Mathematics II Frameworks
Student Edition

Unit 2
Right Triangle
Trigonometry

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Mathematics II
Unit 2

Right Triangle Trigonometry

INTRODUCTION

In this unit students explore the relationships that exist among and between sides and angles of right triangles. Students build upon their previous knowledge of similar triangles and of the Pythagorean theorem to determine the side length ratios in special right triangles and to understand the conceptual basis for the functional ratios sine, cosine, and tangent. They explore how the values of these trigonometric functions relate in complementary angles and how to use these trigonometric ratios to solve problems. Through the work in this unit, students not only develop the skills and understanding needed for the study of many technical areas but also build a strong foundation for future study of trigonometric functions of real numbers in Mathematics IV.

ENDURING UNDERSTANDINGS
- The relationships among the lengths of the legs and the hypotenuse are the same for all right triangles with acute angles of 30 degrees and 60 degrees and can be derived by halving an equilateral triangle using an altitude.
- The relationships among the lengths of the legs and hypotenuse are the same for all right triangles with two acute angles of 45 degree angles and can be derived by halving a square along the diagonal.
- Similar right triangles produce trigonometric ratios.
- Trigonometric ratios are dependent only on angle measure.
- Trigonometric ratios can be used to solve application problems involving right triangles.

KEY STANDARDS ADDRESSED

MM2G1. Students will identify and use special right triangles.
a. Determine the lengths of sides of 30°- 60°- 90° triangles.
b. Determine the lengths of sides of 45°- 45°- 90° triangles.

MM2G2. Students will define and apply sine, cosine, and tangent ratios to right triangles.
a. Discover the relationship of the trigonometric ratios for similar triangles.
b. Explain the relationship between the trigonometric ratios of complementary angles.
c. Solve application problems using the trigonometric ratios.

RELATED STANDARDS ADDRESSED

MM2P1. Students will solve problems (using appropriate technology).
a. Build new mathematical knowledge through problem solving.
b. Solve problems that arise in mathematics and in other contexts.
c. Apply and adapt a variety of appropriate strategies to solve problems.
d. Monitor and reflect on the process of mathematical problem solving.

MM2P2. Students will reason and evaluate mathematical arguments.
   a. Recognize reasoning and proof as fundamental aspects of mathematics.
   b. Make and investigate mathematical conjectures.
   c. Develop and evaluate mathematical arguments and proofs.
   d. Select and use various types of reasoning and methods of proof.

MM2P3. Students will communicate mathematically.
   a. Organize and consolidate their mathematical thinking through communication.
   b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
   c. Analyze and evaluate the mathematical thinking and strategies of others.
   d. Use the language of mathematics to express mathematical ideas precisely.

MM2P4. Students will make connections among mathematical ideas and to other disciplines.
   a. Recognize and use connections among mathematical ideas.
   b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
   c. Recognize and apply mathematics in contexts outside of mathematics.

MM2P5. Students will represent mathematics in multiple ways.
   a. Create and use representations to organize, record, and communicate mathematical ideas.
   b. Select, apply, and translate among mathematical representations to solve problems.
   c. Use representations to model and interpret physical, social, and mathematical phenomena.

UNIT OVERVIEW:

The Georgia Performance Standards address geometry and algebra extensively in grades 6 through 8 with more depth relative to the topics covered than in the previous Quality Core Curriculum. In sixth grade, students use ratios to describe the constant of variation in situations involving direct variations. In the seventh grade students investigate similar figures, learn the basic formal constructions, justify formal construction procedures through properties of congruent triangles, and perform transformations in the plane and in space. In the eighth grade students explore the properties of parallel and perpendicular lines and investigate right triangles, applying the Pythagorean Theorem. In Mathematics I, students discover, prove, and apply properties of triangles, quadrilaterals, and other polygons. In Unit 1 of this course, students solve for sides of triangles using quadratic equations based on the Pythagorean Theorem. In this unit, students build on the extensive geometry background obtained through study of ratios in similar figures, geometric constructions, right triangles, and the Pythagorean Theorem.

Through early content of the unit, students focus on the conceptual basis for the trigonometric ratios through examination of historical and modern applications of using the measures known about one right triangle to solve for corresponding parts of a similar right triangle. Some of these examples involve triangles that students construct with ruler and compass and then measure. In contrast, students follow paths of deductive reasoning about equilateral triangles and squares to determine the relationships among the lengths of the sides in the special right triangles of the

forms 30°- 60°- 90° and 45°- 45°- 90°, respectively.

When the formal definitions of sine, cosine, and tangent are first presented, students use constructed triangles to make by hand a trigonometric table for selected angle values. Only after students have made this table and explored relationships of trigonometric ratios for complementary angles should students begin use calculators to obtain values of the sine, cosine, and tangent. In addition, even after students have transitioned to use of the calculator, they should be required to give exact as well as approximate values when one of the special right triangles is involved.

Once students are using their calculator for finding values of trigonometric functions, they focus on problem solving in real-world contexts. With the aid of calculator, they solve for measures of angles in triangles as well as lengths of sides. Throughout the unit, emphasis is placed on trigonometric ratios as constant relationships in right triangles that are similar to each other and the use of these ratios in conjunction with other geometric knowledge to solve for the unknown parts of given right triangles.

Critical Previously Learned Terms and Properties

Complementary angles: Two angles whose sum is 90° are called **complementary.** Each angle is called the complement of the other.

Right triangle: A right triangle is a triangle in which one of the interior angles is a right angle.
- Each of the non-right angles in a right triangle is an acute angle.
- The acute angles in a right triangle are complementary.
- The side of a right triangle opposite the 90° angle is called the hypotenuse; each of the other sides is called a leg.

Similar triangles: Triangles are similar if they have the same shape but not necessarily the same size.
- Triangles whose corresponding angles are congruent are similar.
- Corresponding sides of similar triangles are all in the same proportion.
- Thus, for the similar triangles shown at the right with angles A, and C congruent to angles A**'**, B**'**, and C**'** respectively, we have that: B,

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}.$$
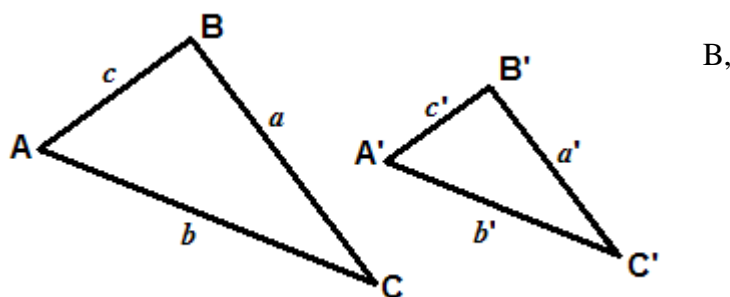
Terms and Definitions

Opposite side: In a right triangle, the side of the triangle opposite the vertex of an acute angle is called the opposite side relative to that acute angle.

Adjacent side: In a right triangle, for each acute angle in the interior of the triangle, one ray forming the acute angle contains one of the legs of the triangle and the other ray contains the hypotenuse. This leg on one ray forming the angle is called the adjacent side of the acute angle.

For any acute angle in a right triangle, we denote the measure of the angle by θ and define three numbers related to θ as follows:

$$\text{sine of } \theta = \sin \theta = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

$$\text{cosine of } \theta = \cos \theta = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$$

$$\text{tangent of } \theta = \tan \theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

Properties, theorems, and corollaries:

- For the **similar triangles**, as shown above, with angles A, B, and C congruent to angles A', B', and C' respectively, the following proportions follow from the proportion between the triangles.

$$\frac{a}{a'} = \frac{b}{b'} \text{ if and only if } \frac{a}{b} = \frac{a'}{b'} ; \qquad \frac{a}{a'} = \frac{c}{c'} \text{ if and only if } \frac{a}{c} = \frac{a'}{c'} ;$$

$$\text{and } \frac{b}{b'} = \frac{c}{c'} \text{ if and only if } \frac{b}{c} = \frac{b'}{c'} .$$

Three separate equalities are required for these equalities of ratios of side lengths in one triangle to the corresponding ratio of side lengths in the similar triangle because, in general, these are three different ratios.

The general statement is that the ratio of the lengths of two sides of a triangle is the same as the ratio of the corresponding sides of any similar triangle.

- In any 30°-60°-90° right triangle, the lengths of the hypotenuse, shorter leg, and longer leg follow the pattern: $2a, \ a, \ a\sqrt{3}$, respectively.

- In any 45°- 45°- 90° triangle, the two legs have the same length and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

- For each pair of complementary angles in a right triangle, the sine of one angle is the cosine of its complement.

- For each pair of complementary angles in a right triangle, the tangent of one angle is the reciprocal of the tangent of its complement.

Tasks:

The remaining content of this framework consists of student learning tasks designed to allow students to learn by investigating situations with a real-world context. The first learning task is intended to launch the unit by examining how constant ratios in similar right triangles were used by Eratosthenes to determine the radius and circumference of the Earth in 200 BC. Its primary focus is to explore the relationships between sides and angles of similar right triangles. In the second task students determine the exact relationships among lengths of the legs and the hypotenuse in 30°- 60°- 90° and 45°- 45°- 90° right triangles. In the third task, students identify right triangles in their environments, use the fixed ratios of side lengths in similar right triangles to solve a modern application problem, and then write and solve their own analogous problems. In this task, students find the needed values of side length ratios by measuring the sides of triangles that they have constructed to have a particular shape or that they have found in the environment.

The fourth task introduces the definitions of the sine, cosine, and tangent trigonometric ratios as functions whose inputs are the degree measures of acute angles in right triangles. In that task, students use construction and measurement of a collection of right triangles to develop a limited table of values for trigonometric ratios. In the fifth task, students explore the relationships between trigonometric ratios for complementary angles. The last learning task of the unit is devoted to a thorough exploration of using sine, cosine, and tangent to solve a wide variety of problems in real world context. The culminating task is designed to demonstrate the type of assessment activities students should be comfortable with by the end of the unit and gives students a chance to apply many of the ideas they have learned throughout the unit.

**Eratosthenes Finds the Circumference of the Earth Learning Task**

As Carl Sagan says in the television series <u>Cosmos</u>:

> Two complex ideas, the wheel and the globe, are grooved into our minds from infancy. It was only 5500 years ago that we finally saw how a rotating wheel could produce forward motion. Recognizing that Earth's apparently flat surface bends into the shape of a sphere was even more recent. Some cultures imagined Earth as a disc, some, box-shaped. The Egyptians said it was an egg, guarded at night by the moon. Only 2500 years ago, the Greeks finally decided Earth was a sphere. Plato argued that, since the sphere is a perfect shape, Earth must be spherical. Aristotle used observation. He pointed to the circular shadow Earth casts on the moon during an eclipse.

> The Greeks had no way of knowing how large the globe might be. The most daring travelers saw Earth reaching farther still beyond the fringe of their journeys. Then, in 200 BC, travelers told the head of the Alexandria Library, Eratosthenes, about a well near present-day Aswan. The bottom of the well was lit by the sun at noon during the summer solstice. At that moment the sun was straight overhead. Eratosthenes realized he could measure the shadow cast by a tower in Alexandria while no shadow was being cast in Aswan. Then, knowing the distance to Aswan, it'd be simple to calculate Earth's radius.

In this task, you will examine the mathematics that Eratosthenes used to make his calculations and explore further the mathematics developed from the relationships he used.

1. Looking at the diagram below, **verify that** the two triangles are similar: the one formed by the sun's rays, the tower, and its shadow, and the one formed by the sun's rays, the radius of the earth, and the distance to Aswan (ignore the curvature of the Earth as Eratosthenes did) Explain your reasoning.
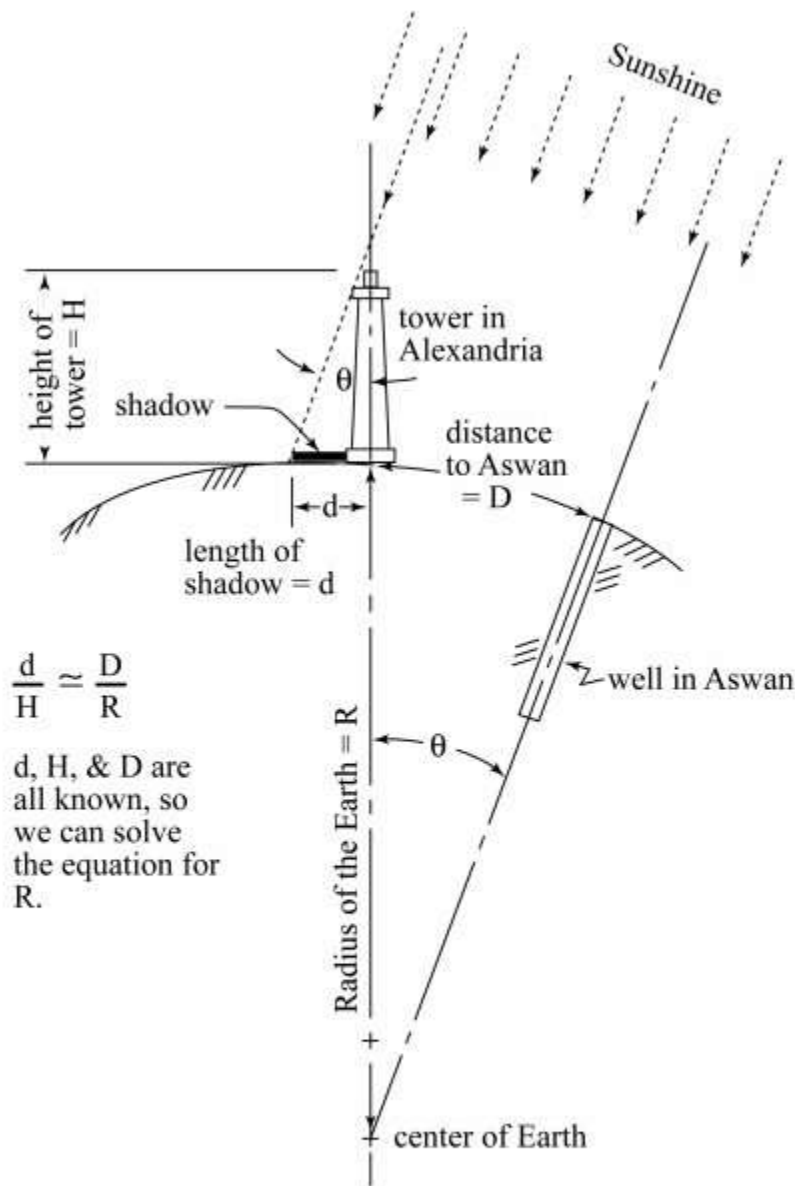

The above diagram is reproduced from the transcript and accompanying diagram for episode 1457 of the radio program The Engines of Our Ingenuity at http://www.uh.edu/engines/epi1457.htm .
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2. Focus on the two similar triangles from the diagram in *Item 1*.

   a. Write a proportion that shows the relationship of the small triangle to the large triangle.

   b. Rearrange the similarity statement in *part a* to match the statement in the given diagram in *Item 1*. Explain why this proportion is a true proportion.

3. Now, look at the triangles isolated from the diagram as shown at the right.

   a. Knowing that these triangles represent the original diagram, where should the right angles be located?

   b. Using the right angles that you have identified, identify the legs and hypotenuse of each right triangle.

   c. Rewrite your proportion from above using the segments from triangle ABC and triangle DEF.
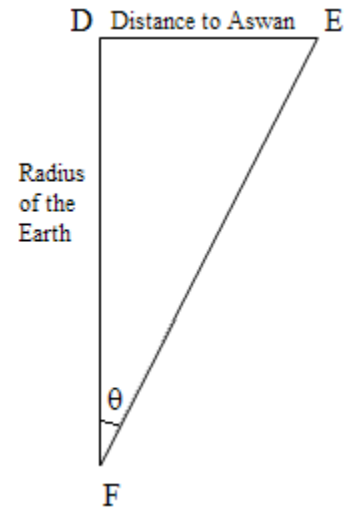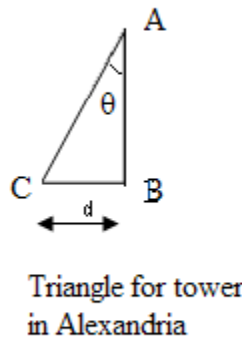
   d. By looking at the triangle ABC, describe how the sides AB and BC are related to angle θ.

   e. By looking at the triangle DEF, describe how the sides DE and DF are related to angle θ.

4. If we rearrange the triangles so that the right angles and corresponding line segments align as shown in the figure below, let's look again at the proportions and how they relate to the angles of the triangles.
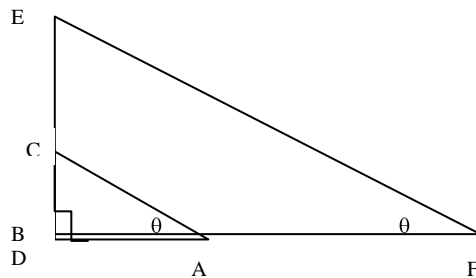
a. By looking at the proportion you wrote in 3c, $\dfrac{\overline{BC}}{\overline{AB}} = \dfrac{\overline{DE}}{\overline{DF}}$ and the answers to 3d

and 3e, when would a proportion like this always be true? Is it dependent upon the length of the sides or the angle measures? Do the triangles always have to be similar right triangles? Why or why not?

Discovering Special Triangles Learning Task

Part 1

1. Adam, a construction manager in a nearby town, needs to check the uniformity of Yield signs around the state and is checking the heights (altitudes) of the Yield signs in your locale. Adam knows that all yield signs have the shape of an equilateral triangle. Why is it sufficient for him to check just the heights (altitudes) of the signs to verify uniformity?

2. A Yield sign from a street near your home is pictured to the right. It has the shape of an equilateral triangle with a side length of 2 feet. If you draw the altitude of the triangular sign, you split the Yield sign in half vertically, creating two 30°-60°-90° right triangles, as shown to the right. For now, we'll focus on the right triangle on the right side. (We could just as easily focus on the right triangle on the left; we just need to pick one.) We know that the hypotenuse is 2 ft, that information is given to us. The shorter leg has length 1 ft. **Why?**
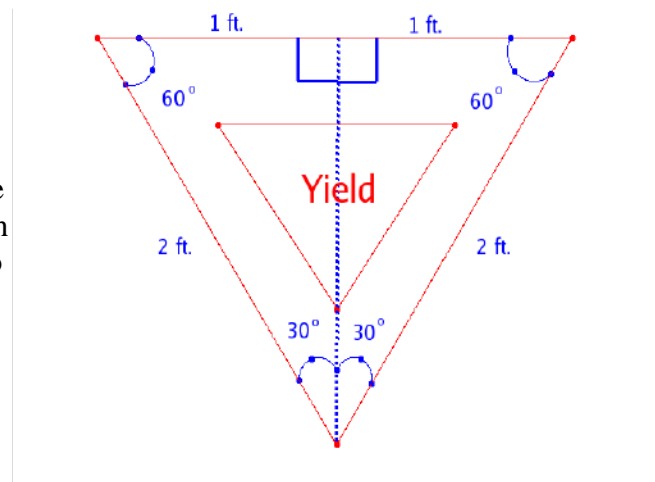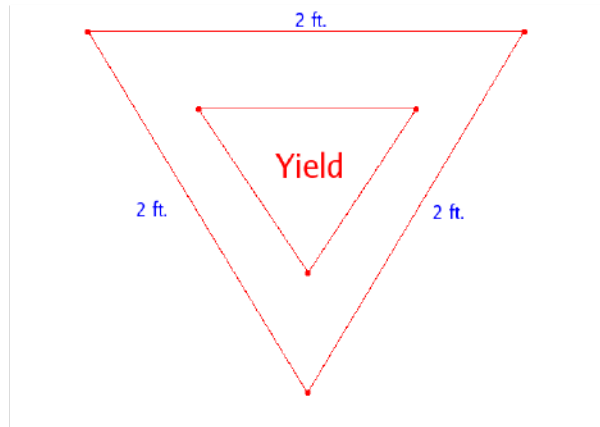
   Verify that the length of the third side, the altitude, is $\sqrt{3}$ ft.

3. The construction manager, Adam, also needs to know the altitude of the smaller triangle within the sign. Each side of this smaller equilateral triangle is 1 ft. long. **Explain why** the altitude of this equilateral triangle is $\sqrt{3}\big/2$.
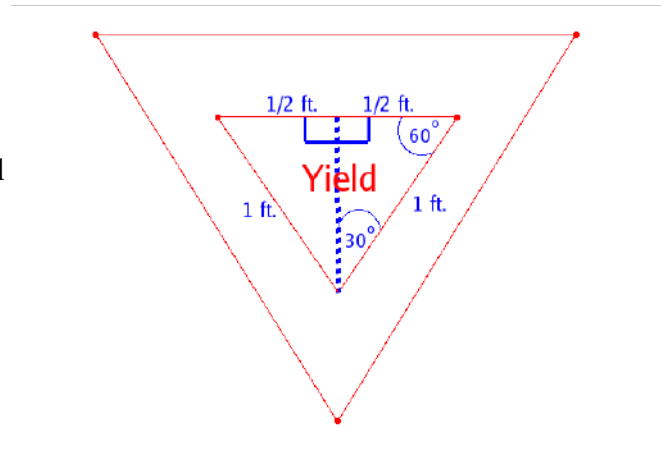
4. Now that we have found the altitudes of both equilateral triangles, we look for patterns in the data. Fill in the first two rows of the chart below, and write down any observations you make. Then fill in the third and fourth rows.

| Side Length of	Each 30°- 60°- 90° right triangle formed by drawing altitude

Equilateral Triangle	Hypotenuse Length	Shorter Leg Length	Longer Leg Length
2			
1			
4			
6			

5. What is true about the lengths of the sides of any 30°-60°-90° right triangle? How do you know?

6. Use your answer for *Item 5* as you complete the table below. Do not use a calculator; leave answers exact.

30°-60°-90° triangle	Δ #1	Δ #2	Δ #3	Δ #4	Δ #5	Δ #6	Δ #7	Δ #8
hypotenuse length	11				$3\sqrt{5}$			
shorter leg length		π		$\frac{12}{5}$		$\sqrt{3}$		$\frac{\sqrt{2}}{2}$
longer leg length			$\frac{\sqrt{3}}{7}$				4	

Part 2

A baseball diamond is, geometrically speaking, a square turned sideways. Each side of the diamond measures 90 feet. (See the diagram to the right.) A player is trying to slide into home base, but the ball is all the way at second base. Assuming that the second baseman and catcher are standing in the center of second base and home, respectively, we can calculate how far the second baseman has to throw the ball to get it to the catcher.
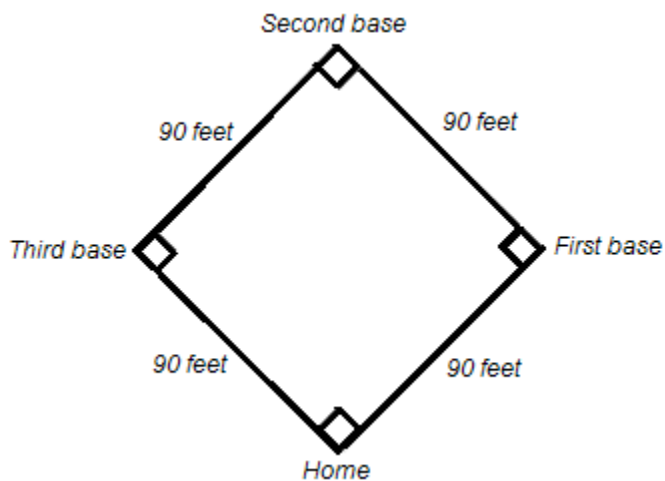
7. If we were to split the diamond in half vertically, we would have two 45°- 45°- 90° right triangles. (The line we would use to split the diamond would bisect the 90° angles at home and second base, making two angles equal to 45°, as shown in the baseball diamond to the right below.) Let us examine one of these 45°- 45°- 90° right triangles. You know that the two legs are 90 feet each. Using the Pythagorean theorem, **verify** that the hypotenuse, or the

displacement of the ball, is $90\sqrt{2}$ feet (approximately 127.3 feet) long.

8. Without moving from his position, the catcher reaches out and tags the runner out before he gets to home base. The catcher then throws the ball back to a satisfied pitcher, who at the time happens to be standing at the exact center of the baseball diamond. We can calculate the displacement of the ball for this throw also. Since the pitcher is standing at the center of the field and the catcher is still at home base, the throw will cover half of the distance we just found in *Item 7*. Therefore, the distance for this second throw is $45\sqrt{2}$ feet , half of $90\sqrt{2}$, or approximately 63.6 feet. If we were to complete the triangle between home base, the center of the field, and first base, we would have side lengths of $45\sqrt{2}$ feet, $45\sqrt{2}$ feet, and 90 feet.

    a. Now that we have found the side lengths of two 45°- 45°- 90° triangles, we can observe a pattern in the lengths of sides of all 45°- 45°- 90° right triangles. Using the exact values written using square root expressions, fill in the first two rows of the table at the right.

| In each 45°- 45°- 90° right triangle		
Leg Length	Other Leg Length	Hypotenuse Length
90 ft.		
$45\sqrt{2}$ ft.		

    b. Show, by direct calculation, that the entries in the second row are related in the same way as the entries in the second row.

9. What is true about the lengths of the sides of any 45°- 45°- 90° right triangle? How do you know?

10. Use your answer for *Item 9* as you complete the table below. Do not use a calculator; leave answers exact.

45°-45°-90° triangle	Δ #1	Δ #2	Δ #3	Δ #4	Δ #5	Δ #6	Δ #7	Δ #8
hypotenuse length			11				$3\sqrt{5}$	
one leg length		π		$\dfrac{\sqrt{2}}{2}$		$\sqrt{3}$		$\dfrac{12}{5}$
other leg length	4				$\dfrac{\sqrt{3}}{7}$			

Finding Right Triangles in Your Environment Learning Task

1. Look around you in your room, your school, your neighborhood, or your city. Can you find right triangles in everyday objects? List at least ten right triangles that you find. Draw pictures of at least three of them, labeling the 90° angle that makes the triangle a right triangle.

2. An older building in the school district sits on the side of a hill and is accessible from ground level on both the first and second floors. However, access at the second floor requires use of several stairs. Amanda and Tom have been given the task of designing a ramp so that people who cannot use stairs can get into the building on the second floor level. The rise has to be 5 feet, and the angle of the ramp has to be 15 degrees.

    a. Tom and Amanda need to determine how long that ramp should be. One way to do this is to use a compass and straightedge to construct a 15°- 75°- 90° triangle on your paper. Such a triangle must be similar to the triangle defining the ramp. ***Explain why the triangles are similar.***
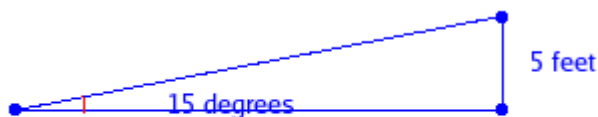
    The diagram is not to scale.

    b. Construct a 15°- 75°- 90° triangle on your paper using straightedge and compass. Use a protractor to verify the angle measurements. (Alternative: If dynamic geometry software is available, the construction and verification of angle measurements can be done using the software.) You'll use this triangle in *part c*.

    c. Use similarity of the ramp triangle and measurements from your constructed triangle to find the length of the ramp. (Save the triangle and its measurements. You'll need them in the next Learning Task also.)

3. Choose one of the types of right triangles that you described in *Item 1* and make up a problem similar to *Item 2* using this type of triangle. Also find an existing right triangle that you can measure, measure the angles and sides of this existing triangle, and then choose numbers for the problem you make up so that the measurements of the existing triangle can be used to solve the problem you make up. Pay careful attention to the information given in the ramp problem, and be sure to provide, and ask for, the same type of information in your problem.

4. Exchange the triangle problems from *Item 3* among the students in your class.
    a. Each student should get the problem and a sketch of the existing right triangle (along with its measurements) from another student, and then solve the problem from the other student.

    b. For each problem, the person who made up the problem and the person who worked the

problem should agree on the solution.

Create Your Own Triangles Learning Task

1. Using construction paper, compass, straightedge, protractor, and scissors, make and cut out nine right triangles. One right triangle should have an acute angle of 5°, the next should have an acute angle of 10°, and so forth, all the way up to 45°. Note that you should already have a constructed right triangle with an angle of 15° that you saved from the *Finding Right Triangles in the Environment Learning Task.* You may use it or make a new one to have all nine triangles.

   As you make the triangles, you should construct the right angles and, whenever possible, construct the required acute angle. You can use the protractor in creating your best approximation of those angles, such as 5°, for which there is no compass and straightedge construction or use alternate methods involving a marked straightedge.

   As you make your triangles, label both acute angles with their measurements in degrees and label all three sides with their measurement in centimeters to the nearest tenth of a centimeter.

Using what we found to be true about ratios from similar right triangles in the Circumference of the Earth Task, we are now ready to define some very important new functions. **For any acute angle in a right triangle, we denote the measure of the angle by θ and define three numbers related to θ as follows:**

$$\textbf{sine of } \boldsymbol{\theta} = \frac{\text{length of leg opposite the vertex of the angle}}{\text{length of hypotenuse}}$$

$$\textbf{cosine of } \boldsymbol{\theta} = \frac{\text{length of leg adjacent to the vertex of the angle}}{\text{length of hypotenuse}}$$

$$\textbf{tangent of } \boldsymbol{\theta} = \frac{\text{length of leg opposite the vertex of the angle}}{\text{length of leg adjacent to the vertex of the angle}}$$

In the figure at the right below, the terms "opposite," "adjacent," and "hypotenuse" are used as shorthand for the lengths of these sides. Using this shorthand, we can give abbreviated versions of the above definitions:

$$\text{sine of } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine of } \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
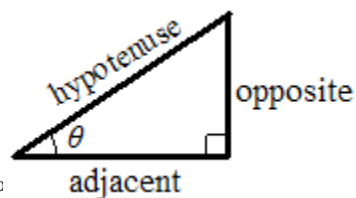
tangent of $\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

2. Using the measurements from the triangles that you created in doing *Item 1* above, for each acute angle listed in the table below, complete the row for that angle. The first three columns refer to the lengths of the sides of the triangle; the last columns are for the sine of the angle, the cosine of the angle, and the tangent of the angle.   Remember that which side is opposite or adjacent depends on which angle you are considering.   (Hint: For angles greater than 45°, try turning your triangles sideways.)
    For the last three columns, write your table entries as fractions (proper or improper, as necessary, but no decimals in the fractions) .

angle measure	opposite	adjacent	hypotenuse	sine (opp/hyp)	cosine (adj/hyp)	tangent (opp/adj)
5°						
10°						
15°						
20°						
25°						
30°						
35°						
40°						
45°						
50°						
55°						
60°						
65°						

70°						
75°						
80°						
85°						

TABLE 1

3. Look back at the *Discovering Special Triangles Learning Task, Item 4*.

    a. Use the lengths in the first row of the table from *Item 4* of that learning task to find the
        values of sine, cosine, and tangent to complete the **Table 2** below.

    TABLE 2.

angle	sine	cosine	tangent
30°			

    b. All right triangles with a 30° angle should give the same values for the sine, cosine, and
        tangent ratios as those in **Table 2**. Why?

    c. Do the values for the sine, cosine, and tangent of a 30° angle that you found for **Table 1**
        (by using measurements from a constructed triangle) agree with the values you found for
        the sine, cosine, and tangent of a 30° angle in **Table 2**? If they are different, why doesn't
        this contradict *part b*?

4. Look back at the *Discovering Special Triangles Learning Task, Item 8*.

    a. Use the table values from *Item 8, part a,* to complete the table below with **exact values** of
        sine, cosine, and tangent for an angle of 45°.

    TABLE 3

angle	sine	cosine	tangent
45°			

    b. How do the values of sine, cosine, and tangent of 45° that you found for Table 1 compare
        to the exact values from *part a*? What can you conclude about the accuracy of your
        construction and measurements?

5. If T is any right triangle with an angle of 80°, approximately what is the ratio of the opposite
    side to the hypotenuse? Explain.

6. If we changed the measure of the angle in *Item 5* to another acute angle measure, how would
    your answer change?

7. Explain why the trigonometric ratios of sine, cosine, and tangent define **functions of θ,
    where 0° < θ < 90°**.

8. Are the functions sine, cosine, and tangent linear functions? Why or why not?

# Discovering Trigonometric Ratio Relationships Learning Task

Now that you have explored the trigonometric ratios and understand that they are functions which use degree measures of acute angles from right triangles as inputs, we can introduce some notation that makes it easier to work with these values.

We considered these abbreviated versions of the definitions earlier.

$$\text{sine of } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine of } \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent of } \theta = \frac{\text{opposite}}{\text{adjacent}}$$
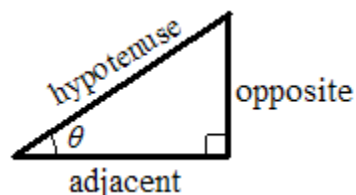
Now, we'll introduce the notation and abbreviate a bit more. In higher mathematics, the following notations are standard.

sine of θ is denoted by $\sin(\theta)$

cosine of θ is denoted by $\cos(\theta)$

tangent of θ is denoted by $\tan(\theta)$

1. Refer back to Table 1 from the *Create Your Own Triangles Learning Task* .

    a. Look up the Table 1 values for sin(10°), cos(10°), and tan(10°). Divide sin(10°) by cos(10°), and simplify the fraction. What else does your answer represent?

    b. Repeat this process of *part a* for several other acute angles in Table 1. What do you notice about the relationship among sine, cosine, and tangent? This relationship holds for any acute angle, even an acute angle not in Table 1. Why is this?

    c. Write the relationship among sine, cosine, and tangent as an **identity equation** (one that is always true) involving sin(θ), cos(θ), and tan(θ).

2. Choose any one of the cut-out triangles created in the *Create Your Own Triangles Learning Task*. Identify the pair of complementary angles within the triangle. (Reminder: complementary angles add up to 90°.) Select a second triangle and identify the pair of complementary angles. Is there a set of complementary angles in every right triangle? Explain your reasoning.

3. Use the two triangles you chose in *Item 2* to complete the table below. What relationships among the values do you notice? Do these relationships hold true for all pairs of

complementary angles in right triangles? Explain your reasoning.

| triangle # and angle	triangle #	θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
1 – smaller angle	1				
1 – larger angle	1				
2 – smaller angle	2				
2 – larger angle	2				

5. Summarize the relationships you stated in *Item 3*.

a. If θ is the degree measure of an acute angle in a right triangle, what is the measure of its complement?

b. State the relationships from *Item 3* as identity equations involving sines, cosines, and/or tangents of θ and the measure of its complement. Use the expression from *part a.*

Find That Side or Angle Learning Task

1. A ladder is leaning against the outside wall of a building.
   The figure at the right shows the view from the end of the
   building, looking directly at the side of the ladder. The
   ladder is exactly10 feet long and makes an angle of 60° with
   the ground. If the ground is level, what angle does the
   ladder make with the side of the building? How far up the
   building does the ladder reach (give an exact value and then
   approximate to the nearest inch)? Hint: Use a known
   trigonometric ratio in solving this problem.]
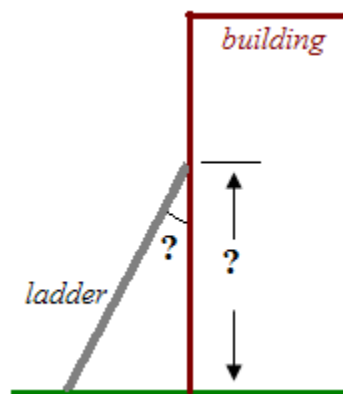
2. One afternoon, a tree casts a shadow that
   is 35.6 feet long. At that time, the angle
   of elevation of the sun is 45°, as shown
   in the figure at the right. We speak of
   the angle of elevation of the sun because
   we must raise, or elevate, our eyes from
   looking straight ahead (looking in the
   horizontal direction) to see the position
   of the sun above us. (Due to the distance
   from the sun to the Earth, rays of
   sunlight are all parallel, as shown in the
   figure.)
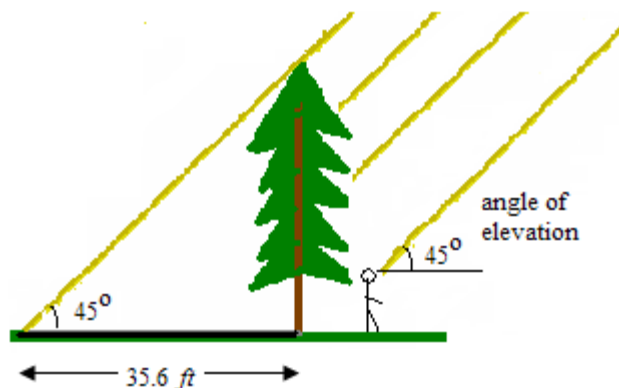
   a. How tall is the tree?

   b. What is tan(45°)?

   c. What's the connection between *part a* and *part b?*

The first two problems in this task involve trigonometric ratios in special right triangles, where
the values of all the ratios are known exactly. However, there are many applications involving
other size angles. Graphing calculators include keys to give values for the sine, cosine, and
tangent functions that are very accurate approximations for all trigonometric ratios of degree
measures greater than 0° and less than 90°. You should use calculator values for trigonometric
functions, as needed, for the remainder of this task.

In higher mathematics, it is standard to measure angles in
radians. Learning more about this method of angle measure
is a topic for Mathematics IV. The issue concerns you now
because you need to make sure that your calculator is in
degree mode (and not radian mode) before you use it for
finding values of trigonometric ratios. If you are using any
of the TI-83/84 calculators, press the MODE button, then use
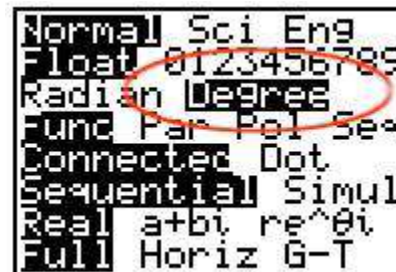
the arrow keys to highlight "Degree" and press enter. The graphic at the right shows how the screen will look when you have selected degree mode. To check that you have the calculator set correctly, check by pressing the TAN key, 45, and then ENTER. As you know, the answer should be 1. If you are using any other type of calculator, find out how to set it in degree mode, do so, and check as suggested above. Once you are sure that your calculator is in degree mode, you are ready to proceed to the remaining items of the task.

3. The main character in a play is delivering a monologue, and the lighting technician needs to shine a spotlight onto the actor's face. The light being directed is attached to a ceiling that is 10 feet above the actor's face. When a spotlight is positioned so that it shines on the actor's face, the light beam makes an angle of 20° with a vertical line down from the spotlight. How far is it from the spotlight to the actor's face? How much further away would the actor be if the spotlight beam made an angle of 32° with the vertical?
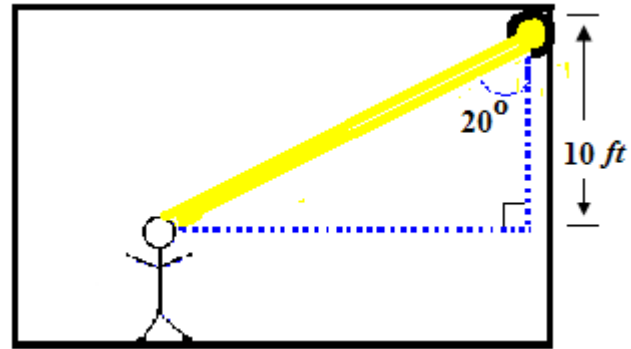
4. A forest ranger is on a fire lookout tower in a national forest. His observation position is 214.7 feet above the ground when he spots an illegal campfire. The angle of depression of the line of site to the campfire is 12°. (See the figure below.)
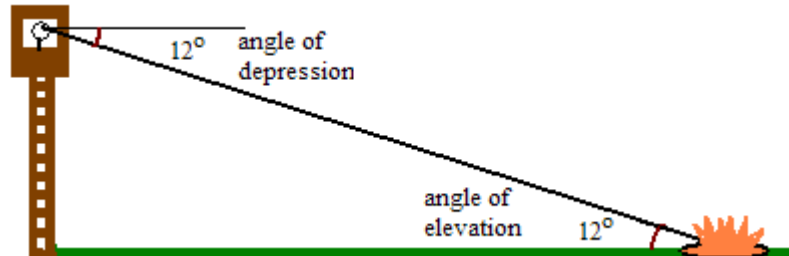
Note that an angle of depression is measured down from the horizontal because, to look down at something, you need to lower, or depress, your line of sight from the horizontal. We observe that the line of sight makes a transversal across two horizontal lines, one at the level of the viewer (such as the level of the forest ranger) and one at the level of the object being viewed (such as the level of the campfire). Thus, the angle of depression is the angle looking down from the fire lookout tower to the campfire, and the angle of elevation is the angle looking up from the campfire to the tower. The type of angle that is used in describing a situation depends on the location of the observer.

a. The angle of depression is equal to the corresponding angle of elevation. Why?

b. Assuming that the ground is level, how far is it from the base of the tower to the

campfire?

Plane

850 ft.

400 ft.

**Airport**
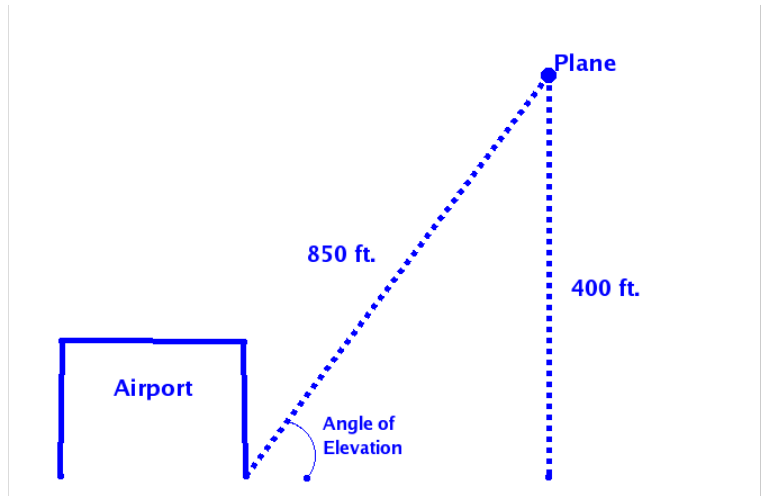
Angle of Elevation

5. An airport is tracking the path of one of its incoming flights. If the distance to the plane is 850 ft. (from the ground) and the altitude of the plane is 400 ft.

    a. What is the sine of the angle of elevation from the ground at the airport to the plane (see figure at the right)?

    b. What are the cosine and tangent of the angle of elevation?

    c. Now, use your calculator to find the measure of the angle itself. Pressing "2nd" followed by one of the trigonometric function keys finds the degree measure corresponding to a given ratio. Press 2nd, SIN, followed by the sine of the angle from *part a*. What value do you get?

    d. Press 2nd, COS , followed by the cosine of the angle from *part b*. What value do you get?

    e. Press 2nd, TAN, followed by the tangent of the angle from *part b*. What value do you get?

    Did you notice that, for each of the calculations in *parts a – e,* the name of the trigonometric ratio is written with an exponent of -1? These expressions are used to indicate that we are starting with a trigonometric ratio (sine, cosine, or tangent, respectively) and going backwards to find the angle that gives that ratio. You'll learn more about this notation in Unit 5. For now, just remember that it signals that you are going backwards from a ratio to the angle that gives the ratio.

    f. Why did you get the same answer each time?

    g. To the nearest hundredth of a degree, what is the measure of the angle of elevation?

    h. Look back at Table 1 from the *Create Your Own Triangles Learning Task*. Is your answer to *part g* consistent with the table entries for sine, cosine, and tangent

6. The top of a billboard is 40 feet above the ground. What is the angle of elevation of the sun when the billboard casts a 30-foot shadow on level ground?

7. The ends of a hay trough for feeding livestock have the shapes of congruent isosceles trapezoids as shown in the figure at the right. The trough is 18 inches deep, its base is 30 inches wide, and the sides make an angle of 118° with the base. How much wider is the opening across the top of the trough than the base?
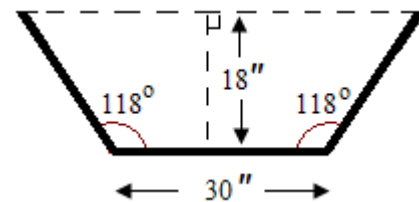
8. An observer in a lighthouse sees a sailboat out at sea. The angle of depression from the observer to the sailboat is 6°. The base of the lighthouse is 50 feet above sea level and the observer's viewing level is 84 feet above the base. (See the figure at the right,which is not to scale.)
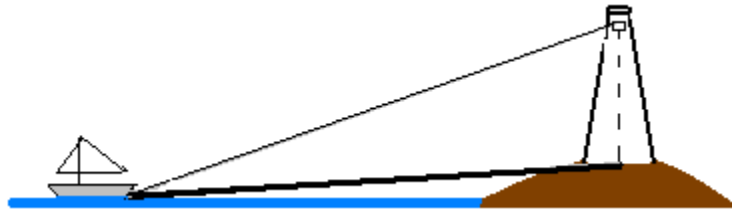
    a. What is the distance from the sailboat to the observer?

    b. To the nearest degree, what is the angle of elevation from the sailboat to the base of the lighthouse?