Section 4.3 – Multiplying Matrices

MATRIX MULTIPLICATION

1. The order makes a difference…AB is different from BA.
2. The number of columns in first matrix must equal number of rows in second matrix.
   In other words, the inner dimensions must be equal.
3. **The answer** will be number of rows in first matrix by number of columns in second matrix.
   In other words, the outside dimensions.

Are the following matrix multiplications possible?

$$\begin{bmatrix} 4 \\ 2 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \end{bmatrix}$$

2 x 1 1 x 2

$$\begin{bmatrix} 3 & 1 \end{bmatrix} \times \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} \square \end{bmatrix}$$

1 x 2 2 x 1

Are the following matrix multiplications possible?

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 5 \end{bmatrix}$$

NO

3 x 2 3 x 2

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & 9 & 9 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 4 \\ 5 & 8 \end{bmatrix}$$

YES

2 x 3 3 x 2

$$\begin{bmatrix} 7 & 0 \\ 2 & \pi \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 \\ 2 & 4 & 3 \end{bmatrix}$$

YES

3 x 2 2 x 3

$$\begin{bmatrix} 3 \\ 1 \\ 8 \end{bmatrix} \times \begin{bmatrix} 2 & 1 & 2 \end{bmatrix}$$

YES

3 x 1 1 x 3

$$\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 5 \\ 4 & 2 \end{bmatrix}$$

YES

2 x 2 2 x 2

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \\ 5 & 7 & 2 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 2 & 7 \end{bmatrix}$$

NO

3 x 3 2 x 2

What is the dimension of the following products, if possible?

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 5 \end{bmatrix}$$

3 x 2 3 x 2

NO

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & 9 & 9 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 4 \\ 5 & 8 \end{bmatrix}$$

2 x 3 3 x 2

YES

2 x 2

$$\begin{bmatrix} 7 & 0 \\ 2 & \pi \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 \\ 2 & 4 & 3 \end{bmatrix}$$

3 x 2 2 x 3

YES

3 x 3

$$\begin{bmatrix} 3 \\ 1 \\ 8 \end{bmatrix} \times \begin{bmatrix} 2 & 1 & 2 \end{bmatrix}$$

3 x 1 1 x 3

YES

3 x 3

$$\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 5 \\ 4 & 2 \end{bmatrix}$$

2 x 2 2 x 2

YES

2 x 2

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \\ 5 & 7 & 2 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 2 & 7 \end{bmatrix}$$

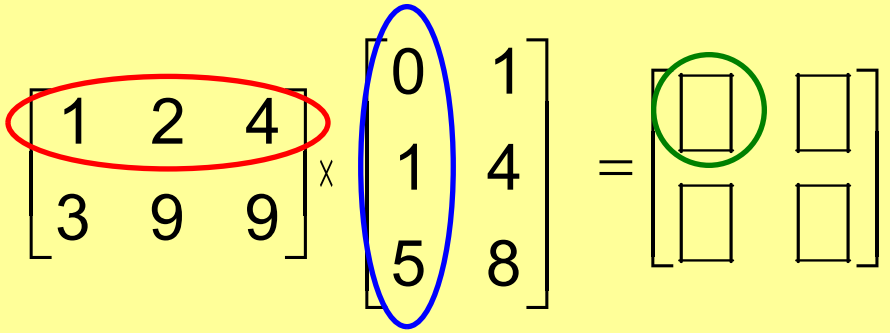
3 x 3 2 x 2

NO

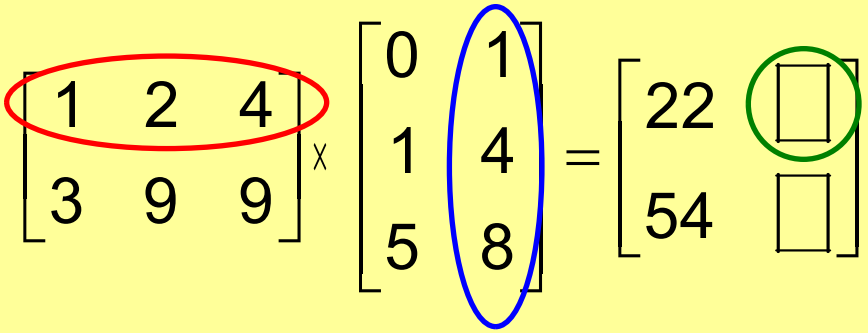
MATRIX MULTIPLICATION

The product of two matrices is found by multiplying the corresponding elements in each row by each column and then adding them together.

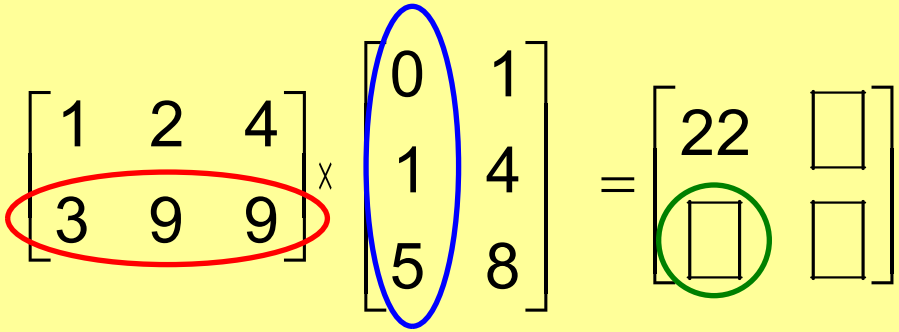
ROW…COLUMN



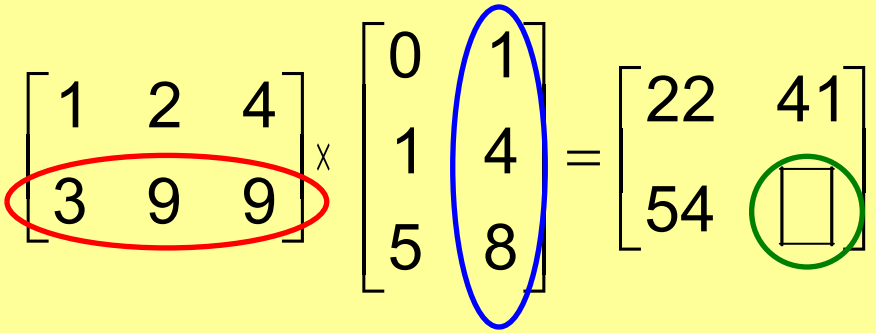
$$(1)(0)+(2)(1)+(4)(5)=22$$



$$(1)(1)+(2)(4)+(4)(8)=41$$



$$(3)(0)+(9)(1)+(9)(5)=54$$



$$(3)(1)+(9)(4)+(9)(8)=111$$

$$\begin{bmatrix} 22 & 41 \\ 54 & 111 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 3 & 7 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 6 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \begin{bmatrix} 42 & 58 \\ 30 & 38 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 1 \\ 8 \end{bmatrix} \times \begin{bmatrix} 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} = \begin{bmatrix} 6 & 3 & 6 \\ 2 & 1 & 2 \\ 16 & 8 & 16 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 5 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \begin{bmatrix} 10 & 12 \\ 0 & 0 \end{bmatrix}$$

MATRIX MULTIPLICATION SUMMARY

1. Is the multiplication possible?
2. If yes, what is the dimension of the product?
3. Create a “blank” matrix.
4. “Multiply/Add” corresponding rows and columns

$$\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & 9 \\ -3 & 4 & -3 \\ 2 & 4 & 7 \end{bmatrix}$$