Mathematical Modeling Prompts

Mathematics is a tool for understanding the world better and making decisions. School mathematics instruction often neglects giving students opportunities to understand this, and reduces mathematics to disconnected rules for moving symbols around on paper. Mathematical modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions (NGA 2010). This mathematics will remain important beyond high school in students' lives and education after high school (NCEE 2013).

The mathematical modeling prompts and this guidance for how to use them represent our effort to make authentic modeling accessible to all teachers and students using this curriculum.

Organizing Principles about Mathematical Modeling

- The purpose of mathematical modeling in school mathematics courses is for students to understand that they can use math to better understand things they are interested in in the world.
- Mathematical modeling is different from solving word problems. It often feels like initially you are not given enough information to answer the question. There should be room to interpret the problem. There ought to be a range of acceptable assumptions and answers. Modeling requires genuine choices to be made by the modeler.
- It is expected that students have support from their teacher and classmates while modeling with mathematics. It is not a solitary activity. Assessment should focus on feedback that helps students improve their modeling skills.

Things the Modeler Does When Modeling with Mathematics (NGA 2010)

1. *Pose a problem* that can be explored with quantitative methods. Identify variables in the situation and select those that represent essential features.
2. *Formulate a model*: create and select geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between variables
3. *Compute*: Analyze these relationships and perform computations to draw conclusions
4. *Interpret* the conclusions in terms of the original situation
5. *Validate* the conclusions by comparing them with the situation. Iterate if necessary to improve the model
6. *Report* the conclusions and the reasoning behind them

It's important to recognize that in practice, these actions don't often happen in a nice, neat order.

When to Use Mathematical Modeling Prompts

A component of this is mathematical modeling prompts. Prompts include multiple versions of a task (the multiple versions require students engage in more or fewer aspects of mathematical modeling), sample solutions, instructions to teachers for launching the prompt in class and supporting students with that particular prompt, and an an analysis of each version showing how much of a "lift" the prompt is along several dimensions of mathematical modeling. A mathematical modeling prompt could be done as a classroom lesson or given as a project. This is a choice made by the teacher.

A mathematical modeling prompt done as a classroom lesson could take one day of instruction or more than one day, depending on how much of the modeling cycle students are expected to engage in, how extensively they are expected to revise their model, and how elaborate the reporting requirements are.

A mathematical modeling prompt done as a project could span several days or weeks. The project is assigned and students work on it in the background while daily math lessons continue to be conducted. (Much like research papers or creative writing assignments in other content areas.) This structure has the advantage of

giving students extended time for more complex modeling prompts that would not be feasible to complete in one class period and affords more time for iterations on the model and cycles of feedback.

Modeling prompts don't necessarily need to involve the same math as the current unit of study. As such, the prompts can be given at any time as long as students have the background to construct a reasonable model.

Students might flex their modeling muscles using mathematical concepts that are below grade level. First of all, learning to model mathematically is demanding—learning to do it while also learning new math concepts is likely to be out of reach. Second of all, we know that in future life and work, when students will be called on to engage in mathematical modeling, they will often need to apply math concepts from early grades to ambiguous situations (Forman & Steen, 1995). This elusive category of problems which are high school level yet draw on mathematics first learned in earlier grades may seem contradictory in a curriculum that takes focus and alignment seriously. However, p. 84 of the standards alludes to such problems, and Table 1 in the high school publisher's criteria (p. 8) leaves room for including such problems in high school materials in column 6.

The mathematical modeling prompts are not the only opportunities for students to engage in aspects of mathematical modeling in the curriculum. Mathematical modeling is often new territory for both students and teachers. Oftentimes within the regular classroom lessons, activities include scaled-back modeling scenarios, for which students only need to engage in a part of the modeling cycle. These activities are tagged with the "Aspects of Modeling" instructional routine, and the specific opportunity to engage in an aspect of modeling is explained in the activity narrative.

How to Prepare and Conduct the Modeling Lesson or Project

- Decide which version of the prompt to give.
- Have data ready to share if you plan to give it when students ask.
- Ensure students have access to tools they might be expected to use.
- If desired, instruct students to use a template for organizing modeling work.
- Whether doing the prompt as a classroom lesson or giving as a project, plan to do the in-class launch in class.
- Decide to what extent students are expected to iterate and refine their model. If you are conducting a one-day lesson, students may not have much time to refine their model and may not engage as much in that part of the modeling cycle. If you conduct a lesson that takes more than one day, or give the task as a project, it is more reasonable to expect students to iterate and refine their model once or even several times.
- Decide how students will report their results. If conducting a one-day lesson, this may be a rough visual display on a whiteboard. If more time is allotted or the task is assigned as a project, you might instruct students to write a more formal report, slideshow, blog post, poster, or create an a mockup of an artifact like a letter to a specific audience, a smartphone app, a menu, or a set of policies for a government entity to consider. One way to scaffold this work is to ask students to turn in a certain number of presentation slides: one that states the assumptions made, one that describes the model, and one or more slides with their conclusions or recommendations.
- Decide how students will be assessed. Prepare a rubric that will be used and share it with them.

Ideas for Setting Up an Environment Conducive to Modeling

- Provide plenty of blank whiteboard or chalkboard space for groups to work together comfortably. “Vertical non-permanent surfaces” are most conducive to productive collaborative work. “Vertical” means on a vertical wall is better than horizontally on a tabletop, and “non-permanent” means something like a dry erase board is better than something like chart paper (Liljedahl 2016).
- Ensure that students have easy access to any tools that might be useful for the task. These might include:
    - A supply table containing geometry tools, calculators, scratch paper, graph paper, dry erase markers, post-its
    - Electronic devices to access digital tools (like graphing technology, dynamic geometry software, or statistical technology)
- Think about how you will help students manage the time that is available to work on the task. For example:
    - For lessons, display a countdown timer for intermittent points in the class when you will ask each group to summarize their progress
    - For lessons, decide what time you will ask groups to transition to writing down their findings in a somewhat organized way (perhaps 15 minutes before the end of the class)
    - For projects, set some intermediate milestone deadlines to help students know if they are on track.

Organizing Students into Teams or Groups

- Mathematical modeling is not a solitary activity. It works best when students have support from each other and their teacher.
- Working with a team can make it possible to complete the work in a finite amount of class time. For example, the team may decide it wants to vary one element of the prompt, and compute the output for each variation. What would be many tedious calculations for one person could be only a few calculations for each team member.
- The members of good modeling groups bring a diverse set of skills and points of view. Scramble the members of modeling teams often, so that students have opportunities to play different roles.

Ways to Support Students While They Work on a Modeling Prompt

- Coach them on ways to organize their work better.
- Provide a template to help them organize their thinking. Over time, some groups may transition away from needing to use a template.
- Remind them of analog and digital tools that are available to them.
- When students get stuck or neglect an important aspect of the work, ask them a question to help them engage more fully in part of the modeling cycle. For example:
    - What quantities are important? Which ones change and which ones stay the same? (identify variables)
    - What information do you know? What information would it be nice to know? How could you get that information? What reasonable assumption could you make? (identify variables)
    - What pictures, diagrams, graphs, or equations might help people understand the relationships between the quantities? (formulate)
    - How are you describing the situation mathematically? Where does your solution come from? (compute)
    - Under what conditions does your model work? When might it not work? (interpret)
    - How could you make your model better? How could you make your model more useful under more conditions? (validate)
    - What parts of your solution might be confusing to someone reading it? How could you make it more clear? (report)

How to Interpret the Provided Analysis of a Modeling Prompt

For any mathematical modeling prompt, different versions are provided. We chose to analyze each version along 5 impactful dimensions that vary the demands on the modeler (OECD 2013). Each version of a mathematical modeling prompt is accompanied by an analysis chart that looks like this:

attribute	DQ	QI	SD	AD	M	mean
lift	0	1	0	0	2	0.6

Each of the attributes of a modeling problem is scored on a scale from 0–2. A lower score indicates a prompt with a “lighter lift” for students and teachers: students are engaging in less open, less authentic mathematical modeling. A higher score indicates a prompt with a “heavier lift” for students and teachers: students are engaging in more open, more authentic mathematical modeling.

This matrix shows the attributes that are part of our analysis of each mathematical modeling prompt. We recognize that not all the attributes have the same impact on what teachers and students do. However, for the sake of simplicity they are all weighted the same when they are averaged.

index	**attribute**	**light lift (0)**	**medium lift (1)**	**heavy lift (2)**
DQ	Defining the Question	well-posed question	elements of ambiguity; prompt might suggest ways assumptions could be made	freedom to specify and simplify the prompt; modeler must state assumptions
QI	Quantities of Interest	key variables are declared	key variables are suggested	key variables are not evident
SD	Source of Data	data is provided	modelers are told what measurements to take or data to look up	modelers must decide what measurements to take or data to look up
AD	Amount of Data given	modeler is given all the information they need and no more	some extra information is given and modeler must decide what is important; or, not enough information is given and modeler must ask for it before teacher provides it	modeler must sift through lots of given information and decide what is important; or, not enough information is given and modeler must make assumptions, look it up, or take measurements
M	The Model	a model is given in the form of a mathematical representation	type of model is suggested in words or by a familiar context; or, modeler chooses appropriate model from a provided list	careful thought about quantities and relationships or additional work (like constructing a scatterplot or drawing geometric examples) is

				required to identify type of model to use

We recognize that there are other features of a mathematical modeling prompt that could be varied. In the interests of not making things too complex, we only included 5 dimensions in the lift analysis. However, one might choose to modify a prompt on one of these dimensions:

- whether the scenario is posed with words, a highly-structured image or video, or real-world artifacts like articles or authentic diagrams
- presenting example for student to explore before they are expected to engage with the prompt, versus the prompt suggesting that the modeler generate examples or expecting the modeler to generate examples on their own
- whether the prompt makes decisions about units of measure or expects the modeler to reconcile units of measure or employ dimensional thinking
- whether a pre-made digital or analog tool is provided, instructions given for using a particular tool, use of a particular tool is suggested, or modelers simply have access to familiar tools but are not prompted to use them
- whether a mathematical representation is given, suggested, or modelers have the freedom to select and create representations of their own choosing