
$$1{,}024 + 1{,}024 \times 0.0675 \quad = \quad 1{,}024(1 + 0.0675) \quad = 1{,}024(1.0675)$$

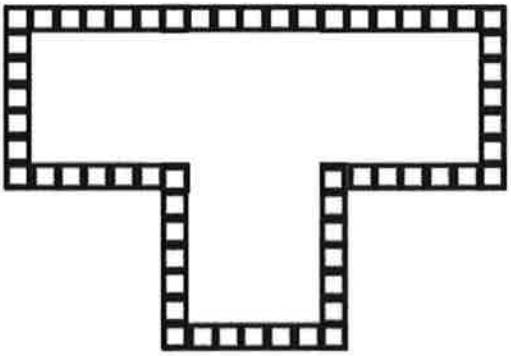
9. A jacket (J) is on sale for 30% off. What is the sale price of the jacket if the original price is $72.50? Solve this problem two different ways. (1pt answer, 2 pts methods)

10. Jamie and Ted both get paid an equal hourly wage of $9 per hour. This week, Ted made an additional $27 dollars in overtime. Write an expression that represents their combined weekly wage if J = the number of hours that Jamie worked this week and T = the number of hours Ted worked this week? Can you write the expression in another way? (2 pts one for each expression)

11. Given a T shaped pool made up of 4 square pools as shown in the picture, write four different expressions to find the total number of tiles in the border. Explain how each of the expressions relates to the diagram and demonstrate that the expressions are equivalent. Which expression do you think is most useful? Explain your thinking. (4pts expressions, 4 pts relationship to diagrams, 4 pts for equivalent expression, 2 pts for useful and why)

B





Answers

1. B (Obj 5 / 7.EE1 / 2)
2. B (Obj 2 / 7.EE1 / 2)
3. Distributive (Obj 1 / 7.EE1 / 2)
   Commutative
   Associative
   Factor
   Arithmetic Fact
   Associative
   Arithmetic Fact

4. No they are not equivalent. (Obj 6 / 7.EE1 / 2)

   $4a^3 + 10$ and $4 + a + a + a + 7 + 3$ substituting 3 = a
   $4(3)^3 + 10$ and $4 + 3 + 3 + 3 + 7 + 3$
   $108 + 10$ and 23
   118 is not equal to 23

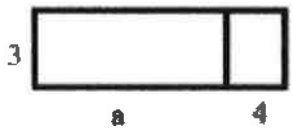
   Exponents are repeated multiplication not repeated addition.

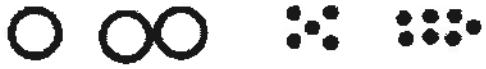
5. Yes they are equivalent (Obj 7 / 7.EE1 / 2)

   $2(3a-2)+4a$ and $10a-4$ substituting 5 = a
   $2(15-2)+20$ and $50-4$
   $26 + 20$ and 46
   46 is equal to 46

   Use the distributive property to multiply 2 times the 3a and -2 to get 6a – 4 then add 4a to the 6a by collecting like terms to get 10a – 4.

6. Possible solutions might include factoring as in $3(a+4)$, or other expressions such as $a+2a+7+5$.
   (Obj 3, 7.EE1, 2)





7. Solution: $w+w+2w+2w$ OR 6w OR $2(w)+2(2w)$.(Obj 8 / 7.EE1 / 2)

8. 1,024 + 1,024 x 0.0675 = 1,024(1 + 0.0675) = 1,024(1.0675) (Obj 4 / 7EE1 / 2)
   Price + Tax Factored out the Price Added the 100% plus the 6.75% and calculating 106.75%

9. Solution: $50.75 (Obj 8 / 7.EE1 / 2)

   Method 1 - $72.50 x .70 = $50.75

   Method 2 - $72.50 x .30 = 21.75 72.50 – 21.75 = $50.75

10. (Obj 8 / 7.EE1 / 2)

One student might say: To find the total wage, I would first multiply the number of hours Jamie worked by 9. Then I would multiply the number of hours Ted worked by 9. I would add these two values with the $27 overtime to find the total wages for the week. The student would write the expression $9J + 9T + 27$.

Another student might say: To find the total wages, I would add the number of hours that Ted and Jamie worked. I would multiply the total number of hours worked by 9. I would then add the overtime to that value to get the total wages for the week. The student would write the expression $9(J + T) + 27$

A third student might say: To find the total wages, I would need to figure out how much Jamie made and add that to how much Ted made for the week. To figure out Jamie's wages, I would multiply the number of hours she worked by 9. To figure out Ted's wages, I would multiply the number of hours he worked by 9 and then add the $27 he earned in overtime. My final step would be to add Jamie and Ted wages for the week to find their combined total wages. The student would write the expression $(9J) + (9T + 27)$

11. 60 tiles (Obj 8 / 7.EE1 / 3)

Add up all of the side lengths and all of the corners separately
5 + 5 + 5 + 5 +5 + 5 + 5 + 5 + 5 + 5 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1
or x + x + x + x + x+ x + x + x + x + x + x + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1
Add up the side lengths then add the total number of corners
5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5+ 10 or x + x + x + x + x + x + x + x + x + x + x +10
Add up the side length with one corner and multiply by the ten.
10(5 + 1) or 10(x + 1)
There are ten sides with the side length connected with two corner, but then all of the corners were counted twice so - 10.
10(5 + 2) -1 0 or 10(x + 2) -10
There are ten sides with the same side length and ten corners
10 (5) + 10 or 10x + 10

x + x + x + x + x+ x + x + x + x + x + x + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1

10x + 10 Collect like terms with the corners

(x+1) + (x + 1) + (x + 1) + (x + 1) + (x+1) + (x + 1) + (x + 1) + (x + 1) + (x+1) + (x + 1) Commutative and Associative
4(x + 1) Collect like terms

Answers may vary. The form 10(x) + 10 is the most useful form because it is efficient and shows the sides and the ten corners separately.

Student Name_____ Evaluation Date_____ Overall Score_____

Mathematical Practice	Novice (1)	Apprentice (2)	Practitioner (3)	Expert (4)
1. Make sense of problems & persevere in solving them	I couldn't get started. I don't know how to begin. I may be feeling frustrated and wanting to give up.	I am stuck. I have part of the solution, but now I don't know what to do, I'm not sure my answer is right and I could use some help. I may be feeling frustrated.	I planned out how to solve this problem and I worked all parts of the problem. I may have felt challenged, but I didn't want to give up-I kept going until the problem was solved.	My solution is effective and inventive! I planned out how to solve this problem and I even showed you other ways to solve it. I checked to make sure my answer was right – I enjoyed this challenge!
2. Reason abstractly & quantitatively	I can't explain how I solved this problem. I don't know what quantities or equations to use. My work on this problem is built on misunderstandings of mathematical ideas.	I'm trying to explain how I solved this problem. My work on this problem may be built on misunderstandings of mathematical ideas. . I'm trying to take numbers out of the problem to work with them. I'm trying to check my work.	I explained how to solve this problem using formal math language. I took numbers out of the problem to work with them, and periodically put them back into the context of the problem to see if my work makes sense.	I clearly explained how to solve this problem using formal math language. I took quantities out of the context of the problem to work with them, and frequently put them back into the context of the problem to see if my work made sense.
3. Construct viable arguments & critique the reasoning of others	I can't describe how I solved the problem. I can't give feedback on someone else's reasoning OR my feedback in unclear, unhelpful, and/or incorrect.	I'm trying to describe how I solved the problem. I may or may not include pictures, tables, graphs, and/or models. I gave feedback on someone else's reasoning, but my feedback may have been unclear, unhelpful, or I may have been unable to back it up with my own work.	I described how I solved the problem using formal math language, pictures, tables, graphs, and/or models. I gave clear, helpful feedback to someone else's reasoning and backed up my feedback with my own work on the problem.	I clearly described how I solved the problem. I included every step so you don't have to guess what I did. I used formal math language, pictures, tables, graphs, and/or models. I gave clear, helpful feedback on someone else's reasoning and backed up my feedback with my own work on the problem.
4. Model with mathematics	I can't use mathematical representations, I don't know what to use for this problem and I can't describe how I would solve this problem.	I'm trying to use mathematical representations to solve the problem and show my solutions, but they may be inaccurate or inappropriate. I'm sharing my ideas, but I'm struggling to use formal math language.	I used appropriate & accurate mathematical representations to solve the problem and show my solutions. I used formal math language to share my ideas.	I used a variety appropriate, accurate, and detailed mathematical representations to solve the problem and show my solutions. I used formal math language to share my ideas.
5. Use appropriate	I don't know what tool to use to solve this problem and/or I	I picked an appropriate tool to use, but I may be struggling to	I picked the correct tool to use based on the needs of the	I selected and used a variety of appropriate tools

Student Name_____ Evaluation Date_____ Overall Score_____

tools strategically	didn't use the tool I picked in a valuable way.	use it in a valuable way to solve the problem.	problem. I planned and monitored my use of the tool.	collaboratively to solve the problem. I planned my use of the tools and monitored my progress.
6. Attend to precision	I can't get the correct answer and/or I can't communicate my answer in a way that is appropriate to the problem. I can't use correct units, labels, or definitions.	I may be getting the correct answer and/or I may be able to communicate my answer is a way that is appropriate to the problem. I may not use correct units, labels, or definitions.	I got the correct answer and I can communicate it using the correct labels, units, and definitions. The precision of my answer is appropriate for the problem.	I got the correct answer and I communicated it using correct labels, units, and definitions. I clearly stated the meaning of the symbols I chose and my answer is complete appropriate for the problem.
7. Look for & make use of structure	I don't see any structure in the problem. I can't make sense of any useful structures/patterns to help me solve the problem.	I may be able to find for structures/patterns in the problem, but I am struggling to use them to help me find a solution.	I found and used structures/patterns in the problem to help me find a solution. I noted these structures/patterns as I found them.	I found, noted, and used structures/patterns in the problem to help me find a solution. I was able to step back from the problem and analyze different perspectives.
8. Look for & express regularity in repeated reasoning	I don't see any regularity or any shortcuts or generalizations I can use to get a solution.	I may notice that some of my calculations are repeated but I may not be able to generalize that observation to help me get a solution.	I noticed some of my calculations were repeating and was able to make a generalization that helped me solve the problem. I evaluated the reasonableness of my work as I progressed.	I noticed some of my calculations were repeating and was able to make a generalization or a formula that helped me solve the problem. I continuously evaluated the reasonableness of my work as I progressed.

D

Adding Integers

Additive Inverse - _____

The Zero Property of Addition - _____

_____

- **We will be using algebra tiles to model the addition of integers.**
  - Yellow tiles represent positive integers (unshaded squares in our diagrams).
  - Red tiles represent negative integers (shaded squares in our diagrams).
  - By the Zero Property of Addition, a yellow tile plus a red tile equals _____.

1. Use algebra tiles to model the addition problem (+5) + (+3). Sketch your model below.

What is the sum of +5 and +3? _____

2. Use algebra tiles to model the addition problem (-2) + (-7). Sketch your model below.

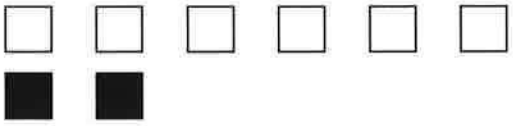
What is the sum of -2 and -7? _____

3. Describe how to find the sum of a pair of integers with like signs. _____

_____

_____

4. Write the addition problem for the model below. _____

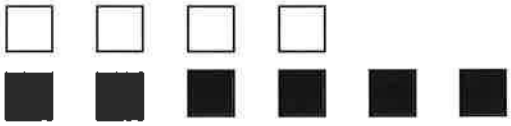


Circle the zero pairs, a positive and a negative tile paired up equals zero.

What is the sum of the above model? _____

D

5. Write the addition problem for the model below. _____



Circle the zero pairs, a positive and a negative chip paired up equals zero.

What is the sum of the above model? _____

6. Use algebra tiles to find the sum of (+3) and (-8). Show your model below.

7. Use algebra tiles to find the sum of (+5) and (-2). Show your model below.

8. Use algebra tiles to find the sum of (-6) and (-3). Show your model below.

9. Use algebra tiles to find the sum of (-6) and (+8). Show your model below.

10. Describe how to find the sum of a pair of integers with different signs. _____

_____

_____

_____

E

Subtracting Integers

1. Use algebra tiles to model the subtraction problem (+5) - (+3). Sketch your model below. Cross out the tiles that you are removing to represent the tiles being subtracted.

What is the difference of +5 and +3? _____

2. Use algebra tiles to model the addition problem (-5) - (-2). Sketch your model below.

What is the difference of -5 and -2? _____

3. Use algebra tiles to model the addition problem (+1) - (+3). Sketch your model below.

What is the difference of +1 and +3? _____

4. Use algebra tiles to model the addition problem (-3) - (-5). Sketch your model below.

What is the difference of -3 and -5? _____

E

5. Use algebra tiles to model the addition problem (+2) - (-3). Sketch your model below.

What is the difference of +2 and -3? _____

6. Use algebra tiles to find the sums and difference in the table below.

(+1) + (-6) =	(-4) + (-1) =	(-1) + (+1) =	(+2) + (+2) =	(-3) + (-7) =
(+1) - (+6) =	(-4) - (+1) =	(-1) - (-1) =	(+2) - (-2) =	(-3) - (+7) =

7. Describe how to rewrite an integer subtraction problem as an integer addition problem.

_____

_____

Rewrite each subtraction problem as an addition problem and evaluate.

8. $-20 - (-18) =$

9. $-20 - (+18) =$

10. $+20 - (-18) =$

11. $+20 - (+18) =$

F

First Task: Do your best to answer the following questions:

a. 78 + 87 =	b. 15 * 4 =	c. 175 – 40 =
d. 6 + (5 *3) =	e. 16 + ⁻4 =	f. 5 – (⁻5)

---

1. With your group, complete the following table. The first row has been done for you as an example

Column Number		*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
a	**b**	*a+b*	*b+a*	*a-b*	*b-a*	*a*b*	*b*a*	*a÷ b*	*b ÷ a*
1	**2**	3	3	-1	1	2	2	0.5	2
2	**2**								
3	**3**								
4	**3**								
4	**4**								
1.5	**3**								
6	**3**								
5	**-5**								
5	**-10**								

2. Look over your table. What observations can you make based on the patterns in the table? What do you notice?

3. What happened in the first two columns? Did the order in which you added make a difference?

4. Based on the table, a classmate proposes a new law of mathematics: "It doesn't matter in what order you add, subtract, multiply or divide numbers. You'll always get the same answer no matter what." Is this law true? If not, give an example of when this law **does not** work.

5. Make a general statement for when the commutative property (switching the order of the numbers) works for a given operation, e.g. addition, subtraction, multiplication, division.

KEY POINT #1: The _____ _____ says that you can switch the order of the numbers for _____ and _____ and still get the same result.

F

Let's discover a new property.

$$4 + (5 + 6) = (4 + 5) + 6$$

6. Simplify the above. Do both sides of the equals sign result in the same number?

7. Pick three other numbers besides 4, 5 and 6. If you replaced 4,5 and 6 with your different numbers would it still be true? Justify your answer with an example.

8. Can you find an example that won't work?

Try using the same numbers, but different operations. Remember – the order of operations states we must do **whatever is within the parentheses first.**

9. **Subtraction:**

$$4 - (5 - 6) \overset{?}{=} (4 - 5) - 6$$

10. **Multiplication:**

$$4 * (5 * 6) \overset{?}{=} (4 * 5) * 6$$

11. **Division:**

$$4 \div (5 \div 6) \overset{?}{=} (4 \div 5) \div 6$$

12. What do you notice? Make a general statement regarding the associative property for the four different operations.

KEY POINT #2: The _____ property says that we can _____ numbers differently when we <u>add</u> and <u>multiply</u> and still get the same result.

G

Summary: Properties of Operations

Property	Variable Definition	Number Example
Commutative Property of Addition	$a+b=b+a$	
Commutative Property of Multiplication	$a \bullet b = b \bullet a$	
Associative Property of Addition	$(a+b)+c=a+(b+c)$	
Associative Property of Multiplication	$(a \bullet b) \bullet c = a \bullet (b \bullet c)$	
Identity Property of Addition	$a + 0 = a$	
Identity Property of Multiplication	$a \bullet 1 = a$	
Inverse Property of Addition	For every a, there is an additive inverse $-a$ such that $a + (-a) = 0$.	
Inverse property of Multiplication	For every a ($a \neq 0$), there is a multiplicative inverse $\frac{1}{a}$ such that $a(\frac{1}{a}) = 1$.	
Symmetric Property	If $a = b$, then $b = a$	
Distributive Property	For every real number a, b, and c: $a(b+c) = ab + ac$, $(b+c)a = ba + ca$, $a(b - c) = ab - ac$, $(b - c)a = ba - ca$	
Multiplication Property of Zero	For every real number n, $n \bullet 0 = 0$	
Multiplication Property of -1	For every real number n, $n \bullet -1 = -n$	

H

Form A

1 Which expression is an example of the associative property?
(1) $(x + y) + z = x + (y + z)$
(2) $x + y + z = z + y + x$
(3) $x(y + z) = xy + xz$
(4) $x \cdot 1 = x$

2 If M and A represent integers, $M + A = A + M$ is an example of which property?
(1) commutative (3) distributive
(2) associative (4) closure

3 Which equation illustrates the commutative property?
(1) $a(1) = a$ (3) $a(b + c) = (ab) = (ac)$
(2) $a + b = b + a$ (4) $(a + b) + c = a + (b + c)$

4 Which property is illustrated by the equation $6 + (4 + x) = 6 + (x + 4)$?
(1) associative property of addition
(2) associative property of multiplication
(3) distributive property
(4) commutative property of addition

Form B

1 Which equation illustrates the commutative property?
(1) $a(1) = a$ (3) $a(b + c) = (ab) = (ac)$
(2) $a + b = b + a$ (4) $(a + b) + c = a + (b + c)$

2 Which property is illustrated by the equation $6 + (4 + x) = 6 + (x + 4)$?
(1) associative property of addition
(2) associative property of multiplication
(3) distributive property
(4) commutative property of addition

3 If M and A represent integers, $M + A = A + M$ is an example of which property?
(1) commutative (3) distributive
(2) associative (4) closure

4 Which expression is an example of the associative property?
(1) $(x + y) + z = x + (y + z)$
(2) $x + y + z = z + y + x$
(3) $x(y + z) = xy + xz$
(4) $x \cdot 1 = x$

I

The Distributive Property

The Distributive Property is a tool you can use to _____ an expression or equation. Let's start with a simple problem:

6(4 + 2)

Using Order of Operations, we know that you start with the parentheses and then do the multiplication.

6(4+2)
6(6)
36

Another way to do this problem is to use what is called the Distributive Property. Formally, it is stated as: $a(b + c) = ab + ac$. When a number a is sitting outside the parentheses, that means you can _____what's in the parentheses by a.

6(4+2)
6 • 4 + 6 • 2
24 + 12
36

Example 1:

6(4x + 2)

Example 2:

5x(3x + 1)

You Try!

J

1: 9(2x + 2)

2: 2(2x + 4)

3: 2(9x + 8)

4: 6(8x + 8)

5: 5(8x + 2)

6: 7(2x + 7)

K

Class Discussions on Properties

How the class discussion works....

Problems are written on large pieces of paper and placed on a table or display them on your smart board. Place enough chairs around the table or smart board so that only half of the class can sit down at one time. The students that are seated are required to take notes on what they see, but are not allowed to participate in the discussions. The students that are standing must participate in the discussions in order to find some place to sit and take notes. After a student explains something that they see, they are allowed to tap someone that is seated and take their place. I find it helpful to use 2 yard sticks to pass around. The stick helps the student point to what they are explaining and the second stick is traveling around finding the student that will speak next.

Suggested Problems

Associative

$(2 + 3) + 7 = 2 + (3 + 7)$

$(-150 + 270) + 30 = -150 + (270 + 30)$

$(7 \times 8) \times 5 = 7 \times (8 \times 5)$

$(36 \times 25) \times 4 = 36 \times (25 \times 4)$

$17 \times 5 \times 5 \times 4 = 17 \times (5 \times 5 \times 4)$

$7w + 5W + 3 + 8 = (7W + 5W) + (3 + 8)$

$3d^2 + 8d^2 + 4d + 8d + 12 - 4 = (3d^2 + 8d^{2)} + (4d + 8d) + (12 - 4)$

Commutative

$74 + 29 + 26 = 74 + 26 + 29$

$5 \times 23 \times 4 = 5 \times 4 \times 23$

$(5)(3w)(4) = (5)(4)(3w)$

$3d + 7 + 8 + 4d = 3d + 4d + 7 + 8$

$3d^2 + 7d + 3 + 8d^2 + 4d + 12 = 3d^2 + 8d^2 + 4d + 7d + 3 + 12$

Possible Probing Questions

How can we recognize this property?
When does it work (ex. Operation)?
Why would we want to use this property?
Which side of the expression is easier to solve? Why?
How would you summarize this property?

K

Distributive

$3(2x + 7) = 6x + 21$

$(3x - 2)\,5 = 15x - 10$

$50 \times 396 = 50 \times 400 - 50 \times 4$

$30 \times 402 = 30 \times 400 + 30 \times 2$

$7 \times 3.99 = 7 \times 4 - 7 \times .01$

$(43 \times 120) + (43 \times -20) = 43\,(120 + -20)$

$(0.8 \times -23) + (0.8 \times -7) = 0.8 \times -30$

Inverse Property

$4 \times \frac{1}{4} = 1$

$2/3 \times 3/2 = 1$

$-1/2 \times -2 = 1$

$3 + -3 = 0$

$-5 + 5 = 0$

Identity Property

$8 + 0 = 8$

$0 + -4 = -4$

$-2 \times 1 = -2$

$3 \times 1 = 3$

$-3 + 3 + 12 = 0 + 12 = 12$

$14 \times 2/3 \times 3/2 = 4 \times 1 = 4$

Zero property

$3 \times 0 = 0$

$6 \times 0 = 0$

$7 \times (-3 + 3) = 0$

Combine like terms

$3x + 7 + 8x + 5 = 11x + 12$

$7x + 8y + 4z + 2x + 3y + 8z = 9x + 11y + 12z$

$4x - 7y + 8x - 3x + -2y = 9x + -9y$

$4y + 5 - 8 + 3y = 7y - 3$

Expand

$32W^3 = 2 \times 2 \times 2 \times 2 \times 2 \times W \times W \times W$

$7 \times 4.99 = 7 \times (5.00 - 0.01)$

$2 + 3 + 17 = 2 + 3 + 7 + 10$

$6 \times 5 = 5 + 5 + 5 + 5 + 5 + 5$

$75d = 5 \times 5 \times 3 \times d$

Factor

$3a + 3b = 3(a + b)$

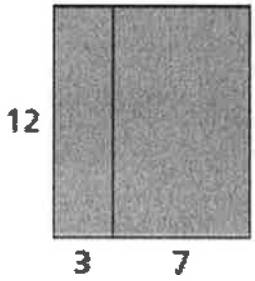
$3x + 6y + 12 = 3(x + 2y + 4)$

$4x + 8y = 4(x + 2y)$

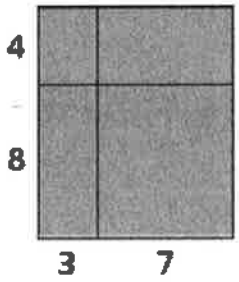
K

The **distributive** property allows you to distribute multiplication over addition and subtraction.

1.



2.



This is the area model for the **distributive** property. To get the factored form think about finding the area of the whole rectangle Width x Length.

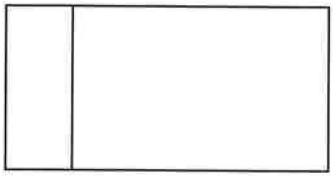
Factored form $12 \times (3 + 7)$ $(4 + 8) \times (3 + 7)$

To get the expanded form think about finding the area of each little rectangle.

Expanded Form $12 \times 3 + 12 \times 7$ $4 \times 3 + 4 \times 7 + 8 \times 3 + 8 \times 7$

K

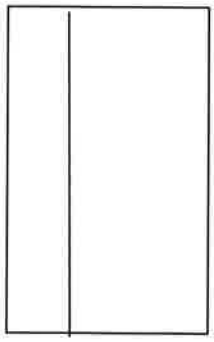
3



2 4

$3(2+4) = 3(2) + 3(4)$

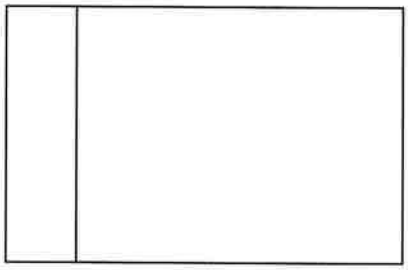
8



2 4

$8(2+4) = 8 \times 2 + 8 \times 4$

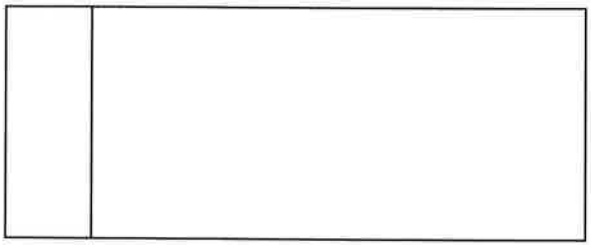
X



3 6

$x(3+6) = 3x + 6x$

4



$4(3+x) = 4(3) + 4x$

L

$6 \times -3 = -3 \times 6$	Commutative Property of Multiplication
$-7 + 4 = 4 + -7$	Commutative Property of Addition
$(3 \times 4) \times 2 = 3 \times (4 \times 2)$	Associative Property of Multiplication
$(-4 + 8) + -9 = -4 + (8 + -9)$	Associative Property of Addition
$0 \times 23 = 0$	Property of Zero

L

$9 + 0 = 9$	Identity Property of Addition
$-34 \times 1 = -34$	Identity Property of Multiplication
$-4(3 + 10) = (-4 \times 3) + (-4 \times 10)$	Distributive Property over Addition
$-4 + 4 = 0$	Opposite Property of Addition
$8(5 - 2x) = 40 - 8x$	Distributive Property over Subtraction

M

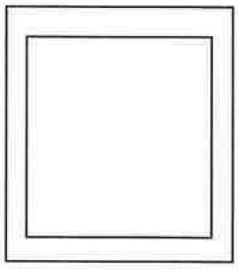
Expressions with Geometric Shapes

Find the Perimeter of a Rectangle

Name the property that allows you to change from one form of the perimeter to the next.

$L + W + L + W = L + L + W + W = 2L + 2W = 2(L + W)$

Write an expression for the number of border tiles N based on the side length S needed to surround a square pool. Explain the different forms of the equation as they relate to the number of tiles around the pool. Name the properties that allow you to change from one form to another. Draw the picture represented by each expression.



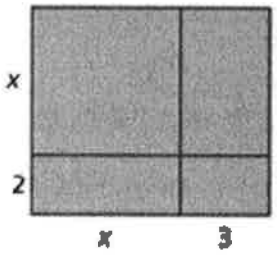
Example

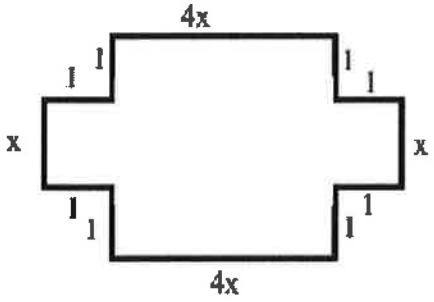
$S + S + S + S + 4 = 4S + 4 = 4(s + 1) = 4(s + 2) - 4$

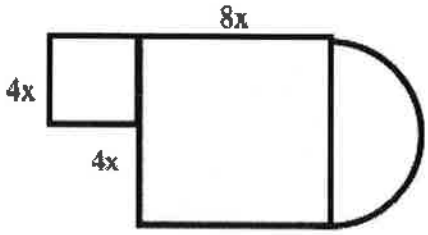
M

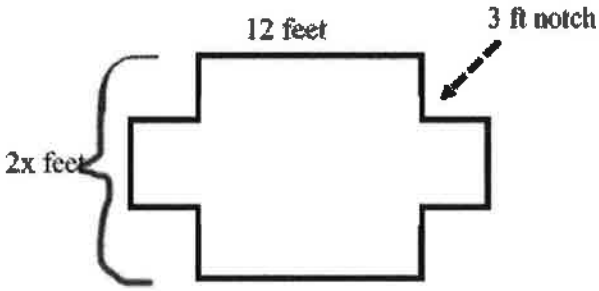
Name _____

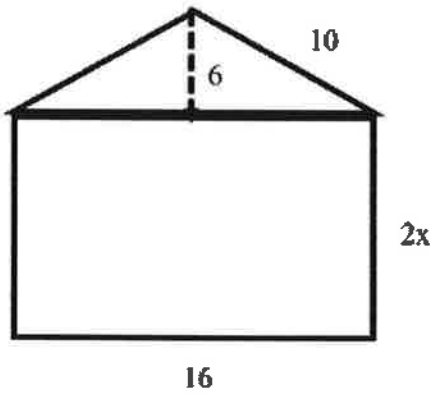
Write a formula for the perimeter and area of the following shapes. Complete have each group put their expression on the board. See if the students can use properties to convert one expression into another.











M

Possible Solutions

1. Perimeter = $2(x + 2) + 2(x + 3)$ or $2(2x + 5)$ or $4x + 10$
   Area = $(x + 2)(x + 3)$ or $x^2 + 2x + 3x + 6$ or $x^2 + 5x + 6$

2. Perimeter = $2(4x) + 2(x + 4)$ or $8x + 2x + 8$ or $10x + 8$
   Area = $4x(x+2) + 2(1x)$ or $4x^2 + 8x + 2x$

3. **Perimeter = $8x + 4x + 4x + 4X + 4X + 8X + \frac{8X\pi}{2}$ OR $64X + 4X\pi$**

   Area = $(4X)(4X) + (8X)(8X) + (4X)^2\pi$

4. Perimeter = $2(12) + 3(8) + 2(2x - 6)$
   Area = $(2x)(12) - 4(9)$

5. Perimeter = $2(2X) + 16 + 2(10)$
   Area = $16(2X) + \frac{1}{2}(16)(6)$

Formative Assessment Properties of Operation Name _____

1. How many of the expressions shown are equivalent to $2a + 2b$?

$2(a + b)$ $a + a + b + b$ $(a + b)2$ $(a)(a) + (b)(b)$

A. 1
B. 2
C. 3
E. 4

2. How many of these expressions are equivalent to $3a + 6$?

$2a + 4 + a + 2$ $3(a + 6)$ $(a)(a)(a) + (2)(3)$ $6a + 3$

A. 0
B. 1
C. 2
D. 3

3. Which of the following completes the equation to show the distributive property?

$(3 \times 12) + (3 \times 4) = 3 \times$ ____

A. (12×4)
B. $(12 - 4)$
C. (12)
D. $(12 + 4)$

4. Find the equivalent expression for: $4(3 + 2)$

A. $(3 + 2) \times (3 + 2) \times 2$
B. $12 + (3 + 2)$
C. $(4 \times 3) + (4 \times 2)$
D. $(4 + 3) \times (4 + 2)$

5. Find the equivalent expression for 7^5

A. $7 + 7 + 7 + 7 + 7$
B. $7 \times 7 \times 7 \times 7 \times 7$
C. $7(7)$
D. 7×5

6. Find the equivalent expression for 3×5

A. $3 + 3 + 3$
B. $5 + 5 + 5$
C. $3 \times 3 \times 3$
D. $5 \times 5 \times 5$

N

7. Which property is shown below? $(9q + 5r) + 3s = 9q + (5r + 3s)$

- **A.** identity
- **B.** associative
- **C.** commutative
- **D.** distributive

8. Using the distributive property, which expression equals the following?

$$(8 - a)4$$

- **A.** $32 - 4a$
- **B.** $12 - 4a$
- **C.** $12 - a$
- **D.** $32 - a$

9. What property is shown below? $^{2}/_{5}(^{8}/_{17} + q) = {}^{16}/_{85} + {}^{2}/_{5}q$

- **A.** commutative
- **B.** associative
- **C.** identity
- **D.** distributive

10. Using the associative property, which expression equals $10 \bullet (17 \bullet q)$?

- **A.** $10 \bullet (q \bullet 17)$
- **B.** $(10 \bullet 17) \bullet q$
- **C.** $(17 \bullet 10) \bullet q$
- **D.** $17 \bullet (10 \bullet q)$

11. Simplify the following expression:

$$6(3y - 7) + 38y$$

- **A.** $18y - 4$
- **B.** $74y - 7$
- **C.** $56y - 42$
- **D.** $18y - 42$

○ **Choices**
for associative, distributive, additive inverse, commutative, factor, expand
problems or arithmetic fact
12 – 15

12. Which property is illustrated in each step?

Step 1 $7x + 7 + 5(x - 3) = 7x + 7 + 5x - 15$
Step 2 $= 7x + 5x + 7 - 15$
Step 3 $= (7 + 5)x + 7 - 15$
Step 4 $= (7 + 5)x + (7 - 15)$
Step 5 $= 12x - 8$

13. Identify the property used in each step.

Step 1 $2x + 9 + 7x - 2 = 2x + 7x + 9 - 2$
Step 2 $= 2x + 7x + (9 - 2)$
Step 3 $= 2x + 7x + 7$
Step 4 $= (2x + 7x) + 7$
Step 5 $= (2 + 7)x + 7$
Step 6 $= 9x + 7$

14. Which property is illustrated in each step?

Step 1 $^1/_8(x + 5) + {}^3/_5 = [(^1/_8)x + {}^5/_8] + {}^3/_5$
Step 2 $= (^1/_8)x + ({}^5/_8 + {}^3/_5)$
Step 3 $= (^1/_8)x + 1\ {}^9/_{40}$

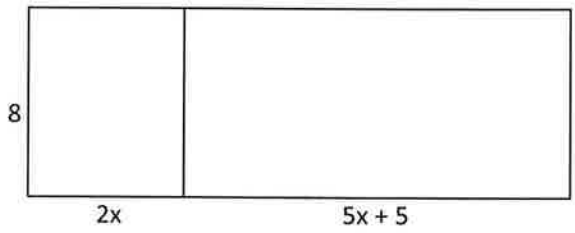
15. Which property is illustrated in each step? Explain why using the properties made working this problem easier. _____

Step 1 $(-300 + 270) + 30 = -300 + (270 + 30)$
Step 2 $= -300 + 300$
Step 3 $= 0$

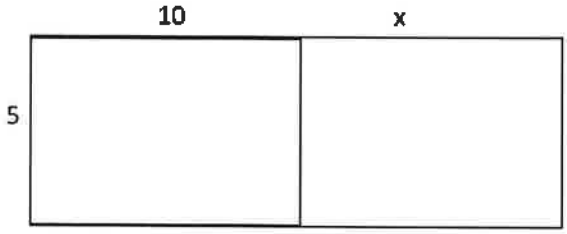
16. Which property is illustrated in each step? Explain why using the properties made working this problem easier. _____

Step 1 $(43 \times 120) + (43 \times -20) = 43\,(120 + -20)$
Step 2 $= 43\,(100 + 20 + -20)$
Step 3 $= 43\,(100 + 0)$
Step 4 S $= 4300$

17. Write two different expressions for the area of the rectangles below.



18. **Write two different expressions for the area of the rectangle below.**



Formative assessments of Properties of Operation

Answers

1. C (Obj 1, 2, 4, 5, 6 / 7.EE1 / 2)
2. B (Obj 1, 2, 4, 5, 6 / 7.EE1 / 2)
3. D (Obj 6 / 7.EE1 / 2)
4. C (Obj 6 / 7.EE1 / 2)
5. B (Obj 6 / 7.EE1 / 2)
6. B (Obj 6 / 7.EE1 / 2)
7. B (Obj 1 / 7.EE1 / 2)
8. A (Obj 1 / 7.EE1 / 2)
9. D (Obj 1 / 7.EE1 / 2)
10. B (Obj 1 / 7.EE1 / 2)
11. C (Obj 2 / 7.EE1 / 2)
12. Distributive, Commutative, factor(distribute) , Associative, Arithmetic fact (Obj 1, 4, 5, 6 / 7.EE1 / 2)
13. Commutative, Associative, Arithmetic Fact, Associative, factor(distribute), Arithmetic fact(Obj 1, 4, 5, 6 / 7.EE1 / 2)

14. Distribute, Associative, Arithmetic Fact. (Obj 1, 4, 5, 6 / 7.EE1 / 2)

15. Associative, Arithmetic fact, Additive Inverse. It is easier to add the two positive number together first, especially since the equal an even 300. (Obj 1, 4, 5, 6 / 7.EE1 / 2)

16. Factor (Distribute), Expand, Additive Inverse, Arithmetic Fact It is easier to add the 20 and -20 first then multiply by 100. (Obj 1, 4, 5, 6 / 7.EE1 / 2)

17. $8(2x + 5x + 5)$ or $8(2x) + 8(5x + 5)$ or $16x + 40x + 40$ or $56x + 40$ (Obj 3 / 7.EE1 / 2)

18. $5(10 + x)$ or $5(10) + 5x$ or $50 + 5x$ (Obj 3 / 7.EE1 / 2)

_____/ 38

The first page is designed to help students recognize the difference between the commutative, associative, and distributive properties.

The second page is included to challenge students who already have a good understanding of the properties. It includes false statements that look like the properties, but they are really false statements. It also includes expanded expressions that are not really one of the properties, but are still true statements.

Suggestions for making your flashcards . I would suggest writing the answers on the back of the cards before you copy them. Students like instant feedback. I would make about 15 class sets of cards that contain just the first page and about 15 sets of cards that contain both pages. The cards are then ready for a challenge class, a low class and a mixed class where you can differentiate based on readiness. Students can work in pairs at first and later race to see who can complete the sorting process first.

8 x -3 = -3 x 8	Commutative Property
-7 + 5 = 5 + -7	Associative Property
(8 x 4) x 2 = 8 x (4 x 2)	Distributive Property
(-3 + 7) + -9 = -3 + (7 + -9)	3 + (4 + 8) = (3 + 4) + 8
3 x -2 = -2 x 3	-4 + (3 + 7) = (-4 + 3) + 7
3 + 2 + 8 = 3 + (2 + 8)	8 + 7 + 13 = 8 + (7 + 13)
3+(4 - 3) = (4 – 3) + 3	7+ (-4 + 3) = (-4 + 3) + 7
-4(3 + 10) = (-4 x 3) + (-4 x 10)	-6(7 + -5) = -6 x 7 + -6 x -5
-7(3 + 2x) = -21 + -14 x	-4(3 + 20) = -12 + -80
8(7 – 2x) = 56 – 16x	-5(8+-3) = -5x8 + -5x-3

D

False Statement	$8 - -3 = -3 - 8$
$-7 \div 5 = 5 \div -7$	$Kw + wy + w^2 = w(k + y + w)$
$(8 - 4) - 2 = 8 - (4 - 2)$	$4(p + q + 20) = 4p + 4q + 80$
$(-3 \div 7) \div -9 = -3 \div (7 \div -9)$	$3 + (1 + 8 + 1) = (3 + 1) + 8 + 1$
$3n \times -2 = -2(3) \times n$	Expanded expressions
$3 \times 2 + 8 = 3 \times (2 + 8)$	$8 + 7 + 13 = 8 + (7 + 3 + 10)$
$3 - (4 + 3) = (4 - 3) + 3$	$7 + (-4 + 3) = (-1 + -3 + 3) + 7$
$-4(3 + 10) = -4 \times 3 \times -4 \times 10$	$-6(7 + -5) = -6 (2 + 5 + -5)$
$-7(3 + 2x) = -21 + -7x$	$-4 (23) = -4(3 + 20)$
$8(7 - 2x) = 56 - 2x$	$5(8.99) = 5x9 + 5x-0.01$

P

7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Finding the total price including tax.

Find the total cost of a game that cost $58.00 with a tax rate = 6.5%/

Method 1	Method 2
58 x .065 = 3.77 tax	100% + 6.5% = 106.5%
58 + 3.77 = 61.77	58 x 1.065 = 61.77

1. Jeans $35 at 5.5% tax

2. M3P player $150 at 7% sales tax

3. Plasma TV at $1,560.00 with a sales tax of 6.725%

Find the sale price of an item.

Find the sale price of a game that cost $32.50 that is on sale at 20% off

Method 1	Method 2	What is happening here???
32.50 x .20 = 6.50 off	100% - 20% = 80%	26.00 x 1.065= $27.69
32.50 – 6.50 = $26.00	32.50 x .80 = $26.00	Why does this not work?
		100% - 20% + 6.5% = 86.5%
		32.50 x .865 = $28.11

1. A sweatshirt for $45.00 at 30% off.

2. A cell phone for $250 at 25% off.

3. A guitar for $390 for 15% off and a tax rate of 7.8%

Q

Matching Situations to Equations

1. Match the following equation to the correct situation.
$3K + K = 4K$

- ○ **A.** The number of stuff animals that Kaylee and Kate have when Kaylee has 3 times the number of stuffed animals as Kate.
- ○ **B.** The number of kites that Kaylee has when she buys 3 more kites.
- ○ **C.** The number of text messages Kaylee and Kristin have when Kaylee has 4 more than Kristin.
- ○ **D.** The kilometers that Kaycee has after running 3 more Kilometers.

2. You go to the supermarket. 1 bag of apples costs 4 dollars. 1 gallon of olive oil costs 10 dollars. You get 6 bags of apples and 6 gallons of olive oil. Which equations could you use to solve this problem?

- ○ **A.** Total cost = $6 \times 4 + 10 = 6 \times 4 + 10 = 24 + 10 = 34$ dollars.
- ○ **B.** Total cost = $6 \times (4 + 10) = (6 \times 4 + 6) \times 10 = 24 + 6 \times 10 = 30 \times 10 = 300$ dollars.
- ○ **C.** Total cost = $4 \times (6 + 10) = 4 \times 16 = 64$ dollars.
- ○ **D.** Total cost = $6 \times (4 + 10) = 6 \times 4 + 6 \times 10 = 24 + 60 = 84$ dollars.

3. Robert has 8 notebooks and his brother has 6. If we double both amount, how many do they now have altogether?

- ○ **A.** We get $2 \times (8 + 6) = 2 \times 8 + 2 \times 6 = 16 + 12 = 28$
- ○ **B.** We get $2 \times 8 + 6 = 16 + 6 = 22$
- ○ **C.** We get $2 \times (8 + 6) = (2 \times 8 + 2) \times 6 = 18 \times 6 = 108$
- ○ **D.** We get $2 \times (2 + 6) = 2 \times 2 + 2 \times 6 = 4 + 12 = 16$

4. Suppose I go to the supermarket and buy ice cream for 12 dollars, milk for 15 dollars and bread for 8 dollars. How much money do I owe the cashier? Which property should I use to make this problem easier to figure out? Show your work using that property.

- ○ **A.** Associative
- ○ **B.** Commutative
- ○ **C.** Distributive
- ○ **D.** Identity

Q

5. Tom took riding lessons last year. She took 26 lessons at $45 each. In addition to the lessons, he had to pay for a riding helmet. He spent a total of $1,177.
How much did he have to pay for the helmet?

- **A.** $19
- **B.** $7
- **C.** $21
- **D.** $3

6. Which of the following can be represented by the equation below?
$A = 0.05s + 200$

- **A.** A weekly income of $200 minus 5% commission on sales, *s*.
- **B.** A weekly income of $600 minus 5% commission on sales, *s*.
- **C.** A weekly income of $200 plus 6% commission on sales, *s*.
- **D.** A weekly income of $0.05 plus 200% commission on sales, *s*.

7. Marty sold refreshments at her mother's garage sale. She sold cups of lemonade for $0.50 each and cookies for $1.00 each. She sold 24 cookies and made a total of $40.50.
How many cups of lemonade did she sell?

- **A.** 16
- **B.** 44
- **C.** 33
- **D.** 30

8. Match the following equation to the correct situation.
$30\%(b + \$17.00) = 30\%(b) + \$11.90$

- **A.** Shawn paid 70% of his bill when he paid $11.90.
- **B.** You had a coupon for 70% off and bought a blouse and a skirt that was originally $17.00.
- **C.** Summer paid $11.90 more than 70% of *b*.
- **D.** The cost for a blouse and the 17 dollar skirt was 30% off.

9. Match the following equation to the correct situation.
$9\%(p) + p = 1.09p$

- **A.** The amount paid for notebook was 9% of p.
- **B.** The amount saved was 9% of p.
- **C.** The amount owed after paying a 9% deposit on p.
- **D.** The amount paid for a shirt with 9% tax on p.

---

10. Match the following equation to the correct situation.
$A - A(6\%) = A(0.94)$

- **A.** The amount donated by a company which gives 6% of A dollars to the school for a fund raiser.
- **B.** The amount owed from borrowing A with 6% interest.
- **C.** The amount Tony owes after putting 6% down on an A home.
- **D.** The amount paid for two balls, one was full price for A, the other was discounted 6%.

11. Chris counted the pennies, nickels, dimes, and quarters in his coin collection. He had 62 pennies, 62 nickels, and 62 dimes. He had $22.72 total.
How many quarters did he have? Which property could I use to make this problem easier? Show your work using that property.

- **A.** Associative
- **B.** Commutative
- **C.** Distributive
- **D.** Identity

12. Which property should I use to make this problem easier to figure out?
$\frac{1}{7}(t + 70) = \frac{1}{7}t + 10$

- **A.** Associative
- **B.** Commutative
- **C.** Distributive
- **D.** Identity

R

Formative Assessment of Expressions In Context Name _____

1. Find the sale price of a phone that cost $450 that is on sale for 20% off. Calculate the final cost two different ways.

2. All varieties of a brand of cookies are $3.50. A person buys 2 peanut butter, 1 chocolate and 3 sugar packages of cookies. Calculate the final cost two different ways.

3. An equilateral triangle has a perimeter of $6x+15$. What is the length of each of the sides of the triangle?

4. What is the final cost of a drum set that originally cost $1,035 and is on sale for 35% off with a tax rate of 6%? Show your work. Calculate the final cost two different ways.

5. Sam wanted to by some sneakers that were on sale for 30% off and he had a coupon for 20% off the sale price. The original price of the sneakers was $55.00. Sam thinks he should be charged $27.50 for the shoes which is 50% off the original price. Explain why that was not the price at the cash register. What was the price that Sam was charged?

R

Answers

1. 1. $360 (Obj 7, 8 / 7.EE2 / 2)

Student understands that a 20% discount is the same as finding 80% of the cost (.80c).

2. $21.00 (Obj 7, 8 / 7.EE2 / 2)

Multiplying 2 x $3.50 to get the cost of the peanut butter cookies, 3 x $3.50 to get the cost of the sugar cookies and 1 x $3.50 for the chocolate cookies and then adding those totals together.

Multiplying $3.50 times 6 will give the same total.

3. $3(2x+5)$, therefore each side is $2x+5$ units long. (Obj 7, 8 / 7.EE2 / 2)

4. $713.12 (Obj 7, 8 / 7.EE2 / 2)

Multiply $1035 to .35 to calculate 35% off, subtract that amount from the original and find the sale price to be $672.75. Then multiply the sale price times .06 to calculate the 6% sales tax and add the amount onto the sale price

100% - 35% = 65% Multiply $1035 time .65 then multiply that answer by 1.06 (100% + 6%) to calculate the final cost

5. $30.80 (Obj 7, 8 / 7.EE2 / 2)

Multiply 55 x .7 to get the sale price of 38.50, then multiply the 38.50 by .8 to get the final price. Sam gets 20% off the sale price not the original price.

S

Discussion Questions

Think about your answers to these questions. Discuss your ideas with your group. Then write a summary of your findings. Use examples and diagrams to create a visual aspect to your explanation.

1. What does it mean to say that two expressions are equivalent?

2. Explain how the Distributive property can be used to write equivalent expressions.

3. Explain how the Commutative property can be used to show that two or more expressions are equivalent. Give an example of how the Commutative property can make a calculation easier.

4. Explain how the Associative property can be used to show that two or more expressions are equivalent. Give an example of how the Associative property can make a calculation easier.

5. Describe a situation in which it is helpful to add expressions to form a new expression. Explain how you can combine the expressions.

Illustrative Mathematics

7.EE Miles to Kilometers

Alignment 1: 7.EE.A

The students in Mr. Sanchez's class are converting distances measured in miles to kilometers. To estimate the number of kilometers, Abby takes the number of miles, doubles it, then subtracts 20% of the result. Renato first divides the number of miles by 5, then multiplies the result by 8.

a. Write an algebraic expression for each method.

b. Use your answer to part (a) to decide if the two methods give the same answer.

Commentary:
In this task students are asked to write two expressions from verbal descriptions and determine if they are equivalent. The expressions involve both percent and fractions. This task is most appropriate for a classroom discussion since the statement of the problem has some ambiguity.

Adapted from Algebra: Form and Function, McCallum et al., Wiley 2010

Solution: Writing and comparing expressions

a. Abby's method starts by doubling m, giving $2m$. She then takes 20% of the result, which we can write $0.2(2m)$. Finally she subtracts this from $2m$, giving

$$2m - (0.2)2m$$

Renato's method starts by dividing m by 5, giving $m \div 5 = \frac{m}{5}$, and then multiplies the result by 8, giving

$$8\left(\frac{m}{5}\right)$$

b. Abby's expression can be simplified as follows:

$$2m - (0.2)2m = 2m - 0.4m = (2 - 0.4)m = 1.6m.$$

(The step where we rewrite $2m - 0.4m$ as $(2 - 0.4)$ uses the distributive property.)

Renato's method gives

$$8 \cdot \frac{m}{5} = 8 \cdot \frac{1}{5} \cdot m = \frac{8}{5} \cdot m = 1.6m.$$

So the two methods give the same answer and the expressions are equivalent.

Student Name	**Teacher Name**
Progress Report Starting Grade:_____ Progress 1: Date_____: Grade:_____ Progress 2: Date_____: Grade:_____ Progress 3: Date_____: Grade:_____	**Parent Contact Log – Contact if student is not complying with Improvement Plan** Phone #_____ Date, Reason, Outcome 1 _____ 2._____ 3. _____ 4.._____ 5.._____
Requirements of Your Improvement Plan: □ Attend afterschool tutoring on: Tues Weds □ Make-up Missing: □ Test □ Quiz □ Classwork □ Homework □ Other:_____	**Agreement to follow this Improvement Plan** Student Signature_____ Teacher Signature_____ Additional Comments (Student or Teacher):

Student Name	**Teacher Name**
Progress Report Starting Grade:_____ Progress 1: Date_____: Grade:_____ Progress 2: Date_____: Grade:_____ Progress 3: Date_____: Grade:_____	**Parent Contact Log** Phone #_____ Date, Reason, Outcome 1 _____ 2._____ 3. _____ 4.._____ 5.._____
Requirements of Your Acceleration Plan:	**Agreement to follow this Acceleration Plan** Student Signature_____ Teacher Signature_____ Additional Comments (Student or Teacher):

If you start off with no money, then earn $20, and then spend the $20 at the movies, how much will you have left? Nothing, of course! Adding $20 to your pocket and then subtracting $20 leaves you zero dollars and an empty pocket. In this case, the operation of subtraction "undoes" the operation of addition. This is an example of an inverse operation.

Lesson Resources | Advance Preparation | Safety

GOALS FOR THIS LESSON:

- Use inverse operations to write a related equation.

Materials

- Student Guide
- Skills Update

GROUNDWORK: Preparing for the Lesson

In this lesson, you will learn how to use opposite actions to solve a problem. What kind of opposite actions can you take in math?

Look at this example to find out.

James adds 7 to a number and gets 18. What can he do to undo what he did?

Answer

1 of 2 Next

↑ Activity Resources

LEARN: Introduction to Inverse Operations

In this lesson you will explore another method for finding the solution to an equation. This method involves using inverse operations

Before you begin, investigate the meaning of the word "inverse" as it applies to this lesson. Consider these two examples:

- Raul walks up the stairs, then down the stairs.
- Lilly opens the door, then shuts the door.

Each example is about taking opposite actions, right? So, you can probably say that inverse operations are like opposites.

What is the inverse, or opposite, of a flag being raised on a pole?

↑ Activity Resources

LEARN: Introduction to Inverse Operations

In math, inverses have to do with these operations: addition, subtraction, multiplication, and division. Some operations are "opposites" of others.

- Addition and subtraction are inverse operations because one operation undoes the other.
- Multiplication and division are also inverse operations because one operation undoes the other.

Do you remember the scenario from the beginning of this lesson? You started off with no money, and then earned $20...

$0.00 + $20.00 = $20.00

...and then spent it all at the movies.

$20.00 – $20.00 = $0.00

As you can see, addition and subtraction are inverse operations because one operation undoes the other. Performing inverse operations takes you right back to where you started from.

Unit 1 | Lesson 9

LEARN: Introduction to Inverse Operations

In mathematics, inverses are operations that undo each other. You can use inverse operations to write related equations. Related equations are equations that communicate the same relationship, but in different ways.

For example, $x + 7 = 19$ and $x = 19 - 7$ are related equations. You can subtract 7 from each side of the first equation to get the second, or you can add 7 to each side of the second equation to get the first.

You can use related equations to solve an equation. The idea is to use the related equation that causes the variable to be alone on one side of the equal sign. Look at the examples here.

Use related equations with inverse operations to solve the equation.

$$k - 5 = 2$$

Answer

WORKED EXAMPLES: Related Equations

Try a couple of problems on your own.

Write a complete set of related equations that contain the given equation.

$x + 4 = 17$

Answer

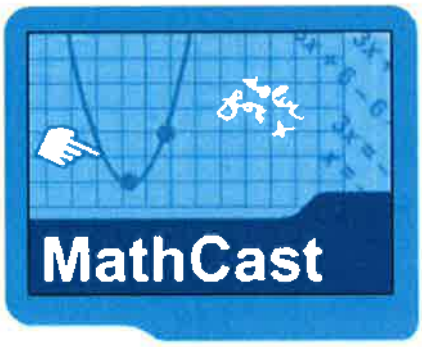
Next

↑ Activity Resources

MATHCAST: Using Related Equations

Now watch how it's done. Click Using Related Equations to see how to solve a typical problem.

This activity has audio. Please use speakers or headphones.



Using Related Equations

↑ Activity Resources

SUMMARY: Related Equations

- Inverse operations can be used to solve an equation.
- Addition and subtraction are inverse operations.
- Multiplication and division are inverse operations.
- Related equations are equations that all communicate the same relationship, but in different ways.

Inverse Operations and Related Equations

$10 \cdot 5 = 50$ $10 = 50 \div 5$	These are related equations.
$k - 5 = 2$ $k = 2 + 5$ $k = 7$	$k - 5 = 2$ and $k = 5 + 2$ are related equations.
$r \cdot 12 = 36$ $r = 36 \div 12$ $r = 3$	$r \cdot 12 = 36$ and $r = 36 \div 12$ are related equations.

Activity Resources

OFFLINE LEARNING: Related Equations

Now go offline to learn more about inverse operations and to practice your skills.

- Review pages 26–28 in the Reference Guide.
- Print and complete the Skills Update.
- Complete Problems 1–9 odd, and 23–26 all on pages 28 and 29 of the Reference Guide.

Materials

- Related Equations Solutions
- Skills Update

Pre-Algebra: A Reference Guide and Problem Sets - pages 28-29

Optional

calculator

Activity Resources

Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title: Using Variables (Writing and Solving Equations), 2 weeks

Designed by: Laura Mayer
District:Moyer Academy

Content Area:Math
Grade Level(s):6th

Summary of Unit

In this unit, students build on their experience in K-5 working with properties of operations with whole numbers, decimals, and fractions, and start to manipulate algebraic expressions to produce different but equivalent expressions for different purposes. Working with numerical expressions will prepare students for future work with algebraic expressions. It is important for students to be precise when stating the meaning of variables while setting up equations. They should specificy if the variable refers to a specific number, or to all numbers in a range.

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

- Include those addressed in Stage 3 and assessed in Stage 2.

Mathematical Practice 1. Make sense of problems and persevere in solving them.

Mathematical Practice 4. Model with mathematics.

Mathematical Practice 5. Use appropriate tools strategically.

Mathematical Practice 8. Look for and express regularity in repeated reasoning.

6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers.

6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the from x + p = q and px = q for cases in which p,q, and x are all nonnegative rational numbers.

Big Idea(s)

- Transferable core concepts, principles, theories, and processes from the Content Standards

-Understanding and demonstrating a process is often more important than obtaining a final answer.

-Real-world problems can be solved by setting up and solving equations using variables to represent unknown quantities.

Unit Enduring Understanding(s)

- Full-sentence, important statements or generalizations that specify what students should understand from the Big Ideas (s) and/or Content Standards and that are transferable to new situations

Number patterns and relationships can be represented using variables, symbols representing numerical values.

Variables are substitutions in expressions and equations for numerical values.

Number operations are always computed in a particular order.

The facts and choice of words in a word problem can offer clues as to which operations are used to solve it.

Unit Essential Questions(s)

- Open-ended questions designed to guide student inquiry and learning

-How much choice do you have when solving math problems?

-How does the inverse operation help me solve equations?

-How can you pull information out of a real-world situation and put it into a math context in order to solve a problem?

Knowledge and Skills

- Needed to meet Content Standards addressed in Stage 3 and assessed in Stage 2

Students will know...

- Solving an equation is a process of finding values that make the equation or inequality true.

- Variables can be used to represent a number, or a set of numbers, when solving real-world problems.

- The two forms of equations: $x+p=q$ and $px=q$ when x, p, and q are all non-negative rational numbers.
- When performing operations in math, everybody agrees to follow the same process for finding solutions (order of operations).
- The order of operations is: parenthases, exponents, multiplication and division, addition and subtraction.

Students will be able to...

- Write expressions and equations
- Read expressions and equations
- Evaluate expressions and equations
- Apply order of operations
- Solve equations
- Solve real-world and mathematical problems

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

Suggested Performance/Transfer Task(s)

- Performance/Transfer tasks as evidence of student proficiency

An effective assessment for ALL students should be designed to include:

*Complex, real-world, authentic applications

*Assessment(s) for student understanding of the Stage 1 elements (Enduring Understandings, Essential Questions, Big Ideas) found in the Content Standards

*Demonstration of high-level thinking with one or more facets of understanding (e.g., explain, interpret, apply, empathize, have perspective, self-knowledge)

Illustrative Mathematics - Busy Day (Appendix A)

The Jonas family had a really busy day. After leaving their home, Mrs. Jonas dropped son Cody at his tennis practice. She then drove daughter Kristin to her soccer game and stayed to watch. After the game, mother and daughter picked up Cody on the way home. Once home, Mrs. Jonas saw that they had driven 20 miles that day. How far are the tennis courts from home?

a. Use any method you like to find the distance between the tennis courts and home.

b. Set up and solve an equation in one variable to find the distance between the tennis courts and home.

Note: The family car can only travel along the main streets (gridlines), and all distances between cross streets are the same distance. Assume Mrs. Jonas took the most direct routes to and from her destinations.

Rubric(s)

- Scoring guide to evaluate performance/transfer tasks used as evidence of student proficiency

An effective scoring guide should:

*Measure what is appropriate for the Content Standard that is assessed.

*Provide opportunities for differentiation of the performance/transfer tasks used as evidence of student proficiency.

Scoring Guide for Performance Task: Busy Day (see Appendix A)

Other Evidence

- Varied evidence that checks for understanding (e.g., tests, quizzes, prompts, student work samples, observations and supplements the evidence provided by the task)

Students will be provided with opportunities to complete varied performance tasks throughout the unit as means of formative assessment to prepare them for the summative performance task.

Student Self-Assessment and Reflection

- Opportunities for self-monitoring learning (e.g., reflection journals, learning logs, pre- and post-tests, self-editing – based on ongoing formative assessments)

-Formative Observations:

Throughout the unit, students will be observed on their application of mathmatical practices highlighted in the unit:

Mathematical Practices 1. Make sense of problems and persevere in solving them.

Mathematical Practice 4. Model with mathematics.

Mathematical Practice 5. Use appropriate tools strategically.

The observations will be shared with the students and parents throughout the unit. These conferences will be centered around the student's progressing score on the Mathematical Practices Rubric (Appendix B). Students will be prompted to reflect on their progress towards using these practices throughout the unit.

-After completing each Formative assessment performance task, students will be given the opportunity to reflect on their work. Students will then do a Think-Pair-Share with a peer and explain their work, defending both their problem solving method and the answers they reached.

**All performance based task work will be kept inside a reflective log that students will add to throughout the year. The teacher will assist students in creating the log and upkeep throughout the year, but it will be the student's responsibility to maintain. At the completion of each task, the students will reflect in writing on the relationship between the task and the learning objective of the lesson.

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

- Instructional activities and learning experiences needed to align with Stage 1 and Stage 2 expectations

Include these instructional elements when designing an effective and engaging learning plan for ALL students:

*Align with expectations of Stage 1 and Stage 2

*Scaffold in order to acquire information, construct meaning, and practice transfer of understanding

*Include a wide range of research-based, effective, and engaging strategies

*Differentiate and personalize content, process, and product for diverse learners

*Provide ongoing opportunities for self-monitoring and self-evaluation

The direct instruction resources needed to implement this course can be found within the k12 Fundamentals of Geometry and Algebra course, Semester A. Use these presentations to guide your information delivery during Direct Instruction. Guided

Practice Examples and additional Independent Practice activities are also embedded within the presentation materials (See example presentation in Appendix R).

Lesson 1: Order of Operations (2-3 days)

Anticipatory Set: Introduce the idea of using conventions to clarify meaning by displaying the following sentences:

"Paul," said the teacher, "is very intelligent."

Paul said the teacher is very intelligent.

- Ask students to talk to a partner about the meaning of each sentence and be ready to share their ideas with the whole class.

- Important ideas to come from this discussion include how the words are the same but the punctuation changes the meaning. Explain that punctuation is something that has developed over time to help us communicate in writing.

Next, ask the students to complete the following number sentence: $4 + 4 \times 4 - 4 = ?$

Many students are unaware of the convention of the order of operations and the answers students offer might vary, including 28, 16, and 0. As students share how they arrived at their answers, record their thinking for the class. Some examples:

28	16	0
$4 + 4 = 8$	$4 \times 4 = 16$	$4 + 4 = 8$
$8 \times 4 = 32$	$4 + 16 - 4 = 16$	$4 - 4 = 0$
$32 - 4 = 28$		$8 \times 0 = 0$

2. Continue by discussing how confusing this could be when we are trying to communicate our thinking in writing. Explain that, just like in using agreed upon punctuation to clarify meaning with written words, we use an agreed upon order of operations to clarify meaning with written math sentences.

Introduce the order of operations for the four basic operations: First: Simplify all operations inside parentheses.

Then: Simplify all exponents, working from left to right.

Next: Perform all multiplications and divisions, working from left to right.

Finally: Perform all additions and subtractions, working from left to right.

3. Ask the students to again complete the number sentence but this time use the agreed upon order, or conventional order. Invite each student to compare his solution with that of another student sitting nearby. Finally, ask for students to share their solutions with the whole class. This time most of the solutions should be the same.

Confirm the process and solution using the order of operations.

28	16	0
$4 + 4 = 8$	$4 \times 4 = 16$	$4 + 4 = 8$
$8 \times 4 = 32$	$4 + 16 - 4 = 16$	$4 - 4 = 0$
$32 - 4 = 28$		$8 \times 0 = 0$

4. Refer back to the record of their thinking and model how to record their original thinking using number sentences. Use parentheses to communicate the order of operations used by each method:

Next, introduce the Four 4s challenge. Note: with students that in the lesson, three different values have already been created using only four 4s—28 and 16 and 0. Ask students to work with a partner to complete the challenge.

Four 4s Challenge

How many of the numbers from 1 to 10 can you create using four 4s?

1 =

2 =

3 =

4 =

5 =

6 =

7 =

8 =

9 =

10 =

Independent Practice: Balancing Expressions (Appendix C)

Reinforcement Activity: Order of Operations Bingo (Appendix D)

Group Work: Who is Correct? (Appendix E)

Formative Assessment: Watch out for Parentheses (Appendix F)

Self-Reflection and Peer-Review: Students will do a Think-Pair-Share with a peer and explain their work, defending both their problem solving method and the answers they reached.

After Performance Task Think-Pair-Share, students will reflect in their journal on the application of their learning to the Performance Task and their mastery of the lesson objectives. Reflective Journal Entries will be graded using the following rubric:

4 points - Reflection is well written with clear organization and contains almost no errors; demonstrates thoughtful analysis of how new learning and performance task related; evidence and details included to demonstrate student's grasp of concepts; submitted on time.

3 points - Reflection is well written with some organization and contains few errors; demonstrates analysis of how new learning and performance task related; some evidence and details included to demonstrate student's grasp of concepts; submitted on time.

2 points - Reflection shows little organization and contains errors; demonstrates attempt at analysis of how new learning and performance task related; little to no evidence and details are included to demonstrate student's grasp of concepts; submitted on time.

1 points - Reflection has no clear organization and contains many errors; little to no analysis of how new learning and performance task related; little to no evidence and details included to demonstrate student's grasp of concepts; and/or not submitted on time.

0 points - No relevant reflection is submitted.

Remind students that during this unit, they will be building their own math equations. In order to build an equation that means what they want it to mean, they have to remember to follow the same order of operations just learned. Much like how the meaning of their words will change if they don't follow the rules for writing and punctuation, the meaning of their equations will change if they don't follow the order of operations.

Lesson 2: Introduction to equations and inequalities (1-2 days)

Task: Writing and Evaluating Expressions - Exploration

The teacher should pair/group the students for this introductory exploration activity. The students should use their prior knowledge of addition and multiplication to arrive at the answers. At this point, they have not been introduced to equations/inequalities as a means to problem solving.

Part I: Mr. Green's Math class is planning a trip to the Theater. It will cost $10 for the school bus and the price of a ticket is $13 dollars per student. What will determine the amount of money the class will have to make?

How will the number of students affect the price?

How will they know how much money they need to make?

What value varies in this example?

How much will it cost if 10 students attend?

How much will it cost if 17 students attend? Draw a model to represent this situation

Students will pair up to read the passage "Variables and Expressions" (Appendix G). They will complete two Frayer Models (Appendix H) from their close reading, one for "variable" and one for "equation."

Once Frayer Model has been completed the teacher should have a class discussion about what was read. Teacher should clarify anything that the students have misunderstood. This is an opportunity for the teacher to emphasize all important pieces of the reading and to ensure that the students have an understanding of the information that was read.

Task: Equations and Their Solutions (Appendix I)

Students will be given an envelope that will have 10 equations and 10 solutions. Students will work with their group to match the solution with the correct equation. Students will have to prove by writing why they think that is the solution to their equation. Students will also be asked to write what strategy they used to come up with their answer.]

Formative Assessment:

Determine whether the given value of each variable is a solution. If no, give a reason why you disagree.

1. $85 = 13x$ for $x = 5$

2. $w + 38 = 210$ for $w = 172$

3. $8y = 88$ for $y = 11$

4. $16 = w \div 6$ for $w = 98$

5. The local pizza shop charged Barbara Jean $172 for 21 medium pizzas. The price of a medium pizza is $8. Determine whether Barbara Jean paid the correct amount of money.

Lesson 3: Equations and Solutions (4 days)

The teacher will bring 5 apples to school (or candies, or anything else exciting to grab their interest). Show the students two of them and have an "unknown" number of apples inside a paper bag. Tell the students that there are five apples total. Only seeing the two, they need to tell how many apples are in the bag.

After eliciting responses from the students, ask them how they knew there were 3 apples in the bag. "What did you do mathematically that made you state I had 3 hidden apples?" Hopefully, students will respond that since they knew I had 5 apples and I showed them 2, or mathematically speaking and writing, $5 - 2 = 3$.

Introduce to students the concept of building equations to answer questions in a more efficient way.

Connecting this thinking to symbolically solving one step equations:

At the board the teacher will write the equation x + 2 = 5 and connect this to the thought process above. "Let the symbol "x" represent my question, How many apples are in the bag? How many apples in the bag is Unknown." Demonstrate symbolically what the students did by subtracting 2 from 5 to get 3. Reminding them that solving equations is like keeping a see- saw in balance you must remember that "whatever you do to one side, you must do to the other." Therefore you need to subtract two from both sides of the equal sign to get the variable alone leaving you with x = 3. Mathematically speaking, "The unknown number of apples in the bag is 3."

Guided Practice: Why does x=2? (Appendix J)

Independent Practice: Evaluating Expressions BINGO (Appendix K)

Direct Instruction: Use the Task: Cups, Equations, and Solutions (Appendix L) to guide students through creating equations. In the grey bar on the side of the paper, have the students write the steps they need to take to write an equation for each scenario.

Move the students from translating pictures into equations, to translating words into equations (Algebraic Expressions for Words) (Appendix M)

Guided Practice/Group Work (Appendix N): Partner students and have them cut up the cards on both worksheets. Randomly distribute the cards and have the students work to translate the math expressions into words and the words into math expressions.

Formative Assessment: Equations in the Real-World (Appendix O)

Reinforcement Activity: Group the students according to the level of mastery demonstrated with the formative assessment. Distribute the Situation Cards (Appendix P) appropriately to each group, differentiating by level of difficulty. Before the activity ends, be sure ALL students have the opportunity to try working on at least ONE "challenging" card.

1. Put students into groups of three and pass out one Situation Card and calculator (optional) to each group. Cards are on different colors to indicate whether they are easy (red), midlevel (green), or challenging (blue). You may use these to vary difficulty or to help specific students. Students must still show all of their work if they use a calculator.

2. Have students work cooperatively to understand the problem, write an equation, and find the solution. They should each be able to explain the solution to someone in another group. For example, if a student has Situation Card 7, she should understand that since baby-sitting pays $7.50 an hour and she baby-sat for 4 hours that means she made $7.50 each of those hours. The unknown is how much she made. 7.50 x 4 = $30 on Saturday night.

3. If students finish early, ask them questions to extend their thinking, or pass out another situation card not being solved by another group.

4. When all groups are finished, regroup students in threes with each person from a different original group. Have them take turns sharing their situation and equation then allowing the other students in the group to solve the problem using the equation. Allow enough time for all students to share and solve.

Again regroup students into threes and instruct them to create a situation of their own. They must have a variable and their equation must be two steps.

Extension Exercises: If students need additional practice, assign them the following scenarios.

1. Mr. White drives 55 km a day for work. How many km will he drive in:

a. 5 days?

b. 8 days?

c. 15 days?

d. Write an expression to represent the number of km he will drive in d days

2. Sean's father is working on a crew that will build a skyscraper. He found out that each story is 13 ft tall. How tall, in feet, would the skyscraper be if it were:

a. 55 stories?

b. 65 stories?

c. 75 stories?

d. Write an expression to represent the height of a skyscraper with f stories

3. 55 figurines of a porcelain doll can be safely shipped in a case. A distributor is investigating to find which size box is the safest to hold the largest number of cases. How many figurines could be shipped in a box that could hold:

a. 750 cases?

b. 1000 cases?

c. 1250 cases?

d. Write an expression to represent the number of figurines can be shipped in a box that holds c cases

4. The rental fee for a bike is $10 plus $3 for each hour the bike is used. How much will it cost if you rent the bike for:

a. 1 hour?

b. 8 hours?

c. 1 day?

d. Write an expression that represents the cost for h hours

5. A wireless service provider charges $29.99 per month for service plus $0.10 for each text message. How much will it cost if:

a. 35 text messages are sent?

b. 105 text messages are sent?

c. 217 text messages are sent?

d. Write an expression to represent the cost if t text messages are sent

Summative Assessment: Busy Day (1 day)

Resources and Teaching Tips

- A variety of resources are included (texts, print, media, web links)
- Help in identifying and correcting student misunderstandings and weaknesses

Common Student Misunderstandings:

-PEMDAS leads to the misconceptions that addition always precedes subtraction and multiplication always precedes division.

Clarifications to make to address weaknesses in student performance:

-Add or subtract, in order from left to right; if subtraction comes before addition in order from left to right, subtract before you add. Multiply or divide, in order from left to right. If division comes before multiplication in order from left to right, divide before you multiply

Throughout the unit, students have access to a variety of resources.

Textbook

Smartboard and Projector

Graphic Organizer Paper

Reference Sheets

Additionally, each formative check for understanding provides an opportunity for the students to identify their own misunderstandings and weaknesses in mastering the content. Guide the students in understanding how to use these formative checks to enhance their learning through the use of rubrics, conferences, and remediation or acceleration plans designed with student input (Appendix Q). While working towards mastery of grade level content standards, continuously refer students back to the standards for mathematical practices. These practice standards are the attitudes and skills that will allow students to achieve mastery of content standards, as well as deep understanding of mathematics.

Differentiation

- Stage 2 and 3 allow students to demonstrate understanding with choices, options, and/or variety in the products and performances without compromising the expectations of the Content Standards.
- Instruction is varied to address differences in readiness, interest, and/or learning profiles.
- Accommodations and differentiation strategies are incorporated in the design of Stage 2 and 3.

☐ Refer to the IEPs of your special education students

☐ Allow students to have tests, quizzes, worksheets, etc read to them.

☐ Allow the use of a calculator

☐ Communicate with the special education teacher about allowing testing in the special education room

☐ Pair and group students with differing ability levels.

☐ Use manipulatives as needed

☐ Allow for extra time on tests, quizzes, and performance tasks

☐ Allow for a word bank and a formula sheet on tests

☐ Allow students to choose to demonstrate understand in written or verbal format as needed.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.
- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning** – the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

(Briefly explain how design principle(s) are embedded within the unit design.)

In alignment with the Common Core State Standards, this unit pushes students to develop 21st Century Skills to meet the learning demands. Students come together as a learning community to analyze real-world applications of the math they are learning, think critically and reflect on the work of their peers as well as their own, and constantly work to apply their knowledge to new, more complex situations. This push towards using developed skills in new ways is an accurate reflection of the higher order thinking required by 21st Century jobs. Students should become excited and engaged by the "hands-off" approach of the teacher. Rather than giving students the formulas and a calculator and having them compute numbers and spit out answers, the teacher should present numbers and problems in a real world context, allowing the students to select the tools, resources, and knowledge needed to arrive at a reasonable answer. This push towards applying mathematical ways of thinking to real world issues will prepare them for the type of work they will encounter in high school, college, and in their careers.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- **8th Grade Technology Literacy** - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century(SETDA, 2003).

In alignment with Mathematical Practice 5 - Use appropriate tools strategically, students will have access to calculators as needed to complete the Performance Tasks. Additionally, when sharing their answers to the Performance Tasks, the teacher may opt to have the students construct a powerpoint presentation as an additional layer of the project.

Content Connections

Content Standards integrated within instructional strategies

Connections to other content areas are made both explicitly and implicitly throughout this unit. Students will complete tasks, outlining the connection between elements of mapping, finances, travel, and math. Additional connections are encouraged at the teacher's discretion. For example, when assigning the students additional practice work, make the work meaningful by aligning the examples with content currently being covered in other classes such as Science, English, and Social Studies.

A

Illustrative Mathematics

6.EE Busy Day

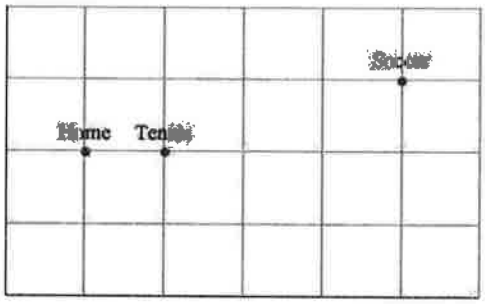
Alignment 1: 6.EE.B

The Jonas family had a really busy day. After leaving their home, Mrs. Jonas dropped son Cody at his tennis practice. She then drove daughter Kristin to her soccer game and stayed to watch. After the game, mother and daughter picked up Cody on the way home. Once home, Mrs. Jonas saw that they had driven 20 miles that day. How far are the tennis courts from home?

a. Use any method you like to find the distance between the tennis courts and home.

b. Set up and solve an equation in one variable to find the distance between the tennis courts and home.

Note: The family car can only travel along the main streets (gridlines), and all distances between cross streets are the same distance. Assume Mrs. Jonas took the most direct routes to and from her destinations.



A

Commentary:
This problem can be solved using simple arithmetical reasoning but is also naturally represented by a simple equation. Thus, it provides a good entry point for students into representing quantities in contexts with variables and expressions and building equations that reflect the relationships presented in the context.

In order for this task to fulfill its instructional promise, students are first asked to solve the problem themselves in any way they see fit. They could then explain their solution strategy to a partner, giving everyone a chance to articulate their reasoning. The teachers might circulate around the room and ask students with different ways of representing the problem to present their approach to the class, starting with the least abstract and ending with the most abstract. If no students use an equation to represent the problem in the first part, they are specifically asked to set up and solve an equation in the second part of the task. Finally, students should be given opportunities to work on problem contexts such as 6.EE Morning Walk where an algebraic approach is both grade-appropriate and clearly more efficient. This will help them see the value of a symbolic approach.

One advantage to using this task to introduce the idea of a variable is that the unknown has a natural visual representation, namely the length of the segments between gridpoints, in particular, the distance between home and the tennis courts.

An extension to this task could be to ask the students: What other questions could you ask about this situation? Some possibilities are:

- How far is it from the tennis courts to the soccer field?
- How many different routes can Mrs. Jonas take on her trip?
- How long does the trip take if we assume an average speed of 35 miles per hour?

Solution: Using an equation

a. There are 5 "blocks" along Mrs. Jonas' route to the soccer field, and she followed the route twice (there and back). So she went a total of 10 "blocks." Since she drove 20 miles, each block is $20 \div 10 = 2$ miles.

b. We want to solve for the distance between home and the tennis courts, so we let d be this distance. From the map, we can keep track of Mrs. Jonas' movements, noticing she covers d miles for each block she travels. Her driving distance can be broken down as follows:

   d - from home to tennis

   $4d$ - from tennis to soccer

   $4d$ - from soccer to tennis

   d - from tennis to home,

   for a total distance of $10d$. Alternatively, this can be thought of as $5d$ from home to soccer and then $5d$ from soccer to home, since the tennis courts are en route between home and soccer. As she drove a total of 20 miles, we have

$$10d = 20$$
$$d = 2$$

   Thus, the tennis courts are 2 miles from home.

Student Name_____ Evaluation Date_____ Overall Score_____

Mathematical Practice	Novice (1)	Apprentice (2)	Practitioner (3)	Expert (4)
1. Make sense of problems & persevere in solving them	I couldn't get started. I don't know how to begin. I may be feeling frustrated and wanting to give up.	I am stuck. I have part of the solution, but now I don't know what to do, I'm not sure my answer is right and I could use some help. I may be feeling frustrated.	I planned out how to solve this problem and I worked all parts of the problem. I may have felt challenged, but I didn't want to give up-I kept going until the problem was solved.	My solution is effective and inventive! I planned out how to solve this problem and I even showed you other ways to solve it. I checked to make sure my answer was right – I enjoyed this challenge!
2. Reason abstractly & quantitatively	I can't explain how I solved this problem. I don't know what quantities or equations to use. My work on this problem is built on misunderstandings of mathematical ideas.	I'm trying to explain how I solved this problem. My work on this problem may be built on misunderstandings of mathematical ideas. . I'm trying to take numbers out of the problem to work with them. I'm trying to check my work.	I explained how to solve this problem using formal math language. I took numbers out of the problem to work with them, and periodically put them back into the context of the problem to see if my work makes sense.	I clearly explained how to solve this problem using formal math language. I took quantities out of the context of the problem to work with them, and frequently put them back into the context of the problem to see if my work made sense.
3. Construct viable arguments & critique the reasoning of others	I can't describe how I solved the problem. I can't give feedback on someone else's reasoning OR my feedback in unclear, unhelpful, and/or incorrect.	I'm trying to describe how I solved the problem. I may or may not include pictures, tables, graphs, and/or models. I gave feedback on someone else's reasoning, but my feedback may have been unclear, unhelpful, or I may have been unable to back it up with my own work.	I described how I solved the problem using formal math language, pictures, tables, graphs, and/or models. I gave clear, helpful feedback to someone else's reasoning and backed up my feedback with my own work on the problem.	I clearly described how I solved the problem. I included every step so you don't have to guess what I did. I used formal math language, pictures, tables, graphs, and/or models. I gave clear, helpful feedback on someone else's reasoning and backed up my feedback with my own work on the problem.
4. Model with mathematics	I can't use mathematical representations, I don't know what to use for this problem and I can't describe how I would solve this problem.	I'm trying to use mathematical representations to solve the problem and show my solutions, but they may be inaccurate or inappropriate. I'm sharing my ideas, but I'm struggling to use formal math language.	I used appropriate & accurate mathematical representations to solve the problem and show my solutions. I used formal math language to share my ideas.	I used a variety appropriate, accurate, and detailed mathematical representations to solve the problem and show my solutions. I used formal math language to share my ideas.
5. Use appropriate	I don't know what tool to use to solve this problem and/or I	I picked an appropriate tool to use, but I may be struggling to	I picked the correct tool to use based on the needs of the	I selected and used a variety of appropriate tools

Student Name_____ Evaluation Date_____ Overall Score_____

tools strategically	didn't use the tool I picked in a valuable way.	use it in a valuable way to solve the problem.	problem. I planned and monitored my use of the tool.	collaboratively to solve the problem. I planned my use of the tools and monitored my progress.
6. Attend to precision	I can't get the correct answer and/or I can't communicate my answer in a way that is appropriate to the problem. I can't use correct units, labels, or definitions.	I may be getting the correct answer and/or I may be able to communicate my answer is a way that is appropriate to the problem. I may not use correct units, labels, or definitions.	I got the correct answer and I can communicate it using the correct labels, units, and definitions. The precision of my answer is appropriate for the problem.	I got the correct answer and I communicated it using correct labels, units, and definitions. I clearly stated the meaning of the symbols I chose and my answer is complete appropriate for the problem.
7. Look for & make use of structure	I don't see any structure in the problem. I can't make sense of any useful structures/patterns to help me solve the problem.	I may be able to find for structures/patterns in the problem, but I am struggling to use them to help me find a solution.	I found and used structures/patterns in the problem to help me find a solution. I noted these structures/patterns as I found them.	I found, noted, and used structures/patterns in the problem to help me find a solution. I was able to step back from the problem and analyze different perspectives.
8. Look for & express regularity in repeated reasoning	I don't see any regularity or any shortcuts or generalizations I can use to get a solution.	I may notice that some of my calculations are repeated but I may not be able to generalize that observation to help me get a solution.	I noticed some of my calculations were repeating and was able to make a generalization that helped me solve the problem. I evaluated the reasonableness of my work as I progressed.	I noticed some of my calculations were repeating and was able to make a generalization or a formula that helped me solve the problem. I continuously evaluated the reasonableness of my work as I progressed.

C

Balancing Expressions

NAME _____

Go to the Activities section of the Illuminations web site (http://illuminations.nctm.org), and access the *Pan Balance – Numbers* activity.

Enter the following expressions in the left pan. Using the order of operations, and *one step at a time*, enter an equivalent expression in the right pan. When the pans balance, you are correct, and the equivalent expressions are shown under Balanced Equations. Record the expressions below.

For example, put $(6 + 3) \div 9$ in the left pan. Then, enter $9 \div 9$ in the right pan:

$$\begin{aligned}(6 + 3) \div 9 &= 9 \div 9 \\ &= 1\end{aligned}$$

If the pans don't balance, make changes until they do. The computer is checking your work, so continue until it is correct. Be sure to record every step that the computer records.

Note that the computer uses * for multiplication and / for division. Press Reset Balance before starting a new problem.

1. $18 \div (5 + 4)$

2. $(5 - 2) \div 3$

3. $16 - 5 \times 2$

4. $(5 + 7) \div (1 + 3)$

5. $5 + 7 \div 1 + 3$

6. $24 \div (1 + 5) + 2$

D

Order of Operations Bingo

Cut into strips, and share just the expressions with the class.

Expression	Answer
$(5\times5)\div(5\times5)$	1
$7\div7+7\div7$	2
$1+1+1\times1$	3
$2+3+4-5$	4
$3\cdot2-1$	5
$2\times4+6-8$	6
$1+2+1\times2+1\times2$	7
$2\times2+2\times2$	8
$3\times3\times3\div3$	9
$1+2+3+4$	10
$2\times3\times4-6-7$	11
$5\times4\times3\div5$	12
$((1+2)\times3)+4$	13

Expression	Answer
$2\times3+4\times5$	26
$4+5\times6-7$	27
$2\times2\times(3+4)$	28
$(3+4\cdot5)+6$	29
$9\cdot8-7\cdot6$	30
$3\times3\times3+4$	31
$4\times4+4\times4$	32
$7+6\times5-4$	33
$10\times9-8\times7$	34
$5+5+5\times5$	35
$(3+3)\times2\times3$	36
$5\times6+7$	37
$(5\cdot4\cdot4-4)\div2$	38

D

$1\times2+3\times4$	14
$5+4+3\times2$	15
$(1+2)\times(3+4)-5$	16
$8+8+8\div8$	17
$6\cdot5-4\cdot3$	18
$4\times5+6-7$	19
$2+((3+4)+5)+6$	20
$1+(2+3)\times4$	21
$6\times7-5\times4$	22
$6+(5\times4)-3$	23
$2\times3\times4$	24
$(1+4)\times(4+1)$	25

$3\times(4+(4+5))$	39
$(4+3+2+1)\times4$	40
$6\times7+8-9$	41
$2\times3\times(3+4)$	42
$5+6+8\times4$	43
$4\times(3\times2\times2-1)$	44
$8+7\times6-5$	45
$2\cdot(3+4\cdot5)$	46
$2\cdot3+4+5\cdot6+7$	47
$54-3\times2$	48
$7\times7\times6\div6$	49
$8+7\times6$	50

D

B (1 – 10)	I (11 – 20)	N (21 – 30)	G (31 – 40)	O (41 – 50)

B (1 – 10)	I (11 – 20)	N (21 – 30)	G (31 – 40)	O (41 – 50)

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Who is Correct?

We need help determining the cost of new floor tiles for our bathroom. Marble tiles cost $7 each, and granite tiles cost $5. We need 40 marble tiles, 15 granite tiles, plus a bag of adhesive that costs $10. Can you write a single mathematical sentence to determine this cost?

Mark wrote: $10 + (7 + 5) \times (40 + 15)$
Doug wrote: $10 + (7 \times 40) + (5 \times 15)$
Andy wrote: $10 + 7 \times 40 + 5 \times 15$

1. Which equation could be used to determine the correct total cost? Show how you know.

2. Andy went on to demonstrate how he knew that he was correct.

$$\begin{aligned} \mathbf{10 + 7 \times 40 + 5 \times 15} &= \mathbf{17 \times 40 + 5 \times 15} \\ &= \mathbf{680 + 75} \\ &= \mathbf{755} \end{aligned}$$

What error did he make? How can you help him correct his mistake?

F

Illustrative Mathematics

6.EE Watch out for Parentheses

Alignment 1: 6.EE.A

Evaluate the following numerical expressions.

a. $2(5 + (3)(2) + 4)$

b. $2((5 + 3)(2 + 4))$

c. $2(5 + 3(2 + 4))$

Can the parentheses in any of these expressions be removed without changing the value the expression?

F

Commentary:
This problem asks the student to evaluate three numerical expressions that contain the same integers yet have differing results due to placement of parentheses. Students from all levels of mathematics often make an error on (c) by first doing the operation of $5 + 3$. Parts (a) and (b) help the student recognize this error.

This type of problem helps students to see structure in expressions and represents an important transition from the work that students do in elementary school with numeric expressions to the work they will be doing in middle school with algebraic expressions. Note that this task was originally written to illustrate 5.OA.1, but an alert reader pointed out that the K–5 Operations and Algebraic Thinking Progression explicitly excludes numeric expressions with nested parentheses in 5th grade. Thus, this task would be appropriate for 5th graders needing an extra challenge or 6th graders getting ready to make the transition to working with expressions that contain variables.

Solution: Evaluating expressions

a. $2(5 + (3)(2) + 4)$ We may evaluate this expression in two ways:

Distributing the lead constant first:

$$2 \cdot 5 + 2 \cdot 3 \cdot 2 + 2 \cdot 4 = 10 + 12 + 8 = 30$$

or distributing the lead constant last:

$$2(5 + 6 + 4) = 2 \cdot 15 = 30.$$

Either way, we first have to multiply $(3)(2) = 6$ before adding any of the terms. The parentheses in the middle are not necessary. Instead of writing $(3)(2)$ we can say $3 \cdot 2$.

b. Notice that in the expression $2((5 + 3)(2 + 4))$ the outer set of parentheses are not necessary:

$$2((5 + 3)(2 + 4)) = 2(5 + 3)(2 + 4).$$

The other parentheses are necessary since they indicate that we first have to perform the additions inside these parentheses:

$$2(5 + 3)(2 + 4) = 2(8)(6) = 96.$$

c. In this expression we complete the operations from the inside out. The inner most addition must occur first, then the inner multiplication, then the secondary addition and finally the outer multiplication:

$$2(5 + 3(2 + 4)) = 2(5 + 3(6)) = 2(5 + 18) = 2(23) = 46.$$

In this expression all parentheses are needed.

Variables

A variable is a symbol that represents a number. Usually we use letters such as n, t, or x for variables. For example, we might say that s stands for the side-length of a square. We now treat s as if it were a number we could use. The perimeter of the square is given by $4 \times s$. The area of the square is given by $s \times s$. When working with variables, it can be helpful to use a letter that will remind you of what the variable stands for: let n be the number of people in a movie theater; let t be the time it takes to travel somewhere; let d be the distance from my house to the park.

Expressions

An expression is a mathematical term or a sum or difference of mathematical terms that may use numbers, variables, or both.

Example: The following are examples of expressions:

2

x

$3 + 7$

$2 \times y + 5$

$2 + 6 \times (4 - 2)$

$z + 3 \times (8 - z)$

Example: Roland weighs 70 kilograms, and Mark weighs k kilograms. Write an expression for their combined weight. The combined weight in kilograms of these two people is the sum of their weights, which is $70 + k$.

Example: A car travels down the freeway at 55 kilometers per hour. Write an expression for the distance the car will have traveled after h hours. Distance equals rate times time, so the distance traveled is equal to $55 \times h$..

Example: There are 2000 liters of water in a swimming pool. Water is filling the pool at the rate of 100 liters per minute. Write an expression for the amount of water, in liters, in the swimming pool after m minutes. The amount of water added to the pool after m minutes will be 100 liters per minute times m, or $100 \times m$. Since we started with 2000 liters of water in the pool, we add this to the amount of water added to the pool to get the expression $100 \times m + 2000$.

To evaluate an expression at some number means we replace a variable in an expression with the number, and simplify the expression.

http://www.mathleague.com/help/algebra/algebra.htm#expressions

Example: Evaluate the expression $4 \times z + 12$ when $z = 15$.

We replace each occurrence of z with the number 15, and simplify using the usual rules: parentheses first, then exponents, multiplication and division, then addition and subtraction.

$4 \times z + 12$ becomes

$4 \times 15 + 12 =$

$60 + 12 =$

72

Example: Evaluate the expression $(1 + z) \times 2 + 12 \div 3 - z$ when $z = 4$.

We replace each occurrence of z with the number 4, and simplify using the usual rules: parentheses first, then exponents, multiplication and division, then addition and subtraction.

$(1 + z) \times 2 + 12 \div 3 - z$ becomes

$(1 + 4) \times 2 + 12 \div 3 - 4 =$

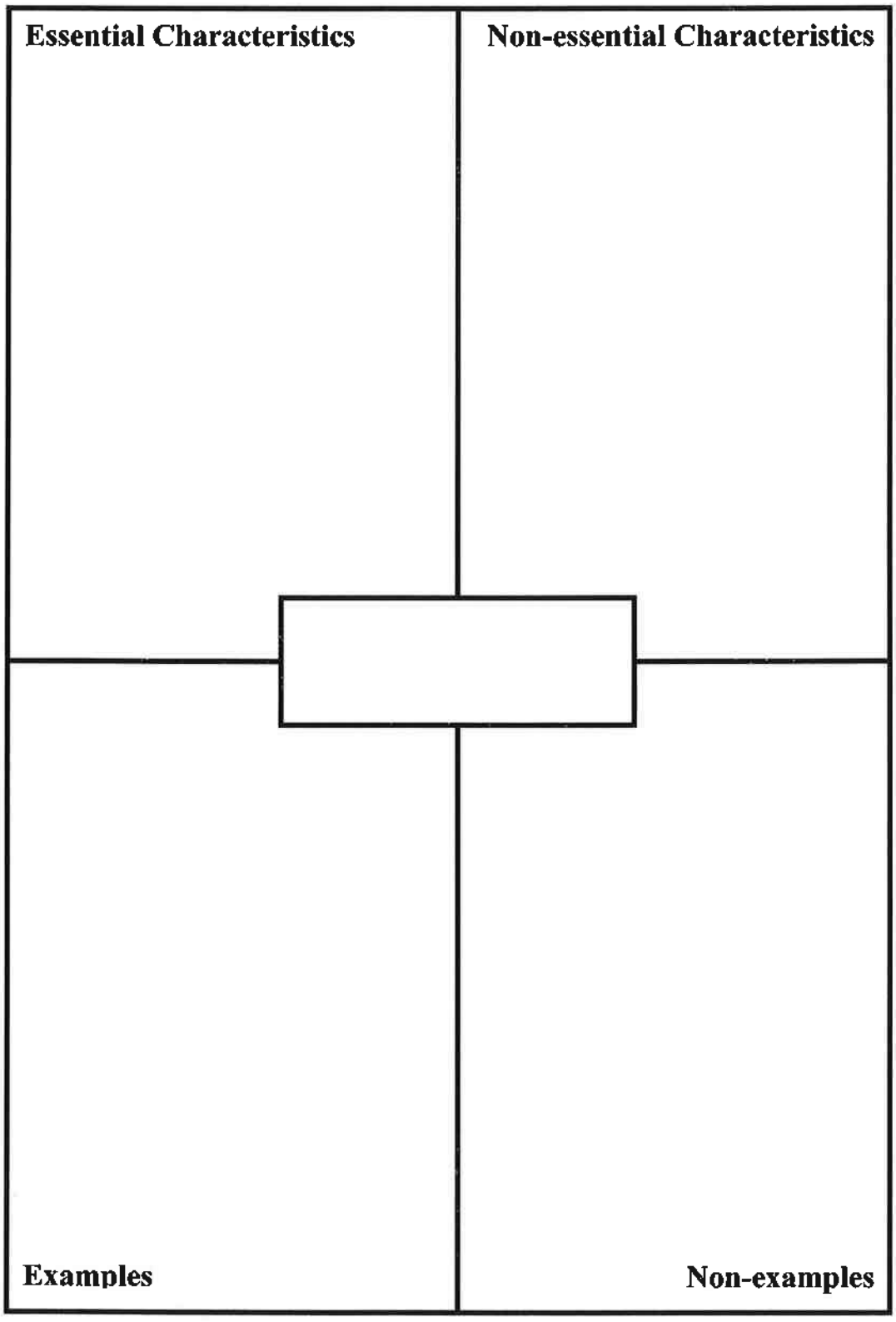
$5 \times 2 + 12 \div 3 - 4 =$

$10 + 4 - 4 =$

10.

H

Frayer Model



Task: Equations and their Solutions

Teacher: Cut the following pieces out and place them in an envelope. Each group should receive an envelope.

$8 + x = 12$	$x = 4$
$\frac{x}{4} = 2$	$x = 8$
$6x = 18$	$x = 3$
$x - 4 = 2$	$x = 6$
$14 = 7x$	$x = 2$
$2 = \frac{x}{6}$	$x = 12$
$\frac{35}{x} = 7$	$x = 5$
$x - 7 = 4$	$x = 11$
$8x = 72$	$x = 9$
$2x = 26$	$x = 13$

Round I

Name: _____ **Date:** _____

Task: Equations and their Solutions

Directions: Your teacher will give your group an envelope with 10 equations and 10 solutions. Pair each equation with its solution. Once you have paired up the equations with their solution, choose five equations/solutions to answer the following questions.

1) Equation 1: _____
Solution : _____
What strategy or strategies did you use to find the solution?

_____

_____

How can you prove that the solution you chose is the correct answer?

_____

_____

2) Equation 2: _____
Solution : _____
What strategy or strategies did you use to find the solution?

_____

_____

How can you prove that the solution you chose is the correct answer?

_____

_____

3) Equation 3: _____
Solution : _____
What strategy or strategies did you use to find the solution?

_____

_____

How can you prove that the solution you chose is the correct answer?

_____

_____

4) Equation 4: _____
Solution : _____
What strategy or strategies did you use to find the solution?

_____

_____

How can you prove that the solution you chose is the correct answer?

_____

_____

5) Equation 5: _____
Solution : _____
What strategy or strategies did you use to find the solution?

_____

_____

How can you prove that the solution you chose is the correct answer?

_____

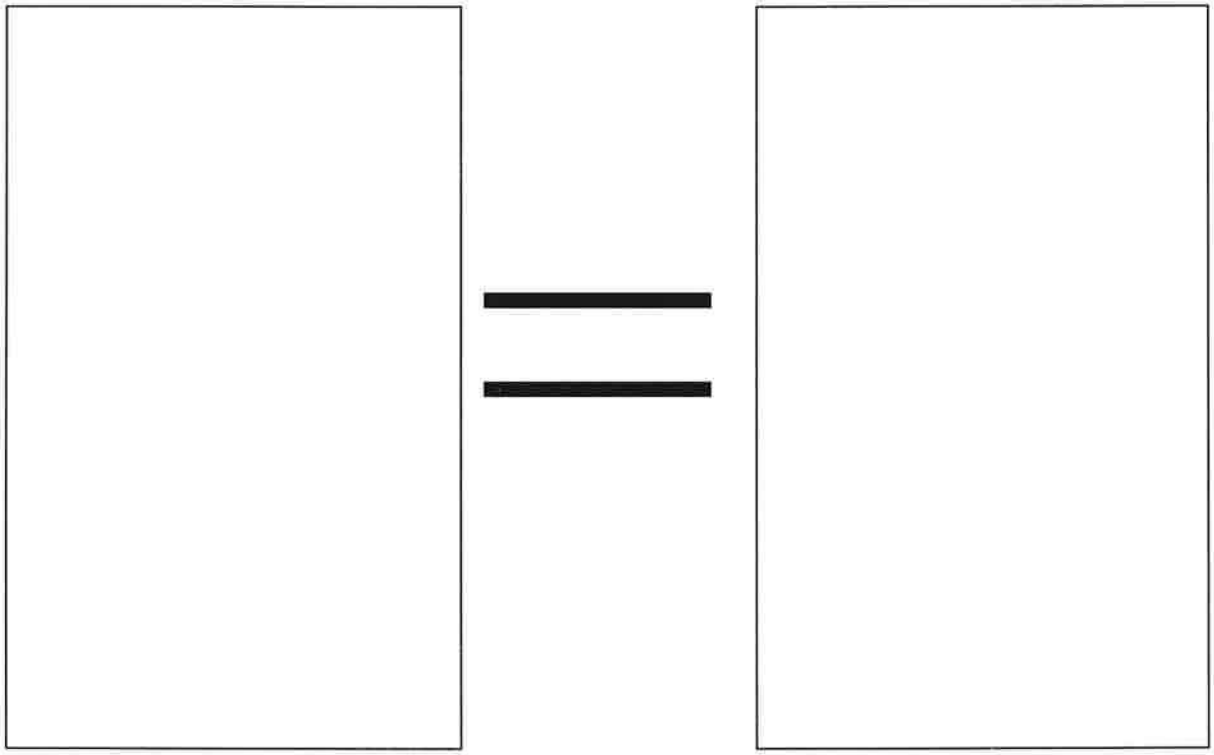
_____

Name: _____ Date: _____

Why does x = 2?

Draw your chips and cups, demonstrating the equation...

$5x + 2 = 12$



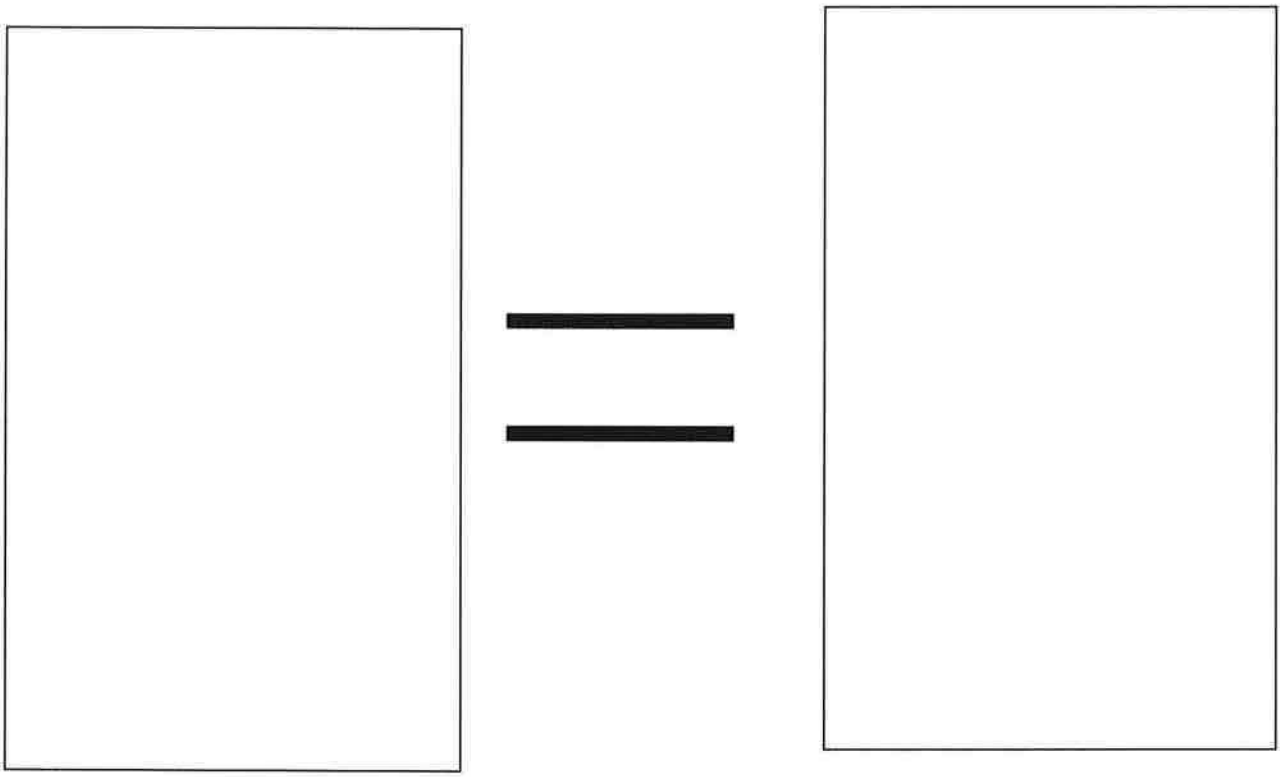
WRITE what you need to do to make it look like...

↓

$5x = 10$?

Draw your chips and cups, demonstrating the equation...

$5x = 10$



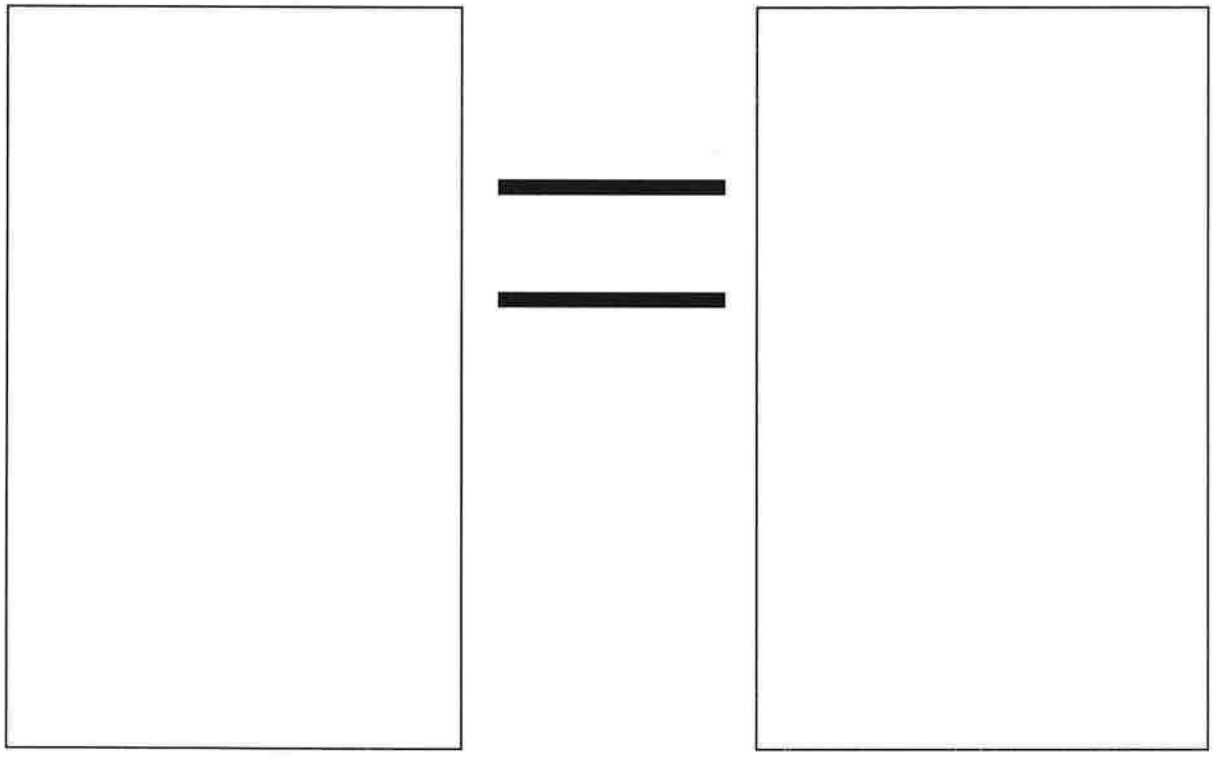
Write what you need to do to make it look like ...

$X = 2$

J

Draw your chips and cups, demonstrating the equation...

$X = 2$



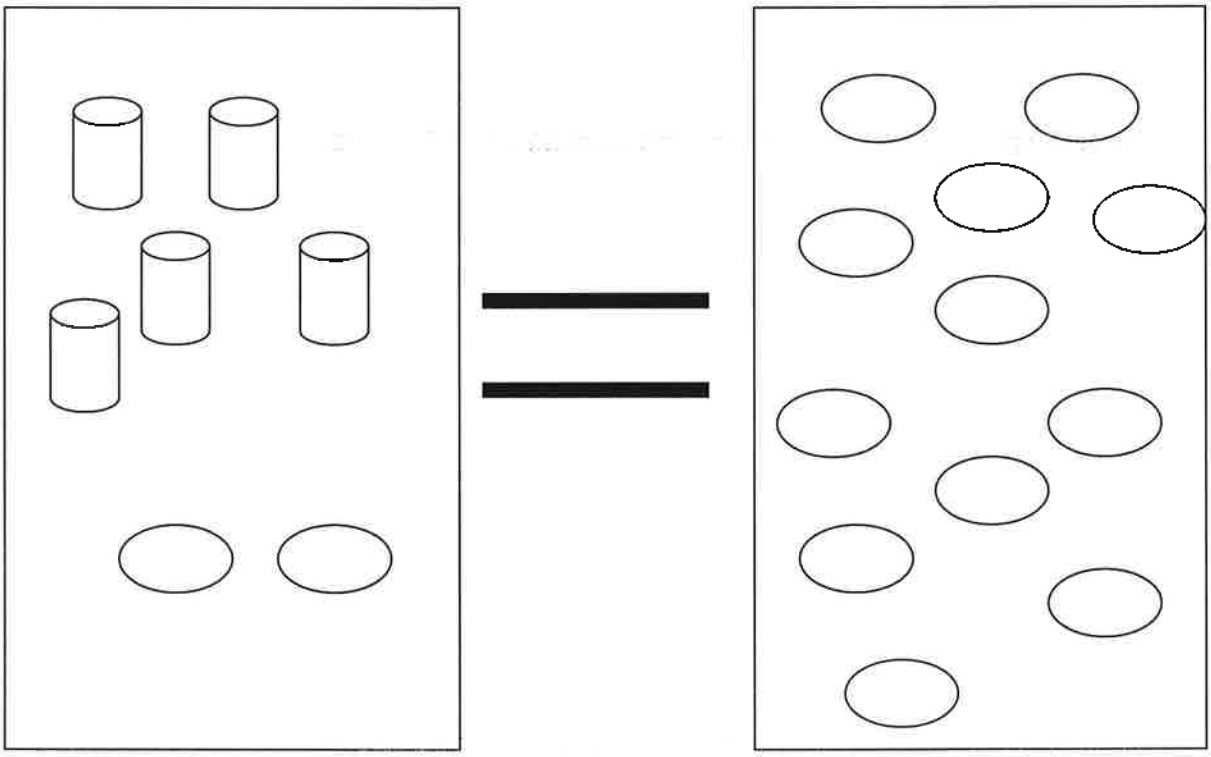
J

Name: _____KEY_____ Date: _____

Why does x = 2?

Draw your chips and cups, demonstrating the equation...

$5x + 2 = 12$



WRITE what you need to do to make it look like...

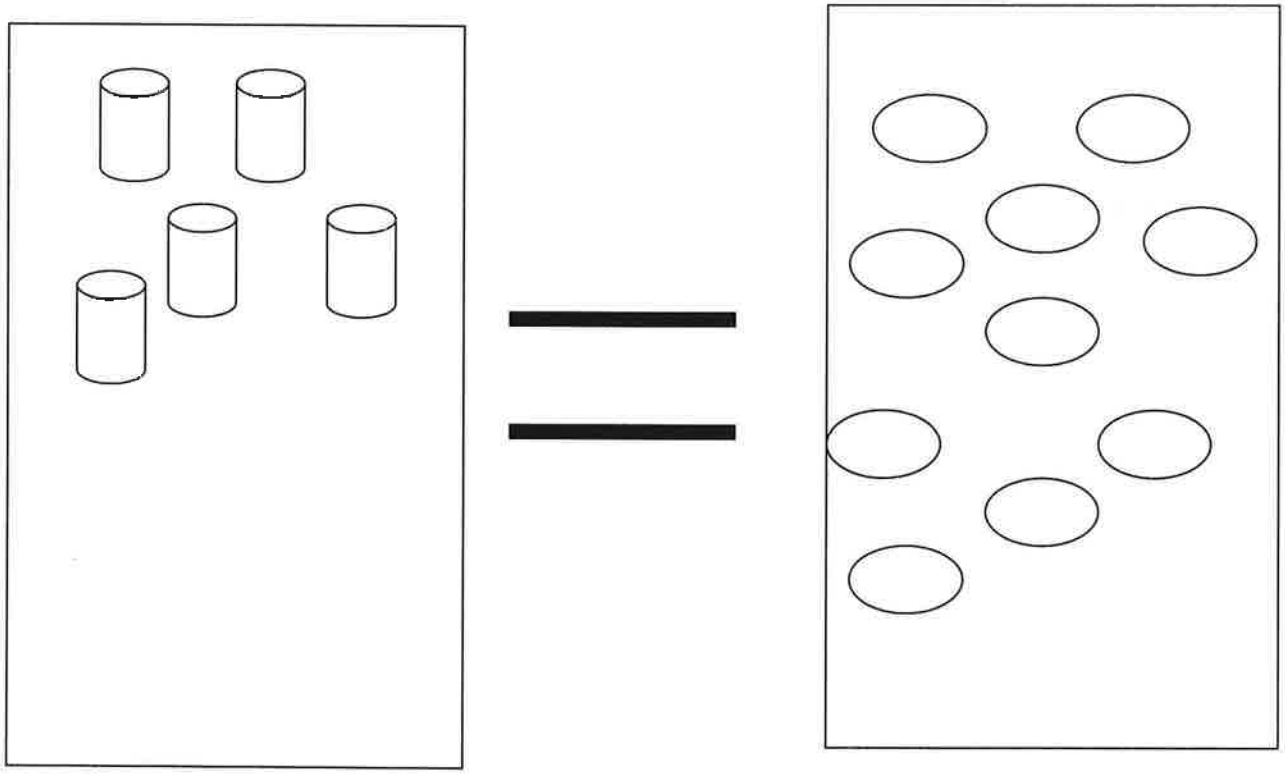
-2 ↓ -2

$5x = 10$?

J

Draw your chips and cups, demonstrating the equation...

$5x = 10$



Write what you need to do to make it look like ...

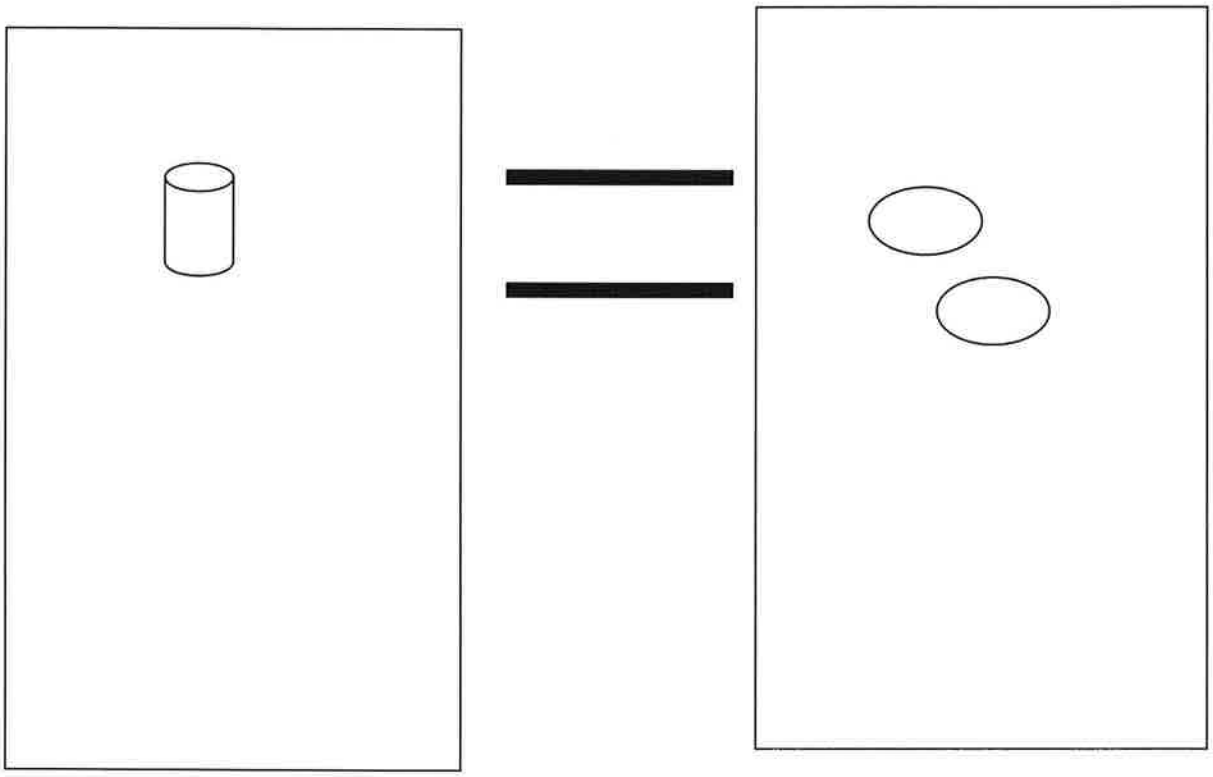
$\div 5$ ↓ $\div 5$

$X = 2$

J

Draw your chips and cups, demonstrating the equation...

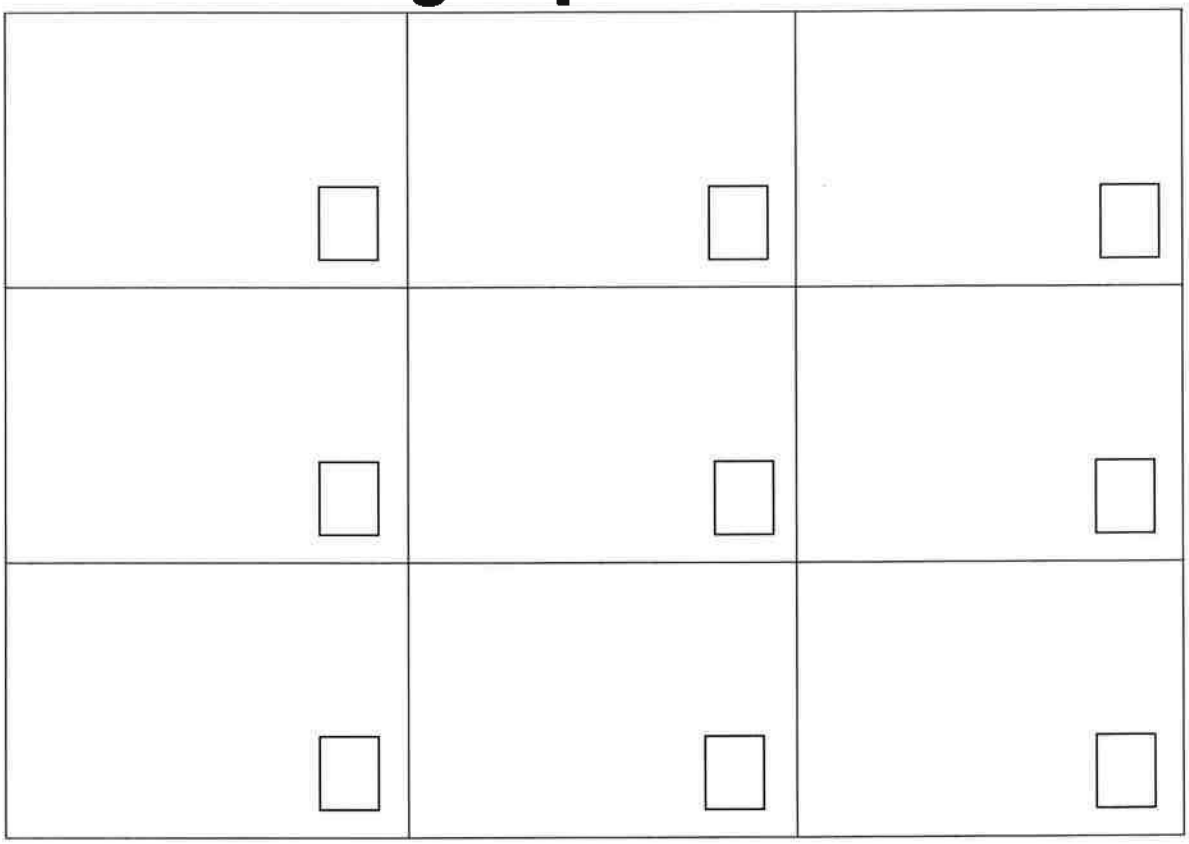
$X = 2$



K

Name_____

Evaluating Expressions BINGO



First: Arrange these numbers in any order the little boxes in each square. Use only one number per square please. **-12, -9, 0, 1, 3, 4, 8, 10, 20**

Next: Evaluate each expression below. Show how you substituted the value given. Write the expressions in the squares above where you have placed their values.

3b, when b = 1

3b, when b = - 4

3b, when b = 0

2m + 3, when m = - 6

2m + 3, when m = - 1

2m + 3, when m = ½

2(5 – *X*), when X = 0

2(5 – *X*), when X = 1

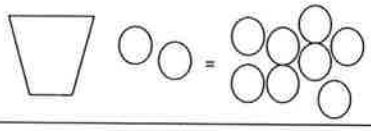
2(5 – *X*), when X = - 5

L

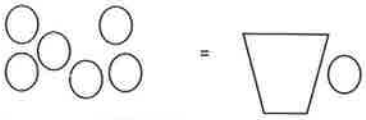
Name: _____ **Date:** _____

Task: Cups, Equations, and Solutions

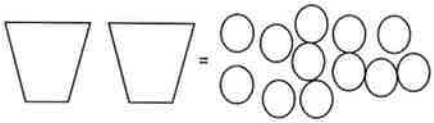
Directions: Write an equation for each picture. Once you have an equation written, find the number of counters (circles)that will be in each cup in order to make the equation true. If an equation has more than one cup, each cup will have the same number of counters.

1) 

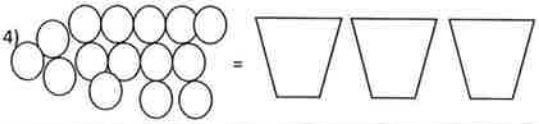
Equation: _____
Solution: _____

2) 

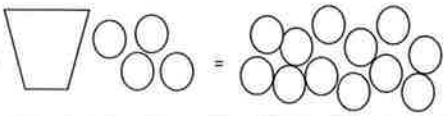
Equation: _____
Solution: _____

3) 

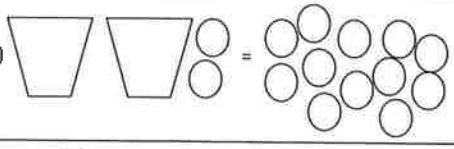
Equation: _____
Solution: _____

4) 

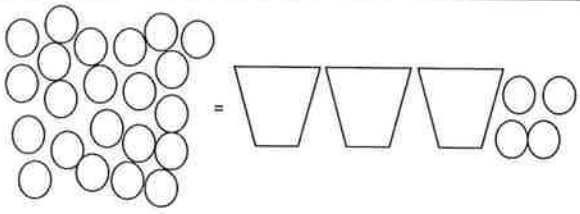
Equation: _____
Solution: _____

5) 

Equation: _____
Solution: _____

6) 

Equation: _____
Solution: _____

7) 

Equation: _____
Solution: _____

M

Algebraic Expressions For Words **Name**_____

Words for Operations: **+, —, x, ÷, power**

+	—	x	÷

Write an algebraic expression for:

1. a number, *c*, increased by 5
2. the quotient of *f* and 4
3. twice 9 decreased by 8
4. the difference between *r* and 2
5. 6 less than a number, *w*
6. the product of q and 10

Write words for each

7. t + 9
8. -6b + 3
9. $\frac{x}{4}$
10. z — 8
11. 5y
12. -9 — s

$3x - 2$	$\frac{x}{3}$	$-3b$	$y \div -8$
$\frac{3x}{2}$	$\frac{4}{c}$	$\frac{m}{5}$	$-8w$
$-3 + f$	$5 + p$	$K + -8$	$2n + 4$
$3y$	$2n$	$2f - 2$	$p - 4$
$5b$	$V - (-3)$	$t - 3$	$5 - p$
$2 - c$	$3 - d$	$-8 + h$	$m + 4$

The quotient of y and -8	The product negative three and b	The quotient of x and 3	two less than three times a number
a number, w, multiplied by -8	the quotient of m and 5	four divided by c	Divide the product of 3 and x by 2
double n and add four	the number, k, plus -8	Five more than p	negative three and f
four less than the number, p	two less than twice f	twice a number, n	three times y
p subtracted from five	a number, t, decreased by three	v minus negative three	five of the number, b
four more than m	the sum of negative 8 and h	the difference between d and three	Two decreased by a number, c

Name: _____ **Date:** _____

Task: Equations in the Real-World

Directions: Below you will find a list of four real-world situations. Using the equations below, write the equation that correctly matches the situation and answer the questions.

$\frac{x}{6} = 18$ $x - 6 = 18$ $6x = 18$ $x + 6 = 18$

1) In 6 years, Rosario will be 18 years old. How old is she now?

Equation: _____
Solution: _____
What strategy did you use to find your equation and your solution?

_____
_____
_____
_____

2) When the students at Essex Middle School are separated into 6 equal groups, there are 18 students in each group. How many students are at the school?

Equation: _____
Solution: _____
What strategy did you use to find your equation and your solution?

_____
_____
_____
_____

3) Mike bought some books that cost $6 each. He spent a total of $18. How many books did he buy?

Equation: _____
Solution: _____
What strategy did you use to find your equation and your solution?

_____
_____
_____
_____

4) From 4:00 to 5:00 P.M., the temperature dropped 6˚F. At 5:00 P.M. the temperature was 18˚ F. What was the temperature at 4:00 P.M.?

Equation: _____
Solution: _____
What strategy did you use to find your equation and your solution?

_____
_____
_____
_____

P

Situation Cards

Situation Card #1

Brianna receives $7.50 per hour when she baby-sits the 3 Steigerwald children. If she baby-sits for 4 hours on Saturday night, write an equation to determine how much she will receive. What will she earn if she baby-sits for five and a half hours?

Situation Card #2

Kolleen and Dallin went ice-skating at Seven Peaks with Kolby and Stephanie on Saturday night. They had to pay for tickets, skate rental, and food. You know that the food and skate rental cost $12 total. You also know they spent $32 that evening. Write an equation to determine the cost of each ticket.

How much did each couple pay?

Situation Card #3

Tomorrow is your birthday. You want to bring a Snickers bar to each member of your class, including your teacher. If each candy bar is $.50 and your mom gave you $12.50 to spend, can you afford enough candy bars for your entire class? Write an equation and solve to find out.

Situation Card #4

Angie and her five siblings want to go to opening night of an upcoming movie. In order to avoid long lines, Angie decides to purchase their tickets on Fandango.com. Tickets are $8 each with a $1 handling fee for each ticket. Her dad gave her $50 to spend. How many of Angie's siblings can she afford to take to the movie? Write an equation and solve to find out.

Situation Card #5

The current temperature is 30 degrees Fahrenheit and is expected to rise 2 degrees per hour for the next several hours. Write an equation that represents the relationship between temperature and time.

After how many hours will the temperature be 55 degrees?

Situation Card #6

Elmbrook Middle School is planning a field trip for the 6th grade and needs to determine how many busses need to be reserved for that day. Your school has 175 6th grade students and 5 teachers. Each bus can hold a maximum of 84 people. Write an equation to determine how many busses the school needs to reserve.

If you divide the students and teacher evenly, how many people should go on each bus?

Situation Cards

Situation Card #7

Mr. Risch's class is sponsoring a walkathon to raise money for science supplies. Three students found sponsors who are willing to pledge the following amounts.

- Ashley's sponsors will pay $10 regardless of how far she walks.
- Caroline's sponsors will pay $3 per mile.
- Donnie's sponsors will make a $5 donation, plus $1 per mile.

After creating equations for each pledge plan, decide which plan will bring in the most money if all of the students in the class are planning on walking 8 miles.

Caroline decides to give a t-shirt to each of her sponsors. She is going to use some of the money she collects from her sponsors to cover the $5 cost of each shirt. If the class decides to use her pledge plan, write an equation to represent the amount of money Caroline will make from each sponsor after paying for the cost of the t-shirts.

Situation Card #8

Emily's mom is hiring a magician for her twelfth birthday party. She obtained several prices for the cost of three magicians.

The Great Cardini charges $75 an hour.

Dante Fantasio charges $100 plus $20 an hour.

Amazing Max charges $150 plus $30 an hour.

Write three equations to determine the cost of each magician for two and a half hours. Based on this information, which magician should Emily's mom hire?

Situation Card #9

Natalie wants to purchase an iPod. An electronics store offers two installment plans for buying the $250 version.

Plan A: A fixed weekly payment of $10.50.

Plan B: A $120 initial payment, followed by $5 per week.

After 12 weeks, how much money will Natalie owe on each plan? Write an equation and solve to find out.

Which plan requires the least number of weeks to pay off the iPod?

Situation Cards Key

Note: Students may represent a variable with any letter.

1. 7.50h = p where h = hours worked and p = payment.
   7.50 x 4 = $30 for 4 hours.
   7.50 x 5.5 = $41.25 for 5.5 hours.

2. 4p + 12 = 32 where p = price of tickets.
   p = $5 per ticket.
   5 x 2 = $10 per couple.

3. This answer will vary based on class size. The following uses a class size of 28.
   .5s = 12.50 where s = number of students.
   28 x .5 = 12.50
   Students should reason that since half of 28 is 14, they would be $2.50 short and therefore unable to buy each child a candy bar.

4. 8p + p = 50 OR 9p = 50 OR 8p + 1p = 50 where p = price of ticket.
   For 6 siblings, you would spend $54, which is $4 too expensive.
   For 5 siblings, you will spend $48, which leaves $4 left over.
   Angie can take 4 siblings with you, or let her 5 siblings go to the movie while she stays home. Even better, her dad may give her an extra $4!

5. 30 + 2h = t where h = hours and t = temperature.
   30 + 2h = 55. h = 12.5 hours

6. 84b = 180 where b = number of busses.
   180/84 = 2.14 busses, so you will need 3 busses for the field trip.
   180/3 busses = 60 people on each bus.

7. p = payment earned and m = miles walked

Ashley's plan p = 10.	For 8 miles, p = $10.
Caroline's plan p = 3m.	For 8 miles, p = $24.
Donnie's plan p = 5 + 1m.	For 8 miles, p = $13.

   Caroline's plan will bring in the most money.
   T-shirt equation using Caroline's plan: p = 3m – 5.

8. c = cost and h = hours worked.

   c = 75h. For 2.5 hours, c = $187.50.

   c = 100 + 20h For 2.5 hours, c = $150.00.

   c = 150 + 30h For 2.5 hours, c = $225.00.

   Emily's mom should hire Dante Fantasio for $150.00.

9. Plan A: 250 = 10.50w where w = weeks.

   Plan B: 250 = 120 + 5x.

   After 12 weeks, Natalie will have paid $126 on plan A and $180 on plan B, which means she still owes $124 on plan A and $70 on plan B.

   After working the two-step equation, it will take 23.8 (or about 24) weeks to pay off plan A and 26 weeks to pay off plan B. Plan A takes the least amount of weeks to pay off.

Student Name	**Teacher Name**
Progress Report Starting Grade:_____ Progress 1: Date_____: Grade:_____ Progress 2: Date_____: Grade:_____ Progress 3: Date_____: Grade:_____	**Parent Contact Log** Phone #_____ Date, Reason, Outcome 1 _____ 2._____ 3. _____ 4.._____ 5.._____
Requirements of Your Acceleration Plan:	**Agreement to follow this Acceleration Plan** Student Signature_____ Teacher Signature_____ Additional Comments (Student or Teacher):

Student Name	**Teacher Name**
Progress Report Starting Grade:_____ Progress 1: Date_____: Grade:_____ Progress 2: Date_____: Grade:_____ Progress 3: Date_____: Grade:_____	**Parent Contact Log – Contact if student is not complying with Improvement Plan** Phone #_____ Date, Reason, Outcome 1 _____ 2._____ 3. _____ 4.._____ 5.._____
Requirements of Your Improvement Plan: □ Attend afterschool tutoring on: Tues Weds □ Make-up Missing: □ Test □ Quiz □ Classwork □ Homework □ Other:_____	**Agreement to follow this Improvement Plan** Student Signature_____ Teacher Signature_____ Additional Comments (Student or Teacher):

R

Unit 1

Lesson 6: Translating Between Words and Math

In this lesson, you will learn how to translate between word phrases and mathematical expressions.

Lesson Overview

- **GET READY:** Matching
- **LEARN:** Going from Words to Expressions
- **WORKED EXAMPLES:** Going from Words to Expressions
- **LEARN:** Going from Expressions to Words
- **WORKED EXAMPLES:** Going from Expressions to Words

GET READY: Matching

Let's explore what you already know about matching operation symbols with words.

Drag the correct symbol to the words or phrases they represent.

sum	quotient	increased by
product	decreased by	difference

+ − • ÷

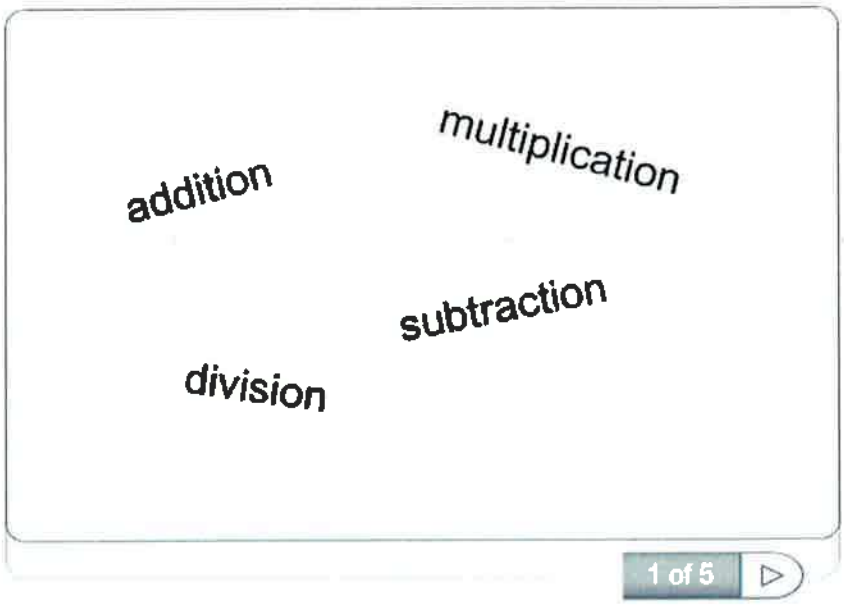
Check

Unit 1 | Lesson 6

LEARN: Going from Words to Expression

To solve an everyday problem, you'll need to translate words into math.

Certain words and phrases can tell you to use addition, subtraction, multiplication, or division.



Activity Resources

Unit 1 | Lesson 6

LEARN: Going from Words to Expressions

To solve an everyday problem, you'll need to translate words into math.

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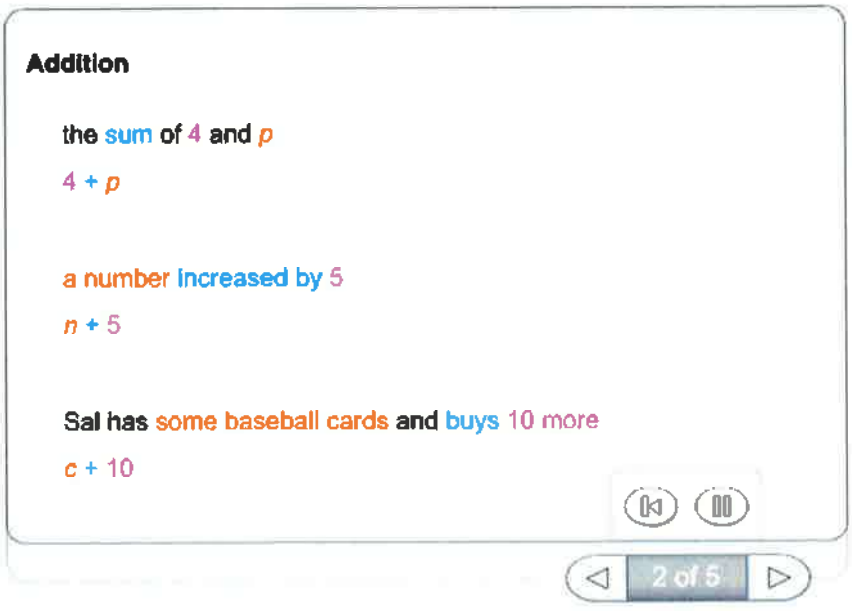


Activity Resources

Unit 1 | Lesson 6

LEARN: Going from Words to Expressions

In your Student Guide, summarize what you just learned:

1. What are some words in a word phrase that might tell you to write an expression involving addition?
2. What are some words in a word phrase that might tell you to write an expression involving subtraction?
3. What are some words in a word phrase that might tell you to write an expression involving multiplication?
4. What are some words in a word phrase that might tell you to write an expression involving division?

↑ Activity Resources

Unit 1 | Lesson 6

WORKED EXAMPLES: Going from Words to Expressions

Click each word or phrase in each statement to see the expression that matches it.

Statement	Expression
the sum of 10 and a number	
18 less than John's age	
Jackie separated her cookies onto 2 dishes.	

↑ Activity Resources

WORKED EXAMPLES: Going from Words to Expressions

Now practice writing expressions from word phrases.

Drag the numbers and symbols to write an expression that describes the word phrase.

10 less than a number

10 | *n* | + | − | = | • | ÷

Check

1 of 2

WORKED EXAMPLES: Going from Words to Expressions

Now see if you can identify which expression correctly translates the given word phrases.

Choose the expression that matches the word phrase.

the product of 8 and a number

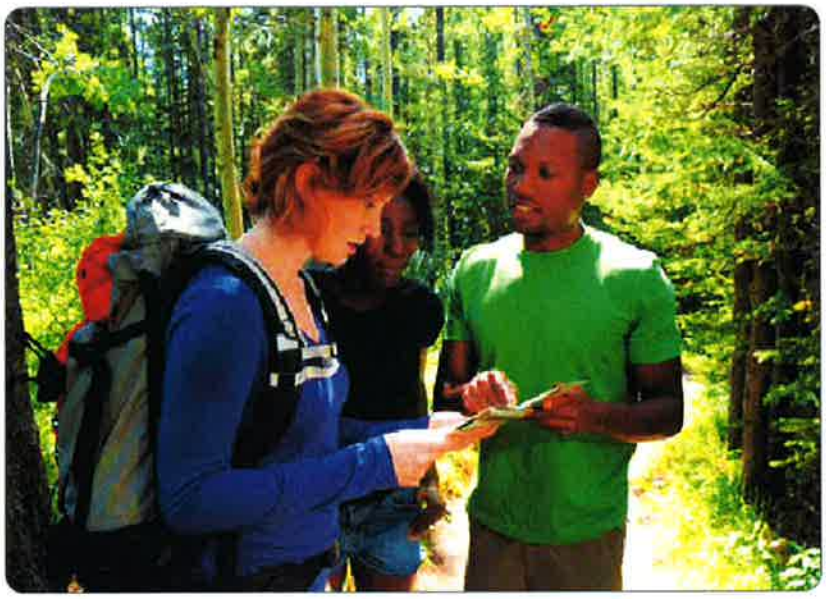
$8 + n$ $8 \cdot n$ $8 \div n$ $8 - n$

Check

1 of 2

LEARN: Going from Expressions to Words

When working with numbers in every day life, we have to figure out what operations need to be done to find an answer. How do we decide whether to add, subtract, multiply or divide?



↑ Activity Resources

Unit 1 | Lesson 6

LEARN: Going from Expressions to Words

It's important to be able to translate from words to math. It's also important to be able to understand what a mathematical expression means in words.

This table summarizes some of the phrases that signal addition, subtraction, multiplication, and division.

Operation	Words or Word Phrases
+	• Increased by • Added to • Sum • Total • Combined with
−	• Decreased by • Less • Less than • Difference
·	• Product • Times
÷	• Quotient • Divided by • Broken into equal parts • Separated into equal parts

R

LEARN: Going from Expressions to Words

Think about a word phrase that matches each expression. Then reveal each answer. Note that the answers you see are not the only word phrases you could write.

Click each part of the expression to reveal the word or phrase that matches it.

Expression	Statement
$15 + x$	
$n - 7$	
$4 \cdot m$	
$15 \div y$	

Unit 1 | Lesson 6

LEARN: Going from Expressions to Words

In your Student Guide, summarize what you just learned:

1. Write a word phrase for the expression $5 - n$.
2. Write a word phrase for the expression $p \bullet 8$.
3. Write a word phrase for the expression $g + 6$.
4. Write a word phrase for the expression $12 \div m$.

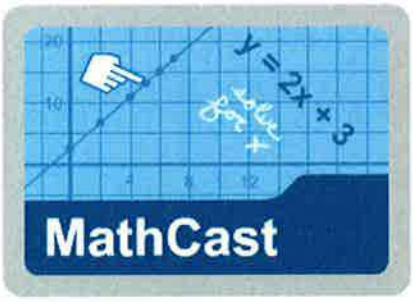
Activity Resources

R

Unit 1 | Lesson 6

WORKED EXAMPLES: Going from Expressions to Words

Click Word and Math Phrases to see how a word phrase is written as a mathematical expression and to watch a mathematical expression translated into a word phrase.



Word and Math Phrases

↑ Activity Resources

Unit 1 | Lesson 6

WORKED EXAMPLES: Going from Expressions to Words

You can use a variable in a mathematical expression to represent an everyday situation.

Choose the situation that can be modeled by the expression 30 times the variable *h*.

$30 \cdot h$

- A. A car travels 30 miles per hour for several hours.
- B. A stack of 30 books is separated into several equal piles.
- C. Carl has $30 and earns more money cutting lawns.

Check

↑ Activity Resources

TRY IT: Translating Between Words and Math

It's time to work on your own. Get your Reference Guide:

- Read pages 20 and 21.
- Complete Problems 1–7 odd, 10, 11, 13, 17–21 odd, 22, 23, 29–35 odd, and 39–45 all on pages 22 and 23.
- When you have finished, go to the next screen to complete the Lesson Checkpoint.

Materials

Translation Between Words and Math Solution Manual

Fundamentals of Geometry and Algebra: A Reference Guide and Problem Sets

Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title: **Exponential Functions, 3 weeks**

Designed by: Laura Mayer
District:Moyer Academy

Content Area:Algebra I
Grade Level(s):9-12

Summary of Unit

Students explore the characteristics of exponentials and are asked to distinguish between linear and exponential relationships represented in various ways. They solve exponential equations and systems of equations that involve exponentials (and linear equations). They write equations for, graph, interpret the parameters of and solve problem involving exponential relationships in context. They will perform vertical transformations of exponential functions and write an expression for the new function.

In alignment with the Common Core State Standards, this unit aids students in their development of 21st Century Skills to meet the learning demands. Students come together as a learning community to analyze real-world applications of the math they are learning, think critically and reflect on the work of their peers as well as their own, and constantly work to apply their knowledge to new, more complex situations. To be well-equipped for the needs of the 21st Century job market, a deep understanding of exponential functions will be essential. Careers in construction, design, engineering, business, and personal finance all use exponential functions to complete job requirements.

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

- Include those addressed in Stage 3 and assessed in Stage 2.

Mathematical Practices 1. Make sense of problems and persevere in solving them.

Mathematical Practices 2. Reason abstractly and quantitatively.

Mathematical Practices 3. Construct viable arguments and critique the reasoning of others.

Mathematical Practices 4. Model with mathematics.

Mathematical Practices 5. Use appropriate tools strategically.

Mathematical Practices 6. Attend to precision.

Mathematical Practices 7. Look for and make use of structure.

Mathematical Practices 8. Look for and express regularity in repeated reasoning.

LE.A.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

Big Idea(s)

- Transferable core concepts, principles, theories, and processes from the Content Standards

- Exponential growth is faster than linear growth

- Exponential functions can be used to model real-life situations and aid in decision making

- When graphing functions, it is important to pay attention to the intercepts, areas of growth and decay, maximums and minimums, and other patterns in order to fully understand how the graph applies to the context of a problem.

Unit Enduring Understanding(s)

- Full-sentence, important statements or generalizations that specify what students should understand from the Big Ideas (s) and/or Content Standards and that are transferable to new situations

- Exponential functions can be used to model real life situations.

- Functions can be modeled with equations, tables and graphs depending on the needs of the problem; different kinds of information can be drawn from each representation.

- When generating a model to represent a real-life problem, it is important to understand which values of that model are applicable to the problem at hand. For instance, when graphing the exponential function that will result from a credit card balance with compounding interest, negative values for x and y do not make sense in the context of the problem: you will not have negative time or a negative balance.

Unit Essential Questions(s)

- Open-ended questions designed to guide student inquiry and learning

1. What is the difference between a linear function and an exponential function?
2. What does it mean to solve an exponential function or system?
3. What effect does altering an exponential function by adding k have on its graph or on a situation?
4. How do we model real life data using exponential functions?
5. What is a limit?

Knowledge and Skills

- Needed to meet Content Standards addressed in Stage 3 and assessed in Stage 2

Students will know...

- how to calculate the rate of change from a graph and table of exponential function values.

- how to identify components of a graph and explain the components in terms of the situation.

- that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

- why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$

- the effects of changing the parameters in a linear or exponential function.

Students will be able to...

- Compare properties of two functions each represented in a different way
- Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$
- Find the solutions approximately, e.g., using technology to graph the functions,
- Make tables of values, or find successive approximations
-Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- Interpret key features of graphs and tables in terms of the quantities
- Sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative)
- Find the value of k given the graphs
- Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- Interpret the parameters in a linear or exponential function in terms of a context.

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

Suggested Performance/Transfer Task(s)

- Performance/Transfer tasks as evidence of student proficiency

An effective assessment for ALL students should be designed to include:

*Complex, real-world, authentic applications

*Assessment(s) for student understanding of the Stage 1 elements (Enduring Understandings, Essential Questions, Big Ideas) found in the Content Standards

*Demonstration of high-level thinking with one or more facets of understanding (e.g., explain, interpret, apply, empathize, have perspective, self-knowledge)

Illustrative Mathematics

F-LE Rumors (Appendix A)

Susanna heard some exciting news about a well-known celebrity.

Within a day she told 4 friends who hadn't heard the news yet.

By the next day each of those friends told 4 other people who also hadn't yet heard the news.

By the next day each of those people told four more, and so on.

a. Assume the rumor continues to spread in this manner. Let "N" be the function that assigns to "d" the number of people who hear the rumor on the "dth" day. Write an expression for N(d).

b. On which day will at least 100,000 people hear the rumor for the first time?

c. How many people will hear the rumor for the first time on the 20th day?

d. Is the answer to (c) realistic? Explain your reasoning

Rubric(s)

- Scoring guide to evaluate performance/transfer tasks used as evidence of student proficiency

An effective scoring guide should:

*Measure what is appropriate for the Content Standard that is assessed.

*Provide opportunities for differentiation of the performance/transfer tasks used as evidence of student proficiency.

Scoring guide for Summative Performance Task (See Appendix A)

Other Evidence

- Varied evidence that checks for understanding (e.g., tests, quizzes, prompts, student work samples, observations and supplements the evidence provided by the task)

Summative Asessment (Appendix B)

Students will be provided with opportunities to complete performance tasks throughout the unit as means of formative assessment to prepare them for the

summative performance task. Additional assessment evidence to be used throughout the unit include formative observations:

Throughout the unit, students will be observed on their application of mathmatical practices highlighted in the unit:

Mathematical Practices 1. Make sense of problems and persevere in solving them.

Mathematical Practices 3. Construct viable arguments and critique the reasoning of others.

Mathematical Practices 6. Attend to precision.

Mathematical Practices 7. Look for and make use of structure.

Mathematical Practices 8. Look for and express regularity in repeated reasoning.

The observations will be shared with the students and parents throughout the unit. These conferences will be centered around the student's progressing score on the Mathematical Practices Rubric (Appendix C)

Student Self-Assessment and Reflection

- Opportunities for self-monitoring learning (e.g., reflection journals, learning logs, pre- and post-tests, self-editing – based on ongoing formative assessments)

After completing each Formative assessment performance task, students will be given the opportunity to reflect on their work. Students will then do a Think-Pair-Share with a peer and explain their work, defending both their problem solving method and the answers they reached.

For example:

1. Students may be given the following task in the unit:

Exponential growth versus linear growth 1 (Appendix D)

Mr. Wiggins gives his daughter Celia two choices of payment for raking leaves:

1. Two dollars for each bag of leaves filled,

2. She will be paid for the number of bags of leaves she rakes as follows: two cents for filling one bag, four cents for filling two bags, eight cents for filling three bags, and so on, with the amount doubling for each additional bag filled.

a. If Celia rakes enough to five bags of leaves, should she opt for payment method 1 or 2? What if she fills ten bags of leaves?

2. Students will be given access to tools and technology from which they can select appropriately to assist them in completing the task. Each student will complete the task independently.

3. Students will pair up, each student given 2 minutes to defend their problem solving method and the answer they reached. Each pair will agree on the most effective way to solve the problem and present that method to the class.

4. Students will be given another step to the problem and time to complete:

b. How many bags of leaves would Celia have to fill before method 2 pays more than method 1?

5. Students will pair up and each will defend their problem solving methods again.

NOTE: At this point in the task, students should be generating equations or a series of steps that would allow them to solve the problem for any amount of sales tax. Students may create a table and/or graph to further illustrate their equation.

NOTE: All performance based task work will be kept inside a reflective log that students will add to throughout the year. The teacher will assist students in creating the log and performing upkeep throughout the year, but it will be the student's responsibility to maintain. At the completion of each task, the students will reflect in writing on the relationship between the task and the learning objective of the lesson.

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

- Instructional activities and learning experiences needed to align with Stage 1 and Stage 2 expectations

Include these instructional elements when designing an effective and engaging learning plan for ALL students:

*Align with expectations of Stage 1 and Stage 2

*Scaffold in order to acquire information, construct meaning, and practice transfer of understanding

*Include a wide range of research-based, effective, and engaging strategies

*Differentiate and personalize content, process, and product for diverse learners

*Provide ongoing opportunities for self-monitoring and self-evaluation

The direct instruction resources needed to implement this course can be found within the k12 MTH303 course, Semester B, Presentations 2.01 - 2.13. Each day of instruction is accompanied by a powerpoint presentation. Use these presentations to guide your information delivery during Direct Instruction, for additional resources when working with small groups, or as a building block for student guided notes. Use these presentations to guide your information delivery during Direct Instruction. Guided Practice Examples and additional Independent Practice activities are also embedded within the presentation materials (See example presentation in Appendix R).

Lesson 1: Recognizing and Identifying Exponential Functions (4-5 days)

Begin with a quick writing assignment where the students respond to how they think a radioactive material decays; is it there one day and gone the next? Is it random? Is there a formulaic process? Circulate around the room and have some students explain what they have written. Open the class up to a discussion. After hearing from a few students, explain that there is in fact a very distinct process to radioactive decay and it involves something called exponential functions.

On the board, put an example of an equation involving exponents that the class is used to. This could be something like: x4 + 3x + 8. Ask for a volunteer to come to the board and solve for x=1, being sure to show work and explain to the class the process it took to arrive at that solution. Have another student explain what the equation equals when x=2, and so on until about x=5. Ask the students to draw a graph with the numbers we have just computed.

Show the class how this process is not unlike the process for determining what the graph of 3x + 24x + 7 is. Begin by solving for x=1, x=2, x=3 and then ask a volunteer to come to the board to compute x=4, another for x=5. These volunteers will need the help from their peers at their desks, who will need to use their calculators since these numbers get very large very fast. Encourage the students at their desks to help their peers arrive at the correct solution. Show how exponentially large these numbers get by exaggerating a drawing of the graph with the values for x=2, x=3, x=4, x=5 going off the board.

Guided Practice:

Students will break into groups of 4. Each group will be given 2 sheets of poster board and some markers. The groups will all be assigned different exponential equations and be asked to use one poster board to chart the values from x=1 to x=4. The other poster board will be used to plot these values either in a bar or line graph; that is left up to the group. An example of this activity is below. The students will make a chart on their poster board:

Equation: 33^x

X Value	Value of Equation
X=1	27
X=2	729
X=3	19683
X=4	531441

The students will take these values and plot them on a graph to be presented to the class.

Students will then present their information to the class, explaining how they arrived at their solutions and showing their graphs, reinforcing the idea of quick growth with exponential functions.

Guided/Independent Practice: Characteristics of Exponential Functions (Appendix E)

**Encourage students to attempt this activity on their own by examining the equations at the top of the activity and exploring other representations of exponential functions using their textbook. When each student has proposed a way to determine if a relationship is exponential from each representation, allow for discussion in small groups or just reveal the answers and conduct a class discussion. Ensure that each student has a copy of the correct answers before proceeding to the next instructional activity.

Independent Practice: Recognizing Exponential Functions (Appendix F)

Students should work individually. After every student has completed the problems, the group should compare answers and come to an agreement about the correct answer. It is essential that the instructor visits groups while they are comparing and reinforces student effort as they are learning to distinguish between exponential patterns and other types of patterns.

Real-World Application Task: Too Hot to Handle, Too Cold to Enjoy (Appendix G)

Group students by differentiated ability for this project. Guide students through the introduction and follow all safety protocols as outlined in the manual.

After Performance Task Think-Pair-Share, students will reflect in their journal on the application of their learning to the Performance Task and their mastery of the lesson objectives. Reflective Journal Entries will be graded using the following rubric:

4 points - Reflection is well written with clear organization and contains almost no errors; demonstrates thoughtful analysis of how new learning and performance task related; evidence and details included to demonstrate student's grasp of concepts; submitted on time.

3 points - Reflection is well written with some organization and contains few errors; demonstrates analysis of how new learning and performance task related; some evidence and details included to demonstrate student's grasp of concepts; submitted on time.

2 points - Reflection shows little organization and contains errors; demonstrates attempt at analysis of how new learning and performance task related; little to no evidence and details are included to demonstrate student's grasp of concepts; submitted on time.

1 points - Reflection has no clear organization and contains many errors; little to no analysis of how new learning and performance task related; little to no evidence and details included to demonstrate student's grasp of concepts; and/or not submitted on time.

0 points - No relevant reflection is submitted.

Formative Assessment: Identifying Exponential Functions (Appendix H)

Lesson 2: Bridge the Gap between Identifying Exponential Functions and Solving Them (1-2 days)

Group work: Drug Filtering (Appendix I)

**Help students visualize this activity as a clear bridge between what they've just covered (recognizing and identifying exponential functions) and what is upcoming (solving exponential functions). Follow all safety protocols as outlined in the manual.

After Performance Task Think-Pair-Share, students will reflect in their journal on the application of their learning to the Performance Task and their mastery of the lesson objectives. Reflective Journal Entries will be graded using the following rubric:

4 points - Reflection is well written with clear organization and contains almost no errors; demonstrates thoughtful analysis of how new learning and performance task related; evidence and details included to demonstrate student's grasp of concepts; submitted on time.

3 points - Reflection is well written with some organization and contains few errors; demonstrates analysis of how new learning and performance task related; some evidence and details included to demonstrate student's grasp of concepts; submitted on time.

2 points - Reflection shows little organization and contains errors; demonstrates attempt at analysis of how new learning and performance task related; little to no evidence and details are included to demonstrate student's grasp of concepts; submitted on time.

1 points - Reflection has no clear organization and contains many errors; little to no analysis of how new learning and performance task related; little to no evidence and details included to demonstrate student's grasp of concepts; and/or not submitted on time.

0 points - No relevant reflection is submitted.

Lesson 3: Solving Exponential Equations and Systems (2-3 days)

Direct Instruction: Guide students through completing question #1 on the Solving Exponential Equations worksheet (Appendix J)

Guided Practice: Students solve questions 2 and 3.

Direct Instruction: Guide students through graphing both functions and recognizing each element of the graph.

Guided Practice: Give students the same table, and two new functions. Have them complete the table with a partner.

Reinforcement Activity: National Debt and Wars (Appendix K)

**Follow the instructions in the manual, students may work individually, in pairs, or small groups.

Open the class up to a discussion at the close of the activity using the extension questions.

Formative Assessment: Solving Exponential Equations and Systems (Appendix L)

Lesson 4: Writing and Transforming Exponential Equations in Context (3-4 days)

Direct Instruction: Introduction to "e"

Introduce the idea of "e". Tell the students the value of e (roughly 2.718) and invite the students to independently research the origin of "e".

Put problems on the board that involve e to various powers of x and ask that the students write down these problems and plug in this information. When finished, students will pick up a penny from me. The intrigue of what we are doing with the penny will serve to introduce radioactive decay.

Tell the students that they are going to use all of this new knowledge and apply it to real-life situations, situations that scientists experience constantly. Explain that every radioactive substance has its own decay constant, its own rate at which it decays. A formula has been developed using this knowledge and using the knowledge of exponential functions that demonstrates how a substance decays. Put this formula on the board, but do not explain it yet.

Ask the students to all stand and when instructed, flip their coins. Those who receive a heads up may remain standing, while those who have flipped a tails must take a seat. The game continues until one person remains.

This will serve as a simplified visual representation of how a material decays. The students have now been given a lot of information about radioactive decay and will hopefully be excited to learn how to apply the formula.

Explain what the variables in the equation mean and do sample problems together. Stress the fact that the exponent in this case is a negative number, explaining why the substance is diminishing exponentially as opposed to growing exponentially.

Give students the base equation for exponential growth: f(x) = a* b^x and walk through applying it to radioactive decay.

Have students apply this equation in context (Appendix M) with a small group.

Using the previous worksheet of Exponentials in Context (Appendix M), introduce the k12 notes on transforming exponential equations and the effects on the asymptote and the y-intercept. Have students practice coming up with their own transformations using their completed examples and share with a partner. Provide

students with large paper and have them graph their "before" and "after" transformations. Have the students do a gallery walk and record what transformations occurred on each group's paper.

Independent Practice: Transformations of Exponentials (Appendix N)

Reinforcement and Exponentials in Context: Predicting Your Financial Future (Appendix O).

**Introduce this concept to students and open the class up for discussion on finances. Try to form an impression of where each student may be in their understanding of financial matters. Use the included manual to guide the activity. Students may work in pairs or small groups.

Formative Assessment: Applications Involving Exponentials (Appendix P)

Summative Assessment and Performance Task (1-2 days) (Appendices A and B)

Resources and Teaching Tips

- A variety of resources are included (texts, print, media, web links)
- Help in identifying and correcting student misunderstandings and weaknesses

Throughout the unit, students have access to a variety of resources.

Algebra I Textbook

Smartboard and Projector

Graphic Organizer Paper

K12 Online Curriculum

Laptop

Reference Sheets

Math in Music video

Each formative check for understanding provides an opportunity for the students to identify their own misunderstandings and weaknesses in mastering the content. Guide the students in understanding how to use these formative checks to enhance their learning through the use of rubrics, conferences, and remediation or acceleration plans designed with student input (Appendix Q). While working towards mastery of grade level content standards, continuously refer students back to the standards for mathematical practices. These practice standards are the attitudes and skills that will allow students to achieve mastery of content standards, as well as deep understanding of mathematics.

Differentiation

- Stage 2 and 3 allow students to demonstrate understanding with choices, options, and/or variety in the products and performances without compromising the expectations of the Content Standards.
- Instruction is varied to address differences in readiness, interest, and/or learning profiles.
- Accommodations and differentiation strategies are incorporated in the design of Stage 2 and 3.

☐ Refer to the IEPs of your special education students

☐ Allow students to have tests, quizzes, worksheets, etc read to them.

☐ Allow the use of a calculator

☐ Communicate with the special education teacher about allowing testing in the special education room

☐ Pair and group students with differing ability levels.

☐ Use manipulatives as needed

☐ Allow for extra time on tests, quizzes, and performance tasks

☐ Allow for a word bank and a formula sheet on tests

☐ Allow students to choose to demonstrate understand in written or verbal format as needed.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures In to provide cross-cultural communicative competence.
- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning** – the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

(Briefly explain how design principle(s) are embedded within the unit design.)

In alignment with the Common Core State Standards, this unit pushes students to develop 21st Century Skills to meet the learning demands. Students come together as a learning community to analyze real-world applications of the math they are learning, think critically and reflect on the work of their peers as well as their own, and constantly work to apply their knowledge to new, more complex situations. This push towards using developed skills in new ways is an accurate reflection of the higher order thinking required by 21st Century jobs. Students should become excited and engaged by the "hands-off" approach of the teacher. Rather than giving students the formulas and a calculator and having them compute numbers and spit out answers, the teacher should present numbers and problems in a real world context, allowing the students to select the tools, resources, and knowledge needed to arrive at a reasonable answer. This push towards applying mathematical ways of thinking to real world issues will prepare them for the type of work they will encounter in college and in their careers.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- **8th Grade Technology Literacy** - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century(SETDA, 2003).

In alignment with Mathematical Practice 5 - Use appropriate tools strategically, students will have access to calculators as needed to complete the Performance Tasks. Additionally, when sharing their answers to the Performance Tasks, the teacher may opt to have the students construct a PowerPoint presentation as an additional layer of the project.

Content Connections

Content Standards integrated within instructional strategies

Connections to other content areas are made both explicitly and implicitly throughout this unit. Additionally, the performance tasks at the end of each lesson connect the math content to Science and Personal Finance. Additional connections are encouraged at the teacher's discretion. For example, when assigning the students additional practice work, make the work meaningful by aligning the examples with content currently being covered in other classes such as Science, English, and Social Studies.

A

Illustrative Mathematics

F-LE Rumors

Alignment 1: F-LE.A.2

Susanna heard some exciting news about a well-known celebrity.

Within a day she told 4 friends who hadn't heard the news yet.

By the next day each of those friends told 4 other people who also hadn't yet heard the news.

By the next day each of those people told four more, and so on.

a. Assume the rumor continues to spread in this manner. Let N be the function that assigns to d the number of people who hear the rumor on the d^{th} day. Write an expression for $N(d)$.

b. On which day will at least 100,000 people hear the rumor for the first time?

c. How many people will hear the rumor for the first time on the 20th day?

d. Is the answer to (c) realistic? Explain your reasoning.

A

Commentary:
This problem is an exponential function example. The other tasks in this set illustrate F.BF.1a in the context of linear (Kimi and Jordan), quadratic (Skeleton Tower), and rational (Summer Intern) functions.

Solution: Solution 1

a. On day 1, Susanna told four friends, so $4 \cdot 1$ people heard the rumor for the first time. On day 2, those four friends each told 4 people, so $4 \cdot 4 = 4^2 = 16$ people heard the rumor for the first time. On day 3, each of those 16 people told 4 others, so $4 \cdot 16 = 4 \cdot 4^2 = 4^3 = 64$ people heard the rumor for the first time. In general, $N(d) = 4^d$ gives the number of people who learn of the rumor on day, d.

b. The problem asks for d given $N(d) = 100{,}000$ We let

$$N(d) = 4^d = 100{,}000.$$

So $d = \log_4(100,000)$ is an exact answer. But most calculators do not compute logarithms with base 4. To obtain a decimal approximation, we take advantage of one of the properties of logarithms, namely that $\log_b(A^p) = p\log_b(A)$:

$$\begin{aligned} \log 4^d &= \log 100{,}000 \\ d \cdot \log 4 &= 5 \\ d &= 5/\log 4 \approx 8.3 \text{ days} \end{aligned}$$

So, the 9th day will be the first day that at least $100{,}000$ people will hear the rumor.

c. The problem asks for $N(d)$ when $d = 20$. We compute

$$N(20) = 4^{20} = 1{,}099{,}511{,}627{,}776.$$

So according to part (a), on the 20th day, $1{,}099{,}511{,}627{,}776$ people will hear the rumor for the first time.

d. The answer in part (c) exceeds the number of people on Earth, so it is unrealistic. Eventually, the number of people hearing a rumor for the first time must cease to increase, because the number of people is finite.

Solution: Approximate Solution

a. As in the previous solution, $N(d) = 4^d$ tells the number of people who learn of the rumor on day d, according to this model. Equivalently, since $4 = 2^2$, we write $N(d) = (2^2)^d = 2^{2d}$, so that we can think in powers of 2.

b. For computer science and for thinking about exponential growth (based on doubling time) a very useful fact is that $2^{10} = 1,024$. Let's approximate that as $2^{10} \approx 1000$. Because $4^5 = 2^{10}$, we can estimate $1,000$ people hear the rumor on the 5th day, so that $64,000$ people hear it on the 8th day, and $256,000$ people hear it on the 9th day, the first day the number exceeds $100,000$ people.

c. From the estimate that $2^{10} \approx 1000$, we have that $2^{20} = 2^{10} \times 2^{10} \approx 1000 \times 1000 = 1$ million and similarly $2^{30} \approx 1$ billion. Because $4^{15} = 2^{30}$, we can estimate that 1 billion people would hear the rumor on the 15th day. That means that 4 billion would hear it on the 16th day, and on the 17th day the number would exceed the population of the planet (which is estimated to reach 7 billion on October 31, 2011). Realistically, there is no reason to proceed to 20 days.

d. The previous answer indicates that this model is not realistic for 20 days. And none of these estimates consider the real-world difficulty of accounting for the people who have already heard the rumor.

B

Exponential Functions **Name:** _____

Summative Assessment **Date:** _____

Answer each question completely and show all work to support your answer.

Determine whether each situation is best represented by a linear function, an exponential function or neither. Explain your reasoning. If the situation is linear or exponential, write a function to describe it.

1.

X	-2	-1	0	1	2
F(x)	.25	.5	1	2	4

2. Sally has a checking account that earns 5% interest compounded monthly when her balance stays over $10,000. She opened the account with $10,000 and has added $3 every month since. She has never used money from the account.

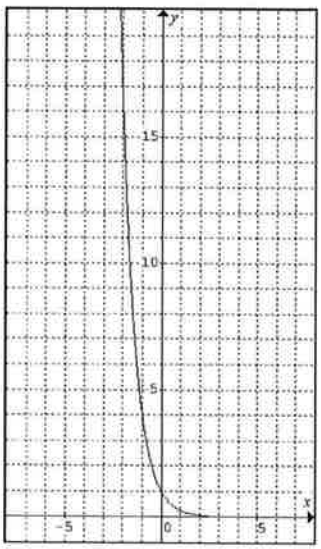
Determine the rate of change in each of the following situations.

3. $g(x) = 15(2)^x$

4.

X	-3	0	3	6	9	12
H(x)	8	1	.125	.0156	.00195	.000244

5.



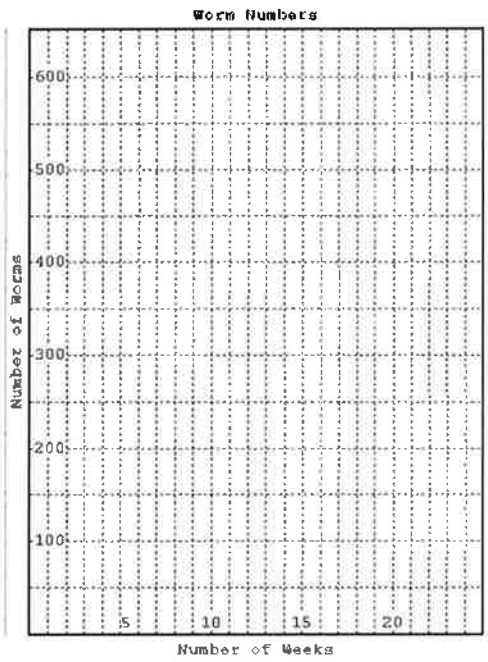
B

Jeremy is preparing to open a bait shop in a few months. The local fish are very attracted to two particular types of worms. He has the equipment needed for the first type of worm, the *splitter*, to grow and reproduce. Each of these worms will divide into two worms every four weeks. He has room to grow thousands of them and has the opportunity to purchase 10 now. The second type of worm, the *rendezvous*, is easy to keep alive, but difficult to get to reproduce. At this time, Jeremy cannot afford the required equipment. However, he can order 120 of these worms now and can get another 40 of them every four weeks. (Hint: Let x stand for the number of 4 week intervals.)

6. Write a function that represents the number of *splitter* worms.

7. Write a function that represents the number of *rendezvous* worms.

8. Graph both functions on the same grid for the intervals indicated.



9. Jeremy orders 10 *splitter* worms now and the maximum number of *rendezvous* worms possible between now and when the store opens. All of the worms survive. After how many **weeks** will he have more *splitter* worms than he does *rendezvous* worms?

Jeremy receives a one-time bonus of 20 non-reproducing *splitter* worms with his original order of 10 *splitter* worms.

10. Will your answer to question #9 change? Why or why not?

11. Write a function to represent the new situation.

12. Describe how the graph of the new function will be different from the original graph representing the number of *splitter* worms.

B

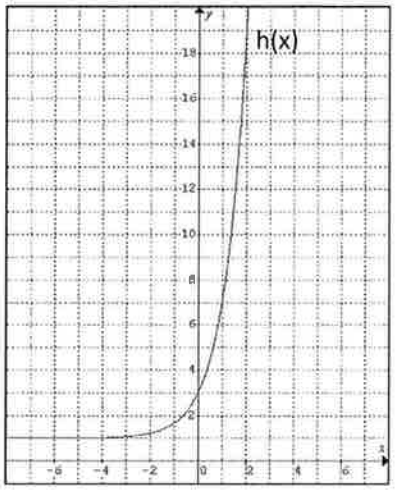
The function $f(x) = 7200\ 1.03^{x}$ represents the population of a town over time.

13. Is the population increasing or decreasing? At what rate?

14. What was the initial population?

Compare the functions described below in terms of their end behavior, intercepts and rate of change.

15. $g(x) = .5^{x}$



B

Exponential Functions Summative Assessment — Answer Key

1. Exponential. For constant increase in x, y increases by a factor of 2. $F(x) = 2^x$
(Obj 1/F.LE.1.c/2 and Obj 2/F.LE.2/2 and Obj 3/F.IF.4/2 and F.BF.1.a/2)

2. Neither. This situation shows characteristics of both exponential and linear patterns.
(Obj 1/F.LE.1.c/2 and Obj 3/F.IF.4/2)

Determine the rate of change in each of the following situations.

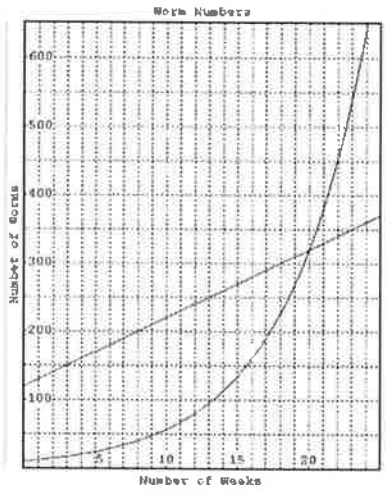
3. The rate of change is 2. (Obj 1/F.IF.6/1)

4. The rate of change is $\frac{8}{1} = \frac{1}{.125} \approx \frac{.125}{.0156} \approx \frac{.0156}{.00195} \approx \frac{.00195}{.000244} \approx 8.$
(Obj 1/F.IF.6/2 and Obj 3/F.IF.4/2)

5. The rate of change is $\frac{4}{16} = \frac{1}{4} = \frac{1/4}{1} = \frac{1}{4}.$
(Obj 1/F.IF.6/3 and Obj 3/F.IF.4/3)

6. $f(x) = 10(2)^x$ (Obj 1/F.LE.1.c/3 and Obj 2/F.LE.2/2 and Obj 3/F.BF,1.a/2)

7. $g(x) = 120 + 40x$ (Obj 2/F.LE.2/2 and Obj 3/F.BF,1.a/2)

8. 

(Obj 1/F.IF.7.e/2 and Obj 2/A.REI.10/2 and Obj 3/F.IF.4/3)

B

9. Since Jeremy will have 320 of each type of worm in the 20th week, he will have more *splitter* worms than he does *rendezvous* worms during the 24th week.
(Obj 1/F.LE.3/2 and Obj 2/A.REI.11/2)

10. Yes. Jeremy will now have 340 *splitter* worms in the 20th week, but he will still have only 320 *rendezvous* worms. Therefore, he will have more *splitter* worms in the 20th week instead of the 24th week.
(Obj 1/F.IF.9/3 and Obj 3/F.IF.4/3 and Obj 3/F.LE.5/3)

11. $f(x) = 10(2)^x + 20$ (Obj 3/F.BF.1.a/3)

12. The new function would have the same shape, but would be translated up 20 units.
(Obj 3/F.IF.4/3 and Obj 3/F.BF.3/3)

13. The population is increasing at 3% per year.
(Obj 3/F.LE.5/2)

14. The initial population was 7200 people.
(Obj 3/F.LE.5/1)

15. Function g has an asymptote of y = 0, a y-intercept of 1, and is decreasing by a factor of one half. Function h has an asymptote of y = 1, a y-intercept of 3, and is increasing by a factor of 3.
(Obj 1/F.IF9/3)

Student Name_____ Evaluation Date_____ Overall Score_____

Mathematical Practice	Novice (1)	Apprentice (2)	Practitioner (3)	Expert (4)
1. Make sense of problems & persevere in solving them	I couldn't get started. I don't know how to begin. I may be feeling frustrated and wanting to give up.	I am stuck. I have part of the solution, but now I don't know what to do, I'm not sure my answer is right and I could use some help. I may be feeling frustrated.	I planned out how to solve this problem and I worked all parts of the problem. I may have felt challenged, but I didn't want to give up-I kept going until the problem was solved.	My solution is effective and inventive! I planned out how to solve this problem and I even showed you other ways to solve it. I checked to make sure my answer was right – I enjoyed this challenge!
2. Reason abstractly & quantitatively	I can't explain how I solved this problem. I don't know what quantities or equations to use. My work on this problem is built on misunderstandings of mathematical ideas.	I'm trying to explain how I solved this problem. My work on this problem may be built on misunderstandings of mathematical ideas. . I'm trying to take numbers out of the problem to work with them. I'm trying to check my work.	I explained how to solve this problem using formal math language. I took numbers out of the problem to work with them, and periodically put them back into the context of the problem to see if my work makes sense.	I clearly explained how to solve this problem using formal math language. I took quantities out of the context of the problem to work with them, and frequently put them back into the context of the problem to see if my work made sense.
3. Construct viable arguments & critique the reasoning of others	I can't describe how I solved the problem. I can't give feedback on someone else's reasoning OR my feedback in unclear, unhelpful, and/or incorrect.	I'm trying to describe how I solved the problem. I may or may not include pictures, tables, graphs, and/or models. I gave feedback on someone else's reasoning, but my feedback may have been unclear, unhelpful, or I may have been unable to back it up with my own work.	I described how I solved the problem using formal math language, pictures, tables, graphs, and/or models. I gave clear, helpful feedback to someone else's reasoning and backed up my feedback with my own work on the problem.	I clearly described how I solved the problem. I included every step so you don't have to guess what I did. I used formal math language, pictures, tables, graphs, and/or models. I gave clear, helpful feedback on someone else's reasoning and backed up my feedback with my own work on the problem.
4. Model with mathematics	I can't use mathematical representations, I don't know what to use for this problem and I can't describe how I would solve this problem.	I'm trying to use mathematical representations to solve the problem and show my solutions, but they may be inaccurate or inappropriate. I'm sharing my ideas, but I'm struggling to use formal math language.	I used appropriate & accurate mathematical representations to solve the problem and show my solutions. I used formal math language to share my ideas.	I used a variety appropriate, accurate, and detailed mathematical representations to solve the problem and show my solutions. I used formal math language to share my ideas.
5. Use appropriate	I don't know what tool to use to solve this problem and/or I	I picked an appropriate tool to use, but I may be struggling to	I picked the correct tool to use based on the needs of the	I selected and used a variety of appropriate tools

Student Name_____ Evaluation Date_____ Overall Score_____

tools strategically	didn't use the tool I picked in a valuable way.	use it in a valuable way to solve the problem.	problem. I planned and monitored my use of the tool.	collaboratively to solve the problem. I planned my use of the tools and monitored my progress.
6. Attend to precision	I can't get the correct answer and/or I can't communicate my answer in a way that is appropriate to the problem. I can't use correct units, labels, or definitions.	I may be getting the correct answer and/or I may be able to communicate my answer is a way that is appropriate to the problem. I may not use correct units, labels, or definitions.	I got the correct answer and I can communicate it using the correct labels, units, and definitions. The precision of my answer is appropriate for the problem.	I got the correct answer and I communicated it using correct labels, units, and definitions. I clearly stated the meaning of the symbols I chose and my answer is complete appropriate for the problem.
7. Look for & make use of structure	I don't see any structure in the problem. I can't make sense of any useful structures/patterns to help me solve the problem.	I may be able to find for structures/patterns in the problem, but I am struggling to use them to help me find a solution.	I found and used structures/patterns in the problem to help me find a solution. I noted these structures/patterns as I found them.	I found, noted, and used structures/patterns in the problem to help me find a solution. I was able to step back from the problem and analyze different perspectives.
8. Look for & express regularity in repeated reasoning	I don't see any regularity or any shortcuts or generalizations I can use to get a solution.	I may notice that some of my calculations are repeated but I may not be able to generalize that observation to help me get a solution.	I noticed some of my calculations were repeating and was able to make a generalization that helped me solve the problem. I evaluated the reasonableness of my work as I progressed.	I noticed some of my calculations were repeating and was able to make a generalization or a formula that helped me solve the problem. I continuously evaluated the reasonableness of my work as I progressed.

D

Illustrative Mathematics

F-LE Exponential growth versus linear growth I

Alignment 1: F-LE.A.3
Not yet tagged

Mr. Wiggins gives his daughter Celia two choices of payment for raking leaves:

1. Two dollars for *each* bag of leaves filled,
2. She will be paid for the number of bags of leaves she rakes as follows: two cents for filling one bag, four cents for filling two bags, eight cents for filling three bags, and so on, with the amount doubling for each additional bag filled.

a. If Celia rakes enough to five bags of leaves, should she opt for payment method 1 or 2? What if she fills ten bags of leaves?

b. How many bags of leaves would Celia have to fill before method 2 pays more than method 1?

D

Commentary

The purpose of this task it to have students discover how (and how quickly) an exponentially increasing quantity eventually surpasses a linearly increasing quantity. Students' intuitions will probably have them favoring Option A for much longer than is actually the case, especially if they are new to the phenomenon of exponential growth. Teachers might use this surprise as leverage to segue into a more involved task comparing linear and exponential growth.

Solution: Table

a. A table of values giving the number of bags of leaves and the amount paid using methods 1 and 2 shows that method 1 pays more up to and including eleven bags.

b. The table also shows that method 2 pays more as soon as Celia rakes at least twelve bags of leaves. We know that method 2 will always pay more, beyond the twelfth bag, because doubling an amount x gives a larger increase than adding 2 as soon as x is greater than 2:

$$2x > x + 2 \quad \text{whenever} \quad x > 2.$$

Number of Bags	Payment Method 1 (dollars)	Payment Method 2 (dollars)
1	2	0.02
2	4	0.04
3	6	0.08
4	8	0.16
5	10	0.32
6	12	0.64
7	14	1.28
8	16	2.56
9	18	5.12
10	20	10.24
11	22	20.48
12	24	40.96

Solution: 2. arithmetic and geometric sequences

The numbers in the second column of the table in the first solution form part of the arithmetic sequence which starts with 2 and increases each time by 2: the n^{th} term in this arithmetic sequence is $2n$ and this is the number in the n^{th} row of the second column of the table. The third column of the table is a geometric sequence which starts at 0.02 and increases by multiples of 2 each time. The n^{th} term in this sequence, found in the n^{th} row of the third column, is

$$\frac{2^n}{100}.$$

The numerator 2^n shows the geometric sequence, while the denominator 100 reflects the fact that the sequence began at $\frac{2}{100}$.

Geometric sequences grow exponentially. Since the multiplier two is larger than one, the geometric sequence grows faster than, and eventually surpasses, the linear arithmetic sequence. To see this more clearly, note that each additional bag of leaves makes Celia two dollars with method 1 while with method 2 it doubles her payment. Hence as soon as payment method 2 is worth more than two dollars (that is after 8 bags of leaves) method 2 pays more than method 1 for every additional bag and so the deficit is quickly made up.

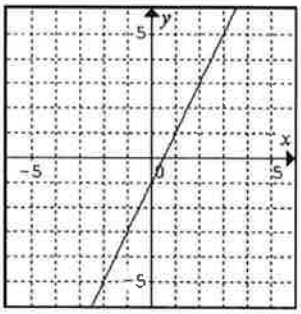
E

Characteristics of Exponential Functions **Name:** _____

Linear Relationships

$Y = 2x - 1$

x	-1	0	1	2
y	-3	-1	1	3

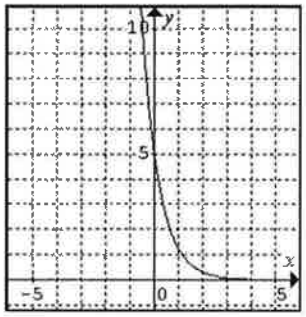


A boy uses a service to find a summer job. He makes $2 per hour, but must pay a flat fee of $1 for the service.

Exponential Relationships

$Y = 5(.25)^x$

x	-1	0	1	2
y	20	5	1.25	.3125

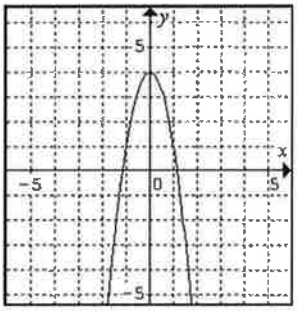


A container holds 5 liters of water. It is evaporating at a rate of 75% every day.

Other Relationships

$Y = -3x^2 + 4$

x	-1	0	1	2
y	1	4	1	-8



An archway in a wall peaks 4 meters above a path through the wall. The arch is 2 meters wide where it meets the path.

Use the relationships above and your textbook to describe how to recognize an exponential function when presented in various forms.

1. (algebraic) _____

_____

2. (table) _____

_____

3. (graph) _____

_____

4. (verbal) _____

_____

E

Activity 1 - Characteristics of Exponential Functions **Answer Key**
(F.LE.1.c /MP1 and MP3 and MP8)

1. An equation or function that contains a variable in the exponent. The base is never one. The base is positive. There may be an additional number multiplied by the base. This number cannot be 0.

2. A relationship in which, for a constant interval of x-values, the y-values are multiplied by a constant percent rate. This rate could be greater than or less than 100%. It could result in an increase or a decrease in y-values.

3. The graph is a curve. The curve could be increasing or decreasing. It has a horizontal asymptote. The asymptote is at y = 0 if the graph is not translated up or down. If there is no number multiplied by the base, the y-intercept is 1.

4. A verbal description would include words that indicate repeated multiplication. Examples include such things as: doubles, half, triples, 30% increase per *(some unit interval)*, or 5% decrease every *(some unit interval)*.

F

Recognizing Exponential Functions

Name: _____

Determine whether each situation is best represented by a linear function, an exponential function or neither. Explain your reasoning.

1. $f(x) = 3^x$

2. $h(x) = 3(2)^x$

3. $k(x) = 2x$

4. $g(x) = (.2)^x$

5. $j(x) = 4(.25)^{-x}$

6.

X	-1	0	1	2	3
H(x)	5	0	5	20	45

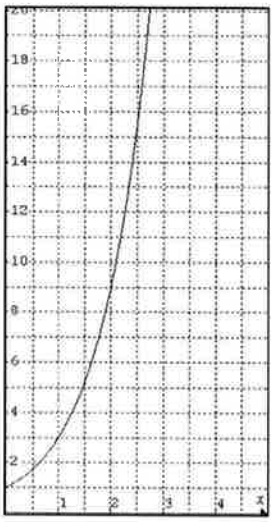
7.

X	0	1	2	3	4
F(x)	16	4	1	.25	.0625

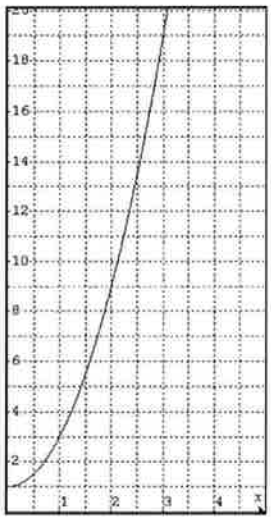
8.

X	-2	-1	0	1	2
J(x)	.02778	.1667	1	6	36

9.



10.



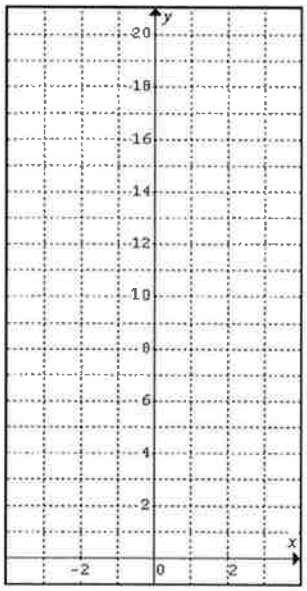
F

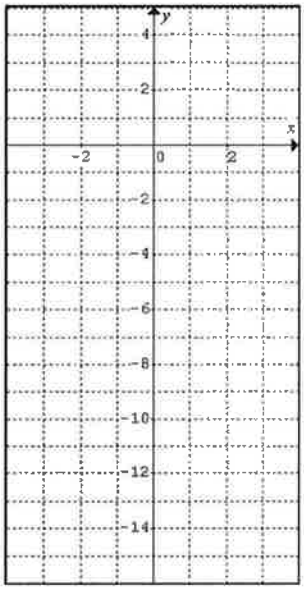
State the rate of change and graph each of the exponential relationships below. Be sure to describe the end behavior and identify asymptotes and intercepts.

11.

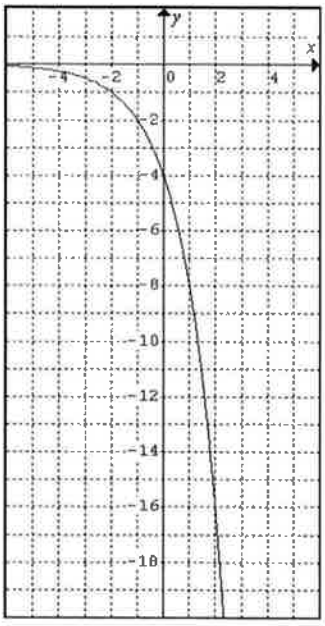
X	-2	-1	0	1
G(x)	.125	.5	2	8

12. $k(x) = -1(2)^x$





13. Estimate the rate of change of the exponential function in the graph.



F

Compare the functions described below where x represents a unit of time.

A. $y = (3)^x$

B.

X	0	1	2	3
G(x)	5	10	15	20

14. Which has the greater y-intercept?

15. Which is greater after 5 units of time pass?

F

Activity 2 - Recognizing Exponential Functions

Answer Key

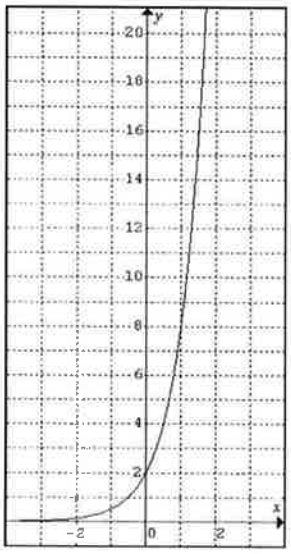
(F.LE.1.c and F.LE.3 and F.IF.6 and F.IF.7.e and F.IF.9/ MP3 and MP 8)

1. Exponential. The exponent is a variable.
2. Exponential. The exponent is a variable.
3. Linear. The variable is raised to the first power.
4. Exponential. The exponent is a variable.
5. Exponential. The exponent is a variable.
6. Neither. There is no common first difference. There is no constant rate per unit interval.
7. Exponential. It decreases by a constant rate of .25 per unit interval.
8. Exponential. It increases by a constant rate of 6 per unit interval.
9. Exponential. It increases by a constant rate of 3 per unit interval.
10. Neither. There is no common first difference. There is no constant rate per unit interval.
11. The rate of change is $\frac{.5}{.125} = \frac{2}{.5} = \frac{8}{2} = 4$.

    End behavior: As $x \rightarrow -\infty$, $G(x) \rightarrow 0$ and as $x \rightarrow \infty$, $G(x) \rightarrow \infty$.

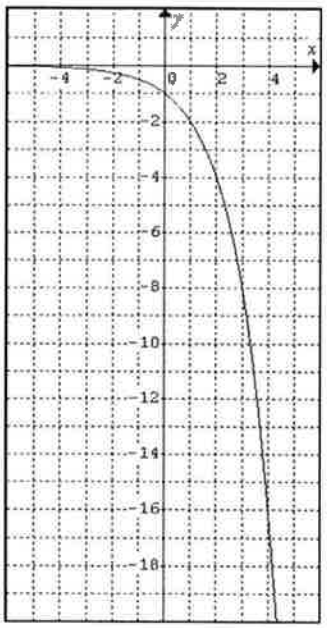
    The horizontal asymptote is at $y = 0$.

    The y-intercept is 2.



F

12. The rate of change is 2.
    End behavior: As $x \to -\infty$, $G(x) \to 0$ and as $x \to \infty$, $G(x) \to -\infty$.
    The horizontal asymptote is at $y = 0$.
    The y-intercept is -1.



13. The rate of change is $\frac{1/2}{1/4} = \frac{1}{1/2} = \frac{2}{1} = \frac{4}{2} = \frac{8}{4} = \frac{16}{8} = 2$.

14. B

15. A

G

ILLUMINATIONS
Resources for Teaching Math

Too Hot To Handle, Too Cold To Enjoy

Predicting the right time to take that first sip of any hot beverage is difficult. Unfortunately, the temperature of hot coffee does not decrease steadily (linearly) over time. If so, it would be easy to predict when to take that first sip. Which function best represents the rate at which coffee cools: linear, quadratic, square root, absolute value, exponential or logarithmic?

Learning Objectives

Students will:

- Collect, graph and analyze real world data
- Choose an appropriate mathematical model for a particular situation
- Make predictions based on their analysis

Materials

Computer projector

Graph paper

Ruler or straight edge

Too Hot To Handle Activity Sheet

Instructional Plan

Have you ever taken a sip of coffee and burned your lip or tongue? Then after you set the coffee aside to cool, it gets too cold. This lesson is designed for students to analyze why this happens and to help predict when to take that first sip. In order to achieve this goal, students will take measurements of hot coffee (actually, *simulated* hot coffee) at various times, and then graph the results. Before they can determine the perfect time for the first sip, however, they must conduct some research to determine what temperature is hot enough to burn.

Lead a class discussion about *Liebeck v. McDonald's Restaurant*, better known as the "McDonald's Coffee Case," a lawsuit regarding a fast food restaurant and a person who was scalded by their coffee. (Note that you may want to exclude the name of the restaurant when discussing the case in class.) The lawyer for the defendant said that McDonald's provided him its operations and training manual, which says its coffee must be brewed at 195–205° and held at 180–190° for optimal taste.

G

Pass out the Too Hot to Handle activity sheet. Tell students that they are going to determine the best time to take the first sip of a hot drink by using a function to model how the temperature of the drink changes over time. Ask them to predict what kind of function will represent this relationship. If they struggle, remind them of the kinds of functions they have studied — linear, absolute value, quadratic, square root, exponential and logarithmic. Do not draw the graphs, however, because they will do this later.

Too Hot To Handle Activity Sheet

If possible, conduct the following experiment in class. Boil a pot of water. Then, place a thermometer in a cup and fill it with hot water. Record the temperature about every 3 minutes. It will take approximately 30 minutes for the coffee to cool to 120°, which is a reasonable temperature at which the first sip might be taken.

As a time-saving alternative, you can show students the following video that shows a mug of hot water cooling, while a thermometer and timer indicate the temperature and elapsed time. Showing the video gives several advantages: the data collection occurs more quickly; all students are able to watch the thermometer throughout the entire process; and, all students will be using the same set of data. One interesting strategy is to require all students to collect at least ten data points, without restriction on which data points they must choose. Although students will collect different sets of data, the resulting graphs should be very similar.

Using either method, students should collect data until the temperature gets to the point where clearly no scalding will occur. Because scalding results partially due to temperature and partially due to the amount of time exposed to the heat, some

G

research should be done to find the scalding temperature. If you are doing the experiment, wait until the temperature gets down to 120° and then gently touch the surface of the water with your finger. If you cannot hold your finger there for very long, then it is still too hot. **Do NOT have a student do this.** If you are using the video, research can be done in class as a teacher-led web exploration or in a lab as individual or group research. Students should determine what temperature would be good for a first sip. A reasonable estimate is approximately 120°.

At this point, students should graph their data. Some students will need guidance through the setup of the axes, scales and intervals. After plotting the data, students should choose the type of function that best models this cooling temperatures. Finally, students should use their scalding temperature research and their graph to determine the best time to take the first sip.

Questions for Students

- Now that you've graphed the data, can you explain why when you set the coffee aside for a few minutes, many times it is too cool when you finally take a sip?

  [Because of the exponential decay, the temperature drops quickly from the starting temperature.]

- If the coffee was left to cool for much longer than 30 minutes, what temperature would it reach? Why? Will the temperature continue to decrease until it reaches 0°?

  [Eventually, the temperature of the coffee will reach an ambient temperature that we call *room temperature*. The asymptote will be somewhere near $y = 70$, since room temperature is about 70°.]

- What is the name for the concept that describes the limit that the temperature will never get to?

  [Exponential decay functions have horizontal asymptotes. In this experiment, there is a final temperature and it is the ambient temperature, however, the temperature just below the ambient temperature will never be reached and that can be considered the asymptote]

- What is the inverse of exponential decay? If you were to heat a liquid and measure the temperature as the liquid gets hotter over time, would the graph look like the inverse of the exponential function?

  [The inverse of exponential decay is logarithmic growth. And, yes, heating a liquid will have logarithmic growth.]

- What factors might affect the cooling rate of hot coffee?

  [Factors that affect cooling include: type of liquid, shape of container, container material, outside air temperature, etc.]

- If you were to design a coffee mug which could help people to avoid scalding themselves, what shape would you use? What shape would you use to keep the coffee the hot the longest? Can a mug be made that prevents scalding and keeps coffee hot or are these conflicting goals?

  [Answers will vary.]

- What other natural phenomena might be represented by exponential decay?

  [An example is depreciation.]

Assessment Options

1. Collect the activity sheets and evaluate students' understanding by looking at their answers to the questions. Answers to the activity sheet can be found in this answer key. Note that the answers to all questions will vary, but if the video is used, the graph in Question 6 will be an exponential function that passes through the point (30, 120) and is asymptotic to the line $y = 70$. Consequently, the answer to Question 10 will be "approximately 30 minutes."
2. Lead a classroom discussion regarding the various mathematical models and how they relate to this lesson. Ask whether anyone thought that the cooling process would be linear and why? Did anyone think that another mathematical model would apply and why?
3. Randomly select some students to present to the class and the class can participate in reviewing and critiquing the presentations. Of particular interest is the time (and temperature) that students decided would be appropriate for taking the first sip.

Extensions

1. Conduct another experiment by putting a thermometer in a beaker of water and recording how the temperature rises as the water is heated. A bunsen burner from a chemistry lab or a sterno can could be used to heat the water.
2. Have students graph their data on semi-log paper. (You and your students can create semi-log paper using Illuminations' Dynamic Paper applet. Click on the Graph Paper tab, and then choose the Semi-Log option.) When you graph an exponential function on semi-log paper, you get a straight line which you usually only see when graphing a linear function. Semi-log paper is used because graphs of exponential functions drop and rise so quickly that it is difficult to measure the change in time accurately and the change in temperature when too much time has passed. With semi-log paper, these changes become magnified.

 Dynamic Paper Applet

A similar result occurs when looking at the graph of the Dow Jones stock index since the Great Depression. It is an exponential function, but many stock sites make it look linear by using a logarithmic scale. Use Yahoo! Finance to change the scale from logarithmic to linear and see the differences in the graphs.

G

3. Ask students to perform an exponential regression using a graphing calculator or Microsoft Excel. But be careful! Because the asymptote is $y = 70$, students will have to do a transformation first by subtracting 70 from each value in the range. Also note that the regression given by most graphing calculators takes the form $y = ab^x$, whereas the form used by Excel is $y = ae^{bx}$, where e is the base of the natural logarithms.
4. Have a class discussion where students attempt to relate what they observed with Newton's Law of Cooling, which states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature (the temperature of its surroundings). The formula is:

$$T = A + Se^{-kt}$$

T = temperature at any instant
A = temperature of surroundings (ambient temperature)
S = initial temperature of the liquid minus the ambient temperature
t = elapsed time
k = constant determined by the conditions

Teacher Reflection

- If you chose to do the experiment in class, could anything have been done to improve safety?
- Was the experiment part of the activity inclusive of all students? What could be done to include more students?
- Did all students set up the graph correctly? What could have been done to prepare students for that part of the lesson?
- Were students able to relate to the lesson? Did they feel that this was practical knowledge that would be useful?

NCTM Standards and Expectations

Algebra 9-12

1. Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
2. Draw reasonable conclusions about a situation being modeled.
3. Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.

Measurement 9-12

1. Analyze precision, accuracy, and approximate error in measurement situations.

This lesson was prepared by Michael Weingarden as part of the Illuminations Summer Institute.

6

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

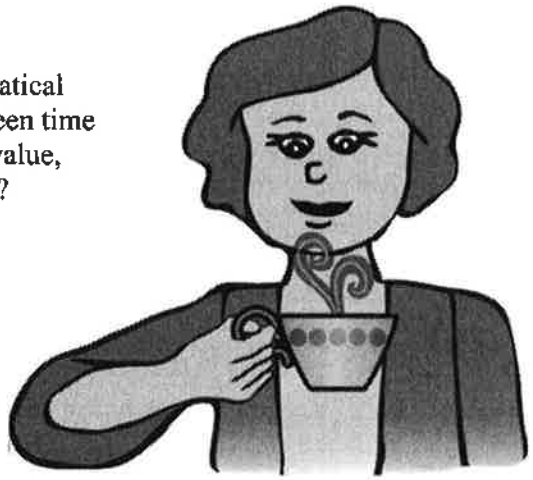
More and Better Mathematics for All Students

G

Too Hot To Handle

NAME_____

This lesson could save your lips and tongue.



1. For a cooling cup of coffee, what kind of mathematical model would best represent the relationship between time and temperature? Is the function linear, absolute value, quadratic, square root, exponential or logarithmic?

2. Use this table to record data points.

TIME (t)	TEMPERATURE (T)

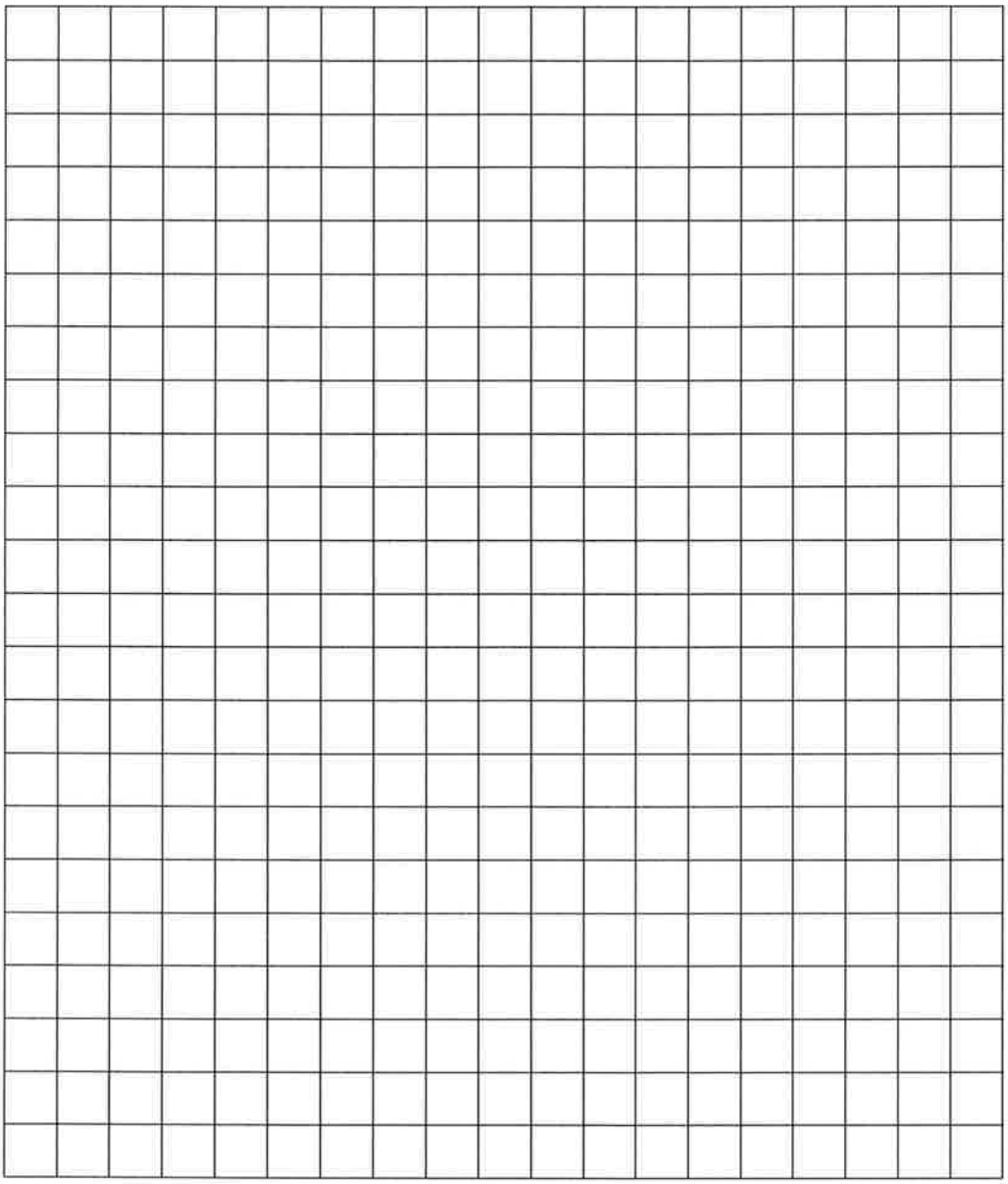
3. To graph the data above, which variable should be used for the domain? Why?

4. To graph the data above, what should the scale for the domain be? For the range?

G

5. What is the scale interval for the x-axis? For the y-axis?

6. Graph the points as a scatterplot.



7. Now that you can see the graph, what type of function models the relationship between temperature and time?

8. What temperature is considered scalding (hot enough to burn flesh)?

9. At what temperature is it safe to take the first sip of coffee?

10. After your coffee is poured, how much time should you wait before you take the first sip?

Name: _____

Identifying Exponential Functions

Date: _____

1. Describe how you know a relationship is exponential from a graph.

_____

_____

Determine whether each situation is best represented by a linear function, an exponential function or neither. Explain your reasoning.

2.

X	-2	-1	0	1	2
F(x)	100	10	1	.1	.01

3. A population increases by 300 people every 2 years.

4. A drug administered intravenously dilutes at a rate of 20% every 8 hours.

5.

X	-20	-10	0	10	20
G(x)	-100	-60	-20	20	60

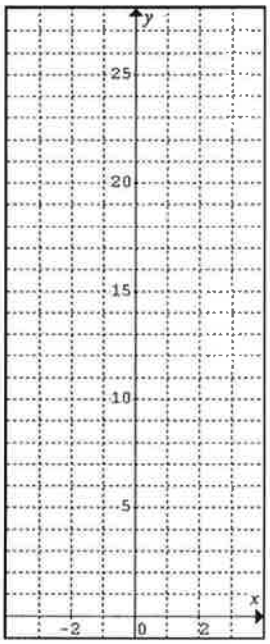
6. $h(x) = 20(4)^x$

7. $j(x) = 15(x)^2$

State the rate of change and graph each of the exponential relationships below. Be sure to identify asymptotes and intercepts.

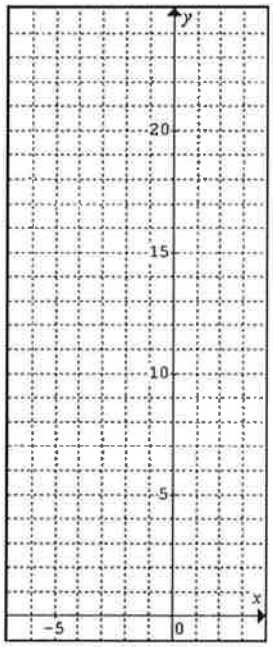
8.

x	F(x)
0	1
1	3
2	9
3	27

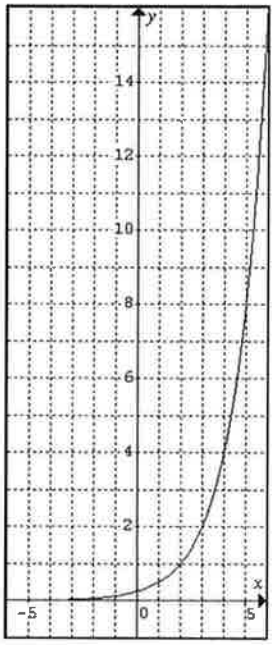


H

9. $y = 3(2)^x$



10. Estimate the rate of change of the exponential function in the graph.



Compare the functions described below where x represents a unit of time.

A. $y = 25(2)^x$

B.

X	0	1	2	3	4
G(x)	5	15	45	135	405

C. Jennifer opens an account by depositing $35. She adds $5 every month thereafter.

11. Which has the greatest y-intercept?

12. Which is greater after 1 unit of time passes?

13. Which is greatest after 5 units of time pass?

H

Identifying Exponential Functions Answer Key

1. You can determine a relationship represented by a graph is exponential by determining if there is a constant percent rate of change per unit interval relative to another. (Obj 1/F.LE.1.c/2)

2. Exponential. The rate of change per unit interval is $\frac{10}{100} = \frac{1}{10} = \frac{.1}{1} = \frac{.01}{.1} = \frac{1}{10}$. (Obj 1/F.LE.1.c/2)

3. Linear. The function changes by the addition of 150 people per year. (Obj 1/F.LE.1.c/2)

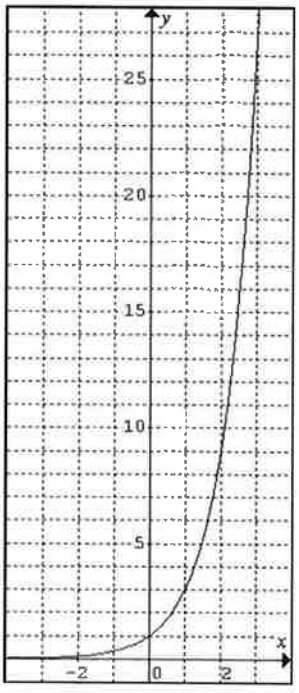
4. Exponential. The rate of change per unit interval is (1 - 0.2) = .8 per every 8 hours. (Obj 1/F.LE.1.c/2)

5. Linear. The function changes by the addition of 4 over each unit interval. (Obj 1/F.LE.1.c/2)

6. Exponential. The function has a variable in the exponent. The rate of change per unit interval is 4. (Obj 1/F.LE.1.c/2)

7. Neither. The variable is not raised to the first power, nor is the exponent a variable. (Obj 1/F.LE.1.c/2)

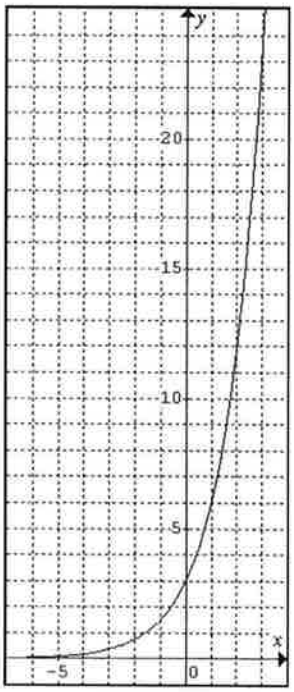
8. The rate of change per unit interval is 3.
   The asymptote is at y = 0.
   The y-intercept is 1.



(Obj 1/F.IF.6/2 and Obj 1/F.IF.7.e/2)

H

9. The rate of change per unit interval is 2.
   The asymptote is at y = 0.
   The y-intercept is 3.



(Obj 1/F.IF.6/2 and Obj 1/F.IF.7.e/2)

10. The rate of change per unit interval is $\frac{1/2}{1/4} = \frac{1}{1/2} = \frac{2}{1} = \frac{4}{2} = \frac{8}{4} = 2$. (Obj 1/F.IF.6/3)

11. C (Obj 1/F.IF.9/2)

12. A (Obj 1/F.IF.9/3 and Obj 1/F.LE.3/3)

13. B (Obj 1/F.IF.9/3 and Obj 1/F.LE.3/3)

I

ILLUMINATIONS
Resources for Teaching Math

Drug Filtering

In this lesson, students observe a model of exponential decay, and how kidneys filter their blood. They will calculate the amount of a drug in the body over a period of time. Then, they will make and analyze the graphical representation of this exponential function.

Learning Objectives

Students will:

- Predict the behavior of a situation represented by an exponential decay model.
- Graph an exponential set of data.

Materials

- Clear 2 liter pitcher
- 250 ml beaker
- Spoon for mixing
- Water
- Food coloring
- Container for discarded colored water
- Drug Filtering Activity Sheet

Instructional Plan

Announce that today you will be modeling the how the body's kidneys filter blood. Pour 1 liter of water into a pitcher, and explain that the water represents some of the blood in your body. To start the discussion, ask, "About how much blood do you have in your body?" [5 liters.]

Put several drops of food coloring into the water. Explain that this food coloring represents 1000 mg of a drug (such as acetaminophen or ibuprofen) that you have taken. Mix the food coloring well with the water. Tell students that after four hours the kidneys will filter out about 25% of this drug. Ask, "How can this be modeled?" Allow students to offer suggestions, and then remove 250 ml of the mixture and replace it with 250 ml of clear water. Then ask, "How many milligrams of the drug remain in my blood?"

[750 mg.]

Before removing any more colored water, ask, "How could the kidney's work be modeled after another four hours?" Many students will respond that you should remove another 250 ml of colored water and replace it with another 250 ml of clear water. Then ask, "If we did this, how many milligrams of the drug would remain in my blood?" Allow students to make a prediction, often 500 mg, without correction. Ask again for one more four-hour period. Most often students will say that again 250 mg of the drug are removed. Finally, ask, "So, if I repeat this process four

I

times, will the drug be completely out of my system?" Most often students will answer, "Yes."

Now, remove 250 mg of the colored water a second time, and replace with 250 ml of clear water. Repeat and ask, "If I do this once more, will all the color will be gone, leaving clear water?" The students can now see that this will not happen. Have students debate whether it was the model or the prediction that was incorrect. Students may realize that the second time you removed 250 mg of colored water there was only 750 mg of the drug in the blood, so replacing a fourth of it only removed $1/4 \times 750 = 187.5$ mg.

Distribute the Drug Filtering activity sheet. Students may work alone or with a partner. Circulate around the room to make sure everyone is engaged in the activity. Help students connect their data to the demonstration. Make sure that students are graphing the model correctly. Check that their data is correct and that they are reading the scale of the graph correctly.

Drug Filtering Activity Sheet

Ask students if the graph representing this situation is a linear graph. [No. It is exponential.] Explain that drug filtering is not linear, because the same amount of the drug is not removed during each four-hour period. Have students look at their graphs. Explain that this is an exponential decay model. Ask students to suggest other scenarios that can be represented by exponential decay models.

Review the answers with the Drug Filtering Answer Key.

Questions for Students

- Do you think the drug will ever be completely out of your blood?

  [No. There is always a small amount of drug in the blood. However, it may be so little that it is undetectable.]

- How would you describe the amount of drug in your blood as time goes on?

  [The amount of drug decreases by a smaller and smaller amount, approaching 0, but never quite reaching 0.]

- Do you think all things leave your body this way?

  [There are some substances that that are filtered so slowly that repeated exposure can cause a toxic build up. These include lead and mercury.]

Assessment Options

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Drug Filtering

NAME_____

1. Assume that your kidneys can filter out 25% of a drug in your blood every 4 hours. You take one 1000-milligram dose of the drug. Fill in the table showing the amount of the drug in your blood as a function of time. The first two data points are already completed. Round each value to the nearest milligram.

TIME SINCE TAKING THE DRUG (HR)	AMOUNT OF DRUG IN YOUR BLOOD (MG)
0	1000
4	750
8	
12	
16	
20	
24	
28	
32	
36	
40	
44	
48	
52	
56	
60	
64	
68	

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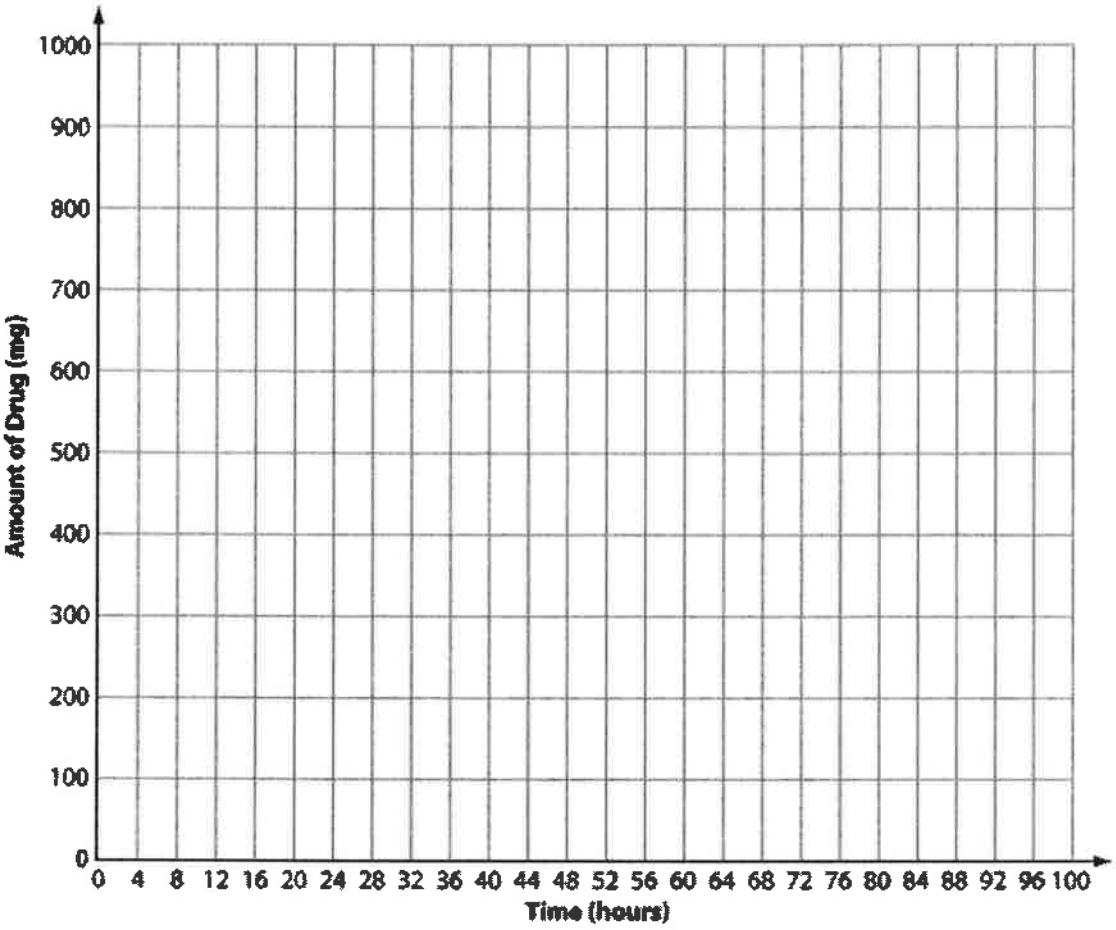
2. Graph the data below.



3. How many milligrams of the drug are in your blood after 2 days?

4. Will you ever completely remove the drug from your system? Explain your reasoning.

5. A blood test is able to detect the presence of the drug if there is at least 0.1 mg in your blood. How many days will it take before the test will come back negative? Explain your answer.

I

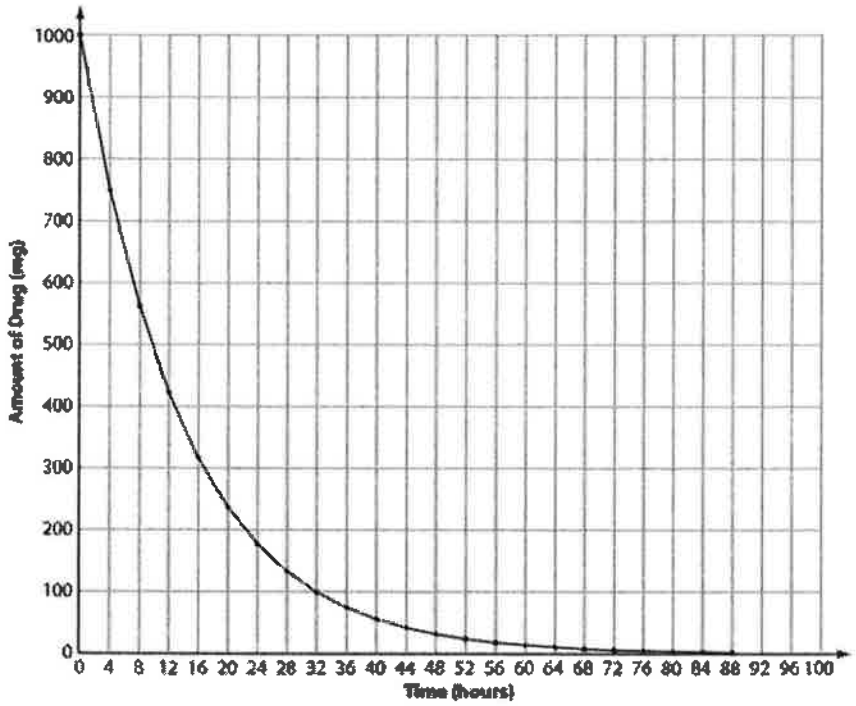
Drug Filtering – Answer Key

1. Assume that your kidneys can filter out 25% of a drug in your blood every 4 hours. You take one 1000-milligram dose of the drug. Fill in the table showing the amount of the drug in your blood as a function of time. The first two data points are already completed. Round each value to the nearest milligram.

Time since Taking the Drug (hr)	Amount of Drug in your Blood (mg)
0	1000
4	750
8	[563]
12	[422]
16	[317]
20	[238]
24	[178]
28	[134]
32	[100]
36	[75]
40	[56]
44	[42]
48	[32]
52	[24]
56	[18]
60	[14]
64	[11]
68	[8]

I

2. Graph the data below.



3. How many milligrams of the drug are in your blood after 2 days?

Since 2 days is equal to 48 hours, check the table or the graph to find the answer. Both representations show that 32 mg remain after 48 hours.

4. Will you ever completely remove the drug from your system? Explain your reasoning.

No. Your kidneys can only remove 25% every 4 hours, so after each four-hour period, 75% of the previous amount remains. Though the numbers get very small, they will never reach 0. Said another way, 75% of a number greater than 0 will always be greater than 0.

5. A blood test is able to detect the presence of the drug if there is at least 0.1 mg in your blood. How many days will it take before the test will come back negative? Explain your answer.

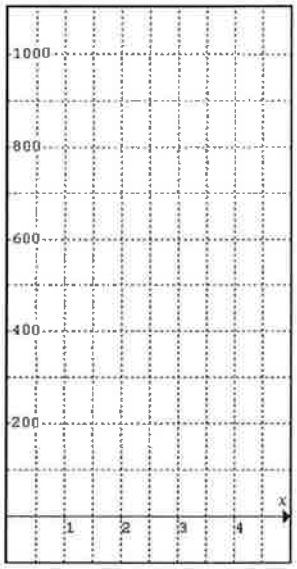
128 hours, or 5 days, 8 hours.
The amount remaining after each four-hour period is 75% of the previous amount.
After n four-hour periods, there are 1000×0.75^n milligrams of drug remaining.
The first time the amount is less than 0.1 is when $n = 42$,
which corresponds to $4 \times 42 = 128$ hours.

J

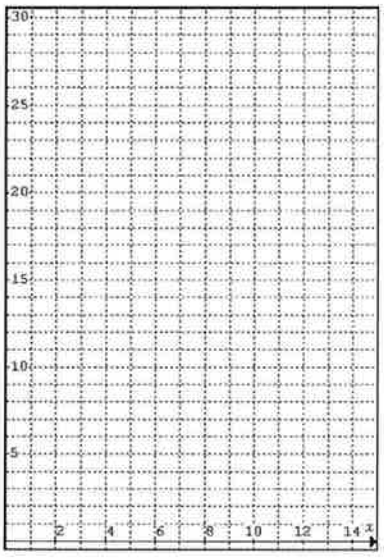
Solving Exponential Equations and Systems Name: _____

Solve each of the following by graphing.

1. Sandra has a new ant farm. She starts with 10 ants and finds that the population grows according to the function $A(x) = 10(4)^x$ where x is the number of months that have passed. During which month will Sandra have 1000 ants in her ant farm?



2. A basketball bounces to a height that is approximately 75% of its original height. If a ball is dropped from a height of 30 feet, how many bounces will it make before it will only reach a height of one foot? The function $b(n) = 30(.75)^n$, where n is the number of bounces, approximates the height the ball reaches on each bounce.



3. There is a linear function and an exponential function that share the points (0,1) and (6, 64). Determine an equation for each function.

J

4. **Use technology to graph and compare the following functions. Address intercepts, asymptotes and end behavior. State the slope or rate of change for each equation. State where the equations cross and indicate over which intervals one function is greater than the other.**

	$F(x) = 20x + 50$	$G(x) = 20(3)^x$
Intercepts		
Asymptotes		
End behavior		
Slope or Rate of Change		
Intersection		
Interval(s) over which the function is greater.		

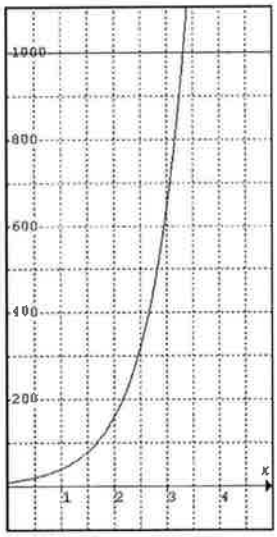
J

Solving Exponential Equations and Systems

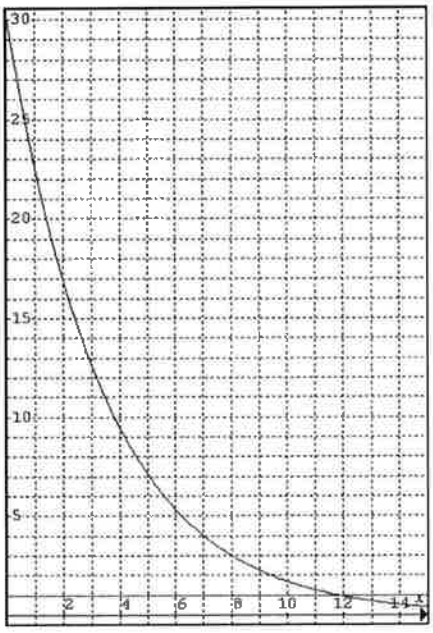
Answer Key

(A.REI.10 and A.REI.11 and F.LE.2/MP3 and MP8)

1. Sandra will have 1000 ants during the fourth month.



2. The basketball will only reach a height of less than one foot on the 12th bounce.



3. Linear function: $y = 10.5x + 1$
   Exponential function: $y = 2^x$

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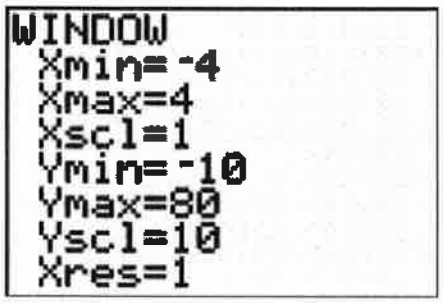
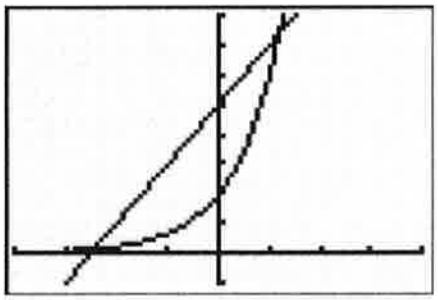
4.

	$F(x) = 20x + 50$	$G(x) = 20(3)^x$
Intercepts	50	20
Asymptotes	none	$Y = 0$
End behavior	As $x \to -\infty$, $F(x) \to -\infty$ And As $x \to \infty$, $F(x) \to \infty$	As $x \to -\infty$, $F(x) \to 0$ And As $x \to \infty$, $F(x) \to \infty$
Slope or Rate of Change	Slope: 20	Rate of Change: 3
Intersection	(-2.431, 1.384) and (1.188, 73.758)	
Interval(s) over which the function is greater.	(-2.431, 1.188)	$(-\infty, -2.431)$ and $(1.188, \infty)$





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ILLUMINATIONS
Resources for Teaching Math

National Debt and Wars

Students will collect information about the National debt, plot the data by decade, and determine whether an exponential curve is a good fit for the data. Then student groups will determine and compare common traits and differences in changes in the National debt in three major eras: the Civil War, World War I, and World War II.

Learning Objectives

Students will:

- Determine if an exponential function adequately describes the changes in the National Debt from its inception;
- Use technology (such as a spreadsheet program) to sketch graphs of functions and bar graphs of percent changes in the National Debt around war times;
- Choose an appropriate scale and compare percent changes in the Debt around three major wars (the Civil War, World War I, and World War II).

Materials

Graphing calculator or spreadsheet program which will calculate curve of best fit
Computers with spreadsheet graphing capability, such as Microsoft Excel®
The National Debt and Wars Activity Sheet
The National Debt and Wars Overhead
Suggested Solutions to the Activity Sheet

Instructional Plan

To be successful with this lesson, students will need the following prior skills and knowledge:

- How to calculate percent change
- How to use technology to find an exponential or linear curve of best fit and interpret the *r* value (correlation)
- How to use a spreadsheet to sketch line and bar graphs
- How to calculate values in a spreadsheet using a formula and "copy that formula down"

If students need help with a spreadsheet, help them record the independent variable values in the left column, then in the next column, use their function to calculate dependent variable values. In the next column, starting with the second piece of data, calculate percent change using the following formula:

$$\frac{new\ value - previous\ value}{previous\ value} \times 100$$

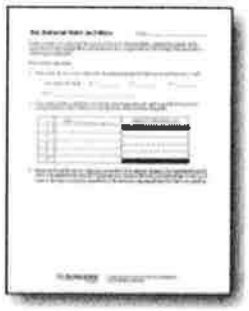
Students should then copy the formula down the column.

K

If some students tend to be the "computer experts" and are doing all of the work while other students are disengaged, try having the "experts" become teachers. Have the students who are sitting back be the ones who work the keyboard and mouse while the experts instruct the workers in what to do.

Pre-Activity

Distribute the National Debt and Wars Activity Sheet to the students.



National Debt and Wars Activity Sheet

The pre-activity questions could be assigned as homework the night before, then discussed as a class at the beginning of the lesson.

Activity I

Students should recognize that the percent change of an exponential growth (decay) curve is constant over uniform intervals. They should also recognize visually the basic shape of an exponential curve.

All groups do Activity I. As groups finish, have them compare results. Did they use the same or different scales? If so, have them explain why they chose their particular scale. Was the data all the same—did some groups pro-rate the data, or did they record the data for the year regardless of the date given by the US Treasury National debt website?

> *Note*: When recording the data by decade, a fairly accurate picture occurs if students do not account for the differences in reporting dates. However, the following processes yield a more accurate set of data to analyze and to standardize the Debt as of January 1 for each year. When the data is given for December 30 or 31, record the data from the preceding year. When the data is given as of June 30 or July 1, estimate the Debt as of January 1 by averaging the two adjacent years. When the data is given as of September 30, estimate the debt as of January 1 by a weighted average of the two adjacent years (0.25 of preceding year + 0.75 of current year). The averaging can be done using spreadsheet formula features. (For 1861, a close look at trends indicate that the Civil War had a huge impact on the debt. Since the Civil War began early in the year, in February, an estimate that follows the trends

K

> of change in preceding years is likely a better estimate that the weighted average.)

The independent variable should be the year, and dependent variable the National debt as of January 1 of the corresponding year.

> *Note:* The activity sheet allows students to research the amount of the National Debt over the history of the United States. If you wish for students to do this, then photocopy the last page of the activity sheet. If you would rather have the students focus on working with the information given to them on the Activity Sheet, it has been provided for them already.

Activity II

In Activity II, assign different groups to work with data from different wars. Initially, students will likely have different scales for the three wars on both the graph of the debt over time and the bar (or column) graph of the percent changes. For final comparison of percent change, use the scale of the percent change for the Civil War, which was the period with the highest percent change.

Students should recognize the following:

- The years preceding, during and after each of the 3 wars produced similar graphs—with fairly level curves leading up to and following the war compared to steep growth in the national debt during the war.
- Piecewise linear functions would be a better curve fit for the data than an exponential curve.
- The percent change in the national debt is variable (another indicator that an exponential function is not a good fit for the data).

After students have completed the Activity II questions on the Activity Sheet, they can work in groups to prepare a presentation of the highlights of their findings. The presentation should include appropriate graphs and tables.

As a class, discuss how wars seemed to be similar or different in their impact on the national debt. To tweak students' comparisons, decide on a scale for the percent change bar chart that could represent the data in all three wars. Rescale those graphs and compare the results. Which war seemed to have to most significant impact? Students should justify how you decided and summarize the main points of the discussion.

Summary

Students should complete the Summary Questions found on the Activity Sheet.

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Sample graphs for the Activity Sheet are available on the Suggested Solutions for the National Debt and Wars Activity Sheet.

Questions for Students

What is the basic shape of an exponential growth curve? An exponential decay curve?

> [An exponential growth curve starts gradually but then speeds up as x gets larger. A decay curve starts at a high value but eventually gets closer and closer to some smaller value.]

It has been said that any curve can be approximated as linear over short intervals. Discuss what this means and whether you think it is true.

> [If a small enough interval is considered, a portion of a curve may appear to be a straight line.]

For an exponential growth (or decay curve), describe the graph of percent change in the dependent values over uniform time intervals.

> [In equal time intervals, the percent change is constant.]

From 1920 to 1930, the debt had declined from $25,952,000,000 to $16,185,000,000. By 1940, the debt had jumped to $42,968,000,000. What was going on in the country economically during the 1920s and 1930s to cause the decline and then increase in the debt?

> [During this time period, the country was suffering from the Great Depression.]

The questions above are also available on The National Debt and Wars Overhead

National Debt and Wars Overhead

Assessment Options

1. Observe student written work and graphs. Is the work comprehensive and accurate? How do you know?

2. Determine whether the presentations identified major points of about the graphs of the wars and exponential growth and percent change.

3. Listen to student discussions. Are students identifying the major math points? Can they identify when exponential growth functions appear to be good

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models and when they do not? Are they discussing the major points of comparisons in changes in the national debt during the three wars?

Extensions

1. Have students explore the national debt and its changes by 4-year presidential terms for different time periods—since the 1960s, between the World Wars, etc.

2. Compare the national debt to Gross Domestic Product (as a ratio) to see how the Debt has changed relative to the GDP standard.

3. Compare the national debt to Per Capita Income (as a ratio) to see how the debt has changed relative to Per Capita Income.

4. Have students investigate potential reasons why particular sharp increases or decreases in the national debt have occurred. *(For example, there was one time in period when a private citizen paid off the national debt.)*

5. Investigate the apparent connection (or lack of connection) to changes in the national debt related to other US wars (War of 1812, Mexican-American War, Indian Wars or Relocation, Iraq I, Iraq II).

Teacher Reflection

- Observe whether all students are engaged. Are different students contributing different strengths? Are students who are strong in historical knowledge interjecting that knowledge? Are students who are proficient in use of technology helping students who need to improve their skills?
- Are all students engaged in the presentation?
- Did students pull from the lesson a greater understanding of exponential growth?
- Did student make connections between math and historical events or economics?
- Was your lesson developmentally appropriate? If not, what was inappropriate? What would you do to change it?

NCTM Standards and Expectations

Algebra 9-12

1. Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
2. Approximate and interpret rates of change from graphical and numerical data.

This lesson prepared by Martha Haehl.

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The National Debt and Wars

NAME _____

In this activity, you will study the history of the U.S. National Debt, compare the impact of the major wars on the national debt, and determine how to represent the debt using a function and by illustrating it graphically.

Pre-Activity Questions

1. Pick values for A_0, b and k and write the related exponential function in the form $A(t) = A_0 b^{kt}$.

   Your choice of values: $A_0 =$ _____ $k =$ _____ $b =$ _____

   $A(t) =$ _____

2. Use a spreadsheet to calculate $A(t)$ for the following values of t and to calculate the percent change between values. Based on your function, fill in the following table.

t	$A(t) =$ _____	PERCENT CHANGE IN $A(t)$
2		
4		
6		
8		
10		

3. Based on the results above, what can you predict about percent change of an exponential growth curve over uniform time periods? Compare your results with curves of other groups to see if you come to the same conclusion regardless of the particular exponential growth curve you made up.

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Activity 1: The National Debt from its Inception

When the U.S. National Bank was first established in 1791, it immediately recorded a debt (i.e., "the national debt") of $75,463,476.52, which predominantly represented the cost of purchasing properties for federal land and the cost of constructing the first federal buildings. Since that time, the debt has had some ups and downs, but overall, it has generally increased.

A Record of the National Debt[1]

YEAR	MONTH & DAY OF RECORD	NATIONAL DEBT (IN DOLLARS)
1791	January 1	75,463,476.52
1801	January 1	83,038,050.80
1811	January 1	48,005,587.76
1821	January 1	89,987,427.66
1831	January 1	39,123,191.68
1841	January 1	5,250,875.54
1851	July 1	68,304,796.02
1861	July 1	90,580,873.72
1871	July 1	2,353,211,332.32
1881	July 1	2,069,013,569.58
1891	July 1	1,545,996,591.61
1901	July 1	2,143,326,933.89
1911	July 1	2,765,600,606.69
1921	June 30	23,977,450,552.54
1931	June 30	16,801,281,491.71
1941	June 30	48,961,443,535.71
1951	June 29	255,221,976,814.93
1961	December 29	296,168,761,214.92
1971	December 31	424,130,961,959.95
1981	December 31	1,028,729,000,000.00[2]
1991	September 30	3,665,303,351,697.03
2001	September 30	5,807,463,412,200.06

[1] Information obtained from the Bureau of Public Debt web site, http://www.publicdebt.treas.gov/, February 2006.

[2] The 1981 record was rounded to the nearest million.

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1. Using a spreadsheet or graphing calculator, input the data above into two columns and graph the curve formed by the points. Sketch the graph below, with appropriate labels for types of quantities and scales.

2. Based on the shape of the curve, what kind of function appears to best represent the data—linear, polynomial, or exponential? Explain how you know.

3. Based on your decision, find the curve of best fit to represent the data. Record the function below along with an explanation of what the independent and dependent variables represent as they relate to the national debt.

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4. How well does the function fit the data?

   a. Plot a graph of the function over the scatterplot of the data. Discuss how well the function appears to model the data. Sketch the graph below.

   b. What is the correlation (r-value) for your curve of best fit? How does that confirm or discount your conclusion from part a?

   c. In your spreadsheet, calculate the percent change in the national debt for each decade. Create a bar graph of the percent changes for each decade. Discuss how the percent change confirms or calls to question your reasoning above on how well your curve fits the data. Attach a copy of the spreadsheet and bar graph.

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Activity 2: The National Debt and Major U.S. Wars

In this activity, each group will explore one of three different major U.S. wars (Civil War, World War I, and World War II) and its apparent impact on the National Debt.

5. The following tables record the National Debt for the three years prior to, the years of, and the three years after each of the major wars.

Civil War (January, 1861 – April, 1865)

Year	Month & Day of Record	National Debt (in Dollars)
1858	July 1	31,972,537.90
1859	July 1	58,496,837.88
1860	July1	64,842,287.88
1861	July 1	90,580,873.72
1862	July 1	524,176,412.13
1863	July 1	1,119,772,138.63
1864	July 1	1,815,784,370.57
1865	July 1	2,680,647,869.74
1866	July 1	2,773,236,173.69
1867	July 1	2,678,126,103.87
1868	July 1	2,611,687,851.19

World War I (August, 1914 – November, 1918)

Year	Month & Day of Record	National Debt (in Dollars)
1911	July 1	2,765,600,606.69
1912	July 1	2,868,373,874.16
1913	July 1	2,916,204,913.66
1914	July 1	2,912,499,269.16
1915	July 1	3,058,136,873.16
1916	July 1	3,609,244,262.16
1917	July 1	5,717,770,279.52
1918	July 1	14,592,161,414.00
1919	July 1	27,390,970,113.12
1920	July 1	25,952,456,406.16
1921	June 30	23,977,450,552.54

World War II (September, 1939 – September, 1945)

Year	Month & Day of Record	National Debt (in Dollars)
1936	June 30	33,778,543,493.73
1937	June 30	36,424,613,732.29
1938	June 30	37,164,740,315.45
1939	June 30	40,439,532,411.11
1940	June 29	42,967,531,037.68
1941	June 30	48,961,443,535.71
1942	June 30	72,422,445,116.22
1943	June 30	136,696,090,329.90
1944	June 30	201,003,387,221.13
1945	June 30	258,682,187,409.93
1946	June 28	269,422,099,173.26
1947	June 30	258,286,383,108.67
1948	June 30	252,292,246,512.99

a. Record the data from above in a two-column table (in your spreadsheet program). In the first column, record the year; in the second column, record the national debt, rounded as appropriate. (You may want to record the values in millions, for example, and write 1,325,789 as 1.3, with the header indicating that the value is in millions of dollars.)

b. Create a graph of the data, appropriately labeling the axes and scales. On the graph and cells of the spreadsheet, identify when the war started and ended. Attach a copy of the spread sheet and graph.

6. Discuss what kind of curve (or curves) could represent the data. Can the curve be well represented by a single function? Are there different curves for different sections of the graph?

7. Reflect upon the apparent impact the war had on the national debt, and discuss this topic with members of your group. Record the highlights of your discussion below.

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Summary Questions:

8. It has been said that any curve can be approximated as linear over short intervals. Discuss what this means and whether you think it is true.

9. For an exponential growth (or decay) curve, describe the graph of percent change in the dependent values over uniform time intervals.

10. On January 1, 1834, the national debt was $4,760,000,000. By January 1, 1835, it had sharply declined to $34,000.

a. What was the decrease in the debt from January 1, 1834 to January 1, 1835?

b. What was the percent decrease during the same time period?

11. From 1920 to 1930, the debt declined from $ $25,952,000,000 to $16,185,000,000. By 1940, the debt jumped to $42,968,000,000. What was going on in the country economically during the 1920's and 1930's that would cause a decline and then an increase in the national debt?

Note to teacher: The following page only needs to be photocopied if you would like students to research the national debt themselves. Otherwise, students may use the data included previously on this activity sheet.

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For each decade from 1791 to 2001, record the national debt, rounded to the nearest million dollars. Data can be obtained from the Bureau of Public Debt web site, http://www.publicdebt.treas.gov.

YEAR	NATIONAL DEBT, IN MILLIONS OF DOLLARS (ROUNDED TO THE NEAREST MILLION)
1791	
1801	
1811	
1821	
1831	
1841	
1851	
1861	
1871	
1881	
1891	
1901	
1911	
1921	
1931	
1941	
1951	
1961	
1971	
1981	
1991	
2001	

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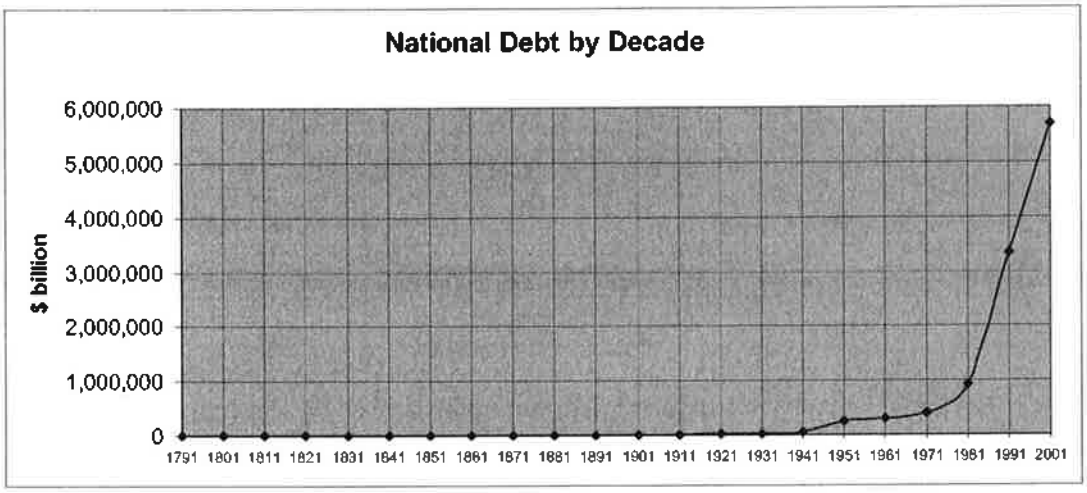
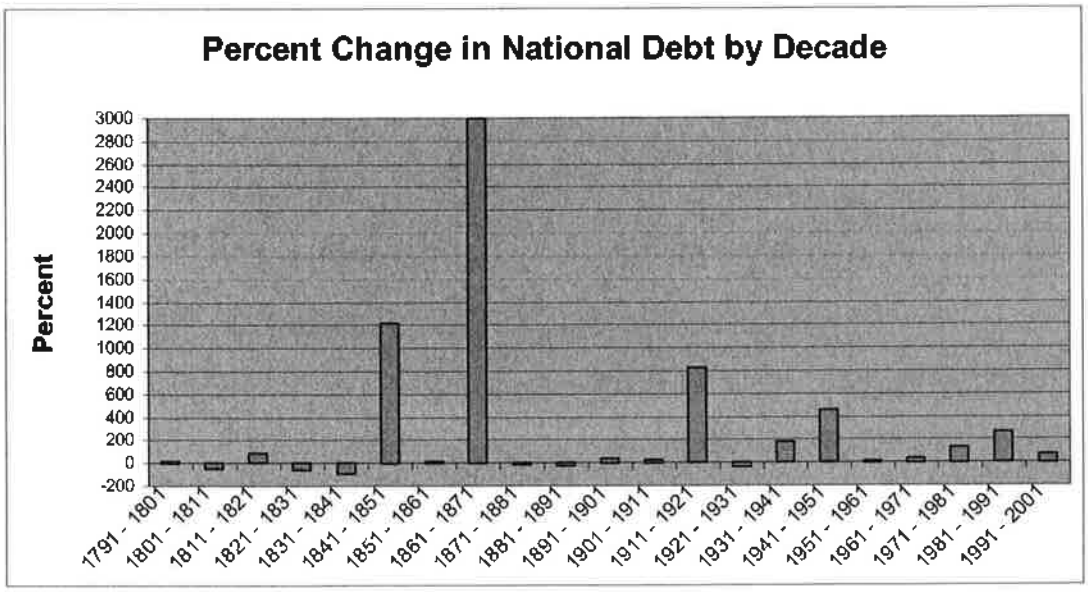
The National Debt and Wars

SUGGESTED SOLUTIONS





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National Debt by Decade

Year	National Debt as of Jan 1 Rounded to Nearest Million $	Comments	10-Year Span	Percent Change
1791	75,000,000			
1801	83,000,000		1791 - 1801	11%
1811	48,000,000		1801 - 1811	-42%
1821	90,000,000		1811 - 1821	88%
1831	39,000,000		1821 - 1831	-57%
1841	5,000,000		1831 - 1841	-87%
1851	66,000,000	Adjust back 6 mo.	1841 - 1851	1218%
1861	78,000,000*	Trend from 7-1-1860	1851 - 1861	18%
1871	2,417,000,000	Adjust back 6 mo.	1861 - 1871	2999%
1881	2,095,000,000	Adjust back 6 mo.	1871 - 1881	-13%
1891	1,549,000,000	Adjust back 6 mo.	1881 - 1891	-26%
1901	2,140,000,000	Adjust back 6 mo.	1891 - 1901	38%
1911	2,709,000,000	Adjust back 6 mo.	1901 - 1911	27%
1921	24,965,000,000	Adjust back 6 mo.	1911 - 1921	822%
1931	16,493,000,000	Adjust back 6 mo.	1921 - 1931	-34%
1941	45,964,000,000	Adjust back 6 mo.	1931 - 1941	179%
1951	256,290,000,000	Adjust back 6 mo.	1941 - 1951	458%
1961	290,628,000,000	12/30/1960 data	1951 - 1961	13%
1971	389,158,000,000	12/30/1970 data	1961 - 1971	34%
1981	930,210,000,000	12/30/1980 data	1971 - 1981	139%
1991	3,341,311,000,000	adjust back 9 mo.	1981 - 1991	259%
2001	5,707,499,000,000	adjust back 9 mo.	1991 - 2001	71%

Throughout the history of the National Debt, the official debt was calculated on various dates—January 1, June 30 or July 1, December 30 or 31, or September 30. The value of the national debt were prorated through linear enterpolation to for the years 1851, 1871 – 1951, and 1991 and 2001 to estimate the debt as of January 1 of the given year.

The value for January 1, 1861 was estimated strictly on pattern of the debt in years prior to the Civil War and was calculated as a 19% increase from the 1860 debt adjusted from July 1 to January 1. This was done because the Civil War, which started in February, 1861, apparently had a great impact on the national debt, so an adjustment based on a strict average between the two July 1 debts would have likely been too high of an estimate.

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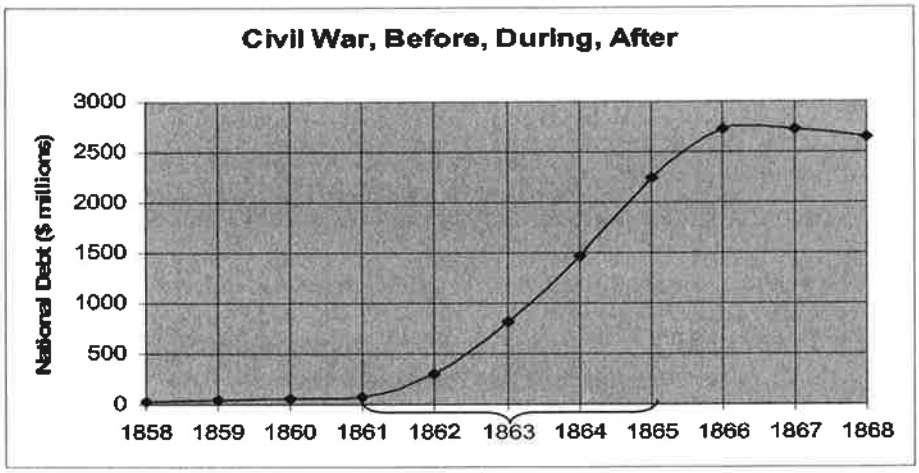
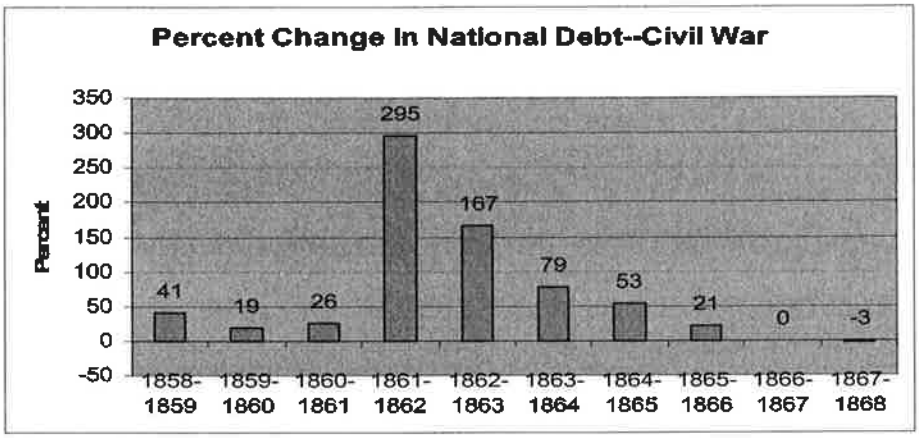
Civil War and National Debt

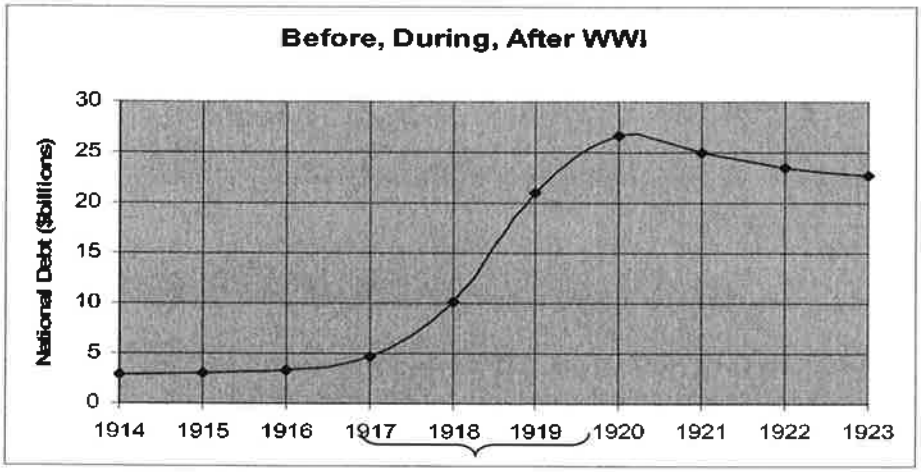


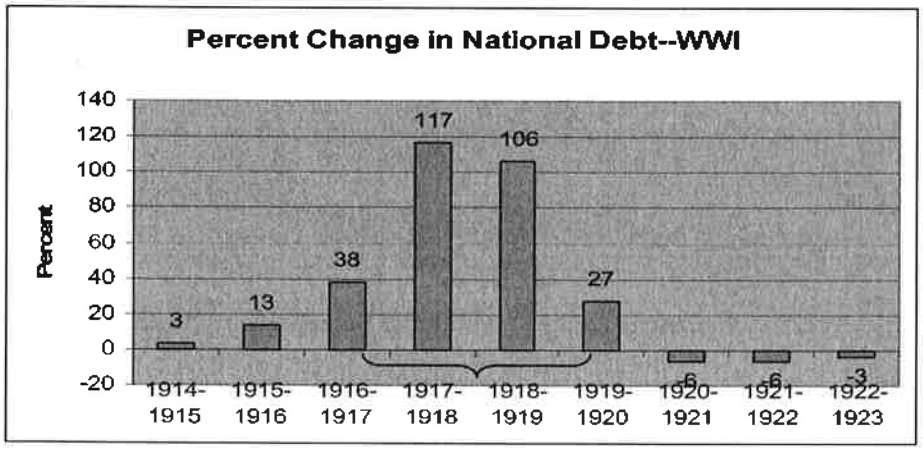


As of Jan 1	In $ Millions	Years	Percent Change
1858	37	1858-1859	41%
1859	52	1859-1860	19%
1860	62	1860-1861	26%
1861	78	1861-1862	295%
1862	308	1862-1863	167%
1863	822	1863-1864	79%
1864	1468	1864-1865	53%
1865	2249	1865-1866	21%
1866	2727	1866-1867	0%
1867	2726	1867-1868	-3%
1868	2645		

Civil War began in Feb 1861, ended April and May, 1865. War years shaded. Values were interpolated to adjust values to Jan 1 from June 30. (1861 debt was estimated based on 19 percent increase from 1860.)

WWI and National Debt





Year	National Debt ($billion)	Years	Percent Change in National Debt
1914	2.9	1914-1915	3%
1915	3	1915-1916	13%
1916	3.4	1916-1917	38%
1917	4.7	1917-1918	117%
1918	10.2	1918-1919	106%
1919	21	1919-1920	27%
1920	26.7	1920-1921	-6%
1921	25	1921-1922	-6%
1922	23.5	1922-1923	-35
1923	22.7		

WWI--US entered war officially March 20, 1917, Ended with Treaty of Versailes June 28, 1919. Some troops were committed prior to the official war time.

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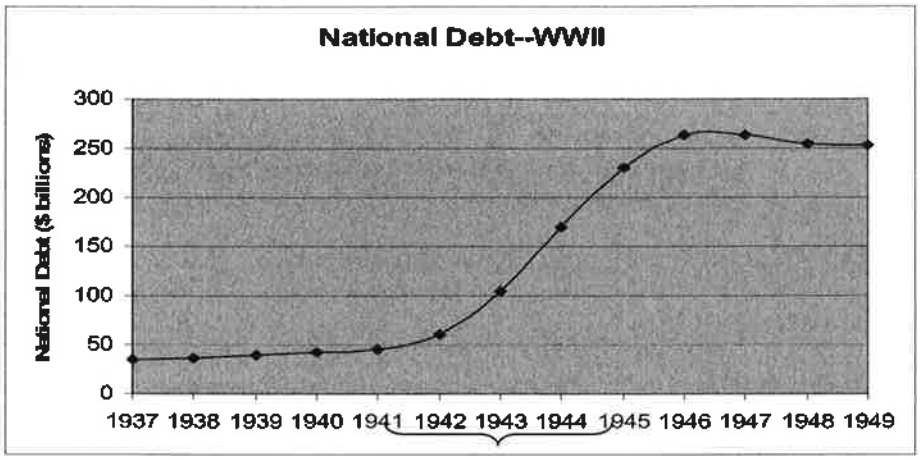
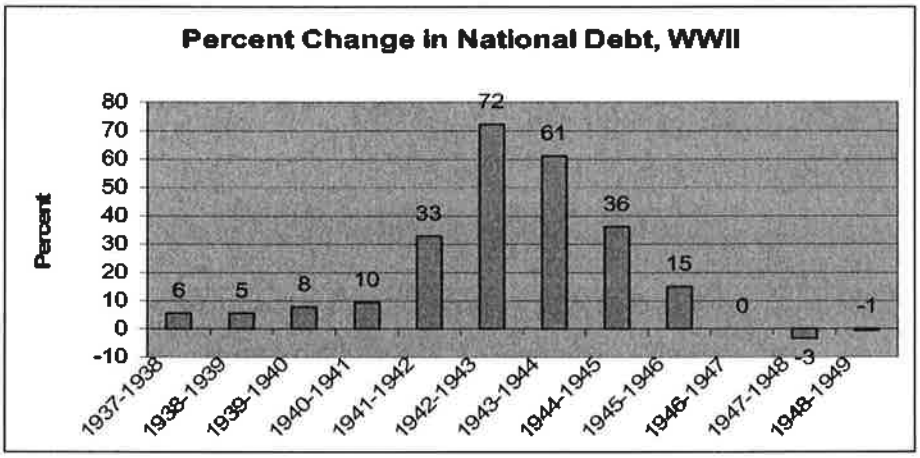
WWII and the National Debt



War Years



Year	National Debt ($billion)	Years	Percent Change in National Debt
1937	35	1937-1938	6%
1938	37	1938-1939	5%
1939	39	1939-1940	8%
1940	42	1940-1941	10%
1941	46	1941-1942	33%
1942	61	1942-1943	72%
1943	105	1943-1944	61%
1944	169	1944-1945	36%
1945	230	1945-1946	15%
1946	264	1946-1947	0%
1947	264	1947-1948	-3%
1948	255	1948-1949	-1%
1949	253		

US entered WWII Dec 7 1941; European treaty May 8, 1945; Japan surrendered Aug 14, 1945.

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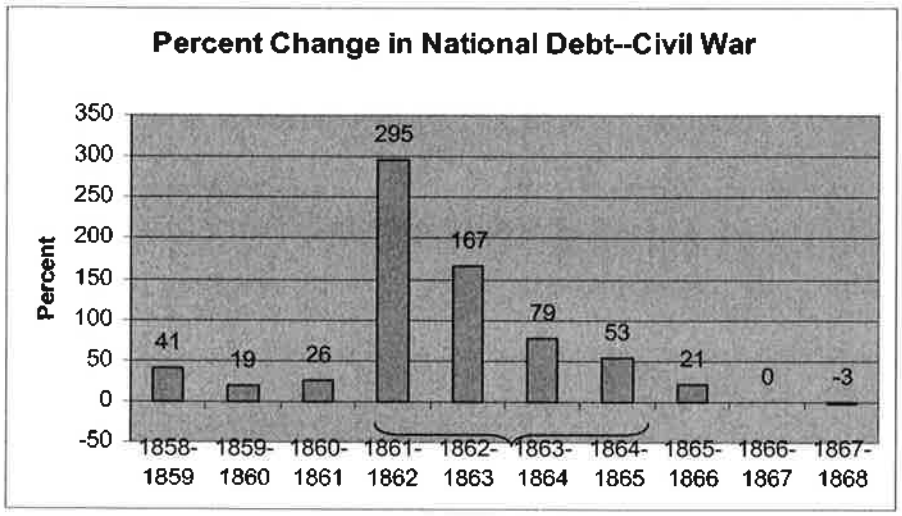
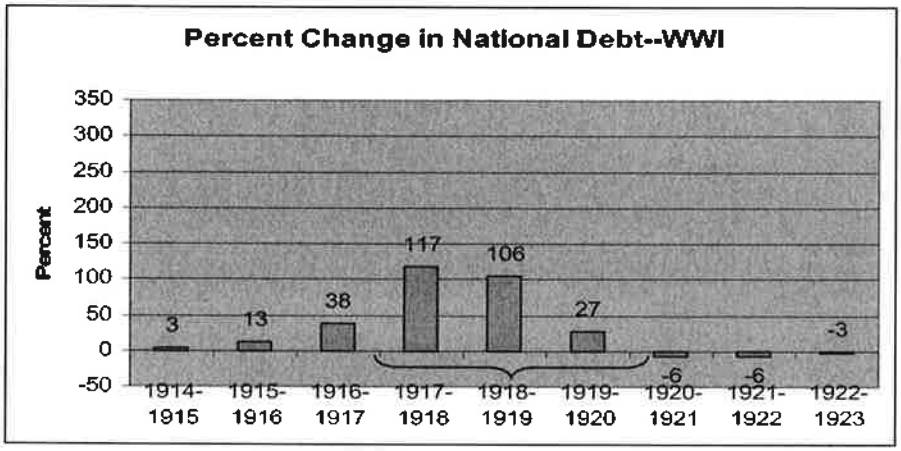
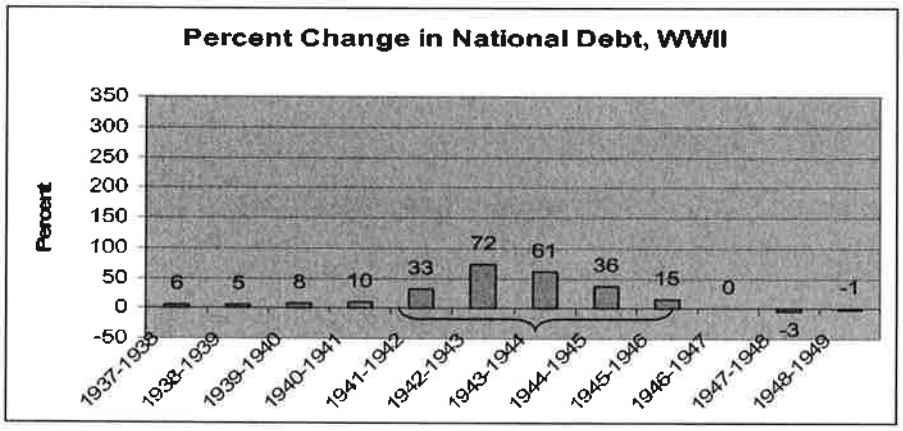
Comparison of Percent Change by War (with same scale graphs)



War Years



War Years



War Years

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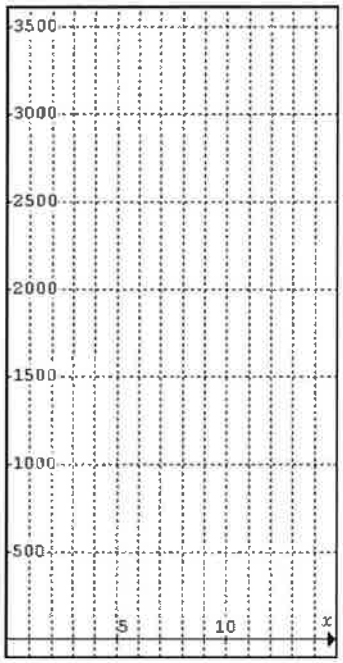
Name: _____

Solving Exponential Equations and Systems **Date:** _____

Solve the following problem by using a graph.

1. The function $f(x) = 3500(0.95)^x$, where x is the time in years, models a squirrel population. Use a graph to determine about how many years will pass before the squirrel population is below 2000.



2. Samuel opened a CD account in 2000 with a $5000 gift from his grandparents. The account earns 3% compounded annually. He leaves the money. The table shows how much money was in the account at the end of the year for the first 4 years. Also in 2000, Samantha opened an account with a $6000 gift from her uncle. She has added $100 to the account every year and has never taken money out of the account. The table also shows how much money was in her account at the end of the year for the first 4 years. Extend the table to determine at the end of what year Samuel will have more money in his account than Samantha has in her account.

Number of years since 2000	Samuel's Account	Samantha's Account
0	5000	6000
1	5150	6100
2	5304.50	6200
3	5463.64	6300
4	5627.50	6400

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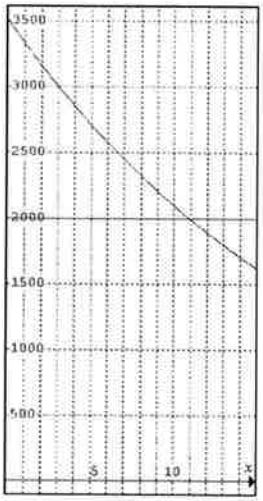
Solving Exponential Equations and Systems Key

1. The squirrel population will be below 2000 after the 11th year.
(Obj 2/A.REI.10/3)



2. Samuel will have more money in his account than Samantha has in her account at the end of the 13th year, or in 2013. (Obj 2/A.REI.11/3 and Obj 2/F.LE.2/3)

Number of years since 2000	Samuel's Account	Samantha's Account
0	5000	6000
1	5150	6100
2	5304.50	6200
3	5463.64	6300
4	5627.50	6400
5	5796.37	6500
6	5970.26	6600
7	6149.37	6700
8	6333.85	6800
9	6523.87	6900
10	6719.58	7000
11	6921.17	7100
12	7128.80	7200
13	7342.67	7300

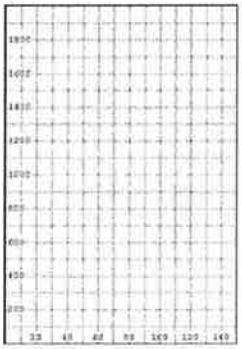
M

Exponentials In Context

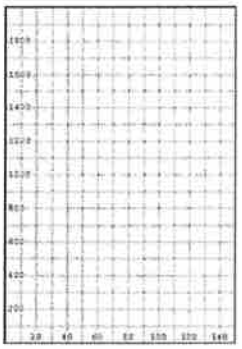
Name: _____

For each of the following situations, write a function. Then graph the function and state the meaning of the y-intercept.

1. $1000 is invested at 2% interest compounded quarterly.



2. $1000 is invested at 2% interest compounded monthly.



3. A culture has 5 bacteria. The number of bacteria triples every 2 hours.

4. Astatine-218 has a half-life of 2 seconds. A scientist is studying the amount of astatine-218 remaining from a 5000-gram sample.

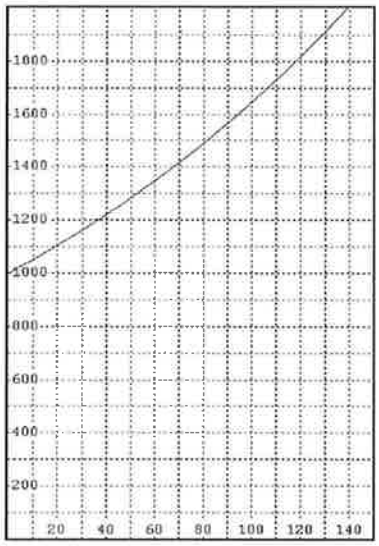
M

Exponentials In Context

Answer Key

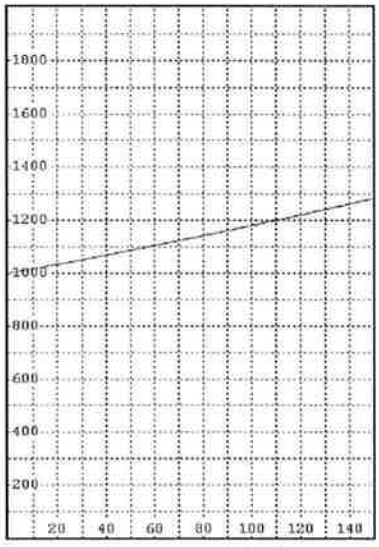
(F.IF.4 and F.BF.1/MP3 and MP8)

1. $y = 1000(1 + (.02/4))^{4x} = 1000(1.005)^{4x}$ where x is the number of years.



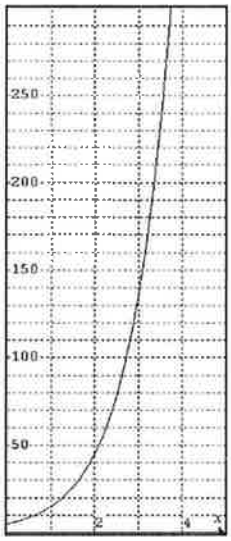
The y-intercept is the $1000 deposited into the account.

2. $Y = 1000(1 + (.02/12))^{12x} = 1000(1.00167)^{12x}$ where x is the number of years.



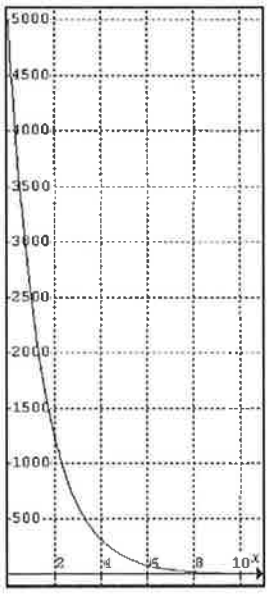
The y-intercept is the $1000 deposited into the account.

3. $Y = 5(3)^x$ where x is the number of 2 hour intervals.



The y-intercept is the 5 original bacteria.

4. $Y = 5000(.5)^x$ where x is the number of 2 second intervals.



The y-intercept represents the 5000-gram sample of astatine-218.

Transformations of Exponentials Name: _____

Rewrite each function to represent the indicated transformation. Then completely describe the effect of the transformation on the asymptote and y-intercept.

1. $f(x) = -2(3)^x$; vertical translation of 7 units.

2. $g(x) = 15^{-x}$; translated down four units

Donald finds 5 different United States stamps on letters in the mail one day. He decides to start collecting US stamps. As more of his friends and family hear about his collection, it grows at a rate of 2% per month.

3. Write a function to represent the situation.

Donald's aunt hears about his collection during the first month and sends to him her collection of 20 stamps from foreign countries.

4. Write a new function to represent the change to the total number of stamps Donald will have.

5. Completely describe the effect on the graph this transformation will have.

N

Transformations of Exponentials — Answer Key

(F.BF.3 and F.LE.5/MP3 and MP8)

1. $f(x) = -2(3)^x + 7$

   The y-intercept changes from -2 to 5. The asymptote moves up 7 units from y = 0 to y = 7.

2. $g(x) = 15^{-x} - 4$

   The asymptote moves down 4 units from y = 0 to y = -4. The y-intercept changes from 1 to -3.

3. $D(x) = 5(1.02)^x$ where x is the number of months that Donald has been collecting stamps.

4. $D(x) = 5(1.02)^x + 20$

5. The shape of the graph will not change. It will be shifted up 20 units. The y-intercept will change from 5 to 25 and represents the total number of stamps Donald has when he starts his collection.

ILLUMINATIONS
Resources for Teaching Math

Predicting Your Financial Future

Students often ask, "When are we ever going to use this?" Compound interest is a topic that provides an inherent answer to this question. In this activity, students use their knowledge of exponents to compute an investment's worth using a formula and a compound interest simulator. Students also use the simulator to analyze credit card payments and debt.

Learning Objectives

Students will:

- Determine the future value of an investment using the formula $A = P(1 + {}^{r}/_{n})^{nt}$
- Decide how much to invest to guarantee a future amount
- Realize how damaging carrying credit card debt can become, even when making the monthly minimum payment

Materials

- Calculator
- Computer with Internet connection
- Savings Account Activity Sheet
- Credit Card Activity Sheet

Instructional Plan

The Savings Account activity sheet begins with a statement of how much certain products have increased in price over time and how hard it is to predict these increases. To get this discussion going, the day before this lesson, have students interview their parents about how much products such as a gallon of gas, a gallon of milk, a loaf of bread, etc cost when they were growing up. Alternatively, have students research the prices of these items 30 years ago. When they return to class, record their answers on the board and compute the percent increase of each item.

To calculate percent increase use the formula $100\left(\frac{\text{price increase}}{\text{original price}}\right)\%$

Students should notice that each item has a different percent increase. Ask students, Why were all the percent increases different? What could be the reason for this?

> [There are several factors that may contribute to price increases. One possible response could be that different items are affected by inflation to different degrees.]

After this discussion, lead students to understand the need to predict the future. Inform them that in terms of invested money, accurate predictions are possible. The purpose of this lesson is to engage and excite students about financial investments, and to educate them about credit card debt.

Distribute the Savings Account activity sheet and have students read the introductory text and work through the first table.


Savings Account Activity Sheet

Here are the correct results for the first table:

Years Investment Has Been in the Bank	Balance at the Start of the Current Year	Interest Earned for the Year (3%)	Balance at the End of the Current Year
1	\$100.00	\$3.00	\$103.00
2	\$103.00	\$3.09	\$106.09
3	\$106.09	\$3.18	\$109.27
4	\$109.27	\$3.28	\$112.55
5	\$112.55	\$3.38	\$115.93

After students complete the first table ask, "Would you want to do this calculation for each investment year from 1 to 80?" [Hopefully, students will say no.] While students may not want to do the hand calculations, tell them you just want the amount in the bank after 80 years. Pose the question, "How can we find this out?" [Possible student responses might be to use spreadsheets, computers, calculator programs, etc.] If students do not suggest it, propose looking for a pattern and developing a formula.

Then, have students complete the algebraic table and work together to generate the formula. The completed table appears below. If students need help with the Simplified Amount column, show them that the entries in this column are based on the factor by grouping method. For example: In the expression $5x(x + 1) + 7(x + 1)$, the term $(x + 1)$ is a common factor of both terms, so it can be factored out. This leaves the simplified expression $(x + 1)(5x + 7)$.

You could also complete the second row, as an example:

$P(1.03) + 0.03P(1.03)$

$= (1.03)(P + 0.03P)$ *** Factored (1.03) from both terms

$= P(1.03)(1.03)$ *** Factored P from the binomial

$= P(1.03)^2$ *** Rewrote the expression using exponents

Years Investment Has Been in the Bank	Balance at the Start of the Current Year	Interest Earned for the Year (3%)	Balance at the End of the Current Year	
			Previous Balance + Interest	Simplified Amount
1	P	$0.03P$	$P + 0.03P$	$P(1.03)$
2	$P(1.03)$			$P(1.03)^2$

		$0.03P(1.03)$	$P(1.03) + 0.03P(1.03)$	
3	$P(1.03)^2$	$0.03P(1.03)^2$	$P(1.03)^2 + 0.03P(1.03)^2$	$P(1.03)^3$
4	$P(1.03)^3$	$0.03P(1.03)^3$	$P(1.03)^3 + 0.03P(1.03)^3$	$P(1.03)^4$
5	$P(1.03)^4$	$0.03P(1.03)^4$	$P(1.03)^4 + 0.03P(1.03)^4$	$P(1.03)^5$

Savings Account Activity Sheet Answers

1.a) This formula calculates the value of the investment, the previous balance plus the interest. If r was used alone, the formula would only calculate the amount of interest earned.

1.b) $6,524.08

2.a) r/n is the interest rate used for each compounding period.

2.b) Earlier we noticed that the exponent in the formula was the number of times interest was assessed over the lifetime of the investment. Since interest is now assessed n times a year and t is measured in years, nt is the number of times interest will be assessed over the life of the investment.

3.a) $6,610.57

3.b) It is greater because interest is being compounded more than once a year.

While this formula is helpful, it cannot handle investments that also have a monthly contribution. To perform those calculations another formula is needed.

To gain an appreciation of the convenience provided by the Compound Interest Simulator, have any students who are interested in using spreadsheets compute the amounts in the savings account after each monthly contribution of $50. Allow these students to observe the number of calculations required to find the amount in the savings account at the end of one year.

Introduce students to the Compound Interest Simulator and how quickly it works to generate these values. Have students use the simulator for Question 4 on the activity sheet.

Savings Account Activity Sheet Answers continued

4.a) 13 years, 11 months

4.b) 1 year, 6 months

When comparing the two individuals who start their investments at different ages, have the students input the given information into the simulator to confirm the given predicted difference and to again demonstrate how quickly the simulator works.

As students are finding their own way to reach $1 million dollars by age 65 in Question 5, record the various strategies on the board as they are discovered. See how many different ways students can come up with. Also comment on each student's graph on the simulator. Ask questions to help students see that each curve grows exponentially. Some of these questions could be:

- Does the graph increase at a constant rate?

  [No, the graph increases at a greater rate as time increases.]

- Is the a straight line or is it curving upward?

  [The graph is curving upward.]

Savings Account Activity Sheet Answers continued

6. 46 years, 3 months

7. This will vary depending on the student's age.

Age 15 = 23.3%, Age 16 = 24.5%, Age 17 = 25.9%, Age 18 = 27.4%

8. Answers will vary.

9. Annual *rate* is the percentage of growth an investment had over a year; the annual *yield* is the amount of money an investment made over a year.

10. You should start investing as early as possible.

Students will now consider credit card debt and how quickly it can get out of control. Distribute the next activity sheet:

Credit Card Activity Sheet

To begin this section of the lesson, you may want to ask if any of your students have a credit card or know someone who has one. Ask those individuals, "What the Annual Percentage Rate is for the card?" The APR is the interest rate you pay for

any purchases made with a credit card if you do not pay the full amount owing by the end of the monthly billing cycle. Be sure to make the observation that this APR is much higher than the interest rates banks offer on savings accounts, which students learned about in Question 8 of the Savings Account activity sheet.

Credit Card Activity Sheet Answers

1. In this situation, the balance on the credit card will reach $8,500 in 7 years, 8 months.

2. Answers will vary.

3. 10 years, 10 months.

4. $1,791.97

5. $2,915

Questions for Students

- For invested money, is the growth linear or exponential?

  [The growth is exponential because the interest earned with each compounding is added to the original amount. This results in receiving interest on the interest on the interest... etc. Linear growth would have a constant rate of increase. Another way to state this is that invested money increases by the same percent each year, so the growth is exponential. If it were increased by the same dollar amount each year, it would be linear.]

- Which is more important, the amount invested or the interest rate?

  [When comparing investments with different percentages of growth, the interest rate is the most important. When comparing actual dollar amounts that investments will be worth in the future, it is a combination of both and depends on the length of the investment. Given enough time, the higher interest rate will always be the best choice. To see a situation where the lower interest rate yields a higher future investment value see the first *Assessment* question below.]

Assessment Options

1. Consider two investments, Plan A and Plan B:

   Plan A: $20,000 investment for 5 years at 10% interest rate
   Plan B: $14,000 investment for 15 years at 15% interest rate with monthly contributions of $50

   Which is the better investment? In what situations would each of these plans be the best option to choose?

2. Have students compare scenarios with varying compounding periods, including annually, monthly, weekly, daily, hourly, etc.

Extensions

1. Doubling Time

   A well known formula for calculating the doubling time of an investment is $^{72}/_{\text{interest rate}}$. This tells us the number of years it will take for an investment to double its value if interest is compounded yearly. If interest is compounded quarterly, then divide the result by 4, and so on. Have students use the simulator and calculate the doubling time for an investment of $1,000 with no monthly contributions. Once that is done, have them change the investment amount to a different amount. The doubling time will remain the same. Tell them that the number 72 is related to this and challenge the class to explain why this doubling formula works regardless of the investment amount.

2. Fill out the following table, plot it, and then run a regression. This is a fun activity to do as a curve-fitting exercise.

Interest Rate	Years for Principal to Double when Compounded Yearly
1%	70
2%	35
3%	...
4%	
...	

   After some trial and error, students will observe that the correct regression to choose is the power regression. The exact equation for the fitted curve is $f(x) = 70.76x^{-0.99}$, but an approximate equation is $y = {}^{72}/_{x}$, hence the name "The Rule of 72." To get better results, more data points are needed. The equation above was arrived at using 4 data points. If a unit on statistics is going to be covered next, this would be a great time to introduce the concept of **sample size**. The larger the sample, the closer the results mimic the expected value. This is also known as the **law of large numbers**.

Teacher Reflection

- What learning styles does this lesson address?
- Did using technology enhance or hinder the mathematical idea being presented?
- What advantages are there in presenting mathematical topics in this setting?
- What were some of the ways your students illustrated that they were actively engaged in the learning process?
- What did you learn, as a teacher, by orchestrating this activity?

NCTM Standards and Expectations

Algebra 9-12

1. Generalize patterns using explicitly defined and recursively defined functions.
2. Use symbolic algebra to represent and explain mathematical relationships.
3. Draw reasonable conclusions about a situation being modeled.
4. Approximate and interpret rates of change from graphical and numerical data.

This lesson was prepared by Patrick Flynn.

Savings Account

NAME _____

The ability to predict the future is an invaluable task when dealing with money. Who would have guessed gas would quadruple in price in a span of ten years? While making predictions about future gas prices is a nearly impossible task, when investing money in a bank account, the future value can be predicted with a great deal of certainty.

For example, suppose you have $2,000 and hide it under your mattress for 40 years. At the end of 40 years, you would still have $2,000. However, if you had invested it in a bank at an interest rate of 4.5%, you would have more than $12,000 at the end of the 40 years. How is this possible? The answer is *compound interest*, which works in the following way. Money is first invested. Then, at regular intervals (for example, monthly, quarterly, yearly), interest is awarded to the account and becomes the investor's money. In this way, interest is earned on previously earned interest – in other words, the interest is *compounded*.

The table below shows banking amounts for a $100 investment that earns 3% interest each year. Complete the table.

Years Investment Has Been in the Bank	Balance at the Start of the Current Year	Interest Earned for the Year (3%)	Balance at the End of the Current Year
1	100.00	3.00	103.00
2	103.00	3.09	106.09
3			
4			
5			

The table below is the same as the one above, but uses algebraic notation. Complete this table and work with your neighbor to develop a formula that could be used to find the bank amount for any year.

Years Investment Has Been in the Bank	Balance at the Start of the Current Year	Interest Earned for the Year (3%)	Balance at the End of the Current Year	
			Previous Balance + Interest	Simplified Amount
1	P	$0.03P$	$P + .03P$	$P \times (1.03)$
2	$P \times (1.03)$	$0.03[P \times (1.03)]$	$P \times (1.03) + 0.03[P \times (1.03)]$	
3				
4				
5				

The formula for the bank amount for any year is $A = P(1+r)^t$, where t = number of years the investment has been in the bank, P = original amount invested, and r = the interest rate expressed as a decimal.

1. a) Why does the formula use $1 + r$?

b) Use this formula to compute the value of the original \$2,000 investment after 40 years at an interest rate of 3% if no additional monthly contributions are made.

Now consider, what happens if the bank compounds the interest four times a year, or twice a year. The formula is changed to $A = P\left(1+\frac{r}{n}\right)^{nt}$, where n is the number of times per year the amount is compounded. (Remember: These formulas are for the situation in which no additional money is being contributed by the investor.)

2. In the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$,

a) What does the fraction $\frac{r}{n}$ represent?

b) What does the exponent nt represent?

3. a) Use this new formula to compute the value of the original \$2,000 investment after 40 years at an interest rate of 3% if the interest is compounded four times a year and if no additional monthly contributions are made.

b) Notice the answer to Question 3.a) is greater than the amount calculated for Question 1 which invested the same \$2,000 for 40 years at 3% interest compounded yearly. Why?

People generally contribute a monthly amount to their savings. This results in the money growing much faster than just waiting for a lump sum to grow to the desired value.

4. **a)** To observe this, use the Compound Interest Simulator to determine how long it will take an investment of \$1,000 at 5% to reach \$2,000 without any monthly contributions.

   b) Now, use the simulator to determine how long it will take the same investment of \$1,000 at 5% to reach \$2,000 when the investor also makes monthly contributions of \$50.

To further observe the effects of compounding interest, imagine we have two people who start saving for retirement.

Person A invests \$2,000 at age 30 and then makes a monthly contribution of \$200 until age 65; the account has an annual interest rate of 4.5%. Person B executes the same plan, but begins at age 40. This means she only has 25 years of investing compared to person A's 35 years.

While ten years may not seem like much, in terms of compound interest, it is. Using the simulator, determine how much her delay of ten year will have cost person B when she retires.

5. Use the simulator to explore how much money you would need to invest to have \$1 million by the time you reach age 65. Try to do this with and without monthly contributions. Try different interest rates as well. Find at least three combinations that yield \$1 million, and record them in the table below.

PRINCIPAL (\$)	MONTHLY CONTRIBUTION (\$)	ANNUAL INTEREST RATE (%)	LENGTH OF INVESTMENT

6. How long will it take an investment of \$10,000 to reach \$1 million if the rate of return is 10% with no additional contributions?

7. What rate of return would be necessary with an investment of $10,000 to make $1 million by the time you reach age 35?

8. Use the Internet to find the current interest rates for online banks and traditional banks (like the one down the street). Use this information to compare the investment results after 20 years on a $40,000 investment.

BANK NAME	INTEREST RATE	BALANCE OF $40,000 INVESTMENT AFTER 20 YEARS

9. Search online or talk to a banker to find the answer to the following question: What's the difference between annual *rate* and annual *yield*?

10. At what age should you start investing your money?

Credit Card

NAME _____

According to the Federal Reserve, in 2007 the average American had $4,000 in credit card debt. How does this happen? Let's say a person decides to join a gym, and buys a membership with a $300 initiation fee plus a $100 monthly charge. After one year, the membership is canceled. This person makes all payments using a credit card, paying only $10 a month, which is the minimum due. In this situation, the gym member would owe a balance of more than $4,000 on the credit card in less than 3 years!

1. Use the Credit Card analyzer in the Compound Interest Simulator to determine how long it will take for a person in the above situation to reach a debt of $8,500 with an annual interest rate of 15.4%.

2. Using the analyzer, vary the initial balance and monthly payment to determine other situations that result in a debt of $4,000.

Now let's determine what most Americans will end up paying if they only make the minimum payment required every month (which is currently close to 4% of the outstanding balance).

3. Use the analyzer to observe how many years it would take to pay off $4,000. Be sure to set the minimum payment percentage to 4% and annual interest rate to 15.4%.

4. How much extra money was paid above the $4,000 initial balance?

5. What if the person put this "extra money" into a savings account at a 4.5% interest rate? How much money would it have grown to if it had been invested for the same amount of time it took to pay off the credit card debt?

P

Name: _____

Applications Involving Exponentials

Date: _____

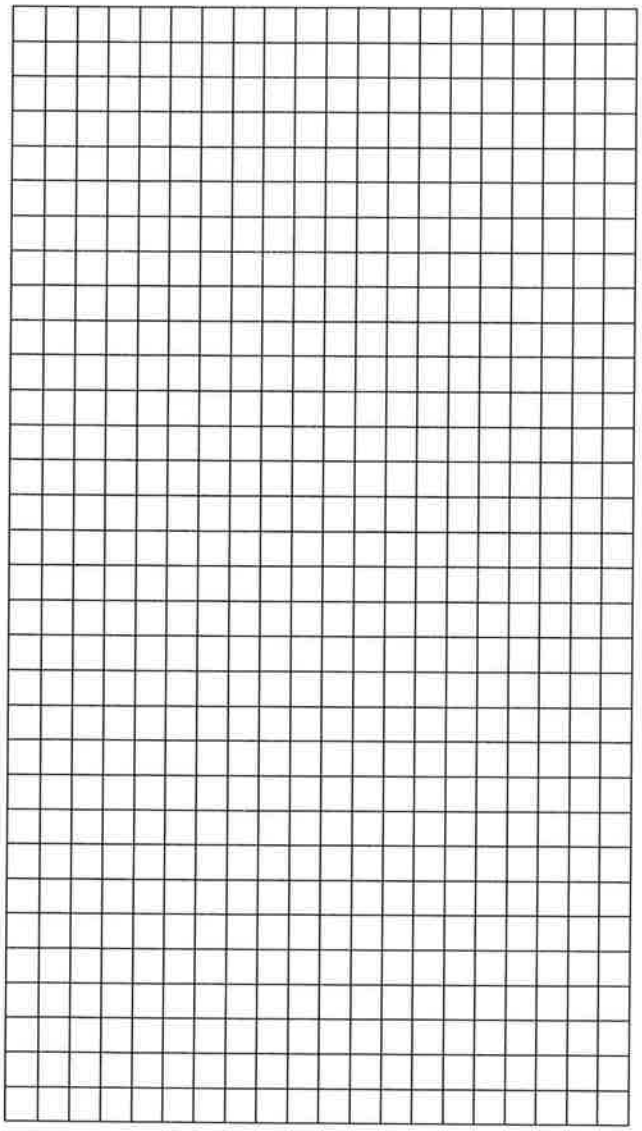
1. The original value of a car is $32,000, and the value decreases by 15% each year. Write a function to represent this situation.

2. Graph the function from problem 1.

3. What does the y-intercept represent?

4. This particular type of car has an optional feature that is worth $3000 more and is not subject to depreciation. Write a new function to represent the addition of this feature at the time of purchase.

5. Completely describe the effect on the graph of this transformation to the function.

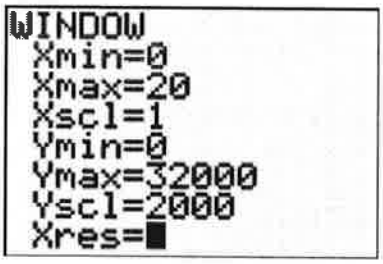


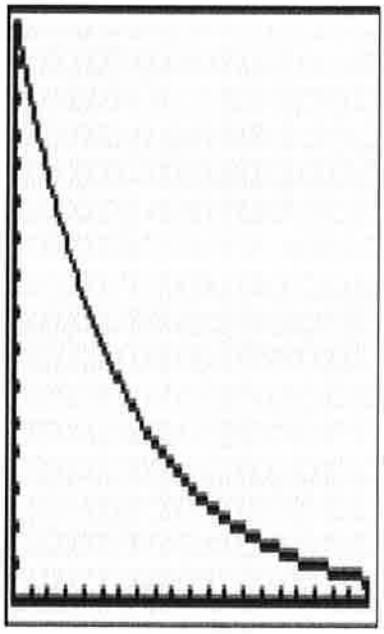
P

Applications Involving Exponentials Answer Key

1. $f(x) = 32{,}000(.85)^x$ (Obj 3/F.BF.1.a/2)

2.





(Obj 3/F.IF.4/3)

3. The y-intercept represents the initial price of the car, $32,000. (Obj 3/F.IF.4/2)

4. $g(x) = 32{,}000(.85)^x + 3000$ (Obj 3/F.LE.5/2)

5. The graph would be translated up $3000. The y-intercept would change to $35,000 and the asymptote would become y = $3000 rather than y = 0. (Obj 3/ F.BF.3/3)

Student Name	**Teacher Name**
Progress Report Starting Grade:_____ Progress 1: Date_____: Grade:_____ Progress 2: Date_____: Grade:_____ Progress 3: Date_____: Grade:_____	**Parent Contact Log – Contact if student is not complying with Improvement Plan** Phone #_____ Date, Reason, Outcome 1 _____ 2._____ 3. _____ 4.._____ 5.._____
Requirements of Your Improvement Plan: □ Attend afterschool tutoring on: Tues Weds □ Make-up Missing: □ Test □ Quiz □ Classwork □ Homework □ Other:_____	**Agreement to follow this Improvement Plan** Student Signature_____ Teacher Signature_____ Additional Comments (Student or Teacher):

Student Name	**Teacher Name**
Progress Report Starting Grade:_____ Progress 1: Date_____: Grade:_____ Progress 2: Date_____: Grade:_____ Progress 3: Date_____: Grade:_____	**Parent Contact Log** Phone #_____ Date, Reason, Outcome 1 _____ 2._____ 3. _____ 4._____ 5._____
Requirements of Your Acceleration Plan:	**Agreement to follow this Acceleration Plan** Student Signature_____ Teacher Signature_____ Additional Comments (Student or Teacher):

Lesson 4: Applications: Growth and Decay

Compound interest has been called the "eighth wonder of the world." This may seem like an exaggeration, until you know that compound interest follows an exponential growth equation. Given time, your money can grow rapidly if left alone in an account with compound interest.

Exponential equations are used to model many real-world problems in finance, science, sociology, and other fields. You will learn about some of these practical applications in this lesson.

GOALS FOR THIS LESSON

- Solve problems involving exponential growth and decay.

In this lesson, you will solve real-world problems that involve exponential growth and decay. Some of these problems use percents.

You can review how to calculate a percent increase or decrease by answering these questions.

Percent of Increase and Decrease

Jorge started the month with a balance of $400 in his checking account, and ended the month with a balance of $450. What is the percent of increase or decrease in the checkbook balance?

Answer

1 of 2 Next

Exponential Growth

Exponential functions are widely used in the real world. However, when you encounter them in real-world applications, chances are they will look different from the ones you have seen so far in this course. They may be written in a different form or use different variables.

Exponential Function

$$f(x) = ab^{x-h} + k$$

where $a \neq 0$, $b > 0$ and $b \neq 1$.

Horizontal asymptote: $y = k$

For example, if you study populations in biology, you might use this exponential growth formula. Notice in this formula that b stands for the initial number of organisms; r stands for the rate of growth; and t stands for time.

Use this formula to write an exponential function that models the growth of the insect population described here.

Exponential Growth Formula

$$y = b(1 + r)^t$$

where y = the total amount, b = the initial amount, t = time, and r = rate of growth

A colony of insects grows at a rate of 5 percent each day. Initially, there were 30 insects. Write an exponential function that models the size of this colony as a function time.

?

Tip

R

Now that you have a function that models the growth of the population over time, you can use it to predict the size of the population at any given time.

A colony of 30 insects grows at a rate of 5 percent each day.

$y = 30 \cdot 1.05^t$

Predict the population after 11 days.

?

Think About It

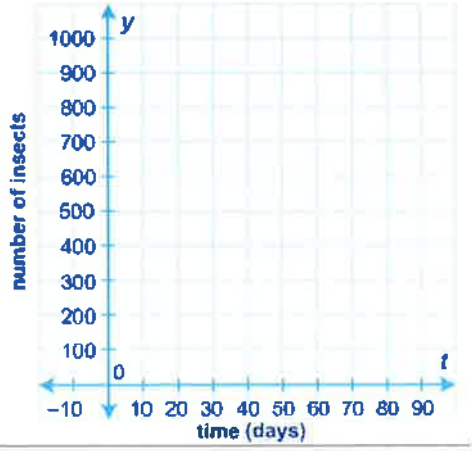
You can also create a table of values and draw a graph to see how the population grows over time.

Drag the slider. As you can see, the insect population increases slowly at first, but then grows more quickly over time. This is typical of exponential growth functions.

A colony of 30 insects grows at a rate of 5 percent each day.

$y = 30 \cdot 1.05^t$

t	$y = 30 \cdot 1.05^t$
10	49
20	80
30	127
40	211
50	344
60	560
70	912
80	1486
90	2422



Think About It

Sociologists use the exponential growth formula with historical data to predict future populations. In this problem, you are not given the rate, r. However, you can determine the rate of growth as shown in the first step of this problem.

A sociologist finds that the 2008 population estimate for California was 36,756,666 and the 2007 population estimate was 36,377,534. Using the exponential growth formula, what does she predict the population of California to be in 2020?

Find the growth rate, r.

$36{,}756{,}666 - 36{,}377{,}534 = 379{,}132$

$$r = \frac{379{,}132}{36{,}377{,}534} \approx 0.0104$$

Substitute values into the exponential growth formula.

The population in 2007 is the initial amount. Since 2020 is 13 years later, use 13 for t.

$y = b(1 + r)^t$

Now consider compound interest. When you earn compound interest, you earn interest on both the initial amount, called the principal, as well as any interest already earned. How often this happens is given by the compounding period. For example, if the interest is compounded quarterly, it is compounded four times a year.

The compound interest formula is similar to the formula for exponential growth that you used in the last problem. The main difference between the two formulas is that the compound interest formula has the variable n.

Compound Interest Formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where A = the total amount, P = principal, t = time in years, r = annual interest rate, n = number of compounding periods per year

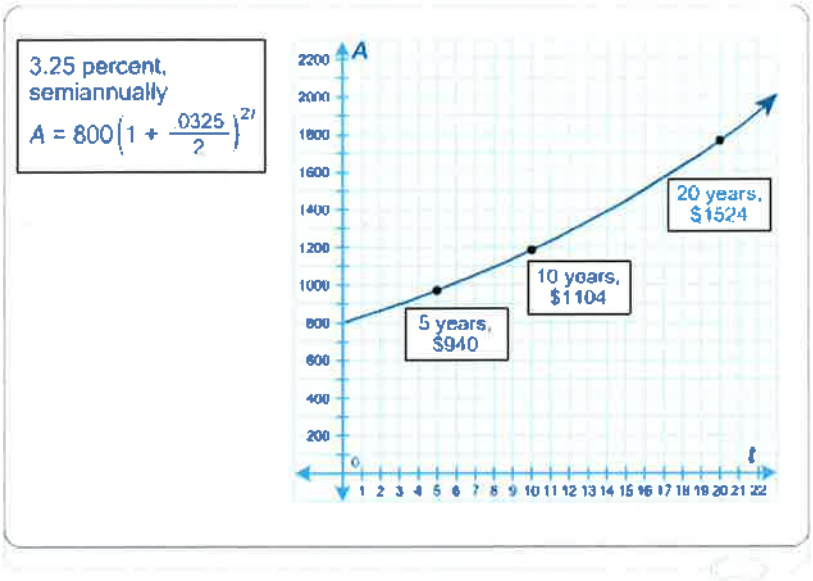
Tanya invests $800 into an account that pays 3.25 percent interest, compounded semiannually. How much will be in the account after 4 years?

?

Think About It

Tanya's money can grow more if she leaves it in an account for more time. It will grow even more in an account with a higher interest rate. Compare how Tanya's $800 dollars will grow over time given the following rates:

- 3.25 percent, compounded semiannually
- 5 percent, compounded semiannually



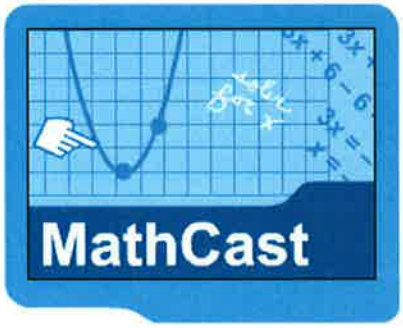
MathCast: Compound Interest

Try to solve this problem.

> After 8 years, what is the total amount of a compound interest investment of $4400 at 3.5%, compounded monthly?

Then click MathCast: Compound Interest to view an explantion.

This activity has audio. Please use speakers or headphones.



MathCast: Compound Interest

Exponential Decay

The general formula for exponential decay is similar to the formula for exponential growth.

Notice that the only difference between this formula and the formula for exponential growth is that r is subtracted from 1, rather than added to it.

Exponential Decay Formula

$$y = b(1 - r)^t$$

where y = the total amount, b = the initial amount, t = time, and r = rate of decay

The tree population in a particular region is 450. The trees are being cut down at a rate of 4.5 percent annually. What is the population of trees in 10 years?

?

Think About It

Try It

Jim bought a used truck in 2006. The table below shows fair market values for the truck for 2007, 2008, and 2009. How much should Jim expect his truck to be worth in 2012?

Year	2006	2007	2008	2009
Value	$12,700	$11,425	$10,270	$9250

Answer

Next

Try It

A scientist starts with 6 bacteria. If the number of bacteria doubles every hour, when will the population exceed 5000?

Answer

Back Next

MTH303B: Algebra II | Unit 2: Exponentials and Logarithms | Lesson 4: Applications: Growth and Decay | Try It

Try It

Sam invests $7500 at 2.8 percent interest, compounded twice a year. What is the amount after 6 years?

Answer

Back Start Over

Summary

Exponential growth models situations where an amount increases by a constant percent. For example, a population may be increasing by 2 percent every year. The formula for compound interest comes from the formula for exponential growth.

Exponential decay models situations where an amount decreases by a constant percent. For example, the value of a car may be decreasing by 15 percent every year. The half-life formula is an example of an exponential decay.

Exponential Growth Formula

$$y = b(1 + r)^t$$

where y = the total amount, b = the initial amount, t = time, and r = rate of growth

Compound Interest Formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where A = the total amount, P = principal, t = time in years, r = annual interest rate, n = number of compounding periods per year

Offline Learning: Applications: Growth and Decay

Now go offline to learn more about exponential growth and decay problems and to practice your skills:

- Read pages 372-375 in the reference guide.
- Complete Problems 1-21 odd on pages 376-377.

The Mid-Unit Test will include questions based on these problems.

MATERIALS

Algebra II: A Reference Guide and Problem Sets - pages 372-377

Student Guide

calculator

Charter School Scope and Sequence Modification

In reviewing the scope and sequence for alignment to the Common Core State Standards and the upcoming Smarter Balanced Assessment, Moyer Academy is requesting a modification to the approved High School K12 MTH123: Algebra I Scope and Sequence.

The following changes have been made to increase alignment to the newly adopted standards, increase student achievement on the new state assessment and, ultimately, better equip Moyer Academy students with the skills and knowledge needed for 21st century college and career readiness:

1. The Semester A: Unit 4: Solving Equations unit has been refocused to exclude any introduction to exponential functions (this has been moved to a new unit). In refocusing this unit, the students will be given greater time to go in depth with linear equations. Fundamental knowledge of linear equations is crucial to further understanding of high school, and real world, math applications. Students will now be given the time to develop the ability to work with, deeply understand, and manipulate linear equations before moving to more complex equations.
2. The Semester B: Unit 7: Logic and Reasoning unit has been removed. The concepts identified for this unit are covered as supporting topics within other units in the upper math classes. This time is instead going to be used to give students a solid introduction to exponential functions.
3. The Semester B: Unit 7: Logic and Reasoning unit has been replaced with the Exponential Functions unit. An area of common misunderstanding and low performance for our students is with manipulating exponential functions and understanding their real world application. Students will now be given an in depth, 3 week unit on exponential functions and their application earlier in their high school math courses. This will properly prepare students to explore new topics in Geometry (the next course in the high school math sequence) and fully develop their ability to look for and make use of structures and patterns (CCSS Math Practices 7 and 8) when applying algebraic knowledge in the Geometry course.

Math Curriculum Scope & Sequence

School: Moyer Academy **Grade or Course: 9th**

Curricular Tool: K12 MTH123: Algebra I **Teacher: _____**

Summary – Students will develop algebraic fluency by learning the skills needed to solve equations and perform manipulations with numbers, variables, equations, and inequalities. Students learn concepts central to the abstraction and generalization that algebra makes possible. Students learn to use number properties to simplify expressions or justify statements; describe sets with set notation and find the union and intersection of sets; simplify and evaluate expressions involving variables, fractions, exponents, and radicals; work with integers, rational numbers, and irrational numbers; and graph and solve equations, inequalities, and systems of equations. They learn to determine whether a relation is a function and how to describe its domain and range; use factoring, formulas, and other techniques to solve quadratic and other polynomial equations; formulate and evaluate valid mathematical arguments using various types of reasoning; and translate word problems into mathematical equations and then use the equations to solve the original problems. This course has a rigorous pace as well as challenging assignments and assessments. It also covers a variety of topics such as number, cost, mixture problems, translating functions, higher degree roots (such as cube roots and fourth roots), and using difficult factoring techniques.

Unit Order By unit title and/or time frame	**Delaware Learning Targets**	**Unit Concepts**	**Essential Questions**
Semester A: Unit 1: Algebra Basics Duration: 2.5 weeks	Mathematical practices-1. Make sense of problems and persevere in solving them. Mathematical Practices – 5. Use appropriate tools strategically. CC.9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients. CC.9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.	Use the order of operations to simplify and evaluate expressions involving addition, subtraction, multiplication, and division of whole numbers. Place grouping symbols in an expression to create a specific value. Determine if two expressions form an equation, and use a replacement set to solve equations.	How are numerical expressions, variable expressions, and equations related? How is the order of operations used to simplify and evaluate expressions? How do you solve a word problem by translate verbal statements into expressions and equations with appropriate operations?

	CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions, CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	Translate a word phrase into an algebraic expression and a word problem into an equation. Describe and apply strategies for solving word problems.	

Semester A: Unit 2: Properties of Real Numbers Duration: 2.5 weeks	Mathematical Practices – 2. Reason abstractly and quantitatively. Mathematical Practices – 3. Construct viable arguments and critique the reasoning of others. Mathematical Practices – 7. Look for and make use of structure. Mathematical Practices – 8. Look for and express regularity in repeated reasoning. CC.9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P. CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.	Identify the graph of a point on a number line. Find the opposite or absolute value of a number. Describe sets using set notation. Find the union or intersection of sets. Determine whether or not a set is closed under a given operation. Compare expressions. Find common factors for a set of expressions. Name the property used to simplify an expression. Simplify an expression involving variable terms and constants. Simplify an expression using the distributive property.	What are the sets(and subsets) and properties of real numbers? How are the properties of real numbers used to rewrite expressions in different ways, and how are these properties useful in simplifying and comparing expressions?
Semester A: Unit 3: Operations with Real Numbers Duration: 2 weeks	Mathematical practices – 1. Make sense of problems and persevere in solving them. Mathematical Practices – 2. Reason	Simplify an expression using number properties. Name the property used to simplify an expression.	How are addition and subtraction, and multiplication and division inverse operations, and how are they used to simplify expressions?

	abstractly and quantitatively. Mathematical Practices – 4. Model with Mathematics CC.9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P. CC.9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. CC.9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	Simplify an expression involving variable terms and constants. Simplify an expression involving variable terms and constants. Simplify an expression involving sums and differences of real numbers. Describe a strategy for adding real numbers. Evaluate a variable expression involving addition or subtraction of real numbers. Evaluate a variable expression involving multiplication or division of real numbers. Simplify an expression involving products and quotients of real numbers. Write an equation to represent a relationship among consecutive integers of a specified type. Solve a word problem involving consecutive integers over a given	What are some strategies used to model word problems using variables, number properties, and operations?

		domain. Provide reasons for steps in a proof involving sums of real numbers. Solve number problems involving consecutive integers.	
Semester A: Unit 4: Solving Equations Duration: 2 weeks	Mathematical Practices – 1. Make sense of problems and persevere in solving them. Mathematical Practices – 4. Model with Mathematics CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. CC.9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V =IR to highlight resistance R. CC.9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	Solve an equation involving addition, subtraction, multiplication, and/or division. Solve an equation involving more than one transformation. Solve an equation that has a variable on both sides. Write an equation that models a word problem involving more than one transformation. Solve a word problem that involves an equation with more than one transformation. Rewrite a formula for a given variable. Solve a word problem involving a transformed	How are inverse operations used to transform and solve equations? How and when is estimation appropriate to solve equations? How are equations used to model real-world situations?

		formula. Use rounding to estimate solutions to an equation. Solve a problem involving the cost of a mix of products with different prices.	
Semester A: Unit 5: Solving Inequalities Duration: 1.5 weeks	Mathematical Practices – 1. Make sense of problems and persevere in solving them. Mathematical Practices – 4. Model with Mathematics CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	Translate a basic word phrase involving inequalities into symbols. Compare and contrast solving equations with solving inequalities. Determine whether an inequality is a true or false sentence. Solve an inequality over a given domain. Graph an inequality in one variable. Use transformations to solve an inequality. Identify the graph of the solution to an inequality, and graph the solution set of an inequality. Find and graph the solution set of a combined	What are the inequality symbols and how are they used to write inequalities for word problems? What are the similarities and differences between solving equations and inequalities? How are the solution sets for one-variable inequalities displayed graphically?

		inequality. Translate an equation or inequality into a word sentence. Write a conjunction or disjunction that is equivalent to a given absolute value sentence. Solve an equation or inequality involving absolute value. Graph the solution of an equation or inequality involving absolute value. Graph an inequality with a restricted domain. Write an inequality that would solve a given word problem. Solve a word problem involving inequalities.	
Semester A: Unit 6: Applying Fractions Duration: 1 week	Mathematical Practices – 1. Make sense of problems and persevere in solving them. Mathematical Practices – 4. Model with Mathematics CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as	Write a ratio in simplest form. Translate a word phrase into a ratio. Write an equation to solve a word problem involving ratios.	What is the relationship between fractions, ratios, and proportions, and how are they used to solve for unknown values in proportional relationships? What is the relationship between fractions, decimals, and percents, and how are percents used when solving

	$(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. CC.9-12.A.CED. 1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. CC.9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.	Solve an equation involving ratios. Solve a word problem involving ratios. Solve for a variable in a proportion. Solve a word problem involving proportions. Write a percent as a fraction in simplest form. Solve for the unknown value in a percent equation. Solve a problem involving discount or markup. Solve a problem involving simple interest. Write an equation for a word problem involving percent. Solve a problem involving percent. Solve a problem involving percent of increase or decrease. Solve a mixture problem.	problems involving discounts, taxes, simple interest, mixtures, and various other applications?
Semester A: Unit 7: Linear	Mathematical Practices – 2. Reason	Identify or find solutions	What are the differences

Equations and Inequalities Duration: 2 weeks	abstractly and quantitatively. Mathematical Practices – 4. Model with Mathematics CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. CC.9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). CC.9-12.A.REI.12 Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. CC.9-12.F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima. CC.9-12.S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. CC.9-12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a	to equations or word problems in two variables over a given domain. Identify points on a graph, given specific criteria. Graph ordered pairs or linear equations in two variables. Find slopes of lines. Write or interpret equations in slope-intercept form. Write equations of lines when given specific criteria. Solve problems involving parallel or perpendicular lines. Graph a linear inequality in two variables. Graph a line when the equation is in point-slope form. Write an inequality in two variables for a given graph.	between graphing two-variable linear equations and two-variable linear inequalities? How can graphing a linear equation or inequality be useful to model a real-world problem? How is the slope and y-intercept interpreted in real-life linear models?

	given line that passes through a given point).		
Semester A: Unit 8: Systems of Equations Duration: 2 weeks	Mathematical Practices – 2. Reason abstractly and quantitatively. Mathematical Practices – 4. Model with Mathematics Mathematical Practices – 5. Use appropriate tools strategically. CC.9-12.A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. CC.9-12.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	Use a graph to solve a system of linear equations. Determine whether a system of linear equations will have 0, 1, or an infinite number of solutions. Write a system of linear equations to solve a word problem. Use substitution to solve a system of linear equations. Use linear combination with or without multiplication to solve a system of equations. Solve a word problem involving a system of equations. Graph a system of two linear inequalities in two variables. Write a system of linear inequalities that corresponds to a given graph.	How does the intersection of two lines on a graph relate to the solution of a system of linear equations, and how many solutions are possible for linear systems of equations? How are substitution, linear combinations, and graphs used to solve systems of linear equations and inequalities in two variables? How are systems of linear equations and inequalities used to model real-world phenomena?

	CC.9-12.A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.		
Semester A: Unit 9: Semester Review and Test Duration: 1 week			
Semester B: Unit 1: Relations and Functions Duration: 2.5 weeks	Mathematical Practices – 2. Reason abstractly and quantitatively. Mathematical Practices – 5. Use appropriate tools strategically. Mathematical Practices – 8. Look for and express regularity in repeated reasoning. CC.9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). CC.9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. CC.9-12.F.IF.7a Graph linear and	Identify whether or not a graph represents a function. Find the range for a relation. Calculate values for functions when given input values. Determine whether a relationship is a linear variation. Find the specific equation for a linear variation. Determine whether a relationship is an inverse variation. Determine whether a relationship is a quadratic variation. Find the specific equation	How do you determine when a relation is a function, and how does the domain relate to the range of a function? How are the differences in equations and graphs for direct, inverse, and quadratic variation, and what are some real-world examples of these?

	quadratic functions and show intercepts, maxima, and minima. CC.9-12.F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. CC.9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. CC.9-12.F.BF.1Write a function that describes a relationship between two quantities. CC.9-12.F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	for a quadratic variation. Graph linear functions, quadratic functions, or functions that exhibit inverse variations. Graph an absolute value function. Find the equation of an absolute value function when given its graph. Write a translation rule that matches a given function and its image under translation.	
Semester B: Unit 2: Rationals, Irrationals, and Radicals Duration: 2.5 weeks	Mathematical Practices – 2. Reason abstractly and quantitatively. Mathematical Practices – 5. Use appropriate tools strategically. Mathematical Practices – 6. Attend to precision. CC.9-12.N.RN. 2 Rewrite expressions involving radicals and rational	Order rational numbers. Find a rational number between two given rational numbers. Write a decimal as a quotient of integers. Simplify sums, differences, products, or quotients of	What is the difference between a rational and irrational number? How is the inverse relationship between squares and square roots used to solve equations, and how is this demonstrated when applying the Pythagorean Theorem? How do you determine when

	exponents using the properties of exponents CC.9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. CC.9-12.A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. CC.9-12.G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	radical expressions. Write a radical with a numerical radicand in simplified radical form. Write a radical with an algebraic radicand in simplified radical form. Determine two consecutive integers between which a square root lies. Evaluate square root expressions or numbers with rational square roots. Solve equations or word problems involving square roots. Use the Pythagorean Theorem or its converse to solve geometric problems. Approximate an irrational square root. Compute cube and higher roots of numbers. Simplify variable expressions involving cube and higher roots.	an approximation is reasonable?
Semester B: Unit 3: Working with Polynomials	Mathematical Practices – 7. Look for and make use of structure.	Write, simplify, or evaluate expressions involving	What are the characteristics of a polynomial?

Duration: 2 weeks	CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. CC.9-12.N.RN. 2 Rewrite expressions involving radicals and rational exponents using the properties of exponents CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	exponents. Determine the degree of a polynomial. Arrange the terms of a polynomial in order by the power of the terms. Simplify polynomials. Add or subtract polynomials. Add or multiply monomials. Find powers of monomials. Multiply polynomials by monomials. Multiply polynomials by polynomials.. Solve word problems involving addition and subtraction of polynomials. Multiply a trinomial by a trinomial. Use a polynomial to represent the area or volume of a figure with binomial side lengths.	How are polynomials and monomials added, subtracted, and multiplied? How are the rules of exponents applied when multiplying monomials and polynomials?

Semester B: Unit 4: Factoring Polynomials Duration: 2 weeks	Mathematical Practices – 7. Look for and make use of structure. Mathematical Practices – 5. Use appropriate tools strategically. CC.9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients. CC.9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P. CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see x^4 - y^4 as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. CC.9-12.A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines. CC.9-12.A.SSE.3c Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $[1.15^{(1/12)}]^{(12t)} ? 1.012^{(12t)}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.	Factor integers and find the greatest common factor of a set of integers. Find the prime factorization of integers. Simplify quotients of monomials. Divide polynomials by monomials. Square binomials. Factor a polynomial that is a difference of squares. Find the greatest monomial factor of a polynomial. Use a common monomial factor to factor a polynomial. Factor a trinomial that is a perfect square. Factor a trinomial with a coefficient of the quadratic term equal to 1. Factor a trinomial with a coefficient of the quadratic term that is not 1. Use two or more factoring	How are the rules of exponents used when factoring and dividing monomials and polynomials? How can recognizing factoring patterns and rewriting an equation in a different form make it easier to find the zeros of a polynomial function?

		techniques to factor a polynomial completely.	
Semester B: Unit 5: Quadratic Equations Duration: 2.5 weeks	Mathematical Practices – 1. Make sense of problems and persevere in solving them. Mathematical Practices – 5. Use appropriate tools strategically. Mathematical Practices – 7. Look for and make use of structure. CC.9-12.A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines. CC.9-12.A.SSE.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. CC.9-12.A.REI. 4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. CC.9-12.A.REI.4b Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.	Determine whether or not quadratic equations have real number solutions. Solve quadratic equations using perfect squares, completing the square, factoring, or the quadratic formula. Write the solutions to quadratic equations in simplest radical form. Calculate the discriminants of quadratic equations. Determine where the vertices of parabolas lie with respect to the x-axes. Use discriminants to classify the nature of the roots of equations.	How do you decide which method is most efficient for solving a particular quadratic equation? How do the x-intercepts of a graph of a parabola relate to its solutions? How are quadratic equations used to model physical phenomena such as area and projectile motion?
Semester B: Unit 6:	Mathematical Practices – 2. Reason	Simplify algebraic	How does rewriting parts of a

Rational Expressions Duration: 1.5 weeks	abstractly and quantitatively. Mathematical Practices – 7. Look for and make use of structure. CC.9-12.A.APR.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. CC.9-12.A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. CC.9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.	fractions. Write equivalent numerical or algebraic fractions. Multiply or divide algebraic fractions with or without factoring. Identify restrictions on variables in algebraic fractions. Find the LCD of two algebraic fractions. Add or subtract algebraic fractions. Divide polynomials. Simplify a product or quotient of three rational expressions. Add or subtract algebraic fractions with at least one trinomial in a denominator.	rational expression (factoring) allow simplification and other operations? How and why do you determine domain restrictions when working with rational expressions?
Semester B: Unit 7: Exponential Functions Duration: 3 weeks	MP 1. Make sense of problems and persevere in solving them. MP2. Reason abstractly and quantitatively.	Calculate the rate of change from a graph and table of exponential function values. Identify components of a	1. What is the difference between a linear function and an exponential function? 2. What does it mean to solve an exponential function or

	MP3. Construct viable arguments and critique the reasoning of others. MP 4. Model with mathematics. MP 5. Use appropriate tools strategically. MP 6. Attend to precision. MP 7. Look for and make use of structure. MP 8. Look for and express regularity in repeated reasoning. LE.A.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the	graph and explain the components in terms of the situation. Expain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ Demonstrate and explain the effects of changing the parameters in a linear or exponential function. Compare properties of two functions each represented in a different way Find the solutions approximately, e.g., using technology to graph the functions, Make tables of values, or find successive approximations Construct linear and	system? 3. What effect does altering an exponential function by adding k have on its graph or on a situation? 4. How do we model real life data using exponential functions? 5. What is a limit?

	coordinate plane, often forming a curve (which could be a line). REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the	exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). Interpret key features of graphs and tables in terms of the quantities Sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative) Find the value of k given the graphs Experiment with cases and	

	value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.	illustrate an explanation of the effects on the graph using technology. Interpret the parameters in a linear or exponential function in terms of a context.	
Semester B: Unit 8: Semester Review and Test Duration: 1 week			

Curriculum Scope & Sequence

School Moyer Academy____ Grade or Course Grade 11 MTH302: Algebra II Teacher _____

Unit Order By unit title and/or time frame	Learning Targets Content Standards, Grade Level Expectations, Proficiency Level Expectations, or Grade Cluster Benchmarks	Theme or Big Idea	Enduring Understandings	Essential Questions
Semester A: Unit 1: Numbers, Expressions, and Equations	**A-SSE** **Interpret the structure of expressions** 1. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. 2. Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2). **A-CED** **Create equations that describe numbers or relationships** 1.Create equations and inequalities in one variable and use them to solve problems. Include	**Expressions and Equations**	**Numbers can be represented in multiple ways.** **Properties of real numbers allow complicated expressions to be rewritten in simpler ways.** **The same operations can be applied in problem situations that seem quite different from one another.** **Being able to compute fluently means making smart choices about which tools to use and when to use them.** **Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations so solutions can be found.** **Inverse operations allow computations to be "undone".**	**How can algebraic expressions and equations be represented?** **How are key words in word problems used to determine the expression, equation, or operation used to solve the problem?** **What are the properties of real numbers?** **How are these properties applied to simplify, rewrite, or compare expressions?**

	equations arising from linear and quadratic functions, and simple rational and exponential functions. 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V =IR to highlight resistance R. **A-REI** **Understand solving equations as a process of reasoning and explain** **the reasoning** 1.Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **Solve equations and inequalities in one variable** 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.			
Semester A: Unit 2: Linear Equations and Systems	**A-SSE** **Interpret the structure of expressions** 1. Interpret expressions	**Linear Equations and Systems**	**A linear equation represents a proportion relationship between**	**What is the relationship between solving an equation graphically and**

	that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. 2. Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2). **A-CED** **Create equations that describe numbers or relationships** 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost		**two variables.** **A system of linear equations can be solved by identifying the point where the lines intersect.**	**algebraically?** **How do you solve a system of equations?** **How are linear models used to represent real-world situations?**

	constraints on combinations of different foods. **A-REI** **Solve systems of equations** 5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. 6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **Represent and solve equations and inequalities graphically** 10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **F-IF** **Interpret functions that arise in applications in terms of the context** 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and			

	sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **Analyze functions using different representations** 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated			

	cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. **S-ID** **Summarize, represent, and interpret data on two categorical and quantitative variables** 6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association. **Interpret linear models** 7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **F-LE** **Construct and compare linear, quadratic, and exponential models**			

	2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **Interpret expressions for functions in terms of the situation they model** 5. Interpret the parameters in a linear or exponential function in terms of a context. **F-BF** **Build new functions from existing functions** 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.			
Semester A: Unit 3: Functions	**A-CED** **Create equations that describe numbers or relationships** 1.Create equations and	**Functions**	**Change is fundamental to understanding functions.** **A function is a special kind of relation in**	**How do you determine if a relation is a function?** **How are the domain**

	inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. **A-CED** **Create equations that describe numbers or relationships** 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. **Represent and solve equations and inequalities graphically** 10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which		**which the value of one variable depends on the value of another variable.** **Functions are used to represent the relationship between unknown quantities.** **Graphs are visual representations of functions/numerical relationships.**	**and range related in graphical and algebraic representations of a function?** **What are unique features of piecewise functions, and how does this relate to their graphs?** **How do functions model real-world phenomena?**

	could be a line). **F-IF** **Understand the concept of a function and use function notation** 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). 2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **Interpret functions that arise in applications in terms of the context** 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features			

	include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. **Analyze functions using different representations** 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. 9. Compare properties of two functions each represented in a different way (algebraically,			

	graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. **F-BF** **Build a function that models a relationship between two quantities** 1.Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.			

	Build new functions from existing functions 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. 4. Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠1. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.			
Semester A: Unit 4: Inequalities	**A-SSE** **Interpret the structure of expressions**	**Inequalities**	**Rules of arithmetic and algebra can be used together with notions**	**How are solving inequalities similar and different from**

	1. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. **A-CED** **Create equations that describe numbers or relationships** 1.Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. **A-REI**		**of equivalence to transform inequalities so solutions can be found.** **Inverse operations allow computations to be "undone".**	**solving equations?** **How are inequalities used to model real-world situations?** **How are the graphs of systems of inequalities different from the graphs of systems of linear equations?**

	Represent and solve equations and inequalities graphically 12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.			
Semester A: Unit 5: Polynomials and Power Functions	**A-SSE** **Interpret the structure of expressions** 1. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. 2. Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2). **A-APR** **Perform arithmetic operations on**	**Polynomials and Power Functions**	**Polynomials are a sum of terms involving non-negative integer powers of a variable. Expressions can be rewritten using number properties to identify relationships.**	**How are polynomials added, subtracted, multiplied, divided, and factored?**

	polynomials 1.Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **Use polynomial identities to solve problems** 4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x2 + y2)2 = (x2 – y2)2 + (2xy)2 can be used to generate Pythagorean triples. **A-CED** **Create equations that describe numbers or relationships** 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A-REI** **Represent and solve equations and inequalities graphically** 10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often			

	forming a curve (which could be a line). **F-IF** **Interpret functions that arise in applications in terms of the context** 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. **F-BF** **Build new functions**			

	from existing functions 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.			
Semester A: Unit 6: Rational Equations	**A-SSE** **Interpret the structure of expressions** 1. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. 2. Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2). **A-APR**	**Rational Equations**	**Expressions can be rewritten using number properties to identify relationships. Graphs are visual representations of functions/numerical relationships. Division by zero is undefined; therefore the domain is restricted in some rational expressions.**	**How is factoring used to solve rational equations? When is it necessary for the domain to be restricted in a rational equation? What is the relationship between a rational equation and its graph?**

	Rewrite rational expressions 6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. 7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. **A-CED** **Create equations that describe numbers or relationships** 1.Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. 3. Represent constraints by equations or inequalities,			

	and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. **A-REI** **Understand solving equations as a process of reasoning and explain the reasoning** 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **F-IF** **Analyze functions using different representations** 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.			

	F-BF **Build new functions from existing functions** 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.			
Semester A: Unit 7: Radicals and Complex Numbers	**N-RN** **Extend the properties of exponents to rational exponents.** 1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5. 2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	**Radicals and Complex Numbers**	**Radicals are inverse operations of exponents.** **Complex numbers contain the imaginary number i, equal to the square root of -1.**	**How are inverse operations used to solve radical equations?** **How are operations and properties with complex numbers different than operations and properties with real numbers?**

	N-CN **Perform arithmetic operations with complex numbers** 1. Know there is a complex number i such that i2 = –1, and every complex number has the form a + bi with a and b real. 2. Use the relation i2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **Represent complex numbers and their operations on the complex plane.** 4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. 5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, (–1 + √3 i)3 = 8 because (–1 + √3 i) has modulus 2 and argument 120°.			

	6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. **Use complex numbers in polynomial identities and equations.** 8. (+) Extend polynomial identities to the complex numbers. For example, rewrite x2 + 4 as (x + 2i)(x – 2i). **A-SSE** **Interpret the structure of expressions** 1. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. 2. Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).			

	Write expressions in equivalent forms to solve problems 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. **A-REI** **Understand solving equations as a process of reasoning and explain the reasoning** 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **F-IF** **Analyze functions using different representations** b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value			

	functions.			
Semester A: Unit 8: Quadratic Functions	**A-SSE** **Interpret the structure of expressions** 1. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. 2. Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2). **Write expressions in equivalent forms to solve problems** 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or	**Quadratic Functions**	**Expressions can be rewritten using number properties to identify relationships. Functions are used to represent the relationship between unknown quantities. Graphs are visual representations of functions/numerical relationships.**	**How does a quadratic equation relate to its graph?** **How is factoring used to solve quadratic equations?** **How is completing the square used to solve quadratic equations?** **How is the quadratic formula used to solve quadratic formulas?** **How do you determine which method is best for solving a quadratic equation?** **How are quadratic equations used to model physical phenomena?**

	minimum value of the function it defines. **A-CED** **Create equations that describe numbers or relationships** 1.Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. **A-REI** **Solve equations and inequalities in one variable** 4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and			

	write them as a ± bi for real numbers a and b. **Represent and solve equations and inequalities graphically** 10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **F-IF** **Interpret functions that arise in applications in terms of the context** 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if			

	the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. **Analyze functions using different representations** 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **F-BF** **Build new functions from existing functions** 3. Identify the effect on the graph of replacing f(x) by			

	f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.			
Semester A: Unit 9: Semester Review and Test				
Semester B: Unit 1: Solving and Graphing Polynomials	**N-CN** **Use complex numbers in polynomial identities and equations.** 9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **A-SSE** **Interpret the structure of expressions** 1. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the	**Graphing Polynomials**	**Polynomials are a sum of terms involving non-negative integer powers of a variable. Expressions can be rewritten using number properties to identify relationships.**	**How do you find the zeroes of a polynomial function?** **How does the equation of the polynomial function relate to key features of its graph?**

	product of P and a factor not depending on P. 2. Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2). **A-APR** **Perform arithmetic operations on polynomials** 1.Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **Understand the relationship between zeros and factors of polynomials** 2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). 3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.			

	A-CED **Create equations that describe numbers or relationships** 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A-REI** **Represent and solve equations and inequalities graphically** 10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **F-IF** **Interpret functions that arise in applications in terms of the context** 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative			

	maximums and minimums; symmetries; end behavior; and periodicity. 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. **Analyze functions using different representations** 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **F-BF** **Build new functions from existing functions** 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of			

	k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.			
Semester B: Unit 2: Exponents and Logarithms	**N-Q Reason quantitatively and use units to solve problems.** 2. Define appropriate quantities for the purpose of descriptive modeling. 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **N-RN Extend the properties of exponents to rational exponents.** 1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5. 2. Rewrite expressions involving radicals and	**Exponents and Logarithms**	**Exponential and logarithmic operations are inverse functions of each other. Graphs are visual representations of functions/numerical relationships.**	**How can logarithms used to solve exponential equations? What are the key features of the graphs of exponential and logarithmic functions? How are the graphs of exponential and logarithmic functions transformed in the plane relative to their equations? How are exponential and logarithmic functions used to describe real-life situations of growth and decay?**

	rational exponents using the properties of exponents. **A-SSE** **Interpret the structure of expressions** 1. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. 2. Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2). **Write expressions in equivalent forms to solve problems** 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression			

	1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. **A-CED** **Create equations that describe numbers or relationships** 1.Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A-REI** **Represent and solve equations and inequalities graphically** 10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **F-IF** **Interpret functions that arise in applications in terms of the context**			

	4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. **Analyze functions using different representations** 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.			

	e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay. **F-BF** **Build new functions from existing functions** 5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. **F-LE** **Construct and compare linear, quadratic, and exponential models and solve problems** 1. Distinguish between			

	situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). 3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. 4. For exponential models, express as a logarithm the solution to abct = d where			

	a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. **Interpret expressions for functions in terms of the situation they model** 5. Interpret the parameters in a linear or exponential function in terms of a context. **F-BF** **Build new functions from existing functions** 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.			
Semester B: Unit 3: Sequences and Series	**A-SSE** **Write expressions in equivalent forms to solve problems** 4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate	**Sequences and Series**	**Sequences and series represent numeric patterns that are governed by specific rules.**	**How can change be described mathematically?** **How are patterns of change related to the behavior of functions?** **How are sequences written recursively and explicitly?** **How are the nth term of arithmetic and**

	mortgage payments. **F-IF** **Understand the concept of a function and use function notation** 3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥1. **F-BF** **Build a function that models a relationship between two quantities** 1. Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. 2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. **F-LE** **Construct and compare linear, quadratic, and exponential models** 2. Construct linear and exponential functions, including arithmetic and			**geometric sequences calculated?** **How are the sums of arithmetic and geometric series calculated?**

	geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).			
Semester B: Unit 4: Counting and Probability	**N-Q** **Reason quantitatively and use units to solve problems.** 2. Define appropriate quantities for the purpose of descriptive modeling. 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **A-APR** **Use polynomial identities to solve problems** 5. (+) Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. **S-IC** **Understand and evaluate random processes underlying statistical experiments** 2. Decide if a specified model is consistent with results from a given data-	**Probability**	**The probability of an event can be used to predict the probability of future events.**	**What are permutations and combinations, and when is it appropriate for each to be used?** **What distinguishes an independent event from a dependent event?** **What is conditional probability?** **How is probability calculated for independent and dependent events?** **How is probability applied to real-world situations including games of chance?** **What is "fair"?**

	generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? **S-CP** **Understand independence and conditional probability and use them to interpret data** 1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). 2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. 3. Understand the conditional probability of A given B as P(A andB)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as			

	the probability of A, and the conditional probability of B given A is the same as the probability of B. 4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. 5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. **Use the rules of probability to compute**			

	probabilities of compound events in a uniform probability model 6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. 7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. 8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. 9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. **S-MD** **Calculate expected values and use them to solve problems** 1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data			

	distributions. 2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. 3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes. 4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly			

	selected households? **Use probability to evaluate outcomes of decisions** 5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fastfood restaurant. b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident. 6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). 7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).			
Semester B: Unit 5: Statistics	**N-Q** **Reason quantitatively and use units to solve problems.**	**Statistics**	**The question to be answered determines the data to be collected and how best to collect**	**What is average?** **What makes a data representation useful and appropriate?**

	2. Define appropriate quantities for the purpose of descriptive modeling. 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **S-ID** **Summarize, represent, and interpret data on a single count or measurement variable** 1. Represent data with plots on the real number line (dot plots, histograms, and box plots). 2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. 3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). 4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets,		**it.** **Basic statistical techniques can be used to analyze data for center and spread in a variety of settings.**	**When might a data representation be misleading?** **How can two data distributions be appropriately compared?** **How can statistics be used to draw conclusions from real-world data?**

	and tables to estimate areas under the normal curve. **Summarize, represent, and interpret data on two categorical and quantitative variables** 5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). 6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association. **Interpret linear models** 8. Compute (using technology) and interpret the correlation coefficient of a linear fit.			

	9. Distinguish between correlation and causation. S-IC **Understand and evaluate random processes underlying statistical experiments** 1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. 2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? **Make inferences and justify conclusions from sample surveys, experiments, and observational studies** 3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. 4. Use data from a sample survey to estimate a population mean or proportion; develop a			

	margin of error through the use of simulation models for random sampling. 5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. 6. Evaluate reports based on data.			
Semester B: Unit 6: Vectors and Matrices	**N-VM** **Represent and model with vector quantities.** 1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, v). **Perform operations on matrices and use matrices in applications.** 6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. 7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. 8. (+) Add, subtract, and multiply matrices of	**Vectors and Matrices**	**Vectors have magnitude and direction.** **Matrices are used to organize data sets and make calculations among data.**	**How is data displayed in a matrix?** **What are the operations and properties that can be performed on matrices and how do they compare to operations and properties of real numbers?** **How are vectors related to matrices?** **How can matrices be used to solve systems of equations?** **What are some real-world applications of matrices?**

	appropriate dimensions. 9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. 10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. 11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. 12. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. **A-REI** **Solve Systems of Equations** 5. Prove that, given a system of two equations in two variables, replacing			

	one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. 6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. 8. (+) Represent a system of linear equations as a single matrix equation in a vector variable. 9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).			
Semester B: Unit 7: Conic Sections	**G-GPE** **Translate between the geometric description and the equation for a conic section** 1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. 2. Derive the equation of a parabola given a focus and directrix. 3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or	**Conic Sections**	**"Slices" of a cone can be used to model natural phenomena.**	**What are the types of conic sections, and what are their equations?** **What are the key features that allow each conic section to be transformed in the plane?** **What are some physical phenomena that are modeled by conic sections and how do they relate to their equations?**

	difference of distances from the foci is constant. **G-GMD** **Visualize relationships between two-dimensional and three-dimensional objects** 4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.			
Semester B: Unit 8: Semester Review and Test				

Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title: Sequences and Series, 3 weeks

Designed by: Laura Mayer
District:Moyer Academy

Content Area:Algebra II
Grade Level(s):9-12

Summary of Unit

Students explore sequences to discover patterns in their various representations (list, table, graph, description, etc.). They will relate arithmetic and geometric sequences to linear and exponential functions, respectively. They will define these types of sequences recursively and explicitly and identify their domains as the set of integers. They will further translate between recursive and explicit forms of a function and interpret parameters in the context of a problem.

In alignment with the Common Core State Standards, this unit pushes students to develop 21st Century Skills to meet the learning demands. Students come together as a learning community to analyze real-world applications of the math they are learning, think critically and reflect on the work of their peers as well as their own, and constantly work to apply their knowledge to new, more complex situations.

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

- Include those addressed in Stage 3 and assessed in Stage 2.

Mathematical Practices 1. Make sense of problems and persevere in solving them.

Mathematical Practices 3. Construct viable arguments and critique the reasoning of others.

Mathematical Practices 5. Use appropriate tools strategically.

Mathematical Practices 6. Attend to precision.

Mathematical Practices 7. Look for and make use of structure.

Mathematical Practices 8. Look for and express regularity in repeated reasoning.

HSF.BF.1b Determine an explicit expression, a recursive process, or steps for calculation from a context.

HSF-LE.A.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

HSF-LE.1c HSF-LE.A.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

HSF-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

HSF-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

HSA.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

Big Idea(s)

- Transferable core concepts, principles, theories, and processes from the Content Standards

1. Students will identify and compare arithmetic and geometric sequences.
2. Students will apply recursive and explicit rules to solve for unknown terms in arithmetic and geometric sequences.
3. Students will write recursive and explicit functions to model situations.
4. Students will translate between explicit and recursive notation.

Unit Enduring Understanding(s)

- Full-sentence, important statements or generalizations that specify what students should understand from the Big Ideas (s) and/or Content Standards and that are transferable to new situations

All arithmetic and geometric sequences can be expressed recursively and explicitly. Some other sequences also can be expressed in both ways but others cannot.

Arithmetic sequences are identifiable by a common difference and can be modeled by linear functions. Infinite arithmetic series always diverge.

Geometric sequences are identifiable by a common ratio and can be modeled by exponential functions.

Identifiable sequences and series are found in many naturally occurring objects.

Unit Essential Questions(s)

- Open-ended questions designed to guide student inquiry and learning

What is the difference between arithmetic and geometric sequences?

How do you use the recursive and explicit rules to find the nth term in an arithmetic or geometric sequence?

How do you find the partial sum of an arithmetic or geometric series?

Knowledge and Skills

- Needed to meet Content Standards addressed in Stage 3 and assessed in Stage 2

Students will know...

The recursive and explicit rules to find terms of sequences.

How to find common differences and nth terms of arithmetic sequences.

How to find common ratios and nth terms of geometric sequences.

The differences between arithmetic and geometric sequences, and if a sequence does not fit either sequence.

Students will be able to...

Use recursive and explicit rules to find terms of sequences.

Find common differences and nth terms of arithmetic sequences.

Find common ratios and nth terms of geometric sequences.

Determine if sequences are arithmetic, geometric, or neither.

Solve problems involving arithmetic and geometric sequences.

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

Suggested Performance/Transfer Task(s)

- Performance/Transfer tasks as evidence of student proficiency

An effective assessment for ALL students should be designed to include:

*Complex, real-world, authentic applications

*Assessment(s) for student understanding of the Stage 1 elements (Enduring Understandings, Essential Questions, Big Ideas) found in the Content Standards

*Demonstration of high-level thinking with one or more facets of understanding (e.g., explain, interpret, apply, empathize, have perspective, self-knowledge)

(Illustrative Mathematics Performance Task) - Population and Food Supply

The population of a country is initially 2 million people and is increasing at 4% per year. The country's annual food supply is initially adequate for 4 million people and is increasing at a constant rate adequate for an additional 0.5 million people per year.

a. Based on these assumptions, in approximately what year will this country first experience shortages of food?

b. If the country doubled its initial food supply and maintained a constant rate of increase in the supply adequate for an additional 0.5 million people per year, would shortages still occur? In approximately which year?

c. If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, would shortages still occur?

Rubric(s)

- Scoring guide to evaluate performance/transfer tasks used as evidence of student proficiency

An effective scoring guide should:

*Measure what is appropriate for the Content Standard that is assessed.

*Provide opportunities for differentiation of the performance/transfer tasks used as evidence of student proficiency.

Scoring guide for Performance Task 1 (Appendix A)

Other Evidence

- Varied evidence that checks for understanding (e.g., tests, quizzes, prompts, student work samples, observations and supplements the evidence provided by the task)

Students will be provided with opportunities to complete performance tasks throughout the unit as means of formative assessment to prepare them for the summative performance task. Additional assessment evidence to be used throughout the unit include:

-Formative Assessment Quizzes

The formative assessment quizzes are designed to give teachers a snapshot of student performance. Each question should take less than a minute and it will be at the teacher's discretion if every formative quiz will be used. The data gathered from these quizzes should be used to identify individual needs and may also be used in the formation of differentiated groupings. Grades should not be assigned to these quizzes and students should be prompted to discuss and defend their answer choices among their peers.

Formative Observations:

Throughout the unit, students will be observed on their application of mathmatical practices highlighted in the unit:

Mathematical Practices 1. Make sense of problems and persevere in solving them.

Mathematical Practices 3. Construct viable arguments and critique the reasoning of others.

Mathematical Practices 6. Attend to precision.

Mathematical Practices 7. Look for and make use of structure.

Mathematical Practices 8. Look for and express regularity in repeated reasoning.

The observations will be shared with the students and parents throughout the unit. These conferences will be centered around the student's progressing score on the Mathematical Practices Rubric (Appendix B)

Student Self-Assessment and Reflection

- Opportunities for self-monitoring learning (e.g., reflection journals, learning logs, pre- and post-tests, self-editing – based on ongoing formative assessments)

After completing each Formative assessment performance task, students will be given the opportunity to reflect on their work. Students will then do a Think-Pair-Share with a peer and explain their work, defending both their problem solving method and the answers they reached.

For example:

1. Students may be given the following task in the unit:

F-LE Algae Blooms

Algae blooms routinely threaten the health of the Chesapeake Bay. Phosphate compounds supply a rich source of nutrients for the algae, Prorocentrum minimum, responsible for particularly harmful spring blooms known as mahogany tides. These compounds are found in fertilizers used by farmers and find their way into the Bay with run-offs resulting from rainstorms. Favorable conditions result in rapid algae growth ranging anywhere from 0.144 to 2.885 cell divisions per day. Algae concentrations are measured and reported in terms of cells per milliliter (cells/ml). Concentrations in excess of 3,000 cells/ml constitute a bloom.

a. Suppose that heavy spring rains followed by sunny days create conditions that support 1 cell division per day and that prior to the rains Prorocentrum minimum concentrations measured just 10 cells/ml. Write an equation for a function that models the relationship between the algae concentration and the number of days since the algae began to divide at the rate of 1 cell division per day.

2. Students will be given access to tools and technology from which they can select appropriately to assist them in completing the task. Each student will complete the task independently.

NOTE: Students will approach solving this problem differently, some choosing to set up equations, others choosing to do a series of connected steps to arrive at the answer.

3. Students will pair up, each student given 2 minutes to defend their problem solving method and the answer they reached. Each pair will agree on the most effective way to solve the problem and present that method to the class.

4. Students will be given another step to the problem and time to complete:

b. Assuming this rate of cell division is sustained for 10 days, present the resulting algae concentrations over that period in a table. Did these conditions result in a bloom?

6. Students will be asked to extend their work to complete the last portion of the task:

c. Concentrations in excess of 200,000 cells/ml have been reported in the Bay. If conditions support 2 cell divisions per day, when will these conditions result in a bloom? When will concentrations exceed 200,000 cells/ml?. Students will pair up and each will defend their problem solving methods again.

NOTE: All performance based task work will be kept inside a reflective log that students will add to throughout the year. The teacher will assist students in creating the log and performing upkeep throughout the year, but it will be the student's responsibility to maintain. At the completion of each task, the students will reflect in writing on the relationship between the task and the learning objective of the lesson.

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

- Instructional activities and learning experiences needed to align with Stage 1 and Stage 2 expectations

Include these instructional elements when designing an effective and engaging learning plan for ALL students:

*Align with expectations of Stage 1 and Stage 2

*Scaffold in order to acquire information, construct meaning, and practice transfer of understanding

*Include a wide range of research-based, effective, and engaging strategies

*Differentiate and personalize content, process, and product for diverse learners

*Provide ongoing opportunities for self-monitoring and self-evaluation

The direct instruction resources needed to implement this course can be found within the k12 MTH303 course, Semester B, Presentations 3.01 - 3.14. Each day of instruction is accompanied by a powerpoint presentation. Use these presentations to guide your information delivery during Direct Instruction, for additional resources when working with small groups, or as a building block for student guided notes. Use these presentations to guide your information delivery during Direct Instruction. Guided Practice Examples and additional Independent Practice activities are also embedded within the presentation materials (See example presentation in Appendix Z).

Lesson 1: Introduction to Patterns and Sequences (1-2 days)

Teacher will open class with a review of exponential rules. Students will complete Simplifying Exponential Expressions (Appendix C)

Introductory Performance Task: Multiplying Cells (Appendix D)

This activity is designed to introduce students to patterns and sequence recognition. This should be a "hands off" activity for the teacher. Partner or group the students as you see fit to ensure a varied assortment of strengths and weaknesses in their math skills. The students should be given 1-2 days to complete the activity.

Upon completion, have groups switch papers and use the rubric to grade each other's work. Students should also make note of whether or not they can see evidence of any of the standards for math practices using the math practices rubric

Lesson 2: Arithmetic and Geometric Sequences (1-2 days)

Anticipatory Set: Students will be given a box of toothpicks. Toothpicks will be used to build a rectangular grid. The grid is filled with squares that have 1 toothpick on each side. Students will build three grids: 1X1, 4X4, and 7X7. Students will answer the following questions: What is the total number of toothpicks used in each grid? How many toothpicks would you need to build a grid that is 12X12? Describe any pattern in the number of toothpicks you have to use for each grid.

Teacher will explain that one way math can be useful in real life is to identify and describe patterns. Instead of taking the time to build a grid that is 12X12 toothpicks, and count each and every toothpick we use, we can look at patterns we see in the smaller grids and use those patterns to find out the number of toothpicks needed to build larger grids.

Direct Instruction: Teacher will guide students through the notes on sequences (Appendix E).

Guided Practice: Arithmetic and Geometric Sequences (Appendix F)

Teacher will do a Think Aloud on solving the first two problems. Students will guide the teacher in solving 3-11.

Students will complete Practice with Sequences (Appendix G) independently. After completion, have students get into groups of 4 and evaluate each other's work for correctness.

Distribute 4 notecards to students to be used as Response Cards. Have them label each card with a letter, A, B, C, D. Using the Smart board, check student's individual understanding with Formative Check 1 (Appendix H). Use student data to inform next steps: group and differentiate reteaching as needed, or move on to next topic.

Lesson 3: Connecting Linear/Exponential Growth to Sequences (1-2 days)

Review linear and exponential growth.

Group students, 2-3 students per group.

Performance Task: Pay it Forward (Appendix I)

Self-Reflection and Peer-Review: Students will do a Think-Pair-Share with a peer and explain their work, defending both their problem solving method and the answers they reached.

After Performance Task Think-Pair-Share, students will reflect in their journal on the application of their learning to the Performance Task and their mastery of the lesson objectives. Reflective Journal Entries will be graded using the following rubric:

4 points - Reflection is well written with clear organization and contains almost no errors; demonstrates thoughtful analysis of how new learning and performance task related; evidence and details included to demonstrate student's grasp of concepts; submitted on time.

3 points - Reflection is well written with some organization and contains few errors; demonstrates analysis of how new learning and performance task related; some evidence and details included to demonstrate student's grasp of concepts; submitted on time.

2 points - Reflection shows little organization and contains errors; demonstrates attempt at analysis of how new learning and performance task related; little to no evidence and details are included to demonstrate student's grasp of concepts; submitted on time.

1 points - Reflection has no clear organization and contains many errors; little to no analysis of how new learning and performance task related; little to no evidence and details included to demonstrate student's grasp of concepts; and/or not submitted on time.

0 points - No relevant reflection is submitted.

Lesson 4: Sequences in Tables and Graphs (2-3 days)

Teacher will give notes: From Geometric Sequences to Tables and Graphs (Appendix J)

Guided Practice: Questions 1-6

Students will practice working with sequences in tables and graphs in a group: The Mice Problem (Appendix K)

Warm-up: Review of Arithmetic vs. Geometric Sequences (Appendix L)

Independent Practice: The Ladybug Invasion (Appendix M)

Formative Assessment: Charity Donations (Appendix N)

Lesson 5: Writing Sequence Rules (1-2 days)

Direct Instruction:

A recursive rule is a rule used to find terms in a sequence that depends on previous terms in the sequence. To calculate a., use the value of previous terms.

Teacher will engage students in guided practice to apply recursive rules to find terms in a sequence. Example: "List the first four terms of the sequence given that a1=7 and an=an-1+4 for n>2"

Teacher will give the following notes:

An explicit rule is a rule that can be used to find the nth term of a sequence without calculating previous terms of the sequence. Only the value of n is needed to calculate an.

Teacher will engage students in guided practice to apply explicit rules to find terms in a sequence. Example: "Find the sixth term of the sequence an=n^2/3"

Students will be given 3 index cards to complete the following vocabulary reinforcement activity:

On one side of the card, the students will put a vocabulary word (sequence, recursive rule, explicit rule) and on the other side the students will put: the definition (in their own words), a complete example. Students will do a Think-Pair-Share on their completed cards, evaluating each other for correctness, and ease of use. Each set of vocabulary cards will be kept and used in later units to build students understanding of interconnected ideas in math.

Teacher will outline the rules for writing arithmetic sequences:

A sequence with a common difference between consecutive terms is called an arithmetic sequence. To find this common difference, select any term of the sequence and subtract the previous term from it.

To find a term of a sequence, given the previous term, use a recursive rule. The nth term of an arithmetic sequence is found by adding the common difference to the preceding term.

Teacher will engage students in guided practice to apply these rules to write arithmetic sequences. (Ex: "For the given sequence, find the common different, write a recursive rule, and find the next term: 27, 31, 35, 39, 43...")

Guided Practice: Writing Sequence Rules (Appendix O)

Students will practice this in differentiated groups:

Begin the activity by showing the following video, which shows a class building popsicle stick bridges: http://www.youtube.com/watch?v=T2ZyqNODJpU. Ask the students to pay particular attention to the students in the class and their modeling of the standards for Mathematical Practice. Students should be able to give you one student that fully demonstrated a standard math practice and one example of a student that needs to improve. Having students visually look for the Math Practice standards in peers will build their awareness of what the practices look like in action, giving them a fuller understanding of how to implement the practice standards in their own investigations.

Provide students with the "Build a Bridge" activity (Appendix P).

Self-Reflection and Peer-Review: Students will do a Think-Pair-Share with a peer and explain their work, defending both their problem solving method and the answers they reached.

Formative Assessment: Writing Sequence Rules (Appendix Q)

Lesson 6: Using Recursive Rules to Create Explicit Rules (3 days)

Direct Instruction:

An explicit rule for an arithmetic sequence can be used to find the nth term in a sequence when only the first term, aR, and the common difference, d, are known.

Teacher will demonstrate how the recursive rule is used to create the explicit rule for an arithmetic sequence. Teacher will engage students in guided practice (EX: Write

an explicit rule for the sequence, and then find the 22nd term in the sequence: 15, 4, -7, -18, -29...)

Step 1: Identify the first term a1, and find the common difference d.

(a1=15, d=-11)

Step 2: Substitute the first term, a1, and the common difference d into the explicit rule.

an=a1+(n-1)*d

an=15+(n-1)*-11

Step 3: Simplify and Solve.

a1=15-11n+11

a1=26-11n

Solve... a22=26-11(22)

a22=-216, so the 22nd term of this sequence is -216.

Performance Task: Honeycombs (Appendix R) in groups of 4.

Self-Reflection and Peer-Review: Students will do a Think-Pair-Share with a peer and explain their work, defending both their problem solving method and the answers they reached.

After Performance Task Think-Pair-Share, students will reflect in their journal on the application of their learning to the Performance Task and their mastery of the lesson objectives. Reflective Journal Entries will be graded using the following rubric:

4 points - Reflection is well written with clear organization and contains almost no errors; demonstrates thoughtful analysis of how new learning and performance task related; evidence and details included to demonstrate student's grasp of concepts; submitted on time.

3 points - Reflection is well written with some organization and contains few errors; demonstrates analysis of how new learning and performance task related; some evidence and details included to demonstrate student's grasp of concepts; submitted on time.

2 points - Reflection shows little organization and contains errors; demonstrates attempt at analysis of how new learning and performance task related; little to no evidence and details are included to demonstrate student's grasp of concepts; submitted on time.

1 points - Reflection has no clear organization and contains many errors; little to no analysis of how new learning and performance task related; little to no evidence and details included to demonstrate student's grasp of concepts; and/or not submitted on time.

0 points - No relevant reflection is submitted.

Lesson 7: Series (2 days)

Direct Instruction: Utilize the k12 notes to guide students during direct instruction and guided practice.

Additional Guided and Independent Practice Opportunities (Appendix S)

Lesson 8: Putting the Pieces Together - Preparing for the Summative Assessment (3 days)

Students will practice writing explicit rules for geometric sequences (Appendix T).

Formative Assessment: Arithmetic and Geometric Sequences (Appendix U)

Performance Task: Algae Blooms (Independent Practice in preparation for Summative Assessment) (Appendix V)

Teacher may use small differentiated groupings, assign different numbers of problems to allow extra time (as recommended in students' IEPs).

If students need additional practice, select from the following:

1. Performance Task: Basketball Bounces (Assessment Variation 1)

(see website below for Task, Rubric, and Associated Video)

http://www.illustrativemathematics.org/illustrations/1306

2. Pay It Forward, Again (Appendix W)

May also assign group project Renaissance Festival (Appendix X) towards the end of the unit for reinforcement.

New Tools in the Students "Toolbox:"

1. Recognizing sequences as arithmetic or geometric.
2. Writing explicit rules for sequences.
3. Writing recursive rules for sequences.

*NOTE: Throughout the year, students will keep a running log of math "tools" they can use to solve problems. When students are presented with performance tasks, students will view these "tools" the same way they view their calculators or rulers: things that they can apply to a problem to come to the solution. The goal is to focus the students on the math behind the concepts and to develop in them the understanding that each new concept they learn is actually a continuation of learning already accomplished, instead of a discrete, standalone topic.

Population and Food Supply (Appendix A)

After Performance Task Think-Pair-Share, students will reflect in their journal on the application of their learning to the Performance Task and their mastery of the lesson objectives. Reflective Journal Entries will be graded using the following rubric:

4 points - Reflection is well written with clear organization and contains almost no errors; demonstrates thoughtful analysis of how new learning and performance task related; evidence and details included to demonstrate student's grasp of concepts; submitted on time.

3 points - Reflection is well written with some organization and contains few errors; demonstrates analysis of how new learning and performance task related; some evidence and details included to demonstrate student's grasp of concepts; submitted on time.

2 points - Reflection shows little organization and contains errors; demonstrates attempt at analysis of how new learning and performance task related; little to no evidence and details are included to demonstrate student's grasp of concepts; submitted on time.

1 points - Reflection has no clear organization and contains many errors; little to no analysis of how new learning and performance task related; little to no evidence and details included to demonstrate student's grasp of concepts; and/or not submitted on time.

0 points - No relevant reflection is submitted.

Resources and Teaching Tips

- A variety of resources are included (texts, print, media, web links)
- Help in identifying and correcting student misunderstandings and weaknesses

Throughout the unit, students have access to a variety of resources.

Algebra I Textbook

Smartboard and Projector

Graphic Organizer Paper

K12 Online Curriculum

Laptop

Reference Sheets

Videos

Each formative check for understanding provides an opportunity for the students to identify their own misunderstandings and weaknesses in mastering the content. Guide the students in understanding how to use these formative checks to enhance their learning through the use of rubrics, conferences, and remediation or acceleration plans designed with student input (Appendix Y). While working towards mastery of grade level content standards, continuously refer students back to the standards for mathematical practices. These practice standards are the attitudes and skills that will allow students to achieve mastery of content standards, as well as deep understanding of mathematics.

Differentiation

- Stage 2 and 3 allow students to demonstrate understanding with choices, options, and/or variety in the products and performances without compromising the expectations of the Content Standards.
- Instruction is varied to address differences in readiness, interest, and/or learning profiles.
- Accommodations and differentiation strategies are incorporated in the design of Stage 2 and 3.

☐ Refer to the IEPs of your special education students

☐ Allow students to have tests, quizzes, worksheets, etc read to them.

☐ Allow the use of a calculator

☐ Communicate with the special education teacher about allowing testing in the special education room

☐ Pair and group students with differing ability levels.

☐ Use manipulatives as needed

☐ Allow for extra time on tests, quizzes, and performance tasks

☐ Allow for a word bank and a formula sheet on tests

☐ Allow students to choose to demonstrate understand in written or verbal format as needed.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.
- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning** – the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

(Briefly explain how design principle(s) are embedded within the unit design.)

In alignment with the Common Core State Standards, this unit pushes students to develop 21st Century Skills to meet the learning demands. Students come together as a learning community to analyze real-world applications of the math they are learning, think critically and reflect on the work of their peers as well as their own, and constantly work to apply their knowledge to new, more complex situations. This push towards using developed skills in new ways is an accurate reflection of the higher order thinking required by 21st Century jobs. Students should become excited and engaged by the "hands-off" approach of the teacher. Rather than giving students the formulas and a calculator and having them compute numbers and spit out answers, the teacher should present numbers and problems in a real world context, allowing the students to select the tools, resources, and knowledge needed to arrive at a reasonable answer. This push towards applying mathematical ways of thinking to real world issues will prepare them for the type of work they will encounter in college and in their careers.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- **8th Grade Technology Literacy** - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century(SETDA, 2003).

In alignment with Mathematical Practice 5 - Use appropriate tools strategically, students will have access to calculators as needed to complete the Performance Tasks. Additionally, when sharing their answers to the Performance Tasks, the teacher may opt to have the students construct a PowerPoint presentation as an additional layer of the project.

Content Connections

Content Standards integrated within instructional strategies

Connections to other content areas are made both explicitly and implicitly throughout this unit. Students will be given a performance tasks that makes these connections clear. Additionally, the performance tasks connect the math content to science, sports, personal finance, and construction design. Additional connections are encouraged at the teacher's discretion. For example, when assigning the students additional practice work, make the work meaningful by aligning the examples with content currently being covered in other classes such as Science, English, and Social Studies.

Illustrative Mathematics

F-LE, A-REI Population and Food Supply

Alignment 1: F-LE.A.2, F-LE.A.3, A-REI.D.11

The population of a country is initially 2 million people and is increasing at 4% per year. The country's annual food supply is initially adequate for 4 million people and is increasing at a constant rate adequate for an additional 0.5 million people per year.

a. Based on these assumptions, in approximately what year will this country first experience shortages of food?

b. If the country doubled its initial food supply and maintained a constant rate of increase in the supply adequate for an additional 0.5 million people per year, would shortages still occur? In approximately which year?

c. If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, would shortages still occur?

Commentary:
In this task students construct and compare linear and exponential functions and find where the two functions intersect (F-LE.2, F-LE.3, A-REI.11).

One purpose of this task is to demonstrate that exponential functions grow faster than linear functions even if the linear function has a higher initial value and even if we increase the slope of the line. This task could be used as an introduction to this idea. The steps in this task require students to find linear and exponential functions from verbal descriptions. If they previously learned how to do this, the task can be completed independently. So they can practice previously learned skills and explore the idea of the dominance of exponential over linear functions at the same time.

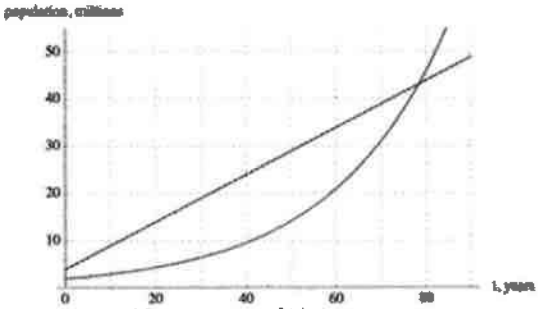
Solution: Finding intersection points

a. We can first express the country's population, $p(t)$, in millions of people, as a function of the time t, measured in years from the initial time. Since we know the initial population $p(0) = 2$ and the annual growth rate is 4%, then $p(t)$ is an exponential function:

$$p(t) = 2(1.04)^t.$$

We are also given that the food supply grows at a constant rate. So we can express the country's food supply at time t, which we call $f(t)$, as a linear function of t. Again, we know the initial value $f(0) = 4$ and the constant rate of change is 0.5 million people per year, so we have:

$$f(t) = 4 + .5t.$$



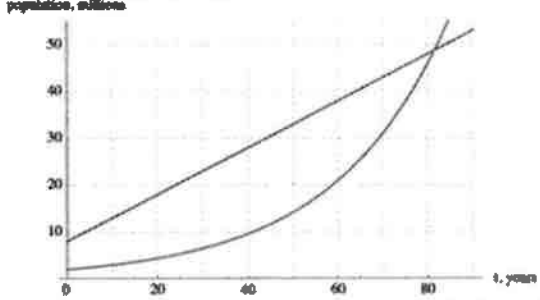
We are looking for the value of t which makes $p(t)$ greater than $f(t)$ for the first time.

We see from the graph that the two functions intersect at around $t = 78$. So after 78 years the food supply is just barely enough for the country's population. After this point, however, we see that $p(t) > f(t)$, so this country will first experience shortages of food after approximately 78 years.

b. If the country doubled its initial food supply, our new function for the food supply would be

$$h(t) = 8 + .5t$$

We would expect food shortages to occur, if at all, later than in part (a).



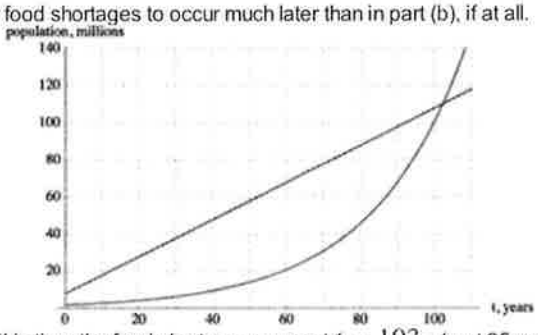
Again looking at the graph, we see that the two functions intersect, and so food shortages would still occur. We find $p(t) = h(t)$ at roughly $t = 81$. So, the country will first experience food shortages after 81 years. So doubling the initial food supply delays the eventual food shortage by only 3 years.

A

c. If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, we have the new food supply function:

$$j(t) = 8 + t$$

We would expect, in this case, for food shortages to occur much later than in part (b), if at all.



Looking at the graph we see that this time the food shortage occurs at $t = 103$, about 25 years later than in part (a).

Examining the behavior of the exponential function more closely we observe, that the slope of the exponential function keeps increasing whereas the slope of any linear function is constant. Even if a linear function has a very large slope, an exponential function will eventually grow even faster and overtake the linear function.

Student Name_____ Evaluation Date_____ Overall Score_____

Mathematical Practice	Novice (1)	Apprentice (2)	Practitioner (3)	Expert (4)
1. Make sense of problems & persevere in solving them	I couldn't get started. I don't know how to begin. I may be feeling frustrated and wanting to give up.	I am stuck. I have part of the solution, but now I don't know what to do, I'm not sure my answer is right and I could use some help. I may be feeling frustrated.	I planned out how to solve this problem and I worked all parts of the problem. I may have felt challenged, but I didn't want to give up-I kept going until the problem was solved.	My solution is effective and inventive! I planned out how to solve this problem and I even showed you other ways to solve it. I checked to make sure my answer was right – I enjoyed this challenge!
2. Reason abstractly & quantitatively	I can't explain how I solved this problem. I don't know what quantities or equations to use. My work on this problem is built on misunderstandings of mathematical ideas.	I'm trying to explain how I solved this problem. My work on this problem may be built on misunderstandings of mathematical ideas. . I'm trying to take numbers out of the problem to work with them. I'm trying to check my work.	I explained how to solve this problem using formal math language. I took numbers out of the problem to work with them, and periodically put them back into the context of the problem to see if my work makes sense.	I clearly explained how to solve this problem using formal math language. I took quantities out of the context of the problem to work with them, and frequently put them back into the context of the problem to see if my work made sense.
3. Construct viable arguments & critique the reasoning of others	I can't describe how I solved the problem. I can't give feedback on someone else's reasoning OR my feedback in unclear, unhelpful, and/or incorrect.	I'm trying to describe how I solved the problem. I may or may not include pictures, tables, graphs, and/or models. I gave feedback on someone else's reasoning, but my feedback may have been unclear, unhelpful, or I may have been unable to back it up with my own work.	I described how I solved the problem using formal math language, pictures, tables, graphs, and/or models. I gave clear, helpful feedback to someone else's reasoning and backed up my feedback with my own work on the problem.	I clearly described how I solved the problem. I included every step so you don't have to guess what I did. I used formal math language, pictures, tables, graphs, and/or models. I gave clear, helpful feedback on someone else's reasoning and backed up my feedback with my own work on the problem.
4. Model with mathematics	I can't use mathematical representations, I don't know what to use for this problem and I can't describe how I would solve this problem.	I'm trying to use mathematical representations to solve the problem and show my solutions, but they may be inaccurate or inappropriate. I'm sharing my ideas, but I'm struggling to use formal math language.	I used appropriate & accurate mathematical representations to solve the problem and show my solutions. I used formal math language to share my ideas.	I used a variety appropriate, accurate, and detailed mathematical representations to solve the problem and show my solutions. I used formal math language to share my ideas.
5. Use appropriate	I don't know what tool to use to solve this problem and/or I	I picked an appropriate tool to use, but I may be struggling to	I picked the correct tool to use based on the needs of the	I selected and used a variety of appropriate tools

Student Name_____ Evaluation Date_____ Overall Score_____

tools strategically	didn't use the tool I picked in a valuable way.	use it in a valuable way to solve the problem.	problem. I planned and monitored my use of the tool.	collaboratively to solve the problem. I planned my use of the tools and monitored my progress.
6. Attend to precision	I can't get the correct answer and/or I can't communicate my answer in a way that is appropriate to the problem. I can't use correct units, labels, or definitions.	I may be getting the correct answer and/or I may be able to communicate my answer is a way that is appropriate to the problem. I may not use correct units, labels, or definitions.	I got the correct answer and I can communicate it using the correct labels, units, and definitions. The precision of my answer is appropriate for the problem.	I got the correct answer and I communicated it using correct labels, units, and definitions. I clearly stated the meaning of the symbols I chose and my answer is complete appropriate for the problem.
7. Look for & make use of structure	I don't see any structure in the problem. I can't make sense of any useful structures/patterns to help me solve the problem.	I may be able to find for structures/patterns in the problem, but I am struggling to use them to help me find a solution.	I found and used structures/patterns in the problem to help me find a solution. I noted these structures/patterns as I found them.	I found, noted, and used structures/patterns in the problem to help me find a solution. I was able to step back from the problem and analyze different perspectives.
8. Look for & express regularity in repeated reasoning	I don't see any regularity or any shortcuts or generalizations I can use to get a solution.	I may notice that some of my calculations are repeated but I may not be able to generalize that observation to help me get a solution.	I noticed some of my calculations were repeating and was able to make a generalization that helped me solve the problem. I evaluated the reasonableness of my work as I progressed.	I noticed some of my calculations were repeating and was able to make a generalization or a formula that helped me solve the problem. I continuously evaluated the reasonableness of my work as I progressed.

C

Exponent Practice Name _____

Simplify each expression.

Practice

2. $(-8m^4)(4m^8)$
5. $3^8 \cdot 3^5$
8. $(-1.5a^5b^2)(6a)$
11. $p^5 \cdot q^2 \cdot p^4$
14. $x^{-9} \cdot x^3 \cdot x^2$
17. $b^7 \cdot b^{13}$
20. $(6r^4s^3)(9rs^2)$
23. $s^7 \cdot t^4 \cdot t^8$
26. $\dfrac{1}{h^7 \cdot h^3}$
29. $r^6 \cdot r^{-13}$

Practice

2. $(2^{-3})^4$
5. $2^5 \cdot (2^4)^2$
8. $(x^5y^3)^3(xy^5)^2$
11. $(3f^4g^{-3})^3(f^2g^{-2})^{-1}$
14. $(a^3b^4)^{-2}(a^{-3}b^{-5})^{-4}$
17. $(m^{-5})^{-3}$
20. $n^6 \cdot (n^{-2})^5$
23. $b^{-9} \cdot (b^2)^4$
26. $(b^{-3})^6$
29. $(x^4y)^3$

Practice

2. $\left(\dfrac{x^3y^{-2}}{z^{-5}}\right)^{-4}$
6. $\left(\dfrac{a^3}{b^2}\right)^4$
10. $\dfrac{7^{-4}}{7^{-7}}$
14. $\dfrac{z^7}{z^{-3}}$
18. $\left(\dfrac{2^3m^4n^{-1}}{p^2}\right)^0$
22. $\dfrac{h^{-13}}{h^{-8}}$
26. $\left(\dfrac{m^{-3}n^4}{n^{-2}}\right)^4$
30. $\left(\dfrac{r^3s^{-1}}{r^2s^6}\right)^{-1}$

C

Practice

2. $-32m^{12}$

5. 3^{13}

8. $-9a^6b^2$

11. p^9q^2

14. $\frac{1}{x^4}$

17. b^{20}

20. $54r^5s^5$

23. s^7t^{12}

26. $\frac{1}{h^{10}}$

29. $\frac{1}{r^7}$

Practice

2. $\frac{1}{2^{12}}$

5. 2^{13}

8. $x^{17}y^{19}$

11. $\frac{27f^{10}}{g^7}$

14. a^6b^{12}

17. m^{15}

20. $\frac{1}{n^4}$

23. $\frac{1}{b}$

26. $\frac{1}{b^{18}}$

29. $x^{12}y^3$

Practice

2. $\frac{y^8}{x^{12}z^{20}}$

6. $\frac{a^{12}}{b^8}$

10. 343

14. z^{10}

18. 1

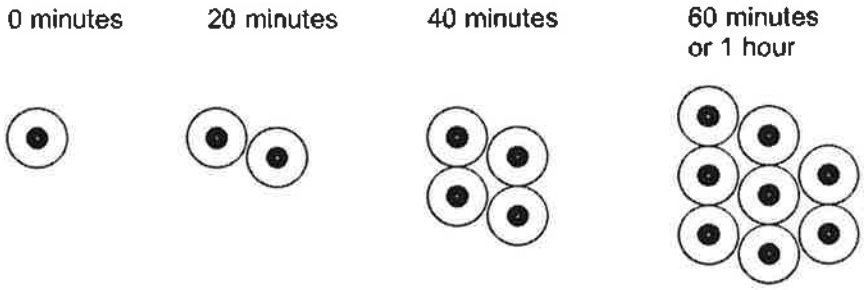
22. $\frac{1}{h^5}$

26. $\frac{n^{24}}{m^{12}}$

30. $\frac{s^7}{r}$

D

Multiplying Cells



Mrs. Lucas's class has a 2-hour science lab.

She gives each student a dish with one cell in it.

She tells the class that in 20 minutes the cell will divide into two cells,

and each 20 minutes after that **each** cell in the dish will divide into two cells.

1. Complete the second row in this table to show how the number of cells increases during the lab.

Time (minutes)	**0**	**20**	**40**	**60**	**80**	**100**	**120**
Number of cells	1	2	4				
Number of cells as a power of 2	2^0	2^1					

2. Olan says that the numbers of cells can be written in the form 2^n.

   Complete the third row in the table to show how the number of cells can be written in this form.

D

3. Linda says that the number of cells after 3 hours will be 2^7 (= 2x2x2x2x2x2x2)

Is she correct? _____

If not, then what is the correct number? _____

Explain how you figured it out.

_____

_____

_____

4. How many cells will be in the dish after 5 hours? _____

Give your answer as a normal number, not as a power of 2.

Show how you figured it out.

5. How long will it take for the number of cells to reach at least 100,000?

Give your answer to the nearest 20 minutes. _____

Show how you figured it out.

D

	Multiplying Cells	Rubric	
	:	Points	Section points
1.	Fills in the table correctly: Time: 0 | 20 | 40 | 60 | 80 | 100 | 120 Number of cells: 1 | 2 | 4 | **8** | **16** | **32** | **64**	2	2
2.	Fills in the table correctly: Time: 0 | 20 | 40 | 60 | 80 | 100 | 120 Number of cells as power of 2: 2^0 | 2^1 | 2^2 | 2^3 | 2^4 | 2^5 | 2^6	2	2
3.	Gives a correct answer: 2^9 (allow 512) Gives a correct explanation such as: 3 hours is 9 lots of 20 minutes and the power of 2 equals the number of 20 minutes which have passed.	1 1	2
4.	Gives a correct answer: **32768** Shows correct work such as: 5 hours = 5x3 *lots of 20-minutes* = 15 *lots of 20-minutes* 2^{15}	1 1	2
5.	Gives a correct answer: **340 minutes or 5 hours 40 minutes** Shows correct work such as: $2^{16} = 32768 \times 2 = 65536$ $2^{17} = 65536 \times 2 = 131072$ 17 x 20	1 1	2
	Total Points		**10**

E

Notes: Growing Sequences

Vocabulary		
Arithmetic Sequence	Common Difference	
Geometric Sequence	Common Ratio	Initial Term

Review of Arithmetic Sequences

An _____ goes from one term to the next by always adding (or subtracting) the same value. For instance, 2, 5, 8, 11, 14, . . . and 7, 3, –1, –5, . . . are arithmetic, since you add 3 in the first sequence and subtract 4 in the second sequence, respectively, at each step.

The number added (or subtracted) at each stage of an arithmetic sequence is called the _____ _____, d, because if you subtract (find the difference of) successive terms, you'll always get this common value. The first term a_1 is often referred to as the _____ _____ of the sequence.

For example, find the initial term, common difference and the next term of the following sequence:

3, 11, 19, 27, 35, . . .

To find the common difference, I have to subtract a pair of terms. It doesn't matter which pair I pick, as long as they're right next to each other:

$11 - 3 = 8$
$19 - 11 = 8$
$27 - 19 = 8$
$35 - 27 = 8$

The start number is _____, so $a_1 =$ _____. The difference is always _____, so ***d*** **=** _____.
Then the next term is _____.

Problem Situation: The Brown Tree Snake

The Brown Tree Snake is responsible for entirely wiping out over half of Guam's native bird and lizard species as well as two out of three of Guam's native bat species. The Brown Tree Snake was inadvertently introduced to Guam bythe US military due to the fact that Guam is a hub for commercial and military shipments in the tropical western Pacific. It will eat frogs, lizards, small mammals, birds and birds' eggs, which is why Guam's bird, lizard, and bat population has been affected. Listed in the table below is the data collected on the Brown Tree Snake's invasion of Guam.

Adapted from Global Invasive Species Database (http://www.issg.org/database/species/ecology.asp?si=54)

E

The number of snakes for the first few years is summarized by the following sequence:

1, 5, 25, 125, 625, . . .

- What are the next three terms of the sequence?
- How did you predict the number of snakes for the 6th, 7th, and 8th terms?
- What is the initial term of the sequence?
- What is the pattern of change?
- Do you think the sequence above is an arithmetic sequence? Why or why not?

For arithmetic sequences, the common difference is *d*, and the In the Brown Tree Snake sequence, the rate of change is not arithmetic as shown below.

1, 5, 25, 125, 625, . . .

5 – 1 = 4
25 – 5 = 20
125 – 25 = 100
635 – 125 = 500

The difference is not a common number; therefore, the sequence is not arithmetic. So, what kind of sequence is this? Strangely enough, the pattern that I see is one of multiplication.

1, 5, 25, 125, 625, . . .

The initial term of the Brown Tree Snake is 1 and the rate of change is that of multiplication by 5 each time in order to generate the next terms of the sequence. This type of sequence is called a geometric sequence. A _____ _____ goes from one term to the next by always multiplying (or dividing) by the same value. So 1, 2, 4, 8, 16, . . . and 81, 27, 9, 3, 1, 1/3, . . . are geometric, since you multiply by 2 in the first sequence and divide by 3 in the second sequence, respectively, at each step.

The number multiplied (or divided) at each stage of a geometric sequence is called the **common ratio *r***, because if you divide successive terms, you'll always get this common value. So, let's determine the common ratio *r* of the Brown Tree Snake Sequence.

1, 5, 25, 125, 625, . . .

5/1 = 5 25/5 = 5 125 /5 = 5 625/5 = 5

The common ratio of the Brown Tree Snake is *r* = 5. Let's now find the initial term and the common ratio of other geometric sequences.

Example 1: 1/2, 1, 2, 4, 8, . . .

Initial term:_____ Common ratio:_____

Example 2: 2/9, 2/3, 2, 6, 18, . . .

Initial term:_____Common ratio:_____

F

Guided Practice: Arithmetic and Geometric Sequences

1. You visit the Grand Canyon and drop a penny off the edge of a cliff. The distance the penny will fall is 16 feet the first second, 48 feet the next second, 80 feet the third second, and so on. What is the total distance the object will fall in 6 seconds?

Solution: Arithmetic sequence: 16, 48, 80, ...
The 6^{th} term is 176. The total distance is $16 + 48 + 80 + 112 + 144 + 176 = 576$ feet.

2. The sum of the interior angles of a triangle is 180°, of a quadrilateral is 360° and of a pentagon is 540°. Assuming this pattern continues, find the sum of the interior angles of a dodecagon (12 sides).

Solution: This sequence is arithmetic and the common difference is 180. The 12-sided figure will be the 10^{th} term in this sequence. The sum of the interior angles of a dodecagon is 1800.

3. After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he suggests that you increase that time by 6 minutes per day. How many weeks will it be before you are up to jogging 60 minutes per day?

Solution: Adding 6 minutes to the weekly jogging time for each week creates the sequence: 12, 18, 24, ... This sequence is arithmetic and it will take 9 weeks to jog 60 minutes per day.

4. You complain that the hot tub in your hotel suite is not hot enough. The hotel tells you that they will increase the temperature by 10% each hour. If the current temperature of the hot tub is 75° F, what will be the temperature of the hot tub after 3 hours, to the *nearest tenth* of a degree?

Solution: Starting temperature is 75°.
If the temperature is increased by 10%, the new temperature will be 110% of the original temperature. The common ratio will be 1.10. There are 4 terms: 75, after 1 hour, after 2 hours, after 3 hours. The temperature will be 99.8 degrees F.

5. A culture of bacteria doubles every 2 hours. If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours?

Solution: There are 500 bacteria to start, doubling every 2 hrs.
Such bacterial growth is a geometric sequence with a common ratio of 2. The number of times the bacteria doubles is 12 times during a 24 hour period.
The starting number is 500 and the common ratio is 2 hours. The total number of bacteria is 2,048,000.

Determine if the sequence is geometric. If it is, find the common ratio.

6. -1, 6, -36, 216, . . .
   Geometric – yes
   Common ratio is -6

7. 4, 16, 36, 64, . . .
   Geometric – no

8. -2, -4, -8, -16, . . .
   Geometric – yes
   Common ratio is 2

9. -1, 1, 4, 8, . . .
   Geometric - no

10. -3, -15, -75, -375, . . .
    Geometric – yes Common ratio is 5

11. 1, -5, 25, -125, . . .
    Geometric - yes
    Common ratio is -5

Practice with Sequences Name _____

For a sequence, write arithmetic and the common difference or geometric and the common ratio. If a sequence is neither arithmetic nor geometric, write neither.

1) 2, 6, 18, 54, 162, ... _____ common _____ = ____

2) 14, 34, 54, 74, 94, ... _____ common _____ = ____

3) 4, 16, 36, 64, 100, ... _____ common _____ = ____

4) 9, 109, 209, 309, 409, ... _____ common _____ = ____

5) 1, 3, 9, 27, 81, ... _____ common _____ = ____

Given the initial term and either common difference or common ratio, write the first 6 terms of the sequence.

6) $a_1 = 7, r = 2$ _____

7) $a_1 = 7, d = 2$ _____

8) $a_1 = 3, r = 5$ _____

9) $a_1 = 4, d = 15$ _____

The water hyacinth is an invasive species from Brazil, which has found its way into North Carolina in the north and inland of the Tar and Neuse river areas. Unchecked, the water hyacinth can lead to clogged waterways, altered water temperature and chemistry, and the exclusion of native plants and wildlife in our own state. Some NC biologists found a region in which 76.9 miles^2 were covered by the water hyacinth. They decided to monitor the area by checking it again every 10 days. Here's the data that they collected:

76.9; 153.8; 315.6; 631.2; 1,262.4, . . .

10) Is the area of the plant growing arithmetically or geometrically? Explain how you know by listing the features of the sequence (common difference or common ratio).

11) How is this problem different from the Brown Tree Snake sequence?

#

1. **Which term of the** sequence is 7.5?

n	1	2	3	4
a_n	60	30	15	7.5

(Points : 1)

- ○ 1st
- ○ 2nd
- ○ 3rd
- ○ 4th

2. **What is the value of a_5?**

n	3	4	5	6
a_n	4	9	16	25

(Points : 1)

- ○ 5
- ○ 6
- ○ 16
- ○ 80

3. Which represents the second term of the sequence: $a_1 = 10$ and $a_n = 0.5(a_n \_ 1)$?
(Points : 1)

- ○ 20
- ○ 15
- ○ 2
- ○ 5

4

4. Which represent the first three terms of the sequence: $a_1 = 2$ and $a_n = 3(a_n \_ 1)^2$?
(Points : 1)

- ○ 2, 12, 27
- ○ 3, 12, 27
- ○ 2, 12, 432
- ○ 3, 12, 432

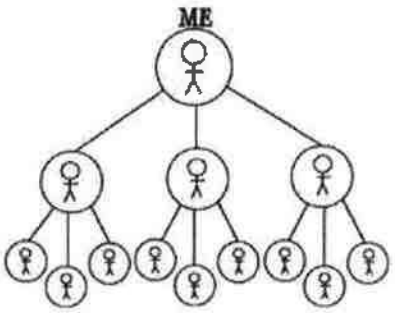
5. Which is the 19th term in the sequence: $a_n = -2(n - 3)$?
(Points : 1)

- ○ 19
- ○ −32
- ○ 32
- ○ −44

Pay It Forward **NAME:** _____ I

In the popular book and movie *Pay It Forward*, Trevor McKinney was given the following social studies assignment: *Think of an idea for world change and put it into practice!*

Trevor's idea was to do something really good for three people. When they asked what they could do to pay him back, he tells them to "pay it forward", which means that they are to do something really nice for three other people.



Trevor figured out that those three people would do something good for a total of nine others. Those nine would do something good for 27 others, and so on. He was sure that before long there would be good things happening to billions of people around the world.

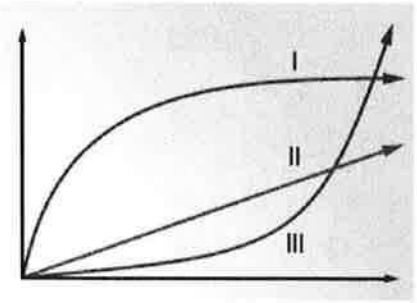
1. How many people would receive a Pay It Forward good deed at each of the next several stages of the process?

Stage 1: Stage 2: Stage 3:

Stage 4: Stage 5: Stage 6:

2. What is your best guess for the number of people who would receive Pay It Forward good deeds at the tenth stage of the process?

3. Which of the graphs below do you think is most likely to represent the pattern by which the number of people receiving Pay It Forward good deeds increases as the process continues over time?



Graph Number: _____

This graph represents _____ growth.

These data sets display properties of _____.

4. Take your values for each stage (from #1) and graph the data. Hint: Your x-axis needs to be the stage number and your y-axis needs to be the number of people receiving Pay It Forward.

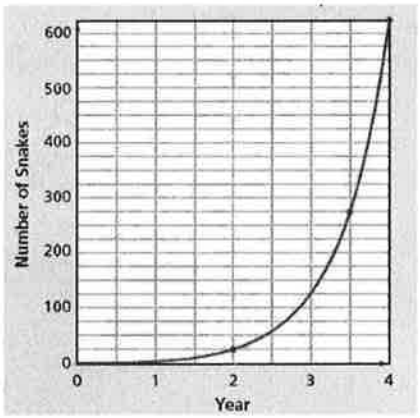
J

From Geometric Sequences to Tables and Graphs

The geometric sequence from the Brown Tree Snake problem (1, 5, 25, 125, 625 . . .) can be written in the form of a table, as shown below:

Year	0	1	2	3	4
# of Snakes	1	5	25	125	625

The Brown Tree Snake was first introduced to Guam in year 0. At the end of year 1, five snakes were found; at the end of year 2, twenty-five snakes were discovered, and so on. Since we now have a table of the information, a graph can be drawn, where the year is the independent variable (x) and the number of snakes is the dependent variable (y). See below:



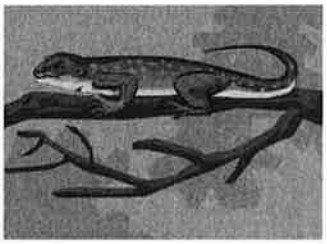
Notice that the graph of the table is not a straight line. Therefore, the graph is not linear in nature, which we already know from the fact that the sequence is not arithmetic. Rather, the graph is curved and moves in a growing fashion very rapidly due to the fact that the common ratio *r* of this sequence is 5. The curved graph of this problem situation is known as an **exponential growth** function. An exponential growth function

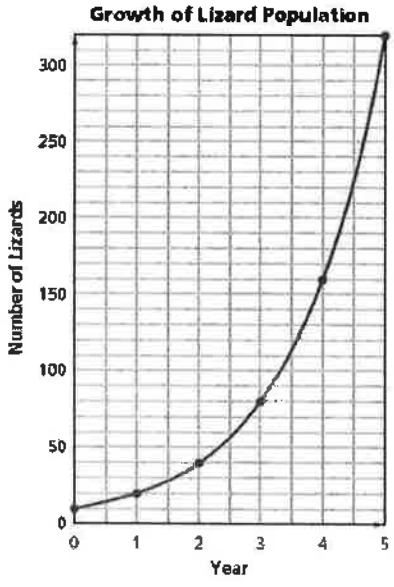
---

Tables and graphs make viewing the data from the problem situation easier to see and we can easily see from either the table or graph that in year 3, the snake population is 125.

Let us look at a similar population growth for a certain kind of lizard in both a table and graph. Use either one or both to answer the questions below.

Year	0	1	2	3	4	5
Number of lizards	10	20	40	80	160	320





Notice from the shape of the graph that the information is exponential in nature.

1. What information does the point (2, 40) on the graph tell you?
2. What information does the point (1, 20) on the graph tell you?
3. When will the population exceed 100 lizards?
4. Explain how to find the common ratio, using either the table or graph.
5. If the information from the table were written as a sequence, what is the initial term?
6. How could we find the 10^{th} term in the table, graph, or sequence?

K

Geometric Sequences to Tables and Graphs

The Mice Problem

A population of mice has a growth factor (otherwise known as the common ratio) of 3. After 1 month, there are 36 mice. After 2 months, there are 108 mice.

1. How many mice were in the population initially (at 0 months)? Explain how you found this number.

2. Write a sequence to show how the mice population is growing.

3. Is this sequence arithmetic or geometric? Explain how you know.

4. Now, put your sequence into the table below.

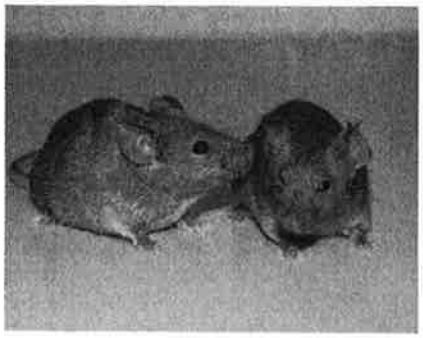
Months	0	1	2	3
Number of Mice				

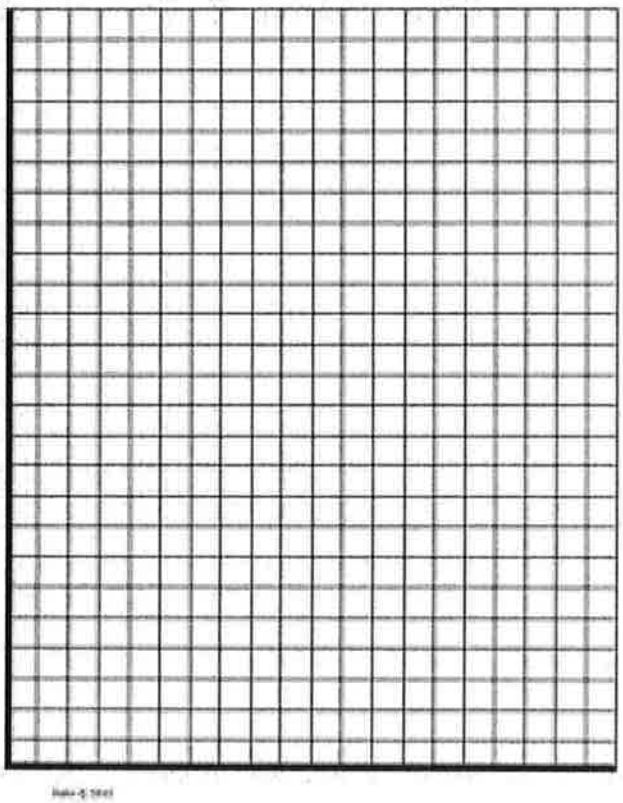
5. Is the graph of the table going to be a straight line or a curve? Explain your answer.

6. Graph the table to make sure of your answer on the graph below. Make sure you label and title the graph below.

   a. What is your scale for the x-axis?

   b. What is your scale for the y-axis?





L

Arithmetic and Geometric Sequences

Determine if the sequence is geometric. If it is, find the common ratio.

1. -1, 6, -36, 216, . . .
2. 4, 16, 36, 64, . . .
3. -2, -4, -8, -16, . . .
4. -1, 1, 4, 8, . . .
5. -3, -15, -75, -375, . . .
6. 1, -5, 25, -125, . . .

7. You visit the Grand Canyon and drop a penny off the edge of a cliff. The distance the penny will fall is 16 feet the first second, 48 feet the next second, 80 feet the third second, and so on.

   Is this sequence arithmetic or geometric?

   What is the initial term?

   What is the rate of change (common ratio or common difference)?

   What is the total distance the object will fall in 6 seconds?

8. A culture of bacteria doubles every 2 hours.

   If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours?

   Is this sequence arithmetic or geometric?

   What is the initial term?

   What is the rate of change (common ratio or common difference)?

The Ladybug Invasion

As a biology project, Tamara is studying the growth of a ladybug population. She starts her experiment with 5 ladybugs. The next month she counts 15 ladybugs.

1. Suppose the ladybug population is growing arithmetically. How many beetles can Tamara expect to find after 2, 3, and 4 months? Write the sequence.

2. What is the common difference?

3. Now put the sequence into a table in the space below.

4. How long will it take the ladybug population to reach 200 if it is growing linearly?

5. Suppose the ladybug population is growing exponentially. How many beetles can Tamara expect to find after 2, 3, and 4 months? Write the sequence.

6. What is the common ratio?

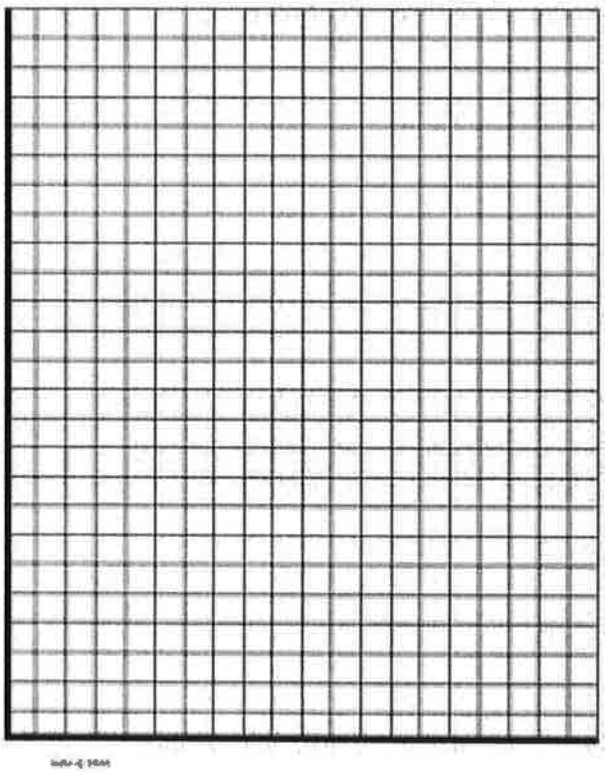
7. Now put the sequence into a table in the space below.

8. How long will it take the ladybug population to reach 200 if it is growing exponentially?

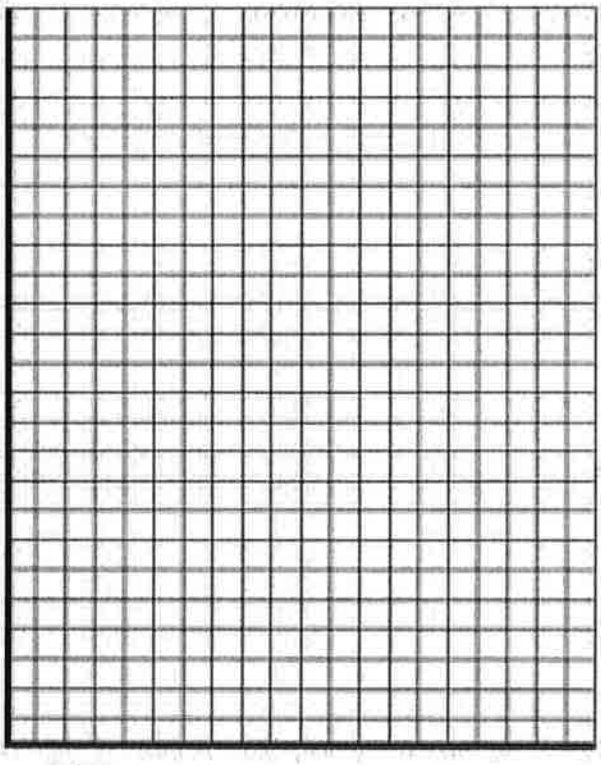
9. Graph both tables on the designated graphs below. Be sure to label your axes.

10. Why does it take the ladybug population longer to reach 200 when it grows linearly?

Linear Growth



Exponential Growth



Charity Donations

Mari's wealthy Great-aunt Sue wants to donate money to Mari's school for new computers. She suggests three possible pans for her donations.

Plan 1: Great-aunt Sue's first plan is give money in the following way: 1, 2, 4, 8, ... She will continue the pattern in this table until day 12. Complete the table to show how much money the school would receive each day.

Day	1	2	3	4	5	6	7	8	9	10	11	12
Donation	$1	$2	$4	$8								

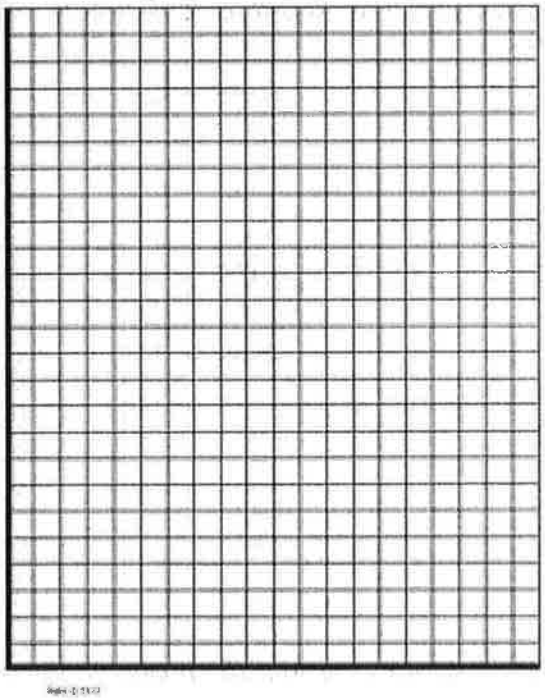
Plan 2: Great-aunt Sue's second plan is to give funds in the following way: 1, 3, 9, 27, ... She will continue the pattern in this table until day 10. Complete the table to show how much money the school would receive each day.

Day	1	2	3	4	5	6	7	8	9	10
Donation	$1	$3	$9	$27						

Plan 3: Great-aunt Sue's third plan is to give money in the following way: 1, 4, 16, 64, . . . She will continue the pattern in this table until day 7. Complete the table to show how much money the school would receive each day.

Day	1	2	3	4	5	6	7
Donation	$1	$4	$16	$64			

Graph each plan on the same graph to the right.



1. How much does each plan give the school on day 6?

2. What is the common ratio (growth rate) for each plan?

   a. Plan 1 _____

   b. Plan 2 _____

   c. Plan 3 _____

3. Which plan should the school choose? Why?

4. Which plan will give the school the **greatest total** amount of money?

N

Jason is planning to swim in a charity swim-a-thon. Several relatives have agreed to sponsor him in this charity event. Each of their donations is explained below.

Grandfather: I will give you $1 if you swim 1 lap, $3 if you swim 2 laps, $5 if you swim 3 laps, $7 if you swim 4 laps, and so on.

Father: I will give you $1 if you swim 1 lap, $3 if you swim 2 laps, $9 if you swim 3 lops, $27 if you swim 4 laps, and so on.

Aunt June: I will give you $2 if you swim 1 lap, $3.50 if you swim 2 laps, $5 if you swim 3 laps, $6.50 if you swim 4 laps, and so on.

Uncle Bob: I will give you $1 if you swim 1 lap, $2 if you swim 2 laps, $4 if you swim 3 laps, $8 if you swim 4 laps, and so on.

5. Decide whether each donation sequence is exponential, linear, or neither.

   a. Grandfather's Plan _____

   b. Father's Plan _____

   c. Aunt June's Plan _____

   d. Uncle Bob's Plan _____

6. Complete the table for each sequence below.

Grandfather's Plan

# of Laps	1	2	3	4	5	6	7	8	9	10
Donation	$1	$3	$5	$7						

Father's Plan

# of Laps	1	2	3	4	5	6	7	8	9	10
Donation	$1	$3	$9	$27						

Aunt June's Plan

# of Laps	1	2	3	4	5	6	7	8	9	10
Donation	$2	$3.50	$5	$6.50						

Uncle Bob's Plan

# of Laps	1	2	3	4	5	6	7	8	9	10
Donation	$1	$2	$4	$8						

Writing Sequence Rules

Name: _____

Write a recursive rule and an explicit rule for each of the following sequences.

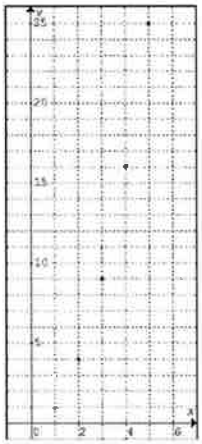
1.

n	1	2	3	4	5	6
sequence	1	3	6	10	15	21

2. 13, 9, 5, 1, -3, -7, -11,...

3.

4.



5. Jeremy was inspired to write 5 poems to his love one day. He plans to write a new poem every day thereafter. Write a rule for the **total number** of poems he has written after **n** days.

6. Samantha made matching friendship bracelets for her closest friends for the first home basketball game. Other people admired them and offered to pay her to make bracelets for them also. She decided to sell the bracelets for $0.75 each at the homecoming game. Write a rule for the **total amount** of money she will collect if she can make three bracelets a day for the next **n** days.

7. A scientist isolates a single bacterium in a Petri dish. She allows it to grow and measures the number of bacteria in the dish every hour. At the beginning of the second hour, there are two bacteria in the dish. This number doubles to four bacteria at the start of the third hour and to eight bacteria at the start of the fourth hour. Write a rule for the number of bacteria in the Petri dish at the beginning of **n** hours.

8. When Billy turned 8 he received a $5000 gift from his great-grandfather. His parents deposited it in a savings account that pays 6% interest annually. Write a rule for the amount of money in the account at the end of **n** years.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____

- Writing Sequence Rules

Answer Key

Remember the following mathematical practices as you view student work, make suggestions/corrections and facilitate group discussion.

- Make sense of problems and persevere in solving them. (MP1)
- Construct viable arguments and critique the reasoning of others. (MP3)
- Look for and make use of structure. (MP7)
- Look for and express regularity in repeated reasoning. (MP8)

(F.BF.1.a and F.BF.2 and F.LE.2/MP1 and MP3 and MP7 and MP8)

1. $a_n = a_{n-1} + n$ for $n \geq 2$ and $a_1 = 1$ $\quad a_n = \frac{1}{2}(n)(n+1)$

2. $a_n = a_{n-1} - 4$ for $n \geq 2$ and $a_1 = 13$ $\quad a_n = -4n + 17$

3. $a_n = a_{n-1} \bullet n$ for $n \geq 2$ and $a_1 = 2$ $\quad a_n = n(n+1)$

4. $a_n = (\sqrt{a_{n-1}} + 1)^2$ for $n \geq 2$ and $a_1 = 1$ $\quad a_n = n^2$

5. $a_n = a_{n-1} + 1$ for $n \geq 2$ and $a_1 = 5$ $\quad a_n = n + 4$

6. $a_n = a_{n-1} + .75(3)$ for $n \geq 2$ and $a_1 = 2.25$ $\quad a_n = 2.25n$

7. $a_n = a_{n-1} \cdot 2$ for $n \geq 2$ and $a_1 = 1$ $\quad a_n = 2^{n-1}$

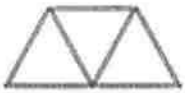
8. $a_n = a_{n-1} \cdot 1.06$ for $n \geq 2$ and $a_1 = 5000$ $\quad a_n = 5000(1.06)^{n-1}$

P

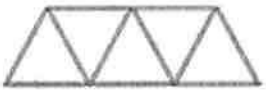
Truss bridges are built using a pattern of steel beams. This pattern has great strength due to weight distribution in the beams. This activity explores the pattern of steel beams in a truss bridge design.

Three beams are joined together in the triangular shape shown below.

One Section



Two Sections



Three Sections

The bridge is made wider by adding more sections as shown above.

There is a pattern in this bridge design. The total number of beams is related to the number of bridge sections. Complete the first three rows in the table to explore the pattern.

Total number bridge sections	**Total number of beams**
1	3
2	
3	

Use the information in the table to predict how many beams a bridge with 4 sections would have. Try to come up with a rule to demonstrate your prediction. What kind of rule should you make?

Determine the total number of beams of the *nth* number of bridge sections. What kind of rule should you make?

Name: _____

Writing Sequence Rules

Date: _____

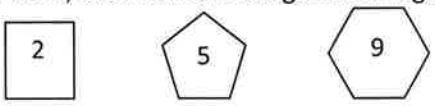
Write a recursive rule and an explicit rule for each of the following sequences.

1.

n	1	2	3	4	5	6
sequence	.25	.5	1	2	4	8

2. 26, 13, 0, -13, -26, -39, -42,...

3. For $n \geq 4$, the number of diagonals in regular polygons.



(Hint: The next term is 14.)

4. A baby weighs 8 pounds at birth. For the first year, the baby's weigh may increase by 10% per month. What would be the baby's weight after n months?

5. Angelica started collecting stamps when she inherited her grandfather's stamp collection of 187 stamps. If she is able to add one stamp to her collection each month, how many stamps will she have in her collection after n years.

Writing Sequence Rules — Answer Key

Write a recursive rule and an explicit rule for each of the following sequences.

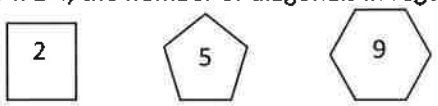
1.

n	1	2	3	4	5	6
sequence	.25	.5	1	2	4	8

$F(n) = 2a_{n-1}$ for $n \geq 2$ given $a_1 = .25$ $f(n) = 2^{n-3}$ (Obj 3/F.LE.2/2 and Obj 4/F.BF.2/2)

2. 26, 13, 0, -13, -26, -39, -42,...
$F(n) = a_{n-1} - 13$ for $n \geq 2$ given $a_1 = 25$ $f(n) = -13n + 39$ (Obj 3/F.LE.2/2 and Obj 4/F.BF.2/2)

3. For $n \geq 4$, the number of diagonals in regular polygons is represented below.



(Hint: The next term is 14.)

$F(n) = a_{n-1} + (n + 2)$ for $n \geq 5$ given $a_4 = 2$ $f(n) = .5(n^2 - 3n)$ (Obj 3/F.LE.2/2 and Obj 4/F.BF.2/2)

4. A baby weighs 8 pounds at birth. For the first year, the baby's weigh may increase by 10% per month. What would be the baby's weight after n months?
$F(n) = 1.1a_{n-1}$ for $n \geq 2$ given $a_1 = 8$ $f(n) = 8(1.1)^{n-1}$ (Obj 3/F.BF.1.a/3 and Obj 4/F.BF.2/3)

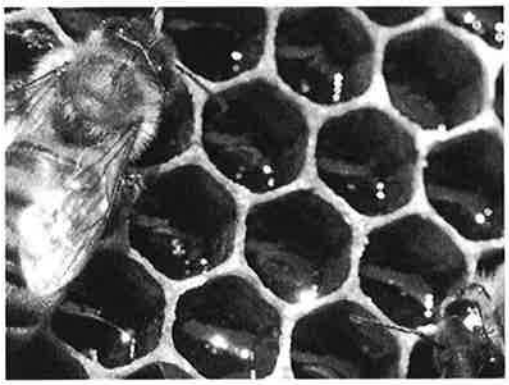
5. Angelica started collecting stamps when she inherited her grandfather's stamp collection of 187 stamps. If she is able to add one stamp to her collection each month, how many stamps will she have in her collection after n years.
$F(n) = a_{n-1} + 12$ for $n \geq 2$ given $a_1 = 187$ $f(n) = 12n + 175$ (Obj 3/F.BF.1.a/3 and Obj 4/F.BF.2/3)

R

Performance Task

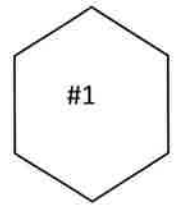
Honeycombs

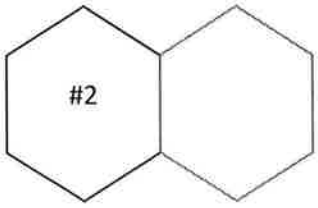
Name:_____ Date:_____



Honeycombs are a network of hexagons that bees create for their hives. They are a wonderful natural shape because they efficiently use material for the amount of area they create and they are strong. Let's build rows of honeycomb shapes starting with a single hexagon.

1. The diagrams below show the first two stages of a honeycomb row. At each stage another hexagon is added to the sequence. Work with your group to draw the next four stages in one row of honeycombs.





R

2. Notice that the first hexagon has six edges, so it took 6 segments to build it. Complete the table to show the number of segments in the next five stages of a honeycomb row.

Honeycomb Edges	
Stage	Total # of Segments
1	6
2	
3	
4	
5	
6	

3. By now, we have talked a lot about the difference between iterative rules and recursive rules. Explain briefly the difference between the two types of rules. Be sure to give examples of when one might be more useful than the other.

4. What is the recursive rule for finding the number of segments in the honeycomb row?_____ Explain how your group arrived at your answer. Also, show all the mathematics you used.

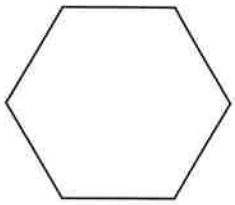
5. What is the iterative rule for finding the number of segments in the honeycomb row? _____ Again, explain how you arrived at your answer. Also, show all the mathematics you used.

6. How many segments will be in the 10^{th} stage of the honeycomb row? ___ The 50^{th} stage?___ Briefly explain why.

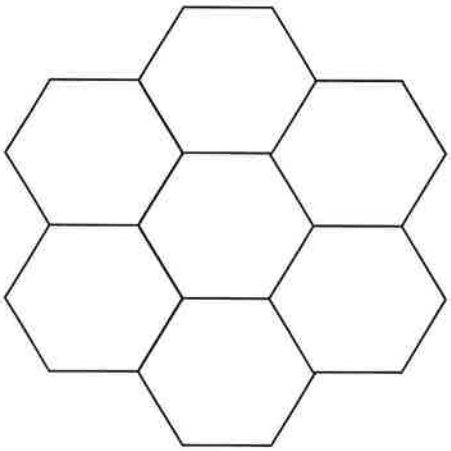
7. Describe what the graph of this pattern would look like.

R

8. Now, let's see how a honeycomb shape can be built starting with a single hexagon and building out:



Now we add new hexagons all around the outside as we start to build our hive:



How many new hexagons did we add? _____

How many do we have all together? _____

9. How many honeycombs do you predict will be in the next formation? _____
Explain how you arrived at your answer.

10. Your group should now either model (using either pattern blocks or hexagon cut-outs) or draw the next three honeycomb formations. Was your prediction in Question 8 right or wrong? Why?

R

11. Now, complete the table below to show the total number of hexagons that are added for each stage and the total number of hexagons in each stage. Note that we are calling the first, single hexagon Stage 0.

Second Honeycomb Exploration		
Honeycomb Stage	Number of hexagons added	Total number of hexagons
0	1	1
1	6	7
2		
3		
4		
5		

12. What is the recursive rule to find the total number of hexagons in each successive stage? Explain how you arrived at your answer. Use the rule to predict the number in the fifth stage.

13. Graph the relationship between the honeycomb stage and the total number of hexagons. Is your graph similar to the graph you were asked about in question 7? If not, why?

R

14. What is the iterative rule to find the total number of hexagons in any stage? This may be difficult or time consuming to determine, so make sure your group has a detailed discussion. What ideas did you try?

15. Using your rule, what would be the number of hexagons in stage 10?

Sigma Notation

1. Determine the number of terms in the series:

a) $\sum_{k=1}^{7} 2(3)^{k-1}$ b) $\sum_{k=4}^{20} 4\left(\frac{1}{2}\right)^{k-1}$ c) $\sum_{k=-3}^{10} 2^{k-5}$ d) $\sum_{k=a}^{b} 2(3)^{k-2}$

2. Determine the given term of the series:

a) $\sum_{k=3}^{5} 2(3)^{k-1}$ find t_1 b) $\sum_{k=-2}^{7} 18\left(\frac{1}{2}\right)^{k}$ find t_2

c) $\sum_{k=a}^{b} 2\left(\frac{1}{2}\right)^{k-1}$ find t_3

3. Evaluate the following:

a) $\sum_{i=1}^{6} 5\left(\frac{1}{3}\right)^{i-1}$ b) $\sum_{m=8}^{20} \frac{1}{16}(2)^{m}$

c) $\sum_{k=-2}^{5} 4(3)^{k-1}$ d) $\sum_{b=5}^{12} 5(2)^{1-b}$

4. Expand the following:

a) $\sum_{k=1}^{5} 5(-2)^{k-1}$ b) $\sum_{k=4}^{7} 3\left(\frac{1}{2}\right)^{k-2}$ c) $\sum_{k=4}^{6} \log_3 k$

5. Solve the following:

a) $\sum_{k=2}^{5} \log_x k = 2$ b) $\sum_{k=0}^{2} x^{k} = 7$

6. Evaluate:

a) $\sum_{k=3}^{5} \log_k k$ b) $\sum_{k=3}^{4} \log_{12} k^2$

7. Write the following using sigma notation:

a) $-2+4-8+16...+1024$ b) $5-\frac{5}{4}+\frac{5}{16}-\frac{5}{64}+\frac{5}{256}-\frac{5}{1024}$

Answers

1. a) 7 b) 17 c) 14 d) b-a+1

2. a) 18 b) 36 c) 2^{-a}

3. a) 7.4897 or $\frac{1820}{243}$ b) 131 056

   c) 485.9259 or $\frac{13120}{27}$ d) .6226 or $\frac{1275}{2048}$

4. a) $5-10+20-40+80$ b) $\frac{3}{4}+\frac{3}{8}+\frac{3}{16}+\frac{3}{32}$

   c) $\log_3 4+\log_3 5+\log_3 6$

5. a) $x=\sqrt{120}$ or $x=10.95$ b) $x=-3,\ 2$

6. a) 3 b) 2

7. a) $\sum_{k=1}^{10}-2(-2)^{k-1}$ b) $\sum_{k=1}^{6}5\left(-\frac{1}{4}\right)^{k-1}$

S

Sequences Practice Name _____

Recursive Rule for arithmetic sequences : $t_1 = \_\_\_, t_n = t_{n-1} + d$

Closed Rule for arithmetic sequences: $t_n = t_1 + d(n-1)$

Matching. Place the letter on the line that defines the given word.

_____1) Sequence	a. A rule that relates one term to the previous term
_____2) Arithmetic Sequence	b. A rule that allows you to calculate any term in the sequence directly
_____3) Recursive Rule	c. A function whose domain is consecutive whole numbers
_____4) Closed Rule	d. A number in the sequence (output)
_____5) Term	e. A sequence that has a common number added to each term

6) Find the next three terms in the sequence.

_____ a) -3, 9, -27, 81, …

_____ b) 3, -3, -9, -15, ….

_____ c) 6, 8, 10, 12, 14, ….

_____ d) 128, 64, 32, 16, …

7) Write a recursive rule for the sequence.

_____ a) 2, 6, 10, 14, …

_____ b) 7, 14, 28, 56, ….

_____ c) 5, 0, -5, -10, ….

_____ d) 1, 4, 16, 64, ….

8) Write a closed rule for the sequence.

_____ a) 5, -10, -25, -40, …

_____ b) 6, 9, 12, 15, …

_____ c) 4, 1, -2, -5, ….

_____ d) 28, 30, 32, 34, ….

_____ e) 18, 14, 10, 6, ….

_____ f) 35, 40, 45, 50, …

9) Write a recursive rule and a closed rule for the sequence.
**Remember to write the 1st term in the sequence for recursive rule.*

	Recursive Rule*	**Closed Rule**
a) -3, -6, -9, -12, …	_____	_____
b) 7, 11, 15, 19, ….	_____	_____
c) -2, 2, 6, 10, ….	_____	_____
d) 5, 7, 9, 11, …	_____	_____
e) 40, 46, 52, 58, …	_____	_____

10) Find the specified term in the sequence.

_____ a) Find the 20^{th} term of the sequence 8, 11, 14, 17, …

_____ b) Find the 100^{th} term of the sequence 9, 6, 3, 0, …

_____ c) Find the 50^{th} term of the sequence -4, -1, 2, 5, …

_____ d) Find the 38^{th} term of the sequence 7, 5, 3, 1, …

_____ e) Find the 72^{nd} term of the sequence 0, 4, 8, 12, ….

ARITHMETIC Recursive and Explicit Worksheet Name _____

Given the following formulas, find the first 4 terms.

1. $t_1 = 0$
   $t_n = t_{n-1} + 6$

2. $t_1 = -4$
   $t_n = t_{n-1} + 2$

3. $t_1 = 8$
   $t_n = t_{n-1} - 4$

4. $t_n = 3n - 1$

5. $t_n = 4n + 3$

6. $t_n = -5n + 2$

7. Write an explicit and recursive formula for the following sequences.

a. -4, -6, -8, -10…

Explicit: _____

Recursive: _____

b. 84, 71, 58, 45…

Explicit: _____

Recursive: _____

c. 19, 13, 7, 1…

Explicit: _____

Recursive: _____

d. 9, 17, 25, 33…

Explicit: _____

Recursive: _____

e. -3, -1, 1, 3…

Explicit: _____

Recursive: _____

f. 110, 88, 66, 44…

Explicit: _____

Recursive _____

Given the recursive formula, write the explicit formula for the sequence.

8. $t_1 = 0$
$t_n = t_{n-1} + 6$

9. $t_1 = -4$
$t_n = t_{n-1} + 2$

10. $t_1 = 8$
$t_n = t_{n-1} - 4$

Given the explicit formula, write the recursive formula for the sequence.

11. $t_n = 3n - 1$

12. $t_n = 4n + 3$

13. $t_n = -5n + 2$

Name: _____

Arithmetic and Geometric Sequences **Date:** _____

1. Write a geometric sequence in recursive form. Translate it into explicit form.

2. Write an arithmetic sequence in explicit form. Translate it into recursive form.

3. State the domain of a sequence with 17 terms.

A fitness club charges a membership fee of $75 for the first month and monthly fee of $25 each month thereafter. The related sequence represents the total amount paid for n months.

4. Determine if this related sequence describes a change which grows by a constant percent rate (geometric) or a change which grows by a constant rate per unit interval (arithmetic). State the common ratio or difference.

5. Write a function that describes the charge for a period of n months.

6. Write a new function, h, that represents a membership fee of $100.

7. Write a new function, g, that represents a monthly fee of $30.

Arithmetic and Geometric Sequences Answer Key

1. Write a geometric sequence in recursive form. Translate it into explicit form.
   Answers will vary. (Obj 1/F.LE.1.c/3 and Obj 3/F.BF.2/2)

2. Write an arithmetic sequence in explicit form. Translate it into recursive form.
   Answers will vary. (Obj 1/F.LE.1.b/3 and Obj 3/F.BF.2/2)

3. State the domain of a sequence with 17 terms.
   {1, 2, 3, . . . , 15, 16, 17} (Obj 2/F.IF.3/2)

A fitness club charges a membership fee of $75 for the first month and monthly fee of $25 each month thereafter. The related sequence represents the total amount paid for n months.

4. Determine if this related sequence describes a change which grows by a constant percent rate (geometric) or a change which grows by a constant rate per unit interval (arithmetic). State the common ratio or difference.
   Arithmetic. The total amount paid increases by a constant rate of $25 each month.
   (Obj 1/F.LE.1.b/2 and Obj 1/F.LE.1.c/2)

5. Write a function that describes the charge for a period of n months.
   F(n) = 25n + 50 (Obj 5/F.LE.5/2)

6. Write a new function, h, that represents a membership fee of $100.
   h(n) = 25n + 75 (Obj 5/F.LE.5/3)

7. Write a new function, g, that represents a monthly fee of $30.
   G(n) = 30n + 45 (Obj 5/F.LE.5/3)

Illustrative Mathematics

F-LE Algae Blooms

Alignment 1: F-LE.A.1.c, F-LE.A.2, F-LE.A.4

Algae blooms routinely threaten the health of the Chesapeake Bay. Phosphate compounds supply a rich source of nutrients for the algae, *Prorocentrum minimum*, responsible for particularly harmful spring blooms known as mahogany tides. These compounds are found in fertilizers used by farmers and find their way into the Bay with run-offs resulting from rainstorms. Favorable conditions result in rapid algae growth ranging anywhere from 0.144 to 2.885 cell divisions per day. Algae concentrations are measured and reported in terms of cells per milliliter (cells/ml). Concentrations in excess of 3,000 cells/ml constitute a bloom.

a. Suppose that heavy spring rains followed by sunny days create conditions that support 1 cell division per day and that prior to the rains *Prorocentrum minimum* concentrations measured just 10 cells/ml. Write an equation for a function that models the relationship between the algae concentration and the number of days since the algae began to divide at the rate of 1 cell division per day.

b. Assuming this rate of cell divison is sustained for 10 days, present the resulting algae concentrations over that period in a table. Did these conditions result in a bloom?

c. Concentrations in excess of 200,000 cells/ml have been reported in the Bay. If conditions support 2 cell divisions per day, when will these conditions result in a bloom? When will concentrations exceed 200,000 cells/ml?

Commentary:
The problem statement describes a changing algae population as reported by the Maryland Department of Natural Resources. In part (a), students are expected to build an exponential function modeling algae concentration from the description given of the relationship between concentrations in cells/ml and days of rapid growth (F-LE.2). The intent of part (b) is for students to gain an appreciation for the exponential growth exhibited despite an apparently modest growth rate of 1 cell division per day. Successful completion of part (c) depends on the application of logarithms (F-LE.4).

As structured, the task is best used in settings with sufficient time given students to process and make sense of the situation presented.

Solution: Solution

a. One cell division per day implies that the number of cell divisions in t days is $1 \frac{\text{cell division}}{\text{day}} \cdot t \text{ days} = t \text{ cell divisions}$
Since, the number of cells doubles with each cell division, their concentration doubles with each cell division. So, the concentration, $C(t)$, of algae cells per milliliter in t days is given by

$$C(t) = 10 \cdot 2^t$$

b. Concentrations of algae (rounded to the nearest whole number) over the 10-day period are given the following table:

Day	1	2	3	4	5	6	7	8	9	10
cells/ml	20	40	80	160	320	640	1280	2560	5120	10,240

Hence, an algae bloom resulted in a little less than nine days.

c. Two cell divisions per day implies that the number of cell divisions in t days is
$2 \frac{\text{cell division}}{\text{day}} \cdot t \text{ days} = 2 \cdot t \text{ cell divisions}$ Again, since a cell division implies the number of cells doubles, and since when the number of cells doubles, the concentration doubles, we have

$$C(t) = 10 \times 2^{2t}$$

To determine when concentrations are high enough to constitute a bloom, solve the following equation for t:

$$3000 = 10 \cdot 2^{2t}$$

Applying logarithms to solve for t:

$$\ln 300 = 2t \cdot \ln 2$$

$$t = \frac{\ln 300}{2 \cdot \ln 2} \approx 4.1 \text{ days.}$$

Hence, an algae bloom is formed under these conditions in a little over 4 days.

Similar procedures are used to determine when concentrations exceed 200,000 cells/ml.

$$200,000 = 10 \times 2^{2t}$$

yields

$$t = \frac{\ln 20,000}{2 \times \ln 2} \approx 7.1 \text{ days.}$$

Hence, the algae bloom exceeds concentrations of 200,000 cells/ml in a little over 7 days.

W

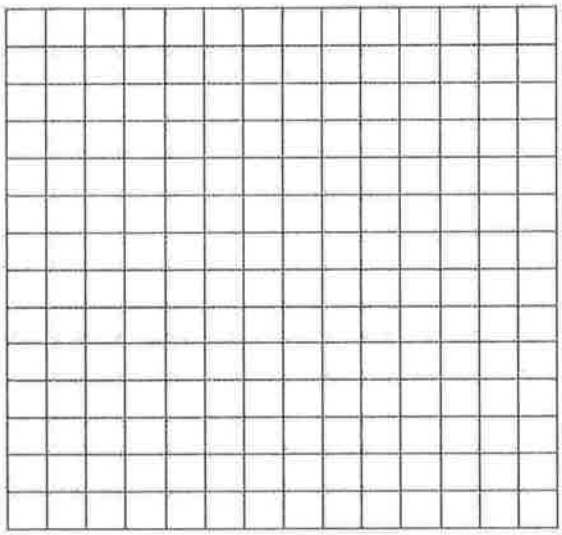
Pay It Forward, Again Name _____

At the beginning of this unit we examined the *Pay It Forward* class project that Trevor McKinney came up with. Let us revisit this situation and take a deeper look at what transpired.

1. Make a table that shows the number of people who will receive good deeds at each of the next seven stages of the *Pay It Forward* process.

Stage of Process	1	2	3	4	5	6	7	8	9	10
Number of Good Deeds	3	9	27							

2. Then plot the data on a graph. Make sure you have accurate axes labels and scales.



3. How does the number of good deeds at each stage grow?

4. What is the common ratio?

5. How is that pattern change shown in the plot of the data?

6. How many stages of the *Pay It Forward* process will be needed before a total of at least 20,000 good deeds will be done?

7. Write a NOW-NEXT rule to illustrate the *Pay It Forward* process.

8. Write an NOW-NEXT rule that would show the number of good deeds at a stage number if each person in the process does good deeds for **two** others.

9. How would the NOW-NEXT rule change if each person in the process does good deed for **four** other people?

Adapted from *Core-Plus Mathematics, Course 1, Glencoe-McGraw-Hill, 2008.*

RENAISSANCE FESTIVAL LEARNING TASK:

As part of a class project on the Renaissance, your class decided to plan a renaissance festival for the community. Specifically, you are a member of different groups in charge of planning two of the contests. You must help plan the archery and rock throwing contests. The following activities will guide you through the planning process.

Group One: Archery Contest[1]

Before planning the archery contest, your group decided to investigate the characteristics of the target. The target being used has a center, or bull's-eye, with a radius of 4 cm, and nine rings that are each 4 cm wide.

1. The Target

a. Sketch a picture of the center and first 3 rings of the target.

b. Write a sequence that gives the radius of each of the concentric circles that comprise the entire target.

c. Write a recursive formula and an explicit formula for the terms of this sequence.

d. What would be the radius of the target if it had 25 rings? Show how you completed this problem using the explicit formula.

e. In the past, you have studied both arithmetic and geometric sequences. What is the difference between these two types of sequences? Is the sequence in (b) arithmetic, geometric, or neither? Explain.

One version of the explicit formula uses the first term, the common difference, and the number of terms in the sequence. For example, if we have the arithmetic sequence 2, 5, 8, 11, 14, …, we see that the common difference is 3. If we want to know the value of the 20^{th} term, or a_{20}, we could think of starting with $a_1 = 2$ and adding the difference, $d = 3$ a certain number of times. How many times would we need to add the common difference to get to the 20^{th} term? _____ Because multiplication is repeated addition, instead of adding 3 that number of times, we could multiply the common difference, 3, by the number of times we would need to add it to 2.

This gives us the following explicit formula for an arithmetic sequence: $a_n = a_1 + (n-1)\,d$.

f. Write this version of the explicit formula for the sequence in this problem. Show how this version is equivalent to the version above.

---

[1] Elements of these problems were adapted from *Integrated Mathematics 3* by McDougal-Littell, 2002.)

Term of a sequence: Each number in a sequence is a term of the sequence. The first term is generally noted as a_1, the second as a_2, …, the *nth* term is noted as a_n. a_n is also referred to as the general term of a sequence.

X

g. Can you come up with a reason for which you would want to add up the radii of the concentric circles that make up the target (for the purpose of the contest)? Explain.

h. Plot the sequence from this problem on a coordinate grid. What should you use for the independent variable? For the dependent variable? What type of graph is this? How does the a_n equation of the recursive formula relate to the graph? How does the parameter d in the explicit form relate to the graph?

i. Describe (using y-intercept and slope), but do not graph, the plots of the arithmetic sequences defined explicitly or recursively as follows:

1. $a_n = 3 + \frac{4}{3}\ n-1$

2. $\begin{cases} a_1 = -2 \\ a_n = a_{n-1} + \frac{1}{2} \end{cases}$

3. $a_n = 4.5 - 3.2\ n-1$

4. $\begin{cases} a_1 = 10 \\ a_n = a_{n-1} - \frac{2}{5} \end{cases}$

2. The Area of the Target: To decide on prizes for the archery contest, your group decided to use the areas of the center and rings. You decided that rings with smaller areas should be worth more points. But how much more? Complete the following investigation to help you decide.

a. Find the sequence of the areas of the rings, including the center. (Be careful.)

b. Write a recursive formula and an explicit formula for this sequence.

c. If the target was larger, what would be the area of the 25th ring?

d. Find the total area of the bull's eye by adding up the areas in the sequence.

e. Consider the following sum: $S_n = a_1 + a_2 + a_3 + \ldots + a_{n-1} + a_{n-2} + a_n$. Explain why that equation is equivalent to $S_n = a_1 + a_1 + d + a_1 + 2d + \ldots + a_n - 2d + a_n - d + a_n$.

Rewrite this latter equation and then write it out backwards. Add the two resulting equations. Use this to finish deriving the formula for the sum of the terms in an arithmetic sequence. Try it out on a few different short sequences.

f. Use the formula for the sum of a finite arithmetic sequence in part (e) to verify the sum of the areas in the target from part (d).

g. Sometimes, we do not have all the terms of the sequence but we still want to find a specific sum. For example, we might want to find the sum of the first 15 multiples of 4. Write an explicit formula that would represent this sequence. Is this an arithmetic

sequence? If so, how could we use what we know about arithmetic sequences and the sum formula in (e) to find this sum? Find the sum.

h. What happens to the sum of the arithmetic series we've been looking at as the number of terms we sum gets larger? How could you find the sum of the first 200 multiples of 4? How could you find the sum of all the multiples of 4? Explain using a graph and using mathematical reasoning.

i. Let's practice a few arithmetic sum problems.

   1. Find the sum of the first 50 terms of 15, 9, 3, -3, …
   2. Find the sum of the first 100 natural numbers
   3. Find the sum of the first 75 positive even numbers
   4. Come up with your own arithmetic sequence and challenge a classmate to find the sum.

j. Summarize what you learned / reviewed about arithmetic sequences and series during this task.

3. Point Values: Assume that each participant's arrow hits the surface of the target.

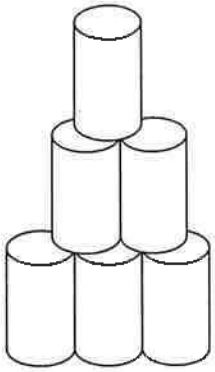
   a. Determine the probability of hitting each ring and the bull's-eye.

Target Piece	**Area of Piece (in cm^2)**	**Probability of Hitting this Area**
Bull's Eye	16π	
Ring 1	48π	
Ring 2	80π	
Ring 3	112π	
Ring 4	144π	
Ring 5	176π	
Ring 6	208π	
Ring 7	240π	
Ring 8	272π	
Ring 9	304π	

   b. Assign point values for hitting each part of the target, justifying the amounts based on the probabilities just determined.

Group Two: Rock Throwing Contest[2]

For the rock throwing contest, your group decided to provide three different arrangements of cans for participants to knock down.



1. For the first arrangement, the tin cans were set up in a triangular pattern, only one can deep. (See picture.)

   a. If the top row is considered to be row 1, how many cans would be on row 10?

   b. Is this an arithmetic or a geometric sequence (or neither)? Write explicit and recursive formulas for the sequence that describes the number of cans in the *nth* row of this arrangement.

   c. It is important to have enough cans to use in the contest, so your group needs to determine how many cans are needed to make this arrangement. Make a table of the number of rows included and the total number of cans.

Rows Included	Total Cans
1	1
2	3
3	
4	
5	
6	
7	
8	

   d. One of your group members decides that it would be fun to have a "mega-pyramid" 20 rows high. You need to determine how many cans would be needed for this pyramid, but you don't want to add all the numbers together. One way to find the sum is to use the summation formula you found in the Archery Contest. How do you find the sum in an arithmetic sequence? _____ Find the sum of a pyramid arrangement 20 rows high using this formula.

---

[2] Adapted from Manouchehri, A. (2007). Inquiry-discourse mathematics instruction. *Mathematics Teacher, 101*, 290–300.

c. Use your answer to (b) to determine the expected number of points one would receive after shooting a single arrow.

d. Using your answers to part (c), determine how much you should charge for participating in the contest OR for what point values participants would win a prize. Justify your decisions.

Student Name	**Teacher Name**
Progress Report Starting Grade:_____ Progress 1: Date_____: Grade:_____ Progress 2: Date_____: Grade:_____ Progress 3: Date_____: Grade:_____	**Parent Contact Log** Phone #_____ Date, Reason, Outcome 1 _____ 2._____ 3. _____ 4.._____ 5.._____
Requirements of Your Acceleration Plan:	**Agreement to follow this Acceleration Plan** Student Signature_____ Teacher Signature_____ Additional Comments (Student or Teacher):

Student Name	**Teacher Name**
Progress Report Starting Grade:_____ Progress 1: Date_____: Grade:_____ Progress 2: Date_____: Grade:_____ Progress 3: Date_____: Grade:_____	**Parent Contact Log – Contact if student is not complying with Improvement Plan** Phone #_____ Date, Reason, Outcome 1 _____ 2._____ 3. _____ 4.._____ 5.._____
Requirements of Your Improvement Plan: □ Attend afterschool tutoring on: Tues Weds □ Make-up Missing: □ Test □ Quiz □ Classwork □ Homework □ Other:_____	**Agreement to follow this Improvement Plan** Student Signature_____ Teacher Signature_____ Additional Comments (Student or Teacher):

Lesson 7: Series and Sigma Notation

Like sequences, series also have many applications, particularly in the fields of physics and computer science. In computer science, series have been used to create software which can generate updated photographs of missing persons and fugitives. In this lesson, you will learn how to write a series using sigma notation, how to write a series in expanded form, and how to find the sum of a sequence. You will also learn how to represent a series using sigma notation.

GOALS FOR THIS LESSON

- Given a series in sigma notation, write a series in expanded form and find the sum.
- Represent a series using sigma notation.

In this lesson, you will learn about the sum of the terms in a sequence. Take a moment to review what you have learned about sequences.

The iterative rule for an arithmetic sequence is:

$$a_n = a_1 + (n-1)d$$

The iterative rule for a geometric sequence is:

$$a_n = a_1 \cdot r^{n-1}$$

Iterative Rules
Match each iterative rule with the sequence.

Iterative Rule	Sequence
1, 5, 9, 13, ...	$a_n = 10 \cdot \left(\frac{1}{5}\right)^{n-1}$
10, 8, 6, 4, 2, ...	$a_n = -2n + 12$
$10, 2, \frac{2}{5}, \frac{2}{25}, \ldots$	$a_n = 5^{n-1}$
1, 5, 25, 125, ...	$a_n = 4n - 3$

Check

Finding a Sum

A series is the sum of consecutive terms of a sequence. For example, consider the sequence 3, 6, 9, 12, 15...The sum 3 + 6 + 9 + 12 is a series, and is represented by S_4: the sum of the first four terms of the sequence. Notice that $S_4 = 30$ (just add the numbers).

In general, finding S_n means adding the sum of the first n terms of a sequence. Try this example.

Given the sequence 44, 41, 38, 35, ..., find each sum:

$s_1 = 44$

$s_2 = 85$

$s_3 =$ ☐

$s_4 =$ ☐

$s_5 =$ ☐

Check

Remember

Sigma Notation:

Learn how use sigma notation to find the sum of a series.

Click next to begin.

cc

1 of 12 Next

Tip

$a_1, a_2, a_3, \ldots a_n$ sequence

$S_n = a_1 + a_2 + a_3 + \ldots + a_n$ series

MTH303B: Algebra II | Unit 3: Sequences and Series | Lesson 7: Series and Sigma Notation | Finding a Sum

$S_n = a_1 + a_2 + a_3 + \ldots + a_n$

$3 \quad 6 \quad 9 \quad 12 \quad 15$ sequence

$S_5 = 3 + 6 + 9 + 12 + 15$ series

$S_5 = 45$

$$S_n = a_1 + a_2 + a_3 + \ldots + a_n$$

$$S_5 = \underbrace{3 + 6 + 9 + 12 + 15}_{\text{expanded form}} \qquad \text{series}$$

cc

Tip

MTH303B: Algebra II | Unit 3: Sequences and Series | Lesson 7: Series and Sigma Notation | Finding a Sum

$$S_n = a_1 + a_2 + a_3 + \ldots + a_n \qquad \text{series}$$

$$\sum_{i=1}^{n} a_i \qquad \text{sigma notation}$$

cc

Tip

2

$$\sum_{i=1}^{n} a_i$$

upper limit: n

algebraic expression: a_i

lower limit: $i = 1$

index: i

sigma notation

"the sum from 1 to n of a_i"

$$\sum_{i=1}^{3} 2i = \underbrace{2 \cdot i}_{i=1} + \underbrace{2 \cdot i}_{i=2} + \underbrace{2 \cdot i}_{i=3}$$

$$\sum_{i=1}^{3} 2i = 2 \cdot 1 + 2 \cdot 2 + 2 \cdot 3$$

MTH303B: Algebra II | Unit 3: Sequences and Series | Lesson 7: Series and Sigma Notation | Finding a Sum

$$\sum_{i=1}^{3} 2i = 2 \cdot 1 + 2 \cdot 2 + 2 \cdot 3$$

$$= \underbrace{2 + 4 + 6}_{\text{expanded form}}$$

$$= 12$$

$$\sum_{i=4}^{6}(2i-4) = \underbrace{(2i-4)}_{i=4} + \underbrace{(2i-4)}_{i=5} + \underbrace{(2i-4)}_{i=6}$$

MTH303B: Algebra II | Unit 3: Sequences and Series | Lesson 7: Series and Sigma Notation | Finding a Sum

$$\sum_{i=4}^{6}(2i-4) = (2\cdot4-4) + (2\cdot5-4) + (2\cdot6-4)$$

$$= \underbrace{4 + 6 + 8}_{\text{expanded form}}$$

$$= 18$$

$\sum_{i=1}^{3} 2i$	Summation notation is made up of four parts: the index i, the lower limit, the upper limit, and the algebraic expression in terms of i.
$\sum_{i=1}^{3} 2i = \underbrace{2 \cdot i}_{i=1} + \underbrace{2 \cdot i}_{i=2} + \underbrace{2 \cdot i}_{i=3}$	The index increases in increments of 1 beginning with the lower limit and ending with the upper limit.
$= \underbrace{2 \cdot 1}_{i=1} + \underbrace{2 \cdot 2}_{i=2} + \underbrace{2 \cdot 3}_{i=3}$	Substitute each value of i into the algebraic expression to find each term of the series.
$= \underbrace{2 + 4 + 6}_{\text{expanded form}}$	Simplify to write the series in expanded form.
$= \underbrace{12}_{\text{sum}}$	Add the terms to find the sum.

cc Back Start Over

Tip

To find the sum of a series written in sigma notation, start by writing the series in expanded form. Then add the terms of the series.

$$\sum_{i=1}^{5} 4i$$

Answer

1 of 3 Next

To find the sum of a series written in sigma notation, start by writing the series in expanded form. Then add the terms of the series.

$$\sum_{m=1}^{7} m^2$$

Answer

Back Next

MTH303B: Algebra II | Unit 3: Sequences and Series | Lesson 7: Series and Sigma Notation | Finding a Sum

To find the sum of a series written in sigma notation, start by writing the series in expanded form. Then add the terms of the series.

$$\sum_{k=2}^{6} (4k - 3)$$

Answer

Back Start Over

Using Sigma Notation

It is often helpful to write a series in condensed form using sigma notation. To write a series using sigma notation,

- Identify the first term of the sequence.
- Identify the type of sequence and the common difference or ratio.
- Write an iterative rule for the sequence.
- Use sigma notation to write the series (substitute the number of terms in the series in for n, then substitute the iterative rule in for a_i).

Write the series using sigma notation.

$$7 + 13 + 19 + 25 + 31 + 37 + 43 + 49$$

Identify the first term, type of sequence, and common difference or common ratio.

The first term is $a_1 = 7$.

The sequence is arithmetic because there is a common difference of $d = 6$ between consecutive terms.

Write an iterative rule.

The iterative rule for an arithmetic sequence is in the form $a_n = a_1 + (n - 1)d$, so the iterative rule for this sequence is:

$a_n = 7 + (n - 1)6$

$a_n = 7 + 6n - 6$

Remember

Take a look at another example involving a geometric series.

Write the series using sigma notation.

$$1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64}$$

Identify the first term, type of sequence, and common difference or common ratio.

The first term is $a_1 = 1$.

The sequence is geometric because there is a common ratio of $r = \frac{1}{4}$ between consecutive terms.

Write an iterative rule.

The iterative rule for a geometric sequence is $a_n = a_1 \cdot r^{n-1}$, so the iterative rule for this

Worked Examples

Write the series in expanded form and find the sum.

$$\sum_{k=3}^{8} (2k-7)$$

Answer

4 of 5 Next

MTH303B: Algebra II | Unit 3: Sequences and Series | Lesson 7: Series and Sigma Notation | Worked Examples

Worked Examples

Write the series using sigma notation.

$$8 + 3 - 2 - 7 - 12 - 17 - 22 - 27 - 32$$

Answer

Back 5 of 5 Next

Worked Examples

Write the series using sigma notation.

$$2 + 6 + 18 + 54 + 162$$

Answer

Back Start Over

Summary

In this lesson, you learned how to write a series in expanded form and then to find the sum by adding the terms.

You also learned how to represent a series using sigma notation by using your knowledge of arithmetic and geometric sequences and their iterative rules.

The sum S_n of the first n terms of a sequence written using sigma notation is:

$$S_n = \sum_{i=1}^{n} a_i = \overbrace{a_1 + a_2 + a_3 + \ldots + a_n}^{\text{expanded form}}$$

The sigma notation is read as "the sum from 1 to n of a_i."

i is the index
1 is the lower limit
n is called the upper limit

Offline Learning: Series and Sigma Notation

Now go offline to learn more about series and sigma notation and to practice your skills:

- Read pages 424–426 in the reference guide.
- Complete Problems 1–25 odd on page 426.
- Complete Problems 2–10 even, and 19–29 odd on page 426 for extra practice (*optional*).

The lesson quiz will be based on these problems.

MATERIALS

Algebra II: A Reference Guide and Problem Sets - pages 424-426

Student Guide

calculator

Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title: Solving Equations, 2 weeks

Designed by: Laura Mayer
District:Moyer Academy

Content Area:Algebra I
Grade Level(s):9-12

Summary of Unit

This unit builds experience in solving linear equations in one variable and applying graphical and algebraic methods to analyze systems of linear equations by asking students to analyze and explain the process of solving an equation. Mathematical fluency develops as students write, interpret and translate between various forms of linear equations and use them to solve problems.

In alignment with the Common Core State Standards, this unit pushes students to develop 21st Century Skills to meet the learning demands. Students come together as a learning community to analyze real-world applications of the math they are learning, think critically and reflect on the work of their peers as well as their own, and constantly work to apply their knowledge to new, more complex situations.

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

- Include those addressed in Stage 3 and assessed in Stage 2.

Mathematical Practices 1. Make sense of problems and persevere in solving them.

Mathematical Practices 3. Construct viable arguments and critique the reasoning of others.

Mathematical Practices 4. Model with Mathematics.

Mathematical Practices 5. Use appropriate tools strategically.

Mathematical Practices 6. Attend to precision.

Mathematical Practices 7. Look for and make use of structure.

Mathematical Practices 8. Look for and express regularity in repeated reasoning.

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

A-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.B.3 Solve linear equations in one variable, including equations with coefficients represented by letters.

N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Big Idea(s)

- Transferable core concepts, principles, theories, and processes from the Content Standards

1. Students will create equations in one variable and use them to solve problems

3. Students will represent constraints by equations and by systems of equations and interpret solutions as viable or non-viable options in a modeling context.

4. Students will rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations

5. Students will explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

6. Students will solve linear equations in one variable, including equations with coefficients represented by letters.

Unit Enduring Understanding(s)

- Full-sentence, important statements or generalizations that specify what students should understand from the Big Ideas (s) and/or Content Standards and that are transferable to new situations

Students will understand that equations are useful mathematically tools for modeling real world problems.

Students will understand that modeling, and solving, real world problems with equations will help them make better decisions in everyday life.

Students will understand that being able to translate between various forms of equations is a useful tool that can be applied to other, more difficult problems.

Unit Essential Questions(s)

- Open-ended questions designed to guide student inquiry and learning

1. Why is it important to formulate algebraic equations in order to solve problems?
2. How does fluency in writing, interpreting, and translating between various forms of linear equations enhance problem solving skills?
3. How are inverse operations used to transform and solve equations?
4. How and when is estimation appropriate to solve equations?
5. How are equations used to model real-world situations?

Knowledge and Skills

- Needed to meet Content Standards addressed in Stage 3 and assessed in Stage 2

Students will know...

The laws of arithmetic can be used to simplify algebraic expressions.

Expressions and equations express relationships between different entities

Solving an equation means finding values for the variable or variables that make each expression within the equation equal.

If they can create a mathematical model for a situation, they can use the model to solve for other problems they might not be able to solve otherwise.

Students will be able to...

Create equations in one variable and use them to solve problems

Represent constraints by equations and by systems of equations and interpret solutions as viable or non-viable options in a modeling context.

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.

Construct a viable argument to justify a solution method

Solve linear equations in one variable, including equations with coefficients represented by letters.

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

Suggested Performance/Transfer Task(s)

- Performance/Transfer tasks as evidence of student proficiency

An effective assessment for ALL students should be designed to include:

*Complex, real-world, authentic applications

*Assessment(s) for student understanding of the Stage 1 elements (Enduring Understandings, Essential Questions, Big Ideas) found in the Content Standards

*Demonstration of high-level thinking with one or more facets of understanding (e.g., explain, interpret, apply, empathize, have perspective, self-knowledge)

Performance Task 1: (Adapted from Mathematics Assessment Project) see Appendix A

Best Buy Tickets

Susie is organizing the printing of tickets for a show her friends are producing. She has collected prices from several printers and these two seem to be the best:

SURE PRINT Ticket printing 25 tickets for $2

BEST PRINT Tickets printed $10 setting up plus $1 for 25 tickets

Susie wants to go for the best buy. She doesn't yet know how many people are going to come. Show Susie a couple of ways in which she could make the right decision, whatever the number. Illustrate your advice with a couple of examples.

Rubric(s)

- Scoring guide to evaluate performance/transfer tasks used as evidence of student proficiency

An effective scoring guide should:

*Measure what is appropriate for the Content Standard that is assessed.

*Provide opportunities for differentiation of the performance/transfer tasks used as evidence of student proficiency.

Scoring guide for Performance Task 1 (See Appendix B)

Other Evidence

- Varied evidence that checks for understanding (e.g., tests, quizzes, prompts, student work samples, observations and supplements the evidence provided by the task)

Students will be provided with opportunities to complete 7 performance tasks throughout the unit as means of formative assessment to prepare them for the summative performance task. Additional assessment evidence to be used throughout the unit include:

-Mid-Unit Formative Assessment (Appendix C)

-Unit Formative Assessment (Appendix D)

The Mid-Unit and Unit Formative Assessments are designed to provide students opportunities to apply the basic skills learned in this unit, which must be mastered to successfully apply to the performance tasks.

Formative Observations:

Throughout the unit, students will be observed on their application of mathmatical practices highlighted in the unit:

Mathematical Practices 1. Make sense of problems and persevere in solving them.

Mathematical Practices 3. Construct viable arguments and critique the reasoning of others.

Mathematical Practices 6. Attend to precision.

Mathematical Practices 7. Look for and make use of structure.

Mathematical Practices 8. Look for and express regularity in repeated reasoning.

The observations will be shared with the students and parents throughout the unit. These conferences will be centered around the student's progressing score on the Mathematical Practices Rubric (Appendix E)

Student Self-Assessment and Reflection

- Opportunities for self-monitoring learning (e.g., reflection journals, learning logs, pre- and post-tests, self-editing – based on ongoing formative assessments)

After completing each Formative assessment performance task, students will be given the opportunity to reflect on their work. Students will then do a Think-Pair-Share with a peer and explain their work, defending both their problem solving method and the answers they reached.

For example:

1. Students will be given the following task in the unit:

Buying a Car

Suppose a friend tells you she paid a total of $16,368 for a car, and you'd like to know the car's list price (the price before taxes) so that you can compare prices at various dealers. Find the list price of the car if your friend bought the car in:

a) Arizona, where the sales tax is 5.6%.

2. Students will be given access to tools and technology from which they can select appropriately to assist them in completing the task. Each student will complete the task independently.

NOTE: Students will approach solving this problem differently, some choosing to set up equations, others choosing to do a series of connected steps to arrive at the answer.

3. Students will pair up, each student given 2 minutes to defend their problem solving method and the answer they reached. Each pair will agree on the most effective way to solve the problem and present that method to the class.

4. Students will be given another step to the problem and time to complete:

b) Suppose your friend bought her car in New York, where the sales tax is 8.25%. Find the list price of the car.

5. Students will pair up and each will defend their problem solving methods again.

6. Students will be asked to extend their work to complete the last portion of the task:

c) Suppose your friend bought her car in a state where the sales tax is r. Find the list price of the car.

NOTE: At this point in the task, students should be generating equations or a series of steps that would allow them to solve the problem for any amount of sales tax. Students may create a table and/or graph to further illustrate their equation.

NOTE: All performance based task work will be kept inside a reflective log that students will add to throughout the year. The teacher will assist students in creating the log and performing upkeep throughout the year, but it will be the student's responsibility to maintain. At the completion of each task, the students will reflect in writing on the relationship between the task and the learning objective of the lesson.

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

- Instructional activities and learning experiences needed to align with Stage 1 and Stage 2 expectations

Include these instructional elements when designing an effective and engaging learning plan for ALL students:

*Align with expectations of Stage 1 and Stage 2

*Scaffold in order to acquire information, construct meaning, and practice transfer of understanding

*Include a wide range of research-based, effective, and engaging strategies

*Differentiate and personalize content, process, and product for diverse learners

*Provide ongoing opportunities for self-monitoring and self-evaluation

The direct instruction resources needed to implement this course can be found within the k12 MTH112 course, Semester A, Presentations 4.01 - 4.13. Use these presentations to guide your information delivery during Direct Instruction. Guided Practice Examples and additional Independent Practice activities are also embedded within the presentation materials (See example presentation in Appendix O).

Lesson Set 1: Transformations to Solve Simple Equations (4 days)

Objective for Day 1: Students will be able to use the Addition and Subtraction Properties of Equality to solve simple equations.

Objective for Day 2: Students will be able to use the Multiplication and Division Properties of Equality to solve simple equations.

Objective for Day 3: Students will be able to perform multiple transformations to solve simple equations.

Objective for Day 4:Students will be able to explain each step in solving a simple equation using multiple transformations.

Anticipatory Set: Students will practice using a simple balance to determine groups of equivalent objects. For example: 5 M&Ms + 7 Skittles + 1 Hershey's Kiss is balanced with 2 pieces of a Kit Kat bar. Students must come up with 4 balanced groupings and fill out the worksheet (Appendix F).

Teacher will explain the relationship between balancing items on a balance and equality (each side of the balance is equal to the other side in terms of weight). Students will be given a new sheet and cut out cards with numbers on them (EX: 5, 5, 5, 10, 25, 25, 50, 70). Students will organize the cards into a new balance sheet, coming up with four examples of balanced numbers (EX: 5,5,5,10 is balanced with 25).

Direct Instruction: Teacher will demonstrate translating the balanced numbers into mathematical equations (5+5+5 = 5+10). Teacher will demonstrate that if 5 is removed from one side of the equation, then the equation is no longer balanced. In order to keep the equation balanced, five would need to be removed, or subtracted, from both sides.

5+5+5 = 5+10
-5 -5
5+5 = 10

Teacher will play k12 lesson 4.01 Video: Transformations. Students will follow along with notes in their notebooks.

Teacher will go back to the model balance, putting up 3 10's cards on one side and 2 10's and 2 5's on the other side:

10+10+10 and 10+10+5+5

Teacher asks "Is this balanced?" [Yes]

Teacher puts a 10's card on only one side, asks "Is this balanced? [No]

Teacher asks students to make it balanced [Put a 10 on the other side]

Teacher writes what the students just did on the board in "math language:"

10+10+10 [+10] = 10+10+5+5 [+10]

Teacher describes this as the Addition Property of Equality, goes through same procedure for Subtraction Property.

Guided Practice: Teacher will guide students in using properties to solve simple equations.

x-5=12

-7=22-y

Teacher will go through k12 lessons on Multiplication and Division transformations, and Multiple Transformations following the same pattern of learning as the Addition and Subtraction transformations.

Students will record their learning in their journal.

Performance Task: Same Solutions (A.REI.A.1) (Appendix G)

Which of the following equations have the same solution? Give reasons for your answer that do not depend on solving the equations for the value of x.

1. x+3 = 5x-4
2. x -3 = 5x + 4
3. 2x + 8 = 5x - 3
4. 10 x + 6 = 2x - 8
5. 10x - 8 = 2x + 6
6. 0.3 + x/10= 1/2x - 0.4

Self-Reflection and Peer-Review: Students will do a Think-Pair-Share with a peer and explain their work, defending both their problem solving method and the answers they reached.

New Tools in the Students "Toolbox:"

1. Solving equations through addition transformations.
2. Solving equations through subtraction transformations.
3. Solving equations through multiplication transformations.
4. Solving equations through division transformations.

*NOTE: Throughout the year, students will keep a running log of math "tools" they can use to solve problems. When students are presented with performance tasks, students will view these "tools" the same way they view their calculators or rulers: things that they can apply to a problem to come to the solution. The goal is to focus the students on the math behind the concepts and to develop in them the understanidng that each new concept they learn is actually a continuation of learning already accomplished, instead of a discrete, stand alone topic.

After Performance Task Think-Pair-Share, students will reflect in their journal on the application of their learning to the Performance Task and their mastery of the lesson objectives. Reflective Journal Entries will be graded using the following rubric:

4 points - Reflection is well written with clear organization and contains almost no errors; demonstrates thoughtful analysis of how new learning and performance task related; evidence and details included to demonstrate student's grasp of concepts; submitted on time.

3 points - Reflection is well written with some organization and contains few errors; demonstrates analysis of how new learning and performance task related; some evidence and details included to demonstrate student's grasp of concepts; submitted on time.

2 points - Reflection shows little organization and contains errors; demonstrates attempt at analysis of how new learning and performance task related; little to no evidence and details are included to demonstrate student's grasp of concepts; submitted on time.

1 points - Reflection has no clear organization and contains many errors; little to no analysis of how new learning and performance task related; little to no evidence and details included to demonstrate student's grasp of concepts; and/or not submitted on time.

0 points - No relevant reflection is submitted.

Lesson Set 2: Transforming Formulas (2 days)

Objective for Day 1: Students will be able to explain a formula as a useful way to represent the relationships between variables and connect their knowledge of simple equations to formulas.

Objective for Day 2: Students will be able to rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Anticipatory Set: Students will be given a bag of sentence strips with sentences on them. Each sentence is a part of a pair that convey the same information in different ways (ex: "I am Jim's parent" and "Jim is my son.") Students will work in pairs to match the coordinating sentences. Teacher will randomly call on pairs to identify sentences they grouped together and defend their reasoning.

Direct Instruction: Teacher will explain the relationship between the introduction and the lesson: just as sentences can be built with different words but mean the same thing, equations can be built with different operations, but mean the same thing.

Teacher will demonstrate this concept using the formula for speed (video provided by k12). S=D/T or D=S*T or T=D/S all using the same information, but transformed to solve for different variables. Teacher will guide students through more examples, spiraling up to examples involving multiple transformations.

Group Work: Performance Task: Changing the Subject (Appendix H)

Performance Task: Equations and Operations (A.CED.A.4) (Appendix I)

Use inverse operations to solve the equations for the unknown variable, or for the designated variable if there is more than one. If there is more than one operation to "undo", be sure to think carefully about the order in which you do them. For equations with multiple variables, it may help to first solve a version of the problem with numerical values substituted in.

1. $5=a-3$
2. $A - B = C$ (solve for A)
3. $6 = -2x$
4. $IR = V$ (solve for R)
5. $x/5=3$
6. $W=A/L$ (solve for A)
7. $7x + 3 = 10$
8. $ax + c = R$ (solve for x)

9. $13 = 15 - 4x$

10. $2h = w - 3p$ (solve for p)

11. $F=GM/r2$ (solve for G)

Self-Reflection and Peer-Review: Students will do a Think-Pair-Share with a peer and explain their work, defending both their problem solving method and the answers they reached.

New Tools in the Students "Toolbox:"

No New Tools - Students will revisit their Addition, Subtraction, Multiplication, and Division cards and edit them to include performing these transformations on variables instead of only on numbers.

After Performance Task Think-Pair-Share, students will reflect in their journal on the application of their learning to the Performance Task and their mastery of the lesson objectives. Reflective Journal Entries will be graded using the following rubric:

4 points - Reflection is well written with clear organization and contains almost no errors; demonstrates thoughtful analysis of how new learning and performance task related; evidence and details included to demonstrate student's grasp of concepts; submitted on time.

3 points - Reflection is well written with some organization and contains few errors; demonstrates analysis of how new learning and performance task related; some evidence and details included to demonstrate student's grasp of concepts; submitted on time.

2 points - Reflection shows little organization and contains errors; demonstrates attempt at analysis of how new learning and performance task related; little to no evidence and details are included to demonstrate student's grasp of concepts; submitted on time.

1 points - Reflection has no clear organization and contains many errors; little to no analysis of how new learning and performance task related; little to no evidence and details included to demonstrate student's grasp of concepts; and/or not submitted on time.

0 points - No relevant reflection is submitted.

Lesson 3: Creating Equations (3-4 days)

Objective for Day 1: Students will be able to persist in developing a model to solve a math problem.

Objective for Day 2: Students will be able to switch fluently between problems written in words and problems written in math.

Objective for Day 3: Students will be able to represent constraints by equations and interpret solutions as viable or non-viable options.

Objective for Day 4: Students will be able to create equations in one variable and use them to solve problems.

Anticipatory Set: Students will be given a performance task as an opener to this lesson. This is done intentionally to reinforce Mathematical Practices 1, 3, 5, and 7. In particular, MP 7 is highlighted, students must "look for and make use of structure." Without direct instruction from the teacher, students should see a pattern between the first two parts of the task, realizing that they are performing the same series of operations to arrive at their answers. They then must "make use of" the identified structure by applying it to a general scenario in part 3. If the teacher deviates from this technique and gives the students the structure, the students would not have opportunity to practice MP 7.

1. Students will be given the following task and a model car

Buying a Car (Appendix J)

Suppose a friend tells you she paid a total of $16,368 for this car, and you'd like to know the car's list price (the price before taxes) so that you can compare prices at various dealers. Find the list price of the car if your friend bought the car in:

a) Arizona, where the sales tax is 5.6%.

2. Students will be given access to tools and technology from which they can select appropriately to assist them in completing the task. Each student will complete the task independently.

NOTE: Students will approach solving this problem differently, some choosing to set up equations, others choosing to do a series of connected steps to arrive at the answer.

3. Students will pair up, each student given 2 minutes to defend their problem solving method and the answer they reached. Each pair will agree on the most effective way to solve the problem and present that method to the class.

4. Students will be given another step to the problem and time to complete:

b) Suppose your friend bought her car in New York, where the sales tax is 8.25%. Find the list price of the car.

5. Students will pair up and each will defend their problem solving methods again.

6. Students will be asked to extend their work to complete the last portion of the task:

c) Suppose your friend bought her car in a state where the sales tax is r. Find the list price of the car.

NOTE: At this point in the task, students should be generating equations or a series of steps that would allow them to solve the problem for any amount of sales tax. Students may create a table and/or graph to further illustrate their equation.

Direct Instruction: Teacher will connect the Anticipatory Set to the work of the lesson: taking real world problems and translating them into equations.

Teacher will demonstrate that there are many ways to express math symbols and expressions in words, for example 11-4 may be written as "Eleven has been reduced by four" or "Eleven minus four" or "Eleven less four" or "Take four from Eleven."

Students will then draw cards from a bag with simple math expressions on them and practice generating 3 ways to translate the math expressions into words.

Think-Pair-Share: students will share their word expressions and have other student translate their words back into the math expression without looking at the original cards.

Teacher will put a chart on the board with math symbols and the indicator words/phrases for the symbol (ex: "+" can be "sum, more than, increased by..."

Guided Instruction: Teacher will open up the class to the students, inviting them to generate as many indicator words and phrases as they can to complete the chart on the board for "+" "-" "/" "x" and "="

Students will post words/phrases using post-it notes. After posting their word/phrase, students will record the completed chart in their notebooks.

Teacher will hand out page 1 (Math only) of the "Math vs. English" handout (Appendix K) and ask students to try solving each equation for x (students should refer back to the transformations learned in prior lessons). Students should be able to quickly solve these problems. Teacher will then hand out page 2 (English words only) and ask the students to solve. Students should struggle a bit in making sense

of the second page, teacher will open class up to a discussion with the following question posted on the board: "Why use math to express ideas instead of English?"

Teacher will present notes on translating words to variable expressions and equations. Teacher will go back to Anticipatory Set Questions and guide students through using the new knowledge of translating words into variable expressions to complete part 3 of the activity correctly. Any students who got the correct equation originally without teacher support will be asked to share with the class their method and thought process behind their answer. Other students will critic the student's approach.

Students will work in groups of 4 on the following Performance Task (A.CED.1):

"Check it Out" (Appendix L)

Student groups will swap papers and analyze other groups work, posing one "I like" and one "I wonder" about the work and returning it to the original group. Students will work to answer the "I wonder."

Performance Task: Teacher will play video "Math in Music"

(http://www.thirteen.org/get-the-math/the-challenges/math-in-music/introduction/20/) for students. Students will be given a sample beat from which they will need to calculate the beats per minute. Students will be given two additional sample beats and will be asked to calculate the beats per minute in the additional pieces. Teacher will instruct students that the recording studio needs an equation they can use to calculate the beats per minute of a piece of music.

Students will come up with an equation to represent the number of beats per minute in a piece of music.

Students will be asked to summarize their knowledge of creating equations and transforming them to solve for a needed quantity. Performance Task (A-CED.4):

Government Purchases (Appendix M)

Self-Reflection and Peer-Review: Students will do a Think-Pair-Share with a peer and explain their work, defending both their problem solving method and the answers they reached.

After Performance Task Think-Pair-Share, students will reflect in their journal on the application of their learning to the Performance Task and their mastery of the lesson objectives. Reflective Journal Entries will be graded using the following rubric:

4 points - Reflection is well written with clear organization and contains almost no errors; demonstrates thoughtful analysis of how new learning and performance task related; evidence and details included to demonstrate student's grasp of concepts; submitted on time.

3 points - Reflection is well written with some organization and contains few errors; demonstrates analysis of how new learning and performance task related; some evidence and details included to demonstrate student's grasp of concepts; submitted on time.

2 points - Reflection shows little organization and contains errors; demonstrates attempt at analysis of how new learning and performance task related; little to no evidence and details are included to demonstrate student's grasp of concepts; submitted on time.

1 points - Reflection has no clear organization and contains many errors; little to no analysis of how new learning and performance task related; little to no evidence and details included to demonstrate student's grasp of concepts; and/or not submitted on time.

0 points - No relevant reflection is submitted.

Resources and Teaching Tips

- A variety of resources are included (texts, print, media, web links)
- Help in identifying and correcting student misunderstandings and weaknesses

Throughout the unit, students have access to a variety of resources.

Algebra I Textbook

Smartboard and Projector

Graphic Organizer Paper

K12 Online Curriculum

Laptop

Reference Sheets

Math in Music video

Each formative check for understanding provides an opportunity for the students to identify their own misunderstandings and weaknesses in mastering the content. Guide the students in understanding how to use these formative checks to enhance their learning through the use of rubrics, conferences, and remediation or acceleration plans designed with student input (Appendix N). While working towards mastery of grade level content standards, continuously refer students back to the standards for mathematical practices. These practice standards are the attitudes and skills that will allow students to achieve mastery of content standards, as well as deep understanding of mathematics.

Differentiation

- Stage 2 and 3 allow students to demonstrate understanding with choices, options, and/or variety in the products and performances without compromising the expectations of the Content Standards.
- Instruction is varied to address differences in readiness, interest, and/or learning profiles.
- Accommodations and differentiation strategies are incorporated in the design of Stage 2 and 3.

☐ Refer to the IEPs of your special education students

☐ Allow students to have tests, quizzes, worksheets, etc read to them.

☐ Allow the use of a calculator

☐ Communicate with the special education teacher about allowing testing in the special education room

☐ Pair and group students with differing ability levels.

☐ Use manipulatives as needed

☐ Allow for extra time on tests, quizzes, and performance tasks

☐ Allow for a word bank and a formula sheet on tests

☐ Allow students to choose to demonstrate understand in written or verbal format as needed.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.
- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning** – the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

(Briefly explain how design principle(s) are embedded within the unit design.)

In alignment with the Common Core State Standards, this unit pushes students to develop 21st Century Skills to meet the learning demands. Students come together as a learning community to analyze real-world applications of the math they are learning, think critically and reflect on the work of their peers as well as their own, and constantly work to apply their knowledge to new, more complex situations. This push towards using developed skills in new ways is an accurate reflection of the higher order thinking required by 21st Century jobs. Students should become excited and engaged by the "hands-off" approach of the teacher. Rather than giving students the formulas and a calculator and having them compute numbers and spit out answers, the teacher should present numbers and problems in a real world context, allowing the students to select the tools, resources, and knowledge needed to arrive at a reasonable answer. This push towards applying mathematical ways of thinking to real world issues will prepare them for the type of work they will encounter in college and in their careers.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- **8th Grade Technology Literacy** - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century(SETDA, 2003).

In alignment with Mathematical Practice 5 - Use appropriate tools strategically, students will have access to calculators as needed to complete the Performance Tasks. Additionally, when sharing their answers to the Performance Tasks, the teacher may opt to have the students construct a PowerPoint presentation as an additional layer of the project.

Content Connections

Content Standards integrated within instructional strategies

Connections to other content areas are made both explicitly and implicitly throughout this unit. Students will watch a video, outlining the connection between music and math, and be given a performance task that makes this connection clear. Additionally, the performance tasks at the end of each lesson connect the math content to science (Ohm's law), Economics (Government Spending), and Personal Finance (Check It Out). Additional connections are encouraged at the teacher's discretion. For example, when assigning the students additional practice work, make the work meaningful by aligning the examples with content currently being covered in other classes such as Science, English, and Social Studies.

Best Buy Tickets

Susie is organizing the printing of tickets for a show her friends are producing.
She has collected prices from several printers and these two seem to be the best.

SURE PRINT
Ticket printing
25 tickets for $2

BEST PRINT
Tickets printed
$10 setting up
plus
$1 for 25 tickets

Susie wants to go for the best buy

She doesn't yet know how many people are going to come.

Show Susie a couple of ways in which she could make the right decision, whatever the number.

Illustrate your advice with a couple of examples.

Please continue your work on the page opposite

Best Buy Tickets (continued)

Best Buy Tickets — Rubric

	Points	Section points
Shows correct reasoning and calculations such as the following: *May solve using algebra*		
Sure Print: The cost for n tickets in dollars is C = 2n/25	2	
Best print: C = 10 + n/25	2	
Method 1: May draw graphs and find the point of intersection, (n = 250).	4	
Method 2 (algebraic)	or	
When the two costs are equal $2n/25 = 10 + n/25$	4	
$n = 250$		
Shows that when n < 250 Sure Print is cheaper When n > 250 Best Print is cheaper	2	10
Or *May decide to solve arithmetically*	or	or
Decides to list costs for different numbers of tickets.	2	
States that the lists show that when n = 250 the costs are equal	1	
States that when n < 250 Sure Print is cheaper When n > 250 Best Print is cheaper	2 x 1	10
Total Points		**10**

Number of tickets	Sure Print	Best Print
50	4	12
100	8	14
150	12	16
200	16	18
250	20	20
300	24	23

5

1. Your teacher says that in this equation: $x + 3 = 11$, $x=7$. Is your teacher correct? Show every step of your calculation for complete credit.

2. Your teacher says that in this equation: $15x = 2$, $x=15/2$. Is your teacher correct? Show every step of your calculation for complete credit.

3. Solve $3x = 24$. Show every step of your calculation for complete credit.

4. By what number must you multiply each side of the equation to produce the equivalent equation $x = 15$?

5. Solve $-1/5x = 6$. Show every step of your calculation for complete credit.

6. Solve $2x + 5 = 27$. Show every step of your calculation for complete credit.

7. Solve $x/7 = 2$. Show every step of your calculation for complete credit.

1. Evaluate $x + y \cdot z$ if $x = 5$, $y = 8$, and $z = 2$. (Points : 4)
   15
   21
   26
   80

2. Evaluate $1.2b$ if $a = 3$ and $b = 1$. (Points : 4)
   Option A: 2
   Option B: 1.2bb
   Option C: 1.2bc
   Option D: 4

3. Simplify. $(5 + 3) \cdot 4$ (Points : 4)
   2
   12
   17
   32

4. Simplify. $24 + 6 \div 3 - 3 \cdot 6$ (Points : 4)
   8
   18
   28
   42

5. Which is a term of the expression? $2x + 5ab - 3$ (Points : 4)
   2
   ab
   4
   –3

6. Write an expression for the phrase.
Six less than twice a number (Points : 4)
   $6 - 2n$
   $2n - 6$
   $5 - x$
   $x - 5$

7. The cost of a concert ticket is $28. There are k people in a group who want to go to the concert. Which of the following expressions describes the total amount of money the group will need to go to the concert? (Points : 4)
   $28 + k$
   $28 - k$
   $28k$
   $28 \div k$

8. Write an equation for the sentence.
Six more than twice a number is twelve. (Points : 4)

$6 - 2n + 12$
$6 + 2n = 12$
$2n - 6 + 12$
$2n - 6 = 12$

9. Which value makes the open sentence true? $3c = 24$ (Points : 4)

$c = 21$
$c = 9$
$c = 8$
$c = 6$

10. Kevin ran 4 miles more than Steve ran. The sum of their distances is 26 miles. Your teacher says that Steve ran 15 miles. Explain, through the use of math and words, whether your teacher is right or wrong. (Points: 10)

11. Translate the word phrase into a variable expression.
The quotient of a number and 3 is decreased by 1 (Points : 4)

12. Write a variable expression that represents the word problem.
Matt gave q quarters and n nickels to his sister. Which equation represents the amount in dollars that Matt gave to his sister? (Points : 4)

13. Jose worked 2.5 hours more than William. Together, they worked a total of 22 hours. Write an equation to represent the amount of time William worked. (Points : 4)

Student Name_____ Evaluation Date_____ Overall Score_____

Mathematical Practice	Novice (1)	Apprentice (2)	Practitioner (3)	Expert (4)
1. Make sense of problems & persevere in solving them	I couldn't get started. I don't know how to begin. I may be feeling frustrated and wanting to give up.	I am stuck. I have part of the solution, but now I don't know what to do, I'm not sure my answer is right and I could use some help. I may be feeling frustrated.	I planned out how to solve this problem and I worked all parts of the problem. I may have felt challenged, but I didn't want to give up-I kept going until the problem was solved.	My solution is effective and inventive! I planned out how to solve this problem and I even showed you other ways to solve it. I checked to make sure my answer was right – I enjoyed this challenge!
2. Reason abstractly & quantitatively	I can't explain how I solved this problem. I don't know what quantities or equations to use. My work on this problem is built on misunderstandings of mathematical ideas.	I'm trying to explain how I solved this problem. My work on this problem may be built on misunderstandings of mathematical ideas. . I'm trying to take numbers out of the problem to work with them. I'm trying to check my work.	I explained how to solve this problem using formal math language. I took numbers out of the problem to work with them, and periodically put them back into the context of the problem to see if my work makes sense.	I clearly explained how to solve this problem using formal math language. I took quantities out of the context of the problem to work with them, and frequently put them back into the context of the problem to see if my work made sense.
3. Construct viable arguments & critique the reasoning of others	I can't describe how I solved the problem. I can't give feedback on someone else's reasoning OR my feedback in unclear, unhelpful, and/or incorrect.	I'm trying to describe how I solved the problem. I may or may not include pictures, tables, graphs, and/or models. I gave feedback on someone else's reasoning, but my feedback may have been unclear, unhelpful, or I may have been unable to back it up with my own work.	I described how I solved the problem using formal math language, pictures, tables, graphs, and/or models. I gave clear, helpful feedback to someone else's reasoning and backed up my feedback with my own work on the problem.	I clearly described how I solved the problem. I included every step so you don't have to guess what I did. I used formal math language, pictures, tables, graphs, and/or models. I gave clear, helpful feedback on someone else's reasoning and backed up my feedback with my own work on the problem.
4. Model with mathematics	I can't use mathematical representations, I don't know what to use for this problem and I can't describe how I would solve this problem.	I'm trying to use mathematical representations to solve the problem and show my solutions, but they may be inaccurate or inappropriate. I'm sharing my ideas, but I'm struggling to use formal math language.	I used appropriate & accurate mathematical representations to solve the problem and show my solutions. I used formal math language to share my ideas.	I used a variety appropriate, accurate, and detailed mathematical representations to solve the problem and show my solutions. I used formal math language to share my ideas.
5. Use appropriate	I don't know what tool to use to solve this problem and/or I	I picked an appropriate tool to use, but I may be struggling to	I picked the correct tool to use based on the needs of the	I selected and used a variety of appropriate tools

Student Name_____ Evaluation Date_____ Overall Score_____

tools strategically	didn't use the tool I picked in a valuable way.	use it in a valuable way to solve the problem.	problem. I planned and monitored my use of the tool.	collaboratively to solve the problem. I planned my use of the tools and monitored my progress.
6. Attend to precision	I can't get the correct answer and/or I can't communicate my answer in a way that is appropriate to the problem. I can't use correct units, labels, or definitions.	I may be getting the correct answer and/or I may be able to communicate my answer is a way that is appropriate to the problem. I may not use correct units, labels, or definitions.	I got the correct answer and I can communicate it using the correct labels, units, and definitions. The precision of my answer is appropriate for the problem.	I got the correct answer and I communicated it using correct labels, units, and definitions. I clearly stated the meaning of the symbols I chose and my answer is complete appropriate for the problem.
7. Look for & make use of structure	I don't see any structure in the problem. I can't make sense of any useful structures/patterns to help me solve the problem.	I may be able to find for structures/patterns in the problem, but I am struggling to use them to help me find a solution.	I found and used structures/patterns in the problem to help me find a solution. I noted these structures/patterns as I found them.	I found, noted, and used structures/patterns in the problem to help me find a solution. I was able to step back from the problem and analyze different perspectives.
8. Look for & express regularity in repeated reasoning	I don't see any regularity or any shortcuts or generalizations I can use to get a solution.	I may notice that some of my calculations are repeated but I may not be able to generalize that observation to help me get a solution.	I noticed some of my calculations were repeating and was able to make a generalization that helped me solve the problem. I evaluated the reasonableness of my work as I progressed.	I noticed some of my calculations were repeating and was able to make a generalization or a formula that helped me solve the problem. I continuously evaluated the reasonableness of my work as I progressed.

Description:

Description:

Description:

Description:

Illustrative Mathematics

A-REI.1-2 Same solutions?

Alignment 1: A-REI.A
Not yet tagged

Which of the following equations have the same solution? Give reasons for your answer that do not depend on solving the equations.

I. $x + 3 = 5x - 4$

II. $x - 3 = 5x + 4$

III. $2x + 8 = 5x - 3$

V. $10x + 6 = 2x - 8$

V. $10x - 8 = 2x + 6$

VI. $0.3 + \frac{x}{10} = \frac{1}{2}x - 0.4$

Commentary:
The purpose of this task is to provide an opportunity for students to reason about equivalence of equations. The instruction to give reasons that do not depend on solving the equation is intended to focus attention on the transformation of equations as a deductive step.

Note that although it is possible to show that two equations are equivalent without solving them, it is more difficult to give reasons why they are not equivalent, even though they do not appear to be. Thus, in the end, confirmation of the solution is achieved by solving the equations.

Solution: Same solutions?

Equations V is Equation I multiplied by 2, and with the left side written on the right. Equation VI is Equation I divided by 10 with the two terms on the left written in the opposite order. So Equations I, V, and VI all have the same solutions.

Equation II has the signs of the constants changed from Equation I, so probably does not have the same solution.

Equation III has two of the terms in Equation I, the x and -4, multiplied by 2 while the other two are not. It does not have the same solutions as equations I or II.

Equation IV is Equation II multiplied by 2, with the constant terms moved to the opposite side of the equation. Equations II and IV have the same solutions.

Confirmation: The solutions to the equations are:

Equation	I	II	III	IV	V	VI
Solution	7/4	$-7/4$	11/3	$-7/4$	7/4	7/4

Edit Tags Update

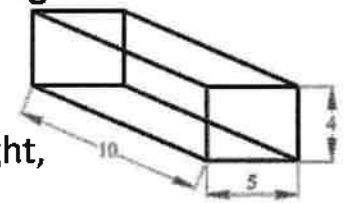
What is a Formula?

A formula is a special type of equation that shows the relationship between different **variables**.

Example: The formula for finding the volume of a box is:

V = hwl

V stands for volume, ***h*** for height, ***w*** for width, and ***l*** for length.



Changing the Subject

The "subject" of a formula is the single variable (usually on the left of the "=") that everything else is equal to.

A formula will have **more than one variable**. These are all equations, but only some are formulas:

$x = 2y - 7$	Formula (relating x and y)
$a^2 + b^2 = c^2$	Formula (relating a, b and c)
$x/2 + 7 = 0$	Not a Formula (just an equation)

What is an Equation?

An equation says that two things are equal. It will have an equals sign "=" like this:

$$x + 2 = 6$$

- That equations says: **what is on the left (x + 2) is equal to what is on the right (6)**

So an equation is like a **statement** "*this* equals *that*"

Change the Subject

Rearrange the volume of a box formula (**V = hwl**) so that the width is the subject:

Start with:	V = hwl
divide both sides by h:	V / h = wl
divide both sides by l:	V / hl = w
swap sides:	w = V / hl

Subject of a Formula

The "*subject*" of a formula is the single variable (usually on the left of the "=") that everything else is equal to.

Example: in the formula

$s = ut + \frac{1}{2} at^2$

"***s***" is the *subject* of the formula

Change the Subject

So now if you have a box with a length of 2m, a height of 2m and a volume of $12m^3$, you can calculate its width:

w = V / hl

$w = 12m^3 / (2m \times 2m) = 12/4 = 3m$

Changing the Subject

One of the very powerful things that Algebra can do is to "rearrange" a formula so that another variable is the subject.

Change the Subject

- Cards
  - Draw one card from the deck and follow the directions on the card.
  - When you finish check with a neighbor to see if he/she agrees.

$F = G\frac{m_1 m_2}{r^2},$ change subject to m_2	$A = \pi r^2,$ change subject to r
$S = 2\pi r\ell + 2\pi r^2,$ change subject to ℓ	$S = 4\pi r^2,$ change subject to r
$V = \frac{4}{3}\pi r^3,$ change subject to r	$A = \frac{1}{2}bh,$ change subject to b
$A = \left(\frac{b_1+b_2}{2}\right)h,$ change subject to b_2	$V^2 = V_0 + 2a(x - x_0),$ change subject to x

$K = \frac{1}{2}mv^2,$ change subject to m	$F_s = -kx,$ change subject to k
$U_s = \frac{1}{2}kx^2,$ change subject to k	$T_s = 2\pi\sqrt{\frac{l}{g}},$ change subject to g
$P = IV,$ change subject to I	$v = v_0 + at,$ change subject to t
$a_c = \frac{v^2}{r},$ change subject to v	$P = \frac{F}{A},$ change subject to A

$\omega = \omega_0 + at,$ change subject to t	$F_M = qv \times B$ change subject to r
$a_n = ar^{(n-1)},$ change subject to a	$a_n = a + (n-1)d,$ change subject to n
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B),$ change subject to $P(A \text{ and } B)$	$Z = \frac{x - \mu}{\sigma},$ change subject to μ
$\frac{y}{2} = 6x + 11,$ solve for x	$x + y = 13,$ solve for y

$w = \frac{2}{3}xyz$ solve for y	$n = 4\frac{r}{xy},$ solve for y
$a^2 + b^2 = c^2,$ solve for b	$S = 4\pi r^2,$ change subject to r
$3x + 6y = 11$ solve for y	$5x + 2y = 12,$ solve for y
$x + 6y = \frac{1}{4},$ solve for y	$3x + y = \frac{2}{3},$ solve for x

Equations and Formulas

Use inverse operations to solve the equations for the unknown variable, or for the designated variable if there is more than one. If there is more than one operation to "undo", be sure to think carefully about the order in which you do them. For equations with multiple variables, it may help to first solve a version of the problem with numerical values substituted in.

1. $5 = a - 3$
2. $A - B = C$ (solve for A)
3. $6 = -2x$
4. $IR=V$ (solve for R)
5. $\frac{x}{5} = 3$
6. $W = \frac{A}{L}$ (solve for A)
7. $7x+3=10$
8. $ax + c = R$ (solve for x)
9. $13 = 15-4x$
10. $2h = w-3p$ (solve for p)
11. $F=\frac{GMm}{r^2}$ (solve for G)

Solutions

1. $5 = a-3$

   Addition is the inverse of subtraction, so add 3 to both sides: $a = 8$

2. $A - B = C$ (solve for A)

   This equation has the same structure as the previous one. Addition is the inverse of subtraction, so add B to both sides: $A = C + B$.

3. $6 = -2x$

   Division is the inverse of multiplication, so divide both sides by -2: $x = -3$.

4. $IR=V$ (solve for R)

   This equation has the same structure as the previous one. Division is the inverse of multiplication, so divide both sides by I: $R = \frac{V}{I}$

5. $\frac{x}{5} = 3$

   Multiplication is the inverse of division, so multiply both sides by 5: $x = 15$

6. $W = \frac{A}{L}$ (solve for A)

   This equation has the same structure as the previous one. Multiplication is the inverse of division, so multiply both sides by L: $A=wL$.

7. $7x+3=10$

   This equation involves multiplication and addition. If we were evaluating for x, the order of operations dictates that we would multiply and then add. To undo these operations, it is easiest to cancel them in the opposite order. (Otherwise we run into issues with distribution.) Thus, we first subtract 3 from both sides and then divide by 7: $x=1$

8. $ax + c = \mathrm{R}$ (solve for x)

This equation has the same structure as the previous one. To undo the multiplication and addition, we first subtract c from both sides and then divide by a: $x = \mathrm{R} - \frac{c}{a}$

9. $13=15-4x$

This equation has multiplication and subtraction. Order of operations dictate that we would evaluate this equation for a given x by first multiplying by -4 and then adding 15, so we will do the inverses in the opposite order. First subtract 15 from both sides, and then divide both sides by -4, to get $\mathrm{x} = \frac{13-15}{-4} = \frac{1}{2}$.

10. $2h = w - 3p$ (solve for p)

This equation has the same structure as the previous one. First subtract w from both sides, and then divide by -3 to get $p = \frac{2h-w}{-3}$.

11. $\mathrm{F}=\frac{GMm}{r^2}$ (solve for G)

First multiply both sides by r^2 then divide both sides by Mm, resulting in $G = \frac{Fr^2}{Mm}$. To create an equation in the same form, simply replace the variables other than G with numbers. For example, the equation $10 = \frac{7x}{3}$ is in the same form. We would solve this by multiplying by 3 and dividing by 7, giving $x=\frac{30}{7}$.

Illustrative Mathematics

A-CED Buying a Car

Alignment 1: A-CED.A.1

Suppose a friend tells you she paid a total of $16,368 for a car, and you'd like to know the car's list price (the price before taxes) so that you can compare prices at various dealers. Find the list price of the car if your friend bought the car in:

a. Arizona, where the sales tax is 5.6%.

b. New York, where the sales tax is 8.25%.

c. A state where the sales tax is r.

Commentary:
The emphasis in this task is not on complex solution procedures. Rather, the progression of equations, from two that involve different values of the sales tax, to one that involves the sales tax as a parameter, is designed to foster the habit of looking for regularity in solution procedures, so that students don't approach every equation as a new problem but learn to notice familiar types.

This task is adapted from *Algebra: Form and Function, McCallum et al., Wiley 2010.*

Solution: Buying a car

a. If p is the list price in dollars then the tax on the purchase is $0.056p$. The total amount paid is $p + 0.056p$, so

$$\begin{aligned} p + 0.056p &= 16{,}368 \\ (1 + 0.056)p &= 16{,}368 \\ p &= \frac{16{,}368}{1 + 0.056} = \$15{,}500. \end{aligned}$$

b. The total amount paid is $p + 0.0825p$, so

$$\begin{aligned} p + 0.0825p &= 16{,}368 \\ (1 + 0.0825)p &= 16{,}368 \\ p &= \frac{16{,}368}{1 + 0.0825} = \$15{,}120.55. \end{aligned}$$

c. The total amount paid is $p + rp$, so

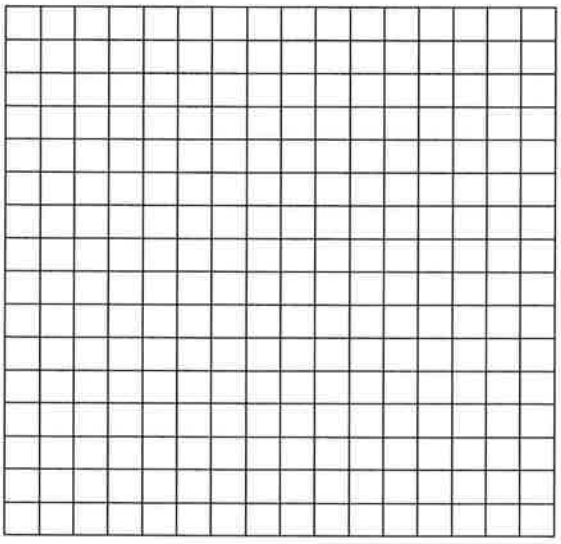
$$\begin{aligned} p + rp &= 16{,}368 \\ (1 + r)p &= 16{,}368 \\ p &= \frac{16{,}368}{1 + r} \text{ dollars.} \end{aligned}$$

Check It Out

A checking account is set up with an initial balance of $4800, and $400 is removed from the account each month for rent (no other transactions occur on the account).

1. Write an equation whose solution is the number of months, *m*, it takes for the account balance to reach $2000.

2. Make a plot of the balance after *m* months for *m* = 1,3,5,7,9,11 and indicate on the plot the solution to your equation in part (1).



1) the difference of ten and five

2) the quotient of fourteen and seven

3) twenty-six decreased by seventeen

4) half of fourteen

5) ninety-seven increased by six

6) the product of x and seven

7) the sum of q and eight

8) six squared

9) twice q

10) the product of eight and twelve

11) the quotient of eighteen and n

12) three cube

Math Only

1) 16-5=

2) 14x2=

3) 23-4=

4) 16/2=

5) 93+7

6) 9x7=

7) 8+4=

8) 4^2=

9) 2x5=

10) 8x13=

11) 18/2=

12) 9^3

Solution:

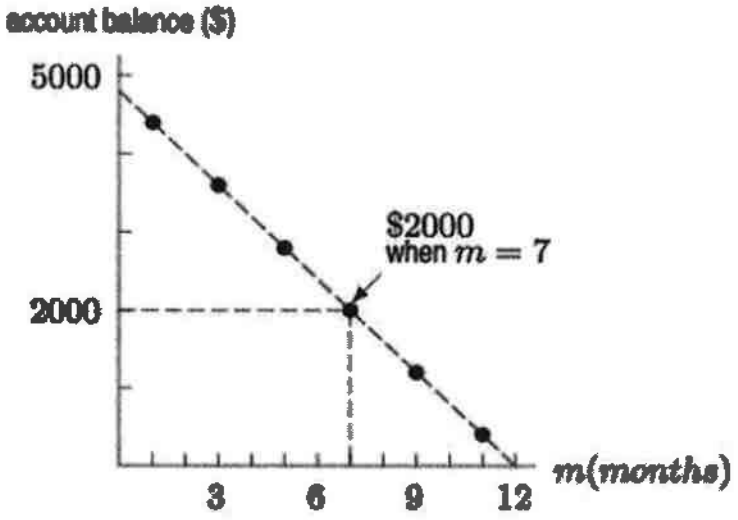
1. Since the account starts with \$4,800 and decreases by \$400 each month, there will be $4800-400m$ dollars left in the account after m months have passed. The question asks us to set up an equation representing the number of months that pass before this quantity equals 2,000, and so the desired equation is
$$4800-400m=2000.$$

2. The following table collects the relevant data points.

Balance in an account 1–11 months after its establishment

m (months)	1	3	5	7	9	11
Balance $4800-400m$	4400	3600	2800	2000	1200	400

As is found in the table or by solving the equation in part (a), the balance in the account is \$2,000 after 7 months. The point $(7, 2000)$, and a dashed line plotting the more general relationship is indicated in the graph below:



Government Purchases

A government buys x fighter planes at z dollars each, and y tons of wheat at w dollars each. It spends a total of B dollars, where $B = xz + yw$. In (a)–(c), write an equation whose solution is the given quantity.

a) The number of tons of wheat the government can afford to buy if it spends a total of $100 million, wheat costs $300 per ton, and it must buy 5 fighter planes at $15 million each.

b) The price of fighter planes if the government bought 3 of them, in addition to 10,000 tons of wheat at $500 a ton, for a total of $50 million.

c) The price of a ton of wheat, given that a fighter plane costs 100,000 times as much as a ton of wheat, and that the government bought 20 fighter planes and 15,000 tons of wheat for a total cost of$90 million.

Solution: Planes and wheat

A government buys x fighter planes at z dollars each, and y tons of wheat at w dollars each. It spends a total of B dollars, where $B = xz + yw$. In (a)–(c), write an equation whose solution is the given quantity:

a) The number of tons of wheat the government can afford to buy if it spends a total of \$100 million, wheat costs \$300 per ton, and it must buy 5 fighter planes at \$15 million each.

We want to find the value of y . We are given B = 100,000,000, w = 300 , x = 5, and z = 15,000,000. So the equation is

$$100{,}000{,}000 = 5 \cdot 15{,}000{,}000 + 300y,$$

or

$$100{,}000{,}000 = 75{,}000{,}000 + 300y.$$

b) The price of fighter planes if the government bought 3 of them, in addition to 10,000 tons of wheat at \$500 a ton, for a total of \$50 million.

We want to find the value of z. We are given that x = 3, y = 10,000, w=500, and B=50,000,000. So the equation is

$$50{,}000{,}000 = 3z + 10{,}000 \cdot 500,$$

or

$$50{,}000{,}000 = 3z + 5{,}000{,}000.$$

c) The price of a ton of wheat, given that a fighter plane costs 100,000 times as much as a ton of wheat, and that the government bought 20 fighter planes and 15,000 tons of wheat for a total cost of\$90 million.

We want to find the value of w. We are given that $x = 20$ and $y = 15{,}000$, $B = 90{,}000{,}000$, and $z = 100{,}000w$. So the equation is

$$90{,}000{,}000 = 20(100{,}000w) + 15{,}000w,$$

which simplifies to

$$90{,}000{,}000 = 2{,}015{,}000w.$$

Student Name	**Teacher Name**
Progress Report Starting Grade:_____ Progress 1: Date_____: Grade:_____ Progress 2: Date_____: Grade:_____ Progress 3: Date_____: Grade:_____	**Parent Contact Log** Phone #_____ Date, Reason, Outcome 1 _____ 2._____ 3. _____ 4.._____ 5.._____
Requirements of Your Acceleration Plan:	**Agreement to follow this Acceleration Plan** Student Signature_____ Teacher Signature_____ Additional Comments (Student or Teacher):

Student Name	**Teacher Name**
Progress Report Starting Grade:_____ Progress 1: Date_____: Grade:_____ Progress 2: Date_____: Grade:_____ Progress 3: Date_____: Grade:_____	**Parent Contact Log – Contact if student is not complying with Improvement Plan** Phone #_____ Date, Reason, Outcome 1 _____ 2._____ 3. _____ 4.._____ 5.._____
Requirements of Your Improvement Plan: □ Attend afterschool tutoring on: Tues Weds □ Make-up Missing: □ Test □ Quiz □ Classwork □ Homework □ Other:_____	**Agreement to follow this Improvement Plan** Student Signature_____ Teacher Signature_____ Additional Comments (Student or Teacher):

Lesson 1: Addition and Subtraction Equations, Part 1

Can you balance a twirling basketball on the end of your finger? Do you eat a balanced diet? Have you seen people scratch their head as they try to balance a checkbook? Balance is an idea that you already think about and can apply to many different situations. Equations must maintain balance to be called "equations."

GOALS FOR THIS LESSON

- Solve addition or subtraction equations.
- Solve addition or subtraction equations involving simplification

Before learning about transformations, take a moment to try a couple of simplification exercises.

Simplify.

$t + 18 - 18$

Answer

1 of 2 Next

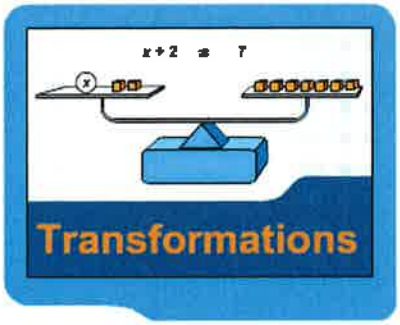
Learn: Transformations

Learn: Transformations

Balance is important, especially when solving problems. You can use addition and subtraction to keep equations balanced.

Click Transformations to begin.

This activity has audio. Use headphones or speakers.



Transformations

Equality Properties

So often in algebra you will see properties that appear to state the obvious. The two new properties you will learn here provide reasons for the process that you have seen. They are the basis for the transformations you have observed.

Examine the addition and subtraction properties of equality. In the future, note that they are used as often as any other property you have learned.

Equality Properties

Addition Property of Equality
If a, b, and c are real numbers and $a = b$, then:

$$a + c = b + c \text{ and } c + a = c + b$$

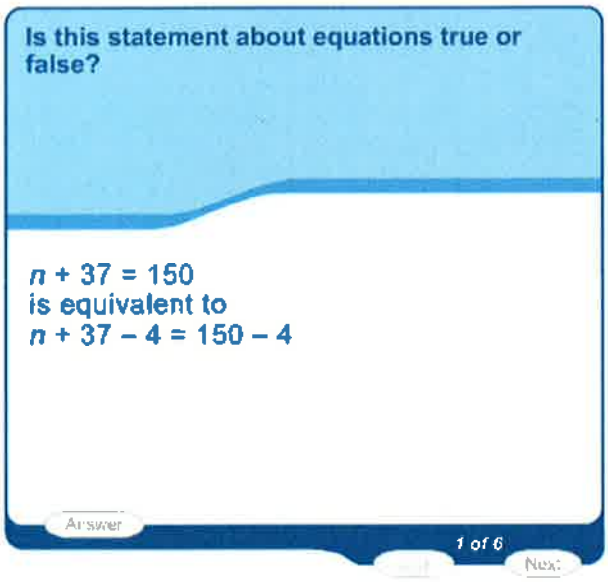
In other words, if you add the same quantity to both sides of an equation, you still have an equation. Both sides of the equation will be equal.

Next

Think About It

Keeping Balance

See if you can figure out which of these statements about equations are true and which are false. If you keep your balance, you should have no trouble staying on the right track.



Worked Examples

Here you can try an example of using transformations to solve an equation. Compare your answers to those that are revealed.

Solve:

$x - 5 = 12$

Rewrite the equation.

$x - 5 = 12$

Add 5 to each side.

$x - 5 + 5 = 12 + 5$ Adding 5 to the left side of the equation combines two key ideas

1) Inverse operations: You are using inverse operations to undo the subtraction of 5. Creating additive inverses on the left side of the equation creates a zero value. In that way, you are isolating the variable.

2) The addition property of equality: This property guarantees that by adding 5 to the right side also, you will have the same

Print

Remember

Here you can try another example of using transformations to solve an equation. Be sure to find the solution yourself first before checking your answer.

Solve:

$$-7 = 22 - y$$

+ **Rewrite the equation.**

Print

Offline Learning

Now go offline to learn more about transforming equations using addition and subtraction and to practice your skills:

- Read pages 99–101 in the textbook through to the solution to Example 3 Part B.
- Review the Do the Math section of the Student Guide.
- Complete Problems 1–7 and 13–16 on page 103.
- Complete the Extension section of the Student Guide. *(optional)*

MATERIALS

textbook - pages 99-103

- Synchronous Session Unit 4 Moderator Guide
- Unit 4 Cluster 1 Solving Equations Whiteboard
- Unit 4 Cluster 2 More Word Problems Whiteboard
- Student Guide
- Square and Rectangular Tiles
- Transforming Equations Activity Guide
- Transforming Equations Answer Key

calculator

Moyer T. Academy

Social Studies Scope and Sequence

2013-2014

Scope and Sequence Grade 6 Global World View

Unit Title	*Unit Essential Question(s)*	*Instructional Benchmark(s) Addressed*	*Dates of Instruction*
Unit One: Mental Mapping	How do you use mental maps to organize information about people, places, and the environment?	**Delaware Geography Standard 1a 6-8**: Students will demonstrate mental maps of the world and its sub-regions, which include the relative location and characteristics of major physical features, political divisions, and human settlements.	
Unit Two: Culture and Patterns	How do geographers use maps to help them understand how cultures affect places, and how places move and change?	**Delaware Geography Standard Three 6-8a:** Students will identify and explain the major cultural patterns of human activity in the world's sub-regions.	
Unit Three: Basic Economics	What is economics and how does it affect me?	**Economics Standard Three 6-8:** Students will demonstrate the ways in which the means of production, distribution, and exchange in different economic systems have a relationship to cultural values, resources, and technologies.	
Unit Four: Regions	How do regions help us to organize and understand geography?	**Geography Standard Four 6-8 a:** Student will understand the process affecting the location of economic activities in different regions of the world.	
Unit Five: Human Ecology	How do humans have an impact on the environment?	**Geography Standard Two 6-8a**: Students will apply a knowledge of the major processes shaping natural environments to understand how different peoples have changed and been affected by, physical environments in the world's sub-regions.	
Unit Six: Technology and Geo-Literacy	How can you use digital media to effectively communicate solutions to global problems?	**Geography Standard Two 6-8a**: Students will apply a knowledge of the major processes shaping natural environments to understand how different peoples have changed and been affected by, physical environments in the world's sub-regions. **NETS (National Educational Technology Standards) K-12.4:** Students use a variety of media and formats to communicate information and ideas effectively to multiple audiences. **NETS K-12.6:** Students employ technology in the development of strategies for solving problems in the real world.	

Scope and Sequence Grade 7 Current Events

Unit Title	*Unit Essential Question(s)*	*Instructional Benchmarks*	*Dates of Instruction*
***Unit One**: Human Conflict and Cooperation*	How can the conditions, motivations, and actions of people cause conflict and cooperation?	**Geography Standard Four 6-8b**: Students will explain how conflict and cooperation among people contribute to the division of the Earth's surface into distinctive cultural and political territories.	
***Unit Two**: Basic Freedoms*	To what extent, should the minority have the right to become the majority and have its voice heard?	**Civics Standard Two 6-8a:** Students will understand that the concept of majority rule does not mean that the rights of minorities may be disregarded and will examine and apply the protections accorded those minorities in the American political system.	
***Unit Three**: History and Research*	How does an historian conduct historical research?	**History Standard Two 6-8a:** Students will master the basic research skills necessary to conduct an independent investigation of historical phenomena. **History Standard Two 6-8b:** Students will examine historical documents, artifacts, and other materials, and analyze them in terms of credibility, as well as the purpose, perspective, or point of view for which they were constructed.	
***Unit Four**: Citizenship*	How does an American citizen actively communicate with local, state, and federal elected officials?	**Civics Standard Four 6-8a:** Students will follow the actions of elected officials, and understand and employ the mechanisms for communicating with them while in office.	
***Unit Five**: Trade*	How does trade with individuals, businesses, and other countries cause an interdependent economy? What policies have been made to influence trade amongst other nations?	**Economics Standard Four 6-8a:** Students will examine how nations with different economic systems specialize and become interdependent through trade and how government policies allow either free or restricted trade.	
***Unit Six**: Eco- Cities*	Why does the sustainable use of resources differ from cities around the country and world?	**Geography Standard Three 6-8a:** Students will identify and explain the major cultural patterns of human activity in the world's sub-regions.	

Scope and Sequence Grade 8- American History Beginnings Through Civil War

Unit Title	*Unit Essential Question(s)*	*Instructional Benchmarks*	*Dates of Instruction*
Unit One: *Principles of the U.S. Constitution*	What are the fundamental principles underlying the Constitution? How did the Constitution create a strong government with roots in history that allowed for change and meet the needs of the people?	**Civics Standard Two 6-8b:** Students will understand the principles and content of major American state papers such as the Declaration of Independence; United States Constitution (including the Bill of Rights); and the Federalist Papers.	
Unit Two: *Division of Power*	How do these powers work together to ensure the goals of the government are met?	**Civics Standard One 6-8b:** Students will analyze the different functions of federal, state, and local governments in the United States and examine the reasons for the different organizational structures each level of government displays.	
Unit Three: *Foundations of Market Economies*	How are scarce resources used to produce and distribute goods and services that satisfy people's needs and wants?	**Economics Standard One 6-8a:** Students will analyze how changes in technology, costs, and demand interact in competitive markets to determine or change the price of goods and services.	
Unit Four: *The Road to the Revolutionary War*	Why might historians have different viewpoints of the same historical event? How might these viewpoints change over time?	**History Standard Three 6-8a:** Students will compare different historians' descriptions of the same societies in order to examine how the choice of questions and use of sources may affect their conclusions.	
Unit Five: *Our Economy and You*	How can I manage money, be a smart consumer, and understand the needs of the marketplace?	**Economics Standard One 6-8a:** Students will analyze how changes in technology, costs, and demand interact in competitive markets to determine or change the price of goods and services.	
Unit Six: *Technology and Historical Research*	How can you use digital media to conduct historical research and effectively communicate your investigation to an audience?	**History Standard Two 6-8a:** Students will master the basic research skills necessary to conduct an independent investigation of historical phenomena. **History Standard Two 6-8b:** Students will examine historical documents, artifacts, and other materials, and analyze them in terms of credibility, as well as the purpose, perspective, or point of view for which they	

		were constructed. **NETS (National Educational Technology Standards) K-12.4:** Students use a variety of media and formats to communicate information and ideas effectively to multiple audiences. **NETS K-12.6:** Students employ technology in the development of strategies for solving problems in the real world	

Scope and Sequence Grade 9 – Civics and Geography

Unit Title	*Unit Essential Question(s)*	*Instructional Benchmarks*	*Dates of Instruction*
***Unit One**: Public Policy*	Why is public policy a major concern of all levels of government?	**Civics Standard Four 9-12a:** Students will develop and employ the skills necessary to work with government programs and agencies. **Civics Standard Four 9-12b:** Students will understand the process of working within a political party, a commission engaged in examining public policy, or a citizen's group.	
***Unit Two**: Civic Duties and Responsibilities*	How is fulfilling our responsibilities as citizens' a way to preserve our rights?	**Civics Standard Three 9-12a:** Students will understand that citizens are individually responsible for keeping themselves informed about public policy issues on the local, state, and federal levels; participating in the civic process; and upholding the laws of the land.	
***Unit Three**: The Field of Geography*	How do geographers show information on maps? Why do geographers use a variety of maps to represent the world?	**Geography Standard One:** Students will develop a personal geographic framework, or "mental map," and understand the uses of maps and other geo-graphics [maps].	
***Unit Four:** Settlement and Consumption Patterns*	How does where you live influence how you live? How do American consumption patterns affect people and the planet?	**Geography Standard Two**: Students will develop a knowledge of the ways humans modify and respond to the natural environment. **Geography Standard Three:** Students will develop an understanding of the diversity of human culture and the unique natures of places.	
***Unit Five**: Urban Sprawl and Population Density*	How does urban sprawl affect people and the planet? How does population density affect people and places?	**Geography Standard Four:** Students will develop an understanding of the character and use of regions and the connections among them.	
***Unit Six**: Extreme Weather Hazards*	How does climate and weather influence human activity in a region?	**Geography Standard Two**: Students will develop a knowledge of the ways humans modify and respond to the natural environment.	

Scope and Sequence Grade 10 Economics

Unit Title	*Essential Question*	*Instructional Benchmarks*	*Dates of Instruction*
Unit One: Personal Choices	How do you analyze the potential costs and benefits of personal economic choices in a market economy?	**Economics One 9-12a:** Students will demonstrate how individual economic choices are made within the context of a market economy in which markets influence the production and distribution of goods and services.	
Unit Two: Humans Interact with the Economy	How do individuals, families, communities, businesses, and governments interact in a market economy?	**Economics Two 9-12a:** Students will develop an understanding of how economies function as a whole, including the causes and effects of inflation, unemployment, business cycles, and monetary and fiscal policies.	
Unit Three: Opportunities and Challenges in Economies	How do changes in different types of economic systems impact the range of economic opportunities and consequences?	**Economics Three 9-12a:** Students will analyze the wide range of opportunities and consequences resulting from the current transitions from command to market economies in many countries.	
Unit Four: International Trade and Economic Systems	How do patterns and results of international trade impact economic systems?	**Economics Four 9-12a:** Students will analyze and interpret the influence of the distribution of the world's resources, political stability, national efforts to encourage or discourage trade, and the flow of investment on patterns of international trade.	

Scope and Sequence Grade 11- United States History Reconstruction to Present Day

Unit Title	*Essential Question*	*Instructional Benchmarks*	*Dates of Instruction*
Unit One: *Developing a National Identity*	How did key economic developments and reform bring important changes to the United States?	**Geography Three 9-12a:** Students will understand the processes, which result in distinctive cultures, economic activity, and settlement form in particular locations across the world.	
Unit Two: *The Union in Crisis*	If some groups of people are denied certain rights, how could this situation be corrected?	**Civics Two 9-12a:** Students will examine and analyze the extra-Constitutional role that political parties play in American politics	
Unit Three: *An Industrial Nation*	How did the growth of industries fuel a second industrial revolution in the United States?	History 2a; History 3a	
Unit Four: *Becoming a World Power*	How did the United States become a power on the world's stage?	**Civics Two 9-12b:** Students will understand that the functioning of the government is a dynamic process which combines the formal balances of power incorporated in the Constitution with traditions, precedents, and interpretations which have evolved over the past 200 years	
Unit Five: *A Modern Nation*	How did some Americans cope with the trauma of war and others enjoy prosperity and peace?	Civics 2a;Civics 2b	
Unit Six: *A Champion of Democracy*	How did World War II and its consequences impact the lives of Americans?	**Economics 9-12a:** Students will develop an understanding of how economies function as a whole, including the causes and effects of inflation, unemployment, business cycles, and monetary and fiscal policies.	
Unit Seven: *A Nation Facing Challenges*	How does exploring the unfamiliar make a society great?	**Civics Two 9-12b:** Students will understand that the functioning of the government is a dynamic process which combines the formal balances of power incorporated in the Constitution with traditions, precedents, and interpretations which have evolved over the past 200 years	
Unit Eight: *Looking Toward the Future*	How has technology or scientific advance have the biggest influence on future generations?	History 1a;History 2b; History 3a; Economics 1a; Civics 2b	

Scope and Sequence Grade 12- Elective Course Offerings (*New* World History Course Offered 2014-2015 SY)

Delaware Model Unit Gallery Template

Unit Title: Eco-Cities

Designed by: Social Studies Specialist
District: Maurice J. Moyer Academy powered by K12

Content Area: Social Studies
Grade Level(s): 7

Unit Duration: 3-4 weeks

Summary of Unit

Cities are urban ecosystems which include both nature and humans, in a largely human built environmental context and that urban ecosystems have emergent properties that cannot easily be seen by simply looking at the different functional parts of a city: The whole is more than the sum of the parts. The historical development of urban ecosystems is important to understand in relationship to food production, food surpluses, exploitation of forests, irrigation waters, and other resources that can lead to catastrophic consequences for some cities. Over time cities have become the centers of population, but that the size and number of modern urban ecosystems is unprecedented. The key to the growth of large cities worldwide has been the human exploitation of fossil fuels.

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

Geography Standard Three 6-8a: Students will identify and explain the major cultural patterns of human activity in the world's sub-regions.

Big Idea(s)

- Urban Ecosystems
- Humans Interaction with the Environment
- Principles of Ecology
- Historical Development of Ecosystems
- Energy Use

Unit Enduring Understanding(s)

- Throughout history cities have been the centers of population
- Human Exploitation of Fossil Fuels
- Positive Environmental Activities

Unit Essential Questions(s)

- Why does the sustainable use of resources differ from cities around the country and world?

Knowledge and Skills

Students will know…

- The importance of food surpluses to the historical development of urban ecosystems
- How the exploitation of forests, irrigation waters, and other resources led to catastrophic events for some early cities

Students will be able to…

- Think critically about assumptions and misconceptions of city infrastructures
- Identify problems and develop solutions to current environmental issues
- Analyze environmental issues

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

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Suggested Performance/Transfer Task(s)

In Production

Rubric(s)

In Production

Other Evidence

- Varied evidence that checks for understanding (e.g., tests, quizzes, prompts, student work samples, observations and supplements the evidence provided by the task)

Student Self-Assessment and Reflection

The Checks for Understanding at the end of each instructional strategy should be used as formative assessments. Students must do the summarizing themselves for the learner to construct meaning. Research shows that getting students cognitively active with processing information yields significant learning effect size.

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

Lesson One

Essential Question(s)

- Why is a city different from a small town or village?
- How many people live in our city (or the nearest large city)?
- How can you bring awareness to your peers and community about the impact human activity has on large cities?

Delaware State Standard

Geography Standard Three 6-8a: Students will identify and explain the major cultural patterns of human activity in the world's sub-regions.

Instructional Strategies

Activating strategies are powerful tools that teachers use to engage students at the beginning of a lesson by activating prior knowledge or providing an experience that will support information to be learned.

Activating Strategy

Have students discuss what they already know about the content. Be sure to create questions to guide your discussions. Break students into groups and assign group leaders to facilitate. Encourage sharing and remembering**.**

Strategy One: Gathering Information

Think/Pair/Share

Have students go to capital cities and cities of 100,000 and more inhabitants, a page on the United Nations website. Here they can find out the population of cities around the globe, as well as one not too far from them Find the United States on the document and look at the table that appears.

Show the class that the population of the "city proper" is often much less than the "urban agglomeration." For example, the city of Atlanta has about 396 thousand people, but the Atlanta metropolitan area has 3.3 million people!

Setting the Stage: Background Information

An ecosystem is a community of living things interacting with nonliving things. Examples include forests, lakes, soils, and coral reefs. A city is an urban ecosystem. People are among the living things, and the buildings, streets, and other structures that people build are among the nonliving things. The word urban has a Latin root that means "city." Did the Latin scholars just make up the word?

Role Play Strategy "Act It Out"- Students will perform a brief skit about the word urban.

Latin Scholar 1: "Come on Augustus! We've got to come up with a word to describe this big group of houses, temples, markets, roads and common baths."
Latin Scholar 2: "Umm…."
LS1: "Look, I know how creative you are, I'll just write down the next word you say and we can go get some lunch!"
LS2: "Er…."
LS1: "Ur! That's IT! It's short, easy to pronounce, and…"
LS2: "Um, Romulus… you spelled it wrong."
LS1: "No worries. Let's get out of here!"

Tell students that actually this conversation never happened. In real life, there was an ancient city named Ur located in what is now the country of Iraq.

Explain to students that Urban ecosystems occupy only about 2% of the land surface area of the planet, but provide a home for half of the world's population, this is roughly about 3 billion people. The proportion of people living in cities is even higher in the developed regions of the world. In the United States, more than three-quarters of the people live in urban areas. More than two-thirds of the people of Europe, Russia, Japan, and Australia live in such areas. Where are the cities?

Check for Understanding

Based on the population data that you gathered, how rapidly do you think large cities like Philadelphia are growing? Why?

Rubric

2- This response gives a valid explanation with examples.

1- This response gives an invalid explanation with no examples.

Strategy Two: Extending /Refining

Analyzing Perspectives

Write the perspective from an ecologists point of view to why it is best to conserve energy use.

Check for Understanding

Rubric

2- This response gives a valid explanation with examples.

1- This response gives an invalid explanation with no examples

Strategy Three: Application

Differentiated Assignment: Focus Groups

Have students discuss environmental topics in a focus group and write a position paper relating your stance to other members of the group. Divide students into groups of four for a focus group and discuss a topic relevant to the day's lesson. Groups are graded for cooperation, respect, assertiveness, staying on task, and mutual effort to understand one another. Group work counts 20% of each student's grade. The students then write a position paper. Each paper must include the following:

1. Clearly and briefly identifies the writer's precise stance of the topic.
2. Specifically compares one's personal stance on each topic to other members of the group.
3. Is written in a brief, clear, simple format
4. Counts 80% of grade

Additional Strategies for Differentiation;

1. Prepare a discussion format as a whole class activity
2. Give examples of possible topics
3. Give an example of a position paper
4. Allow alternative to writing paper (perhaps a speech).

Focus Groups: Writer's Checklist

	Yes	Needs Work	No
Have we shown cooperation, respect, and mutual effort to understand other students in the group?			
Have we clearly, briefly, and precisely identified our own stance on the topic?			
Position Paper			
Have I clearly,briefl, and precisely identified my stance on the topic?			
Did I specifically compare my own stance on each topic to other members of the group?			
Have I used correct mechanics and usage?			
Have I made my format neat and orderly, complete and accurate, and organized with correct spacing?			

Focus Group Scoring Rubric

Category	4 points each	3 points each	2 points each	1 point each
Cooperation	**Shows cooperation, respect, and mutual effort to understand other students in the group.**	**Mostly shows cooperation, respect, and mutual effort to understand other students in the group.**	**Only somewhat shows cooperation, respect, and mutual effort to understand other students in the group.**	**Does not show cooperation, respect, and mutual effort to understand other students in the group.**
Stance	**Clearly, briefly, precisely, identifies his/her own stance on the topic.**	**Briefly, precisely, identifies his/her own stance on the topic.**	**Has difficult indentifying his/her stance on the topic.**	**Does not identify his/her stance on the topic.**
Comparison	**Specifically compare stance on each topic to other members of the group.**	**Compares stance on each topic to other members of the group.**	**Attempts to compare stance on each topic to other members of the group but**	**Attempts to compare stance on each topic to other members of the group,but comparison was**

			comparison was not always clear.	**unclear.**
Mechanics	**Mechanics are correct (spelling, punctuation, and capitalization)**	**Mechanics are mostly correct (spelling, punctuation, and capitalization)**	**Mechanics are only somewhat correct (spelling, punctuation, and capitalization)**	**Mechanics are mostly incorrect (spelling punctuation and capitalization)**

Summarizing Strategy
Learning Logs

During the last 5 minutes of the lesson or class, have students respond in writing to a set of questions. They may explain how to do some skill, summarize the lesson, describe an event or place, state the most important points, or respond to given questions.

Example prompts for Learning Logs and Summarization

Today, I learned….

I would like to learn more about ….

Three things I wonder about…

Something new I learned today..

How did your thinking change?

Lesson Two

Essential Question

How large is your carbon footprint?

Delaware State Standard

Geography Standard Three 6-8a: Students will identify and explain the major cultural patterns of human activity in the world's sub-regions.

Activating Strategy

To activate prior knowledge and to foster some discussion, show students a short clip from National Geographic called "How your T-Shirt Can Make A Difference" This video can be found at http://on.natgeo.com/WtTpKo.

Strategy One: Gathering Information

Give One! Get One!

Have the students list as many words as they can think of that relate to the environment on their own paper. Limit the time for this to two minutes.

Have students walk around the room meeting with each other. Students need to give one of their words to each student they meet with. The word should be new to their peer, and they can't provide a new word, the students should create a new one together.

Have students continue meeting with each other until students have about seven new words. Working in pairs, have students compile their lists into two categories. Students may use resources to assist them in defining the words. and coming up with their categories. Discard the words that do not fit into the categories.

Have the pairs make a graffiti board by choosing one of the categories they created and make a poster that displays the category and the words that belong in that category. Students may use/draw pictures to illustrate their words. Make sure students sign their names, and then put the posters on bulletin boards.

Check for Understanding

Rubric

2- This response gives a valid explanation with examples.

1- This response gives an invalid explanation with no examples

Strategy Two: Extending/Refining

Constructing Support

Check for Understanding

Rubric

2- This response gives a valid explanation with examples.

1- This response gives an invalid explanation with no examples

Strategy Three: Application

Culminating Activity

Take your students on a tour of the website for the **Baltimore Ecosystem Study**. This study aims to understand metropolitan Baltimore as an ecological system by bringing together researchers from the biological, physical, and social sciences. These people are collecting new information—and analyzing and synthesizing existing information—on how the "built" and wild ecosystems of Baltimore work. As a part of the National Science

Foundation's Long-Term Ecological Research Network, the researchers also seek to understand how Baltimore's ecosystems change over long time periods.

Divide your class into teams and have the teams study selected research and education projects described on the website and report out to the class.

Ask questions such as the following:

- How are the different scientists involving history in their work?
- Are there any research findings that you think your local Mayor and the City Council should know about?
- What is the difference between a research finding and an action recommendation?

Summarizing Strategy

P-M-I (Plus- Minus- Intriguing)

In the Plus row describe/list what you think are the positive things about the lesson/reading. In the Minus row describe/list what you think are some of the negative things about the lesson/reading. In the Intriguing row describe/list some of the things you think are neat or different, or even some things you don't fully understand and want to learn more about.

Lesson Three

Essential Question

Is human activity bringing about alarming global warming scenarios and related catastrophes?

Delaware State Standard

Geography Standard Three 6-8a: Students will identify and explain the major cultural patterns of human activity in the world's sub-regions.

Activating Strategy

Provide a visual of the following scenario:

What could you do to solve the problem?

Strategy One: Gathering Information

Brainstorming

Have your class brainstorm and try to come up with a list of features of urban ecosystems that make them so complex (and unpredictable)

and discuss how these might have changed (or might change) over time.

Some suggestions include:
- Water cycles and extremes such as floods and droughts (think pavement, runoff…)
- Nutrient cycles such as nitrogen and phosphorus (think fertilizer, dog and other animal droppings, etc.)
- Energy flow through natural and human food webs (think grass clippings, autumn leaves, auto fuel, etc.)
- Geographic and climate setting
- Formal and informal political arrangements
- Administrative hierarchies
- The division of labor
- The interaction of different professions
- Modern and ancient technologies
- Communications such as person-to-person, and others such as signs, radios, etc.
- Spiritual influences
- Race and ethnicity
- Immigrant populations
- Gender relations
- Domesticated animals and pets
- Invasive species
- Military defenses present and past
- Economic power and influence

Check for Understanding

Rubric

2- This response gives a valid explanation with examples.

1- This response gives an invalid explanation with no examples

Strategy Two: Extending/Refining

Compare and Contrast

Check for Understanding

Rubric

2- This response gives a valid explanation with examples.

1- This response gives an invalid explanation with no examples

Strategy Three: Application

Collaborative Teams

Divide the class into teams and equip each team with four or six sheets of flip-chart type paper, tape, and colorful markers. Have them tape the paper

together to make a giant poster, and ask each team to draw a representation of what they learned in this lesson. They can have some planning time, but should be given a deadline by which their poster needs to be finished. Everyone needs to participate in drawing or coloring, and put their names around the outside of the work. It does not have to be artistic, just expressive. Then, allow about five minutes per team for a designated speaker or two to explain what they drew and how it represents their understanding of urban ecosystems.

Summarizing Strategy

Pass it on

Students in groups. Give each group an envelope with 4 note cards in each envelope. Each group creates a question based on content learned. Write the answer on one of the note cards. Groups rotate the envelopes answering the another group's questions. Students may add to an answer using one of the note cards in the envelopes.

Resources and Teaching Tips

Design Principles for Unit Development

21st Century Learning:

- Analyze, access, manage, integrate, evaluate and create information in a variety of forms and media
- Develop, implement, and communicate new ideas to others

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

Opportunities to integrate technology are included in this unit. For instance, readings can be accessed electronically if student computers are available. The unit assessment is on the computer and projected from the computer, as well as 3 of the quizzes. The review sessions also require students to interact with the smart board. Also, websites to go beyond the lesson are available through the K12 website.

Content Connections

Content Standards integrated within instructional strategies

HS Social Studies Curriculum Unit One

Unit Title: Developing a National Identity

Designed by: **Social Studies Specialist**
Charter: Maurice J. Moyer Academy ***powered by K12***

Content Area: United States History
Grade Level(s): 11

Unit Duration: 3-4 weeks

Summary of Unit

This unit for 11th grade examines how Americans took pride in their new nation and eagerly sought to change their lives and society through religious movements and reforms in education and other institutions. Students learn that as European immigrants flooded eastern cities, and Americans increasingly migrated to the western regions of the country, economic development brought about important changes to the United States. They will understand that the growth of factories and manufacturing established the North as an industrial power, while southern economies continued to rely mainly on agriculture.

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

Geography Standard Three 9-12a: Students will understand the processes, which result in distinctive cultures, economic activity, and settlement form in particular locations across the world.

Big Idea(s)

- Reform
- Immigration and Migration
- Economic Development

Unit Enduring Understanding(s)

- Economic activities often produce regional patterns of land use.
- Reform movements affect society and culture.

Unit Essential Questions(s)

How did key economic developments and reform bring important changes to the United States?

Knowledge and Skills

Students will know…

- Sectional divisions over economic issues and slavery gradually weakened nationalism.
- Reform movements brought improvements in American society.
- Expansion of the United States led to conflict.

Students will be able to…

- Analyze primary source documents to make inferences about economic activity.
- Interpret maps and geo-graphics to gather information about resources and economic activity.
- Identify the cause and effect that the Supreme Court ruling had on federalism.
- Make generalizations about immigration and migration.
- Compare and contrast immigration of the mid-nineteenth century to immigration today.
- Interpreting visuals to investigate societal problems.

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

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Suggested Performance/Transfer Task(s) – Summative Assessment

Standardized Test Practice

Directions: Write your answer for each statement or question on the answer sheet provided for you. Choose the letter of the word or expression that best completes the statement or answers the question.

Part I- Multiple Choice

1. James Monroe's presidency was called "the era of good feelings" because it was characterized by _____.
   A. a new spirit of isolationism
   B. economic growth and feelings of nationalism
   C. territorial growth and feelings of sectionalism

D. major victories in wars with foreign powers

2. The Missouri Compromise resolved a dispute over_____.
   A. the spread of slavery into new states
   B. European colonization in the Americas
   C. Increasing immigration and urban reform
   D. the border between U.S. and Spanish territory

3. How did President Andrew Jackson's policies affect Native Americans?
   A. created an independent Cherokee state.
   B. increased civil rights for Native Americans
   C. increased the available land for Native Americans
   D. forced most Native Americans out of the Southeast

4. What region of the United States was most affected by the Industrial Revolution?
   A. North
   B. South
   C. Northwest
   D. Southwest

5. Improvements in transportation in the early 1800s_____.
   A. included canals and steamboats
   B. included canals and automobiles
   C. included steamboats and electric trains
   D. included automobiles and electric trolleys

6. What crop came to define the South's economy in the early to mid-1800s?
   A. rice
   B. cotton
   C. tobacco
   D. sugar cane

7. A religious movement that swept the United States during the early 1800s is known as the
   A. utopian movement
   B. temperance movement
   C. Second Great Awakening

D. Transcendentalist movement

8. A major transcendentalist who wrote about the principle of civil disobedience was_____.
   A. Dorothea Dix
   B. Ralph Waldo Emerson
   C. Horace Mann
   D. Henry David Thoreau

9. During what year did Irish immigration to the United States peak?

Irish Immigration To The United States	
1845	44,821
1846	51,752
1847	105,536
1848	112,934
1849	159,389
1850	164,004
1851	221,253
1852	159,548
1853	162,649
1854	101,606
1855	49,627
Total	1,333,128

Source: *Historical Statistics of the United States: Colonial Times to 1970*

A. 1849
B. 1850
C. 1851
D. 1852

10. The abolition movement formed to _____.
    A. end slavery in the United States.
    B. improve American public education.
    C. gain suffrage for all American women.

D. improve the conditions of American prisons.

11. The Seneca Falls Convention was significant because it was the nation's first_____.
    A. abolitionist convention
    B. labor rights convention
    C. women's rights convention
    D. immigrants rights convention

12. What social issue is William Lloyd Garrison referring to in the following passage?

> "I will be as harsh as the truth, and as uncompromising as justice. On this subject I do not wish to think, or speak, or write with moderation. No! No!...I am in earnest—I will not equivocate--- and I will not excuse--- I will not retreat a single inch--- and I will be heard."
> ~ The Liberator, 1831, William Lloyd Garrison

13. In the early to mid- 1800s many Americans believed in manifest destiny, the idea that Americans_____.
    A. were destined to be the world's greatest people.
    B. Had the right to spread across the entire continent
    C. Were destined to advance democracy and freedom
    D. Had an obligation to advance capitalism worldwide

14. What event led to mass migration to California in 1849?
    A. the California gold rush
    B. the Mexican-American War
    C. creation of the Pony Express
    D. creation of the California Trail

15. What country used the mission system as a method for settling California and Texas?
    A. Spain
    B. France
    C. Great Britain
    D. United States

16. In which of the following conflicts did William Travis and Sam Houston play important roles?
    A. the Mexican Cession
    B. the Texas Revolution
    C. the Spanish-American War
    D. the Mexican-American War

17. During the Mexican- American War, the uprising that created the Republic of California was_____.
    A. El Brazito
    B. the Bear Flag Revolt
    C. the Battle of Sacramento
    D. the Battle of San Pasqual

18. What large territory did the United States acquire after the Mexican-American War?
    A. Mexican Cession
    B. Republic of Texas
    C. Republic of California
    D. Territory of New Mexico

Part II – Constructed Response

Writing Prompt

The North and the South developed different ways of life and economies during the period between 1815 and 1860. Recall what you have learned about each region during the unit. Then write a five-paragraph essay in which you compare and contrast economic developments in the two regions during this period.

Writing Rubric for CCSS History/Social Studies

http://www.doe.k12.de.us/commoncore/contentareas/files/Gr6-8_Content_Info_Exp.pdf

Suggested Performance/Transfer Task(s) – Authentic Assessment

Planning for how students will indicate learning and understanding of the concepts in the unit. How will the teacher assess the learning? Provide a culminating activity that students will do with the unit's concepts and skills.

~ Max and Julia Thompson's Learning Focused,2009

Essential Question

How did key economic developments and reform bring important changes to the United States?

Prior Knowledge	Now that you have learned how economic development and reform can bring about change, you are ready to exhibit the important changes that happened to the United States.
Problem	A number of private investors are reluctant to fund a railroad project, however some government officials don't think that every project rejected by private investors should be on the fast track for government subsidy.
Role/Perspective	You and other United States government officials must find an alternative funding source for private investors to fund a railroad project and present a proposal to a group of private investors.
Product/Performance	Have students work in small groups to develop two proposals and create a presentation. Two government officials draft proposal A and the other two government officials draft proposal B. Once your draft is completed, compare your findings and make adjustments to both proposals. Next decide funding, how budget sources will be allocated, and what subcommittees will oversee these budget sources that will be included in your proposal. Prepare to present your proposal to an audience of your peers. Present in the form of a two-page written document and use media technology (i.e., Powerpoint, Prezi, etc.).
Criteria for Exemplar	Be sure to include: • A brief summary of the problem being addressed • Purpose of Proposal • Solution to the Problem • Rationale of Proposal -Describe compromise of your solution • Impact of Proposal -Identify funding resources and spending categories -Create visual aids (i.e. charts and graphs)

Scoring Rubric

Scoring Category Your proposal provides:	Score Point 3	Score Point 2	Score Point 1
Summary of Problem	**The student clearly defines the problem and states how they plan to solve the problem. Provides research and reports that support their plan.**	**The student defines the problem and states how they plan to solve the problem. Provides some research and reports that support their plan.**	**The student does not clearly define the problem and state how they plan to solve the problem. Provides minimal research and reports that support their plan.**
Address Purpose of Proposal	**The student clearly presents thoughtful development of the purpose that offers in-depth insight. Provides a brief statement of what the proposal would accomplish.**	**The student presents a well- developed purpose that offers some insight. Provides a brief statement of what the proposal would accomplish.**	**The student presents satisfactory development of the purpose. Provides a statement of what the proposal would accomplish, however the statement is somewhat unclear.**
Solution to the Problem	**The student clearly describes what federal laws exist and how your proposal is necessary to provide private investors with alternative funding for the railroad project and/or explains no law exist and the problem this causes for private investors.**	**The student gives a brief description of what federal laws exist and how your proposal is necessary to provide private investors with alternative funding for the railroad project and/or explains no law exist and the problem this causes for private investors.**	**The student provides a simplistic description of what federal laws exist and how your proposal is necessary to provide private investors with alternative funding for the railroad project and/or explains no law exist and the problem this causes for private investors.**
Rationale	**The student describes in detail the rationale of the their proposal and discusses why conflicts occur, and explains why compromise is an important part of the developing a proposal for the railroad project. The student explains how many economic decisions are politically motivated and describes the impact that the proposal will have on economic development.**	**The student vaguely describes the rationale of the their proposal and discusses why conflicts occur, and explains why compromise is an important part of the developing a proposal for the railroad project. The student addresses how many economic decisions are politically motivated and adequately describes the impact that the proposal will have on economic development.**	**The student describes the rationale of the their proposal , however does not discuss why conflicts occur, nor explain why compromise is an important part of the developing a proposal for the railroad project. The student does not provide details that explain how many economic decisions are politically motivated and inaccurately describes the impact that the proposal will have on economic development.**

Impact of Proposal	**The student provides a logical and well-structured developed argument that focuses on the major points of the problem. Strengths of the proposal are clearly illustrated using visual media technology and objectively discussed.**	**The student provides a logical developed argument that focuses on the major points of the problem. Strengths of the proposal are illustrated using visual media technology and objectively discussed.**	**The student does not provide a clear logical and well-structured developed argument that focuses on the major points of the problem. Strengths of the proposal are incomplete and not clearly illustrated using visual media technology and objectively discussed.**

Student Self-Assessment and Reflection

The Checks for Understanding at the end of each instructional strategy should be used as formative assessments. Students must do the summarizing themselves for the learner to construct meaning. Research shows that getting students cognitively active with processing information yields significant learning effect size.

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Lesson One

Essential Question

- To what extent, did nationalism give way to sectionalism as the two sections disagreed over issues such as slavery, economic policy, and the issue of states' rights?

Delaware State Standards Addressed

- Civics Standard Two 9-12a
- Civics Standard Three 9-12 a

Instructional Strategies

Activating strategies are powerful tools that teachers use to engage students at the beginning of a lesson by activating prior knowledge or providing an experience that will support information to be learned.

Activating Strategy – Brainstorming

**Using this strategy, provides background knowledge to students that may have limited or no prior knowledge*

As a whole class have students brainstorm what they already know about slavery, economic policy, and states' rights. Let students think of as many ideas as possible. Try to limit conversation and judgment about individual ideas. Include all ideas, even if they are off topic.

Adaptations of brainstorming:

- After brainstorming, have students categorize ideas
- Post brainstorming lists and revise as the lesson goes on.
- During brainstorming, assign categories and labels. Encourage students to think of as many different categories as possible. Use web to help connect the categories and ideas to the topic.
- Post different categories or subtopics around the room and have students move to each and add what they know. You may want to use different colors for different groups or pairs.

Introduce Lesson

The teacher tells students that in this lesson they will learn about the challenges that faced the young nation, actions of the Supreme Court that helped define the country, and about the increasingly divisive issue of slavery.

Strategy 1: Gathering Information

Think/Pair/Share

Using a web graphic organizer, have each student generate a list of traits that they associate with the people of young United States. How would they describe these Americans? Would they use the terms like aggressive, young, eager, and energetic, or would they use terms like soft-spoken, traditional, and dedicated to peaceful negotiation?

Once students have generated their lists, students should pair with another student to discuss their list. The class will then debrief giving students time to share their character trait list of young America. From the individual lists, generate one class list and ask students to think aloud about key events that match the character traits listed.

Check for Understanding- Journal Reflection

- Based on what you know, how would you describe the impact that the character traits and attitudes of young American citizens had on the emerging national identity?

Rubric

2- This response gives a valid and accurate description of the impact that the character traits and attitudes of young American citizens had on the emerging national identity.

1- This response gives an invalid description and accurate description of the impact that the character traits and attitudes of young American citizens had on the emerging national identity.

Setting the Stage: Background Information

Between 1803 and 1815, a series of wars fought by or against France under the French emperor, Napoleon, had seriously threatened the monarchies of Europe. Soon after Napoleon's defeat in 1815, the major European powers, including Great Britain and Russia, formed a loose alliance known as the Concert of Europe. Their goals were to keep balance of power in Europe and to repress revolutionary ideas.

At the same time, revolutions were breaking out in South America, as colonies declared their independence from Spain. Although the United States declared neutrality, it supplied the rebels with ships and supplies. In 1822, President James Monroe was the first to give diplomatic recognition to the new nations. But both Great Britain and the United States were worried that France would send troops to reconquer Spain's colonies.

John Quincy Adams, Monroe's secretary of state, was an experienced diplomat who had been living abroad since he was a teenager. He was worried about territorial threats from other European nations. Russia, for example claimed much of the Pacific Coast of North America. Adams wanted to stand up to the monarchs of Europe. He declared " that the American continents are no longer subjects for any new European colonial establishments." He also said that the United States should act on its own, instead of following like " a rowboat in the wake of a British man-of-war." Those brave words led to the statements made in the Monroe Doctrine, which declared the Americas off limits to European colonization.

Strategy Two: Extending and Refining
Menu Strategy

This menu strategy is designed to give students some choice in how they demonstrate their understanding of the content. The directions for using the menu strategy can be modified based on your needs and expectations. Students might choose one of the four options if all of the options provide information you need to assess their learning and answer the essential question.

Choose one of the following tasks to answer the Essential Question:

<table>
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Mastery Task

Use a Fact-Reason graphic organizer to write down key facts from the placards about the following topics:

- A New American Culture
- Nationalism Influences Domestic Policy
- Nationalism Guides Foreign Policy
- The Missouri Compromise

Example of Graphic Organizer:

<table>
<tr><th colspan="2">Fact- Reason</th></tr>
<tr><th>Write Key Facts From Reading</th><th>Explain Why Important To You</th></tr>
<tr><td></td><td></td></tr>
</table>

Once you have gathered your facts, choose one topic and prepare a lesson.

</td>
<td>

Interpersonal Tasks

Read the Supreme Court case Gibbons vs. Ogden and the American System. Conduct research to learn more about this case, the opinions of the justices, and the effect of the case on American law and policy. Use the information from their research to conduct a debate on Gibbons vs. Ogden. Using the constructing support graphic organizer, write a position statement, three reasons for your position and six facts that support your reason. At the conclusion of the debate, you should write a newspaper article on ways that Gibbons vs.Ogden might contribute to a successful program of national improvements in the area of roads and canals.

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Understanding Task

Read the quote by Alexis de Tocqueville called America "a land of wonders." Now look at the painting *A View of the Mountain Pass Called the Notch of the White Mountain* by Thomas Cole. Write a letter that explains the relationship between the Cole painting and de Tocqueville's comments.

</td>
<td>

Self- Expressive Task

Create a cartoon or comic strip that illustrates how American culture resembled the culture described by Alexis de Tocqueville.

</td>
</tr>
</table>

Check for Understanding – Journal Reflection

Write a paragraph explaining why Alexis de Tocquiville's description of American does or does not describe Americans today. Give details that support your position.

Rubric

2- This response gives a valid explanation with details that support the position.

1- This response gives an invalid explanation that lacks details that support the position.

Strategy Three: Application

Problem-Cause-Effect-Solution Paper

Divide the class in half. One group prepares arguments supporting the admission of Missouri as a free state, and the other half as a slave state. Have volunteers from each group use their group's arguments to conduct a classroom debate on the admission of Missouri to the Union as a slave or free state. Then have each student write a one page problem-solution paper about the Missouri Compromise explaining why it is important to maintain a balance of power. Have students consider the following questions in their papers: Was the Missouri Compromise a long or short -term solution? What aspects of the Missouri Compromise might be unsatisfactory to some people?

Writing Format:

A critical problem for_____is… This a problem because… In addition… This problem resulted from several causes. First… Next…Finally…

Several important effects have been attributed to the problem of _____. On the positive side, one sees… Some of the negative consequences are…..

_____ (the problem) could be solved/ resolved in a number of ways. For example,….. This would help solve the problem because… Another way to solve the problem is… This solution seems plausible because… A final solution…

Strategy Four: Summarizing/Closure
3-2-1 Strategy

Choose a summarizing strategy that encourages students to be cognitively active in summarizing for themselves what was important, what they have learned, and why it is important. Students must do the summarizing strategies themselves for the learner to construct meaning.

At the end of each lesson (class), have students write 3 examples of content, 2 examples of how the content is used and 1 reason why the content is important.

Lesson Two

Essential Question

- How did certain issues divide Americans over time?

Delaware State Standards Addressed

- Civics Standard Two 9-12a
- Civics Standard Three 9-12 a

Activating Strategy

Show a short video clip of President Obama's 1st inaugural reception to hook students and as a way to activate prior knowledge.

Introduce Lesson

The Age of Jackson and explain that many issues divided American during the 1820's. Jackson's fame and popularity as a military hero contributed to his success in elections and during his presidency.

Strategy One: Gathering Information
Give One! Get One!

Have the students list as many words as they can think of that relate to democracy on their own paper. Limit the time for this to two minutes.

Have students walk around the room meeting with each other. Students need to give one of their words to each student they meet with. The word should be new to their peer, and they can't provide a new word, the students should create a new one together.

Have students continue meeting with each other until students have about seven new words. Working in pairs, have students compile their lists into two categories. Students may use resources to assist them in defining the words

and coming up with their categories. Discard the words that do not fit into the categories.

Have the pairs make a graffiti board by choosing one of the categories they created and make a poster that displays the category and the words that belong in that category. Students may use/draw pictures to illustrate their words. Make sure students sign their names, and then put the posters on bulletin boards.

Check for Understanding- Journal Reflection

Suppose you could run for president, what would you do that would define your role in the oval office?

Rubric

2- This response gives a valid explanation with examples of what you would do to define your presidency.

1- This response gives an invalid explanation with no examples of what you would do to define your presidency.

Setting the Stage: Background Information

Andrew Jackson won the presidency in 1828 as the candidate of the common man. Rough-hewn voters in the West and South, especially thought of him as one of their own. So when the new president threw open the doors of the White House to anyone who wanted to attend his inaugural reception in 1829, thousands showed up to get a glimpse of their hero—and of the White House.

An estimated 20,000 well-wishers pushed and shoved their way into the White House staterooms. They trampled the carpets with muddy boots and climbed on the upholstered sofas and chairs. They broke china, smashed glassware, and bloodied more than a few noses. Finally, harried servants brought tubs of punch, ice cream, and lemonade outside, as people climbed through open windows to escape the riotous scene. The new president himself fled to the safety of a hotel.

Jackson's opponents denounced the day as "the reign of King Mob." One of Jackson's colleagues, however, was more forgiving. He called it " a proud day for the people."

Strategy Two: Extending and Refining

Provide each student with a piece of blank unlined paper. Model how to fold and set up their paper. Students will use the foldable for structured notes about the Indian Removal Act.

I **Identify an issue**	D **Decide your position**	E **Examine the reasons**	A **Argue with facts**

As you provide your students with a brief lecture (no more than 15-20 minutes) about the Indian Removal Act, guide students as they use the foldable to take structured notes about the issues surrounding the Indian Removal Act, underlying reasons for this legislation, and facts that will support their position.

Check for Understanding –Journal Reflection

How did the Cherokee adopt aspects of European and American culture?

Rubric

2- This response gives a valid explanation with details and accurate facts.

1- This response gives an invalid explanation with minimal details and inaccurate facts.

Strategy Three: Application

Differentiated Assignment

Students will research the life and contributions of a historical figure such as; Alexis de Tocqueville, Noah Webster, James Monroe, John Quincy Adams, Adams- Onis Treaty, Andrew Jackson, John Calhoun, Henry Clay, John Marshall, George, Caitlin, or Davy Crockett.

Alone or with a partner, students select a person or two people who made a significant contribution during the rise of nationalism and Andrew Jackson's presidency. Students should prepare to present as the historical figure to deliver a presentation in another classroom. During the presentation, the character should appear in correct dress, with appropriate language, gestures, and actions as that person. Presentation should include the following:

1. A brief review of the person's life (when and where the person lived and interesting personal details)

2. An explanation of the person's major contributions
3. A demonstration or hands- on activity that would help students better understand those contributions.
4. An explanation of why this person is significant and should be recognized
5. Use of high quality visuals and audience participation throughout.

Additional Strategies for Differentiation:

6. Demonstrate the presentation of a famous person to the class.
7. Provide a list of examples to choose from
8. Allow students to work in groups or pairs.
9. Require only some of the four expected criteria for success.
10. Provide dress and props for students.

Become a Historical Figure: Performer's Checklist

Questions to Consider	**Yes**	**Needs Work**	**No**
Have I given interesting information about the person's life?			
Have I explained the person's major contributions?			
Did I lead the class in a hands-on activity that students can do to better understand his/her contributions?			
Have I explained why this person is famous?			
Does my presentation have pictures or items that represent the person's life?			
Have I made sure my presentation is based on fact?			

Materials:

- Character Dress
- List of facts
- Props
- Scheduled Appearances

Becoming a Historical Figure: Scoring Rubric

Famous Person Criteria	**4 points each**	**3 points each**	**2 points each**	**1 point each**
Purpose	The details are clear and accurate.	Several details are clear but may be inaccurate.	Minimal details are given	No details are given.
Presentation	Makes eye contact, speaks clearly and concisely.	Makes eye contact most of the time and speaks clearly.	Does not make eye contact very often and speaks clearly and concisely most of the time.	Does not make eye contact and does not speak clearly or concisely
Support	Gives good examples, answers questions correctly in the persona of the historical figure.	Gives several concrete examples, answers most questions correctly in the persona of the historical figure.	Gives two or three concrete examples, answers some questions correctly in the personal of the historical figure.	Gives little or not examples, answers questions incorrectly in the persona of the historical figure.

Summarizing Strategy/Closure

The important thing about the (students can either write about the *Rise of Nationalism* or the *Age of Jackson)*_____.

It is

_____
_____.

It is

_____
_____.

It is

_____
_____.

*But the important things about*_____ *is*_____.

Lesson Three

Essential Question(s)

- Why was the national bank a source of controversy?
- How would the question of nullification and secession affect the nation?

Delaware State Standards

- Civics Standard Two 9-12a
- Civics Standard Three 9-12 a

Activating Strategy

Anticipation Guides provide an effective way to activate background knowledge before reading or introducing a new concept/skill to students. Identify five or more major concepts to be learned in the materials. Keep the following guidelines in mind when writing the Anticipation Guide statements**:**

- Think about what your students already know about the topic or concept, paying particular attention to common misconceptions.
- Write statements, which reflect large segments of texts rather than specific details. That will force students to read and synthesize the information rather than simply skim to find the answers.
- Make sure you create both true and false statements.
- Use word statements to provoke critical thinking.

Display or handout the Anticipation Guide and have students complete it prior to reading (viewing, listening).

Have students read the selection and then revisit the Anticipation Guide to see if they have changed their minds.

Ask students to return to their small groups and develop a consensus about the answers. Remind them to provide evidence to convince others.

Discuss the reading as a whole class using the Anticipation Guide, and then have students write in their journals about how their beliefs have changed.

Introduce Lesson

Under the Indian Removal Act, Jackson made the decision to relocate the five Native American Nations even some viewed it as a harsh punishment. After 16, 000 Cherokee were forced to leave their homes, Jackson continued making decisions that had a direct impact on the economic development of young America. The Battle over the National Bank was a hotly contested issue of Jackson's presidency.

Strategy One: Gathering Information

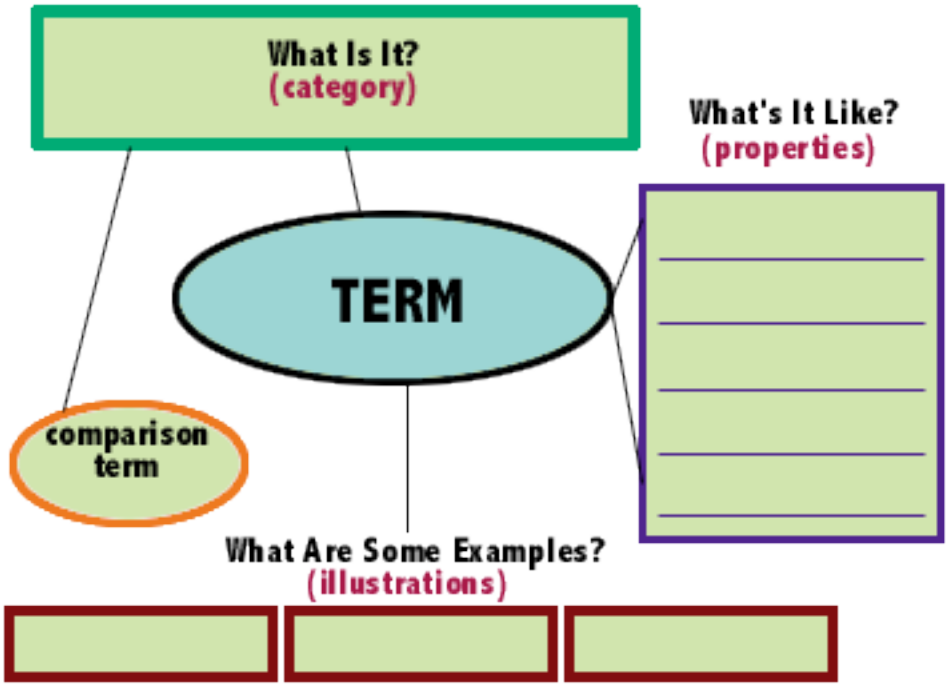
Concept of Definition Map

To help students visualize the components of the term ***controversy***, use a Concept of Definition Map. The map includes three relationships essential to a rich definition:

What is it? (Category)

What is it like? (Properties/Characteristics)

What are some examples? (Illustrations)



Model the concept using a familiar concept. Once students have completed the Concept of Definition Map, ask them to write a Student Friendly Explanation using the information on the map.

Next, as part of a pre-reading discussion, ask students to brainstorm what they already know using the Concept of Definition Map. After students read, have them work in pairs to fill in additional information and make corrections as necessary. Students then present their maps for discussion.

Check for Understanding- Journal Reflection

Why did Jackson oppose the Second Bank of the United States?

Rubric

2- This response gives a valid explanation with details that support the point of view of Andrew Jackson.

1- This response gives an invalid explanation that lacks details that support the point of view of Andrew Jackson.

Strategy Two: Extending and Refining
Analyzing Perspectives

Students complete an Analyzing Perspectives graphic organizer for the issue "Nullification Crisis". Students select among the following perspectives: Andrew Jackson, Robert Hayne, Daniel Webster, John Calhoun and Henry Clay. Then have students role play the various participants appearing on a talk show to share their perspective of the "Nullification Crisis."

Check for Understanding

How would the question of nullification and secession affect the nation?

Rubric

2- This response gives a valid explanation with details and accurate facts.

1- This response gives an invalid explanation with minimal details and inaccurate facts.

Strategy Three: Application
Document Based Question (DBQ)- Shorter Version

Provide a visual of the following writing prompt:

Many great names in American history are closely connected with an idea or an event- George Washington and the Revolution, Abraham Lincoln and the Civil War, Martin Luther King and Civil Rights. Andrew Jackson's name is tied very closely to democracy, but is it historically fair and accurate to do so?

Students read the background essay that provides an overview of Jackson's life and the political climate of the day. Have students quickly skim the 11 documents to get a sense of what they are about. Make sure each student has a clear definition of democracy written down before they analyze the documents.

Each student should read the documents slowly. Modeling how to closely read each document. Have students use a Document Analysis Sheet to record:

a. What or who is the source?
b. What is the issue being discussed? (The Bank, Indian Removal, etc.)
c. Summarize in your own words the main argument or idea being presented in each document.

Next,have students clarify the different issues addressed by the 11 documents. They then must make a judgement as to how democratic Jackson was on each issue. Make sure students compare his actions to their original definition of democracy. Encourage students to make a final judgement of Jackson. Explain to the class that they should consider the following questions:

Did he move the country towards democracy?

Is it reasonable to argue that in some areas he did and in others he did not? Is it even possible that some of his actions may have been both democratic and undemocratic?

Resource: The DBQ Project *Document Based Questions in American History, 2008*

Summarizing Strategy/Closure

Use the acrostic (National Identity) to create detailed statements that answer the Essential Question(s):

Why was the national bank a source of controversy?

How would the question of nullification and secession affect the nation?

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Additional Teacher Resources:
http://www.pbs.org/kcet/andrewjackson/edu/highschool2.html

Video Resources

- Indian Removal
- Cherokee Nation Appeals to the Supreme Court
- Nullification, Part 1
- Nullification, Part 2

Primary Sources:

- Cherokee Alphabet
- Cherokee Phoenix
- Cherokee Phoenix on Farming Methods
- Cherokee Phoenix Introduction
- Chief John Ross
- Constitution of the Cherokee Nation
- Jackson's Message to Congress on Indian Removal
- Jackson's Proclamation on Nullification
- John Marshall's Decision on Worcester v. Georgia
- Letter from Chief John Ross
- Sequoyah

South Carolina Ordinance of Nullification

Design Principles for Unit Development

- **International Education** – the ability to appreciate the richness of our own cultural heritage and that of other cultures in order to provide cross-cultural communicative competence.
- **Universal Design for Learning** – the ability to provide multiple means of representation, expression, and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning –** the ability to use skills, resources, and tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions, make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL, 2007)

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- Analyze, access, manage, integrate, evaluate and create information in a variety of forms and media
- Develop, implement, and communicate new ideas to others

Content Connections

Content Standards integrated within instructional strategies

- Civics Standard Two 9-12a
- Civics Standard Three 9-12 a
- History Standard Two 9-12b
- Economics Standard One 9-12a