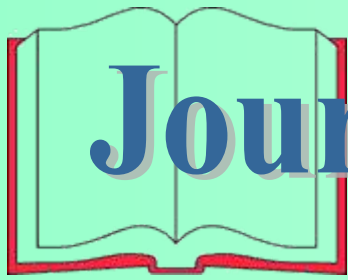


Monday,  
December 13,  
2010

<b>Unit 4</b>	Matrices
<b>Lesson 1</b>	Matrix Operations
Essential Question	What are the similarities and differences between matrices and real numbers?
Standards	M.ALGII.2.4
Objectives	
Vocabulary	

EQ: What are the similarities and differences between matrices and real numbers?



# Journal Topic

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## Unit 4: Lesson 1

Describe how you find your seat in a stadium when you go to a sports game or a concert.

### What we're learning today:

- What matrices are
- How to add, subtract and multiply matrices.

EQ: What are the similarities and differences between matrices and real numbers?



# Discussion

## The Matrix

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EQ: What are the similarities and differences between matrices and real numbers?



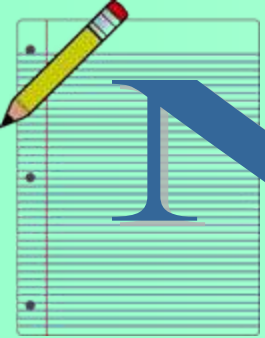
# Discussion

## The Matrix

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$$\begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,m} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,m} \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

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Matrix  
(Plural:  
Matrices)

A matrix is a rectangular array of numbers. Matrices are named using capital letters.

Example:

$$A = \begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 1 \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Discussion

## The Matrix

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$$\begin{array}{l} \text{Row 1} \\ \text{Row 2} \\ \vdots \\ \text{Row } n \end{array} \left[ \begin{array}{cccc} a_{1,1} & a_{1,2} & \cdots & a_{1,m} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,m} \end{array} \right]$$

EQ: What are the similarities and differences between matrices and real numbers?



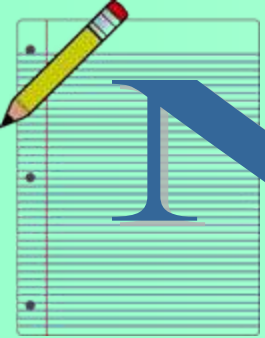
# Discussion

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## The Matrix

$$\begin{array}{cccc} \text{Column 1} & \text{Column 2} & \cdots & \text{Column } m \\ \left[ \begin{array}{cccc} a_{1,1} & a_{1,2} & \cdots & a_{1,m} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,m} \end{array} \right] \end{array}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

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## Dimensions of a Matrix

The dimensions of a matrix are the number of rows and the number of columns in the matrix.

Examples:

$$\begin{matrix} & 2 \times 3 \\ \begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 1 \end{bmatrix} \end{matrix}$$

$$\begin{matrix} 4 \times 1 \\ \begin{bmatrix} 2 \\ -3 \\ 4 \\ 12 \end{bmatrix} \end{matrix}$$

EQ: What are the similarities and differences between matrices and real numbers?





# Discussion

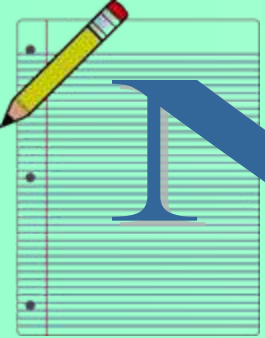
## The Matrix

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Entry  
(Element)

$$\begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,m} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,m} \end{bmatrix}$$
Three red arrows originate from the word 'Entry' and point to the elements  $a_{1,1}$ ,  $a_{2,2}$ , and  $a_{1,m}$  in the matrix. The arrow to  $a_{1,1}$  points to the top-left element. The arrow to  $a_{2,2}$  points to the element in the second row and second column. The arrow to  $a_{1,m}$  points to the top-right element.

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

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Matrix Entries

AKA: Matrix  
Elements

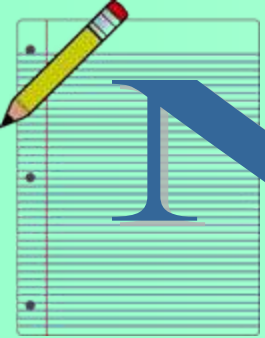
The numbers inside a matrix are called entries or elements. The location or “address” of an entry is the number of the row and column where the entry is located.

Example:

The entry at row 2, column 3 is 1

$$\begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 1 \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

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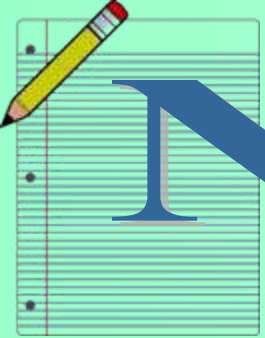
## Matrix Equality

Two matrices are equal if and only if they have identical dimensions and all corresponding entries are equal.

Example:

$$\begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 1 \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

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## Matrix Addition and Subtraction

It is only possible to add or subtract two matrices, if they have identical dimensions. To find the sum, add corresponding entries. To find the difference, subtract corresponding entries.

Examples:

$$\begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -8 & 1 \\ 9 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -8 & 8 \\ 5 & 10 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 3 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ -2 & 7 \end{bmatrix} = \text{Not Possible}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Practice

Monday,  
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2010

Find the difference without a calculator:

$$\begin{bmatrix} 5 & -3 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 7 \\ 9 & 2 \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Practice

Monday,  
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2010

Use your calculator to find each sum or difference:

$$A = \begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 1 & 3 \\ 9 & -8 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 5 & -3 \\ 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} -4 & 7 \\ 9 & 2 \end{bmatrix}$$

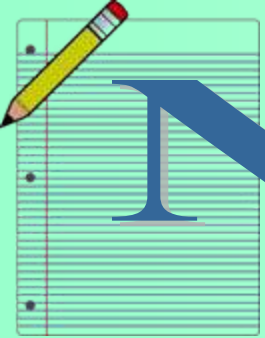
1)  $A + B$

2)  $B + A$

3)  $(A + B) + A$

4)  $A + (B + A)$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

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2010

## Scalar Multiplication

A scalar is a real number. To multiply a scalar by a matrix, multiply the scalar by every entry in the matrix.

Example:

$$3 \cdot \begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Practice

Monday,  
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2010

Use your calculator to find each product:

$$A = \begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 1 & 3 \\ 9 & -8 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 5 & -3 \\ 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} -4 & 7 \\ 9 & 2 \end{bmatrix}$$

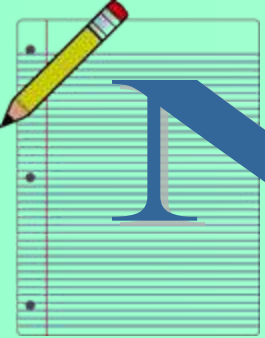
1)  $5 \cdot B$

2)  $-3 \cdot C$

3)  $\frac{1}{2} \cdot D$

EQ: What are the similarities and differences between matrices and real numbers?





# Notes

Monday,  
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2010

## Matrix Multiplication

It is only possible to multiply two matrices when the number of columns in the first matrix is equal to the number of rows in the second matrix.

Example:

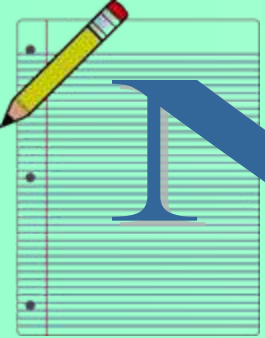
$$\begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} \quad \text{Not Possible}$$

$2 \times 3 \cdot 2 \times 3$

$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} \quad \text{Possible}$$

$2 \times 2 \cdot 2 \times 3$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

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2010

## Matrix Multiplication

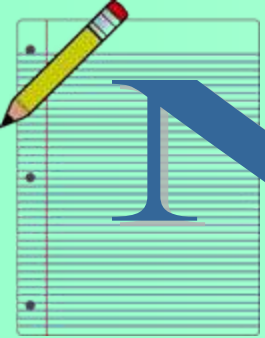
The dimensions of the product of two matrices will be the number of rows in the first matrix and the number of columns in the second matrix.

Example:

$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 15 & 66 \\ 18 & 30 & 153 \end{bmatrix}$$

$$2 \times 2 \cdot 2 \times 3 = 2 \times 3$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
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2010

Matrix  
Multiplication

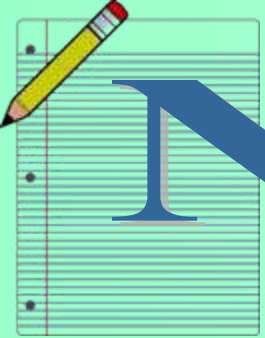
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 1

Column 1

$$\begin{bmatrix} 3 \cdot 6 + 1 \cdot -12 \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

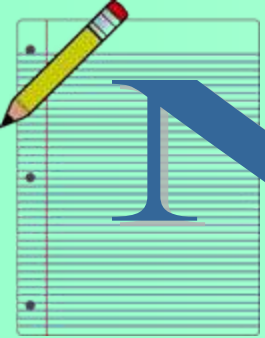
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 1

Column 1

$$\begin{bmatrix} 18 - 12 \\ \phantom{18 - 12} \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

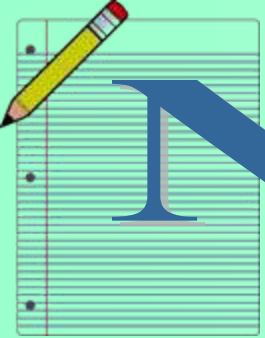
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 1

Column 1

$$\begin{bmatrix} 6 \\ \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

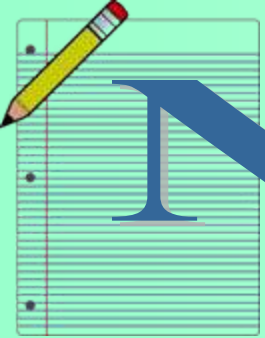
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 1

Column 2

$$\begin{bmatrix} 6 & 3 \cdot 0 + 1 \cdot 15 \\ \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

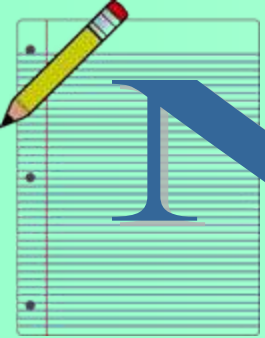
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 1

Column 2

$$\begin{bmatrix} 6 & 0 + 15 \\ \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
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2010

Matrix  
Multiplication

$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

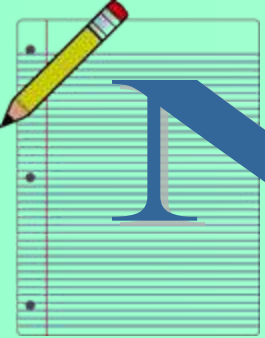
Row 1

Column 2

$$\begin{bmatrix} 6 & 15 \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?





# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

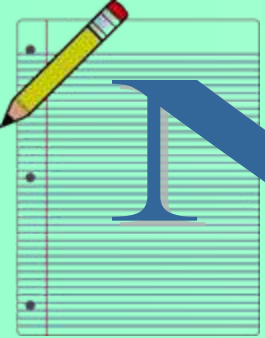
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 1

Column 3

$$\begin{bmatrix} 6 & 15 & 3 \cdot 21 + 1 \cdot 3 \\ & & \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

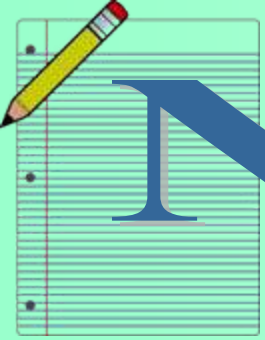
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 1

Column 3

$$\begin{bmatrix} 6 & 15 & 63 + 3 \\ & & \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

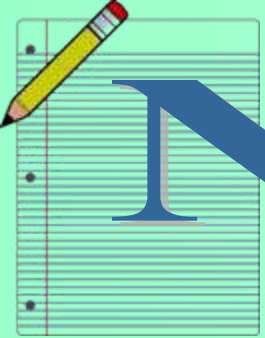
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 1

Column 3

$$\begin{bmatrix} 6 & 15 & 66 \\ & & \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

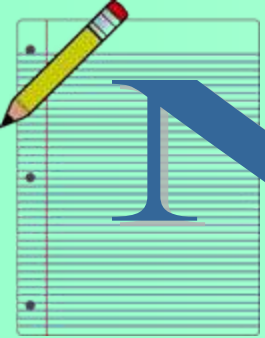
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 2

Column 1

$$\begin{bmatrix} 6 & 15 & 66 \\ 7 \cdot 6 + 2 \cdot -12 \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

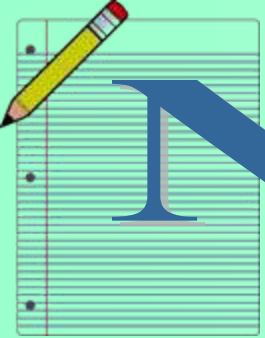
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 2

Column 1

$$\begin{bmatrix} 6 & 15 & 66 \\ 42 - 24 \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

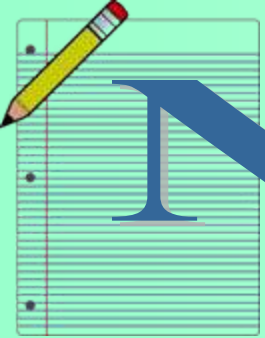
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 2

Column 1

$$\begin{bmatrix} 6 & 15 & 66 \\ 18 & & \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

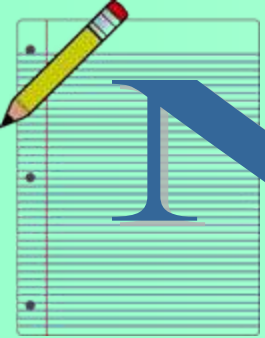
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 2

Column 2

$$\begin{bmatrix} 6 & 15 & 66 \\ 18 & 7 \cdot 0 + 2 \cdot 15 & \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

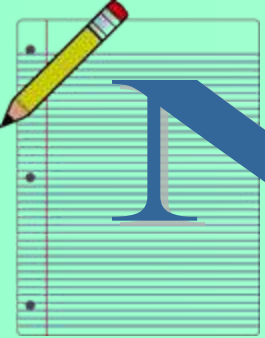
Row 2

Column 2

$$\begin{bmatrix} 6 & 15 & 66 \\ 18 & 0 + 30 & \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?





# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

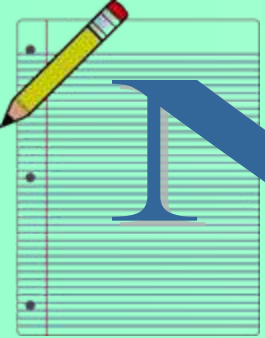
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 2

Column 2

$$\begin{bmatrix} 6 & 15 & 66 \\ 18 & 30 & \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

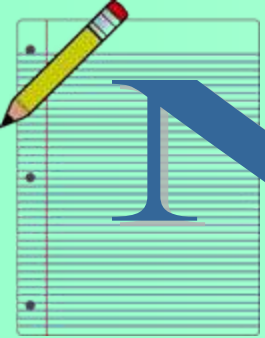
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 2

Column 3

$$\begin{bmatrix} 6 & 15 & 66 \\ 18 & 30 & 7 \cdot 21 + 2 \cdot 3 \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

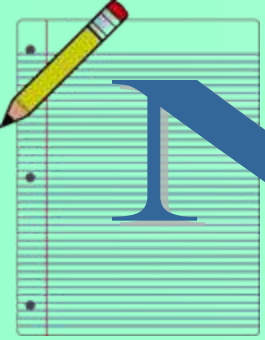
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 2

Column 3

$$\begin{bmatrix} 6 & 15 & 66 \\ 18 & 30 & 147 + 6 \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

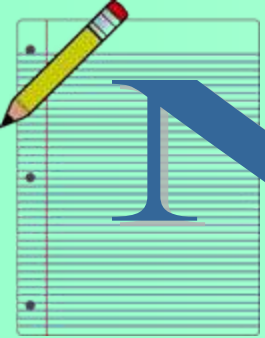
$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

Row 2

Column 3

$$\begin{bmatrix} 6 & 15 & 66 \\ 18 & 30 & 153 \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Notes

Monday,  
December 13,  
2010

Matrix  
Multiplication

$$\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix} =$$

$$\begin{bmatrix} 6 & 15 & 66 \\ 18 & 30 & 153 \end{bmatrix}$$

EQ: What are the similarities and differences between matrices and real numbers?



# Practice

Monday,  
December 13,  
2010

$$\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 5 & 1 \\ 2 & -3 \end{bmatrix} =$$

EQ: What are the similarities and differences between matrices and real numbers?



# Practice

Monday,  
December 13,  
2010

Use your calculator to find each product:

$$A = \begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 1 & 3 \\ 9 & -8 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 5 & -3 \\ 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} -4 & 7 \\ 9 & 2 \end{bmatrix}$$

- 1) C·D
- 2) D·C
- 3) D·A
- 4) A·C
- 5) C·A

EQ: What are the similarities and differences between matrices and real numbers?



# Class Work

Monday,  
December 13,  
2010

- Worksheet: Multiplying Matrices #1

EQ: What are the similarities and differences between matrices and real numbers?





# Home Work

Monday,  
December 13,  
2010

## Worksheet: Multiplying Matrices #2

EQ: What are the similarities and differences between matrices and real numbers?